

Title: PSI 2018/2019 - Quantum Field Theory III - Lecture 3

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Abstract:

Infinitesimal conformal transformations generated by CKV:

$$\xi^M = \underset{\substack{\uparrow \\ P_\mu}}{a^M} + \underset{\substack{\uparrow \\ M_{\mu\nu}}}{\omega^M{}_\nu} x^\nu + \underset{\substack{\uparrow \\ D}}{\lambda} x^M + \underset{\substack{\uparrow \\ K_\mu}}{b^M} x^2 - 2x^M b_\mu x^\mu$$

$\mathbb{R}_2$ conformal transformation

$$I: x^\mu \rightarrow \frac{x^\mu}{x^2}$$

$$\begin{cases} I P_\mu I = K_\mu \\ I K_\mu I = P_\mu \\ I M_{\mu\nu} I = M_{\mu\nu} \\ I D I = -D \end{cases}$$

$$C \perp D \Rightarrow I = -D$$

CFT: QFT invariant under $SO(2,D)$

- Any local QFT $\ni$ a canonical operator.
- Translational invariance:

CFT: QFT invariant under $SO(2,D)$

- Any local QFT $\ni$ a canonical operator.

① Translational invariance: $\Rightarrow T_{\mu\nu}$ (EM tensor) $\xRightarrow{\text{Noether's}} \partial^\mu T_{\mu\nu} = 0$

② Lorentz invariance $\Rightarrow T_{\mu\nu} = T_{\nu\mu}$

$$\delta S = \frac{1}{2} \int d^D x \, T_{\mu\nu} \delta g^{\mu\nu}$$

$$(\partial^\mu \xi^\nu + \partial^\nu \xi^\mu)$$

CAUTION
NO OPEN FLAMES OR SMOKING MATERIALS
ALLOWED IN THE AREA OF THE BOARD
IT IS PROHIBITED TO LEAVE
THE BOARD UNATTENDED
WHILE IN USE

$$\delta S = \frac{1}{2} \int d^D x \, T_{\mu\nu} \delta g^{\mu\nu}$$

$$= \int d^D x \, T_{\mu\nu} \left(\partial^\mu \xi^\nu + \partial^\nu \xi^\mu \right)$$

$$\stackrel{\substack{\text{conf} \\ \text{transf}}}{=} \int d^D x \, T_{\mu\nu} \frac{1}{2} \eta^{\mu\nu} \partial \cdot \xi$$

$$\sim \int d^D x \, T_{\mu}^{\mu}$$

$$\delta S = \frac{1}{2} \int d^D x \, T_{\mu\nu} \delta g^{\mu\nu}$$

$$= \frac{1}{2} \int d^D x \, T_{\mu\nu} (\partial^\mu \xi^\nu + \partial^\nu \xi^\mu)$$

$$\stackrel{\substack{\uparrow \\ \text{conformal} \\ \text{transf}}}{=} \int d^D x \, T_{\mu}^{\mu} (\partial \cdot \xi)$$

formal
algebra

Poincaré
algebra

$v=0$

$$\delta S = \frac{1}{2} \int d^D x \, T_{\mu\nu} \delta g^{\mu\nu}$$

$$(\partial^\mu \xi^\nu + \partial^\nu \xi^\mu)$$

$$\frac{1}{2} g^{\mu\nu} \partial_\mu \xi_\nu$$

$$\stackrel{\text{confocal}}{\sim} \int d^D x \, T_\mu^\mu (\partial \xi)$$

Dilation $\xi^\mu = \lambda X^\mu$
 $\partial \cdot \xi = \lambda = \text{constant}$

D inv $\Rightarrow \boxed{T_\mu^\mu = \partial_\mu L^\mu}$

formal
algebra

Poincaré
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$v=0$

$$\delta S = \frac{1}{2} \int d^D x \, T_{\mu\nu} \delta g^{\mu\nu}$$

$$\delta g^{\mu\nu} = (\partial^\mu \xi^\nu + \partial^\nu \xi^\mu)$$

$$\stackrel{\text{conf}}{\sim} \int d^D x \, T_{\mu}^{\mu} (\partial \cdot \xi)$$

• Dilation $\xi^\mu = \lambda X^\mu$
 $\partial \cdot \xi = \lambda = \text{constant}$

$D \text{ inv} \Rightarrow \boxed{T_{\mu}^{\mu} = \partial_{\mu} L^{\mu}}$

• Special conformal ξ_{μ}
 $\partial \cdot \xi \sim \text{linear in } x$

$$\delta S = \frac{1}{2} \int d^D x \, T_{\mu\nu} \delta g^{\mu\nu}$$

$$\begin{aligned} & \left(\partial^\mu \xi^\nu + \partial^\nu \xi^\mu \right) \\ & \stackrel{\parallel}{=} \frac{2}{D} \eta^{\mu\nu} \partial \cdot \xi \end{aligned}$$

$\uparrow$
conformal
transf

$$\sim \int d^D x \, T_\mu^\mu (\partial \cdot \xi)$$

• Dilation $\xi^\mu = \lambda X^\mu$
 $\partial \cdot \xi = \lambda = \text{constant}$

D inv $\Rightarrow \boxed{T_\mu^\mu = \partial_\mu L^\mu}$

• Special conformal K_μ
 $\partial \cdot \xi \sim \text{linear in } x$

$$\boxed{T_\mu^\mu = \partial_\mu \partial_\nu L^{\mu\nu}}$$

$$\partial^\mu j_\mu = 0$$

$$Q = \int j_0 d^{D-1}x$$

$$\tilde{j}_\mu = j_\mu + \partial_\nu \sigma^{[\mu\nu]}$$

$$\partial^\mu \tilde{j}_\mu = 0 \quad \hat{Q} = Q$$

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$$\partial^\mu \hat{j}_\mu = 0 \quad \hat{Q} = Q$$

By an improvement transformation

$$T_{\mu\nu} \rightarrow \hat{T}_{\mu\nu}$$

$$T_\mu{}^\mu = \partial_\mu \partial_\nu L^{\mu\nu} \rightarrow \hat{T}_\mu{}^\mu = 0$$

$$\partial^\mu j_\mu = 0 \quad \hat{j}_\mu = j_\mu + \partial_\nu \sigma^{[\mu\nu]}$$

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By an improvement transformation $T_{\mu\nu} \rightarrow \hat{T}_{\mu\nu}$

$$T_\mu{}^\mu = \partial_\mu \partial_\nu L^{\mu\nu} \rightarrow$$

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By an improvement transformation

$$T_{\mu\nu} \rightarrow \hat{T}_{\mu\nu}$$

$$T_{\mu\nu} = \partial_\mu \partial_\nu L^{\mu\nu} \rightarrow$$

$$\boxed{\hat{T}_{\mu\nu} = 0}$$

$$\left(\hat{T}_{\mu\nu} = \frac{1}{2} m^2 \phi^2 \right)$$

$$\partial^\mu j_\mu = 0$$

$$Q = \int j_0 d^{D-1}x$$

$$\hat{j}_\mu = j_\mu + \partial_\nu \sigma^{[\mu\nu]}$$

$$\partial^\mu \hat{j}_\mu = 0 \quad \hat{Q} = Q$$

$$\mathcal{L}_{\text{flat}} \rightarrow \mathcal{L}_{\text{curved}} + \frac{1}{2} R_{\mu\nu} L^{\mu\nu}$$

$\eta \rightarrow g$

By an improvement transformation

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$$\partial^\mu j_\mu = 0$$

$$Q = \int j_0 d^{D-1}x$$

$$\tilde{j}_\mu = j_\mu + \partial_\nu \Theta^{[\mu\nu]}$$

$$\partial^\mu \tilde{j}_\mu = 0 \quad \hat{Q} = Q$$

$$\mathcal{L}_{\text{flat}} \rightarrow \mathcal{L}_{\text{curved}} + \frac{1}{2} R_{\mu\nu} L^{\mu\nu}$$

$$\eta \rightarrow g$$

$$R_{\mu\nu\rho\sigma} \Theta^{\mu\nu\rho\sigma}$$

By an improvement transformation

$$T_{\mu\nu} \rightarrow \hat{T}_{\mu\nu}$$

$$T_{\mu}^{\mu} = \partial_\mu \partial_\nu L^{\mu\nu} \rightarrow$$

$$\boxed{\hat{T}_{\mu}^{\mu} = 0}$$

$$\left(\hat{T}_{\mu}^{\mu} = \frac{1}{2} m^2 \phi^2 \right)$$

Operators in a CFT

- In representations of conformal algebra.

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Label by quantum numbers include

$M_{\mu\nu}$: spin

D Δ (scaling dimension)

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- In representations of conformal algebra

Label by quantum numbers include

1. Primary (transform simply under CFT)

2. Descendants

$M_{\mu\nu}$: spin

D Δ (scaling dimension)

Operators in a CFT

- In representations of conformal algebra.

Label by quantum numbers include

- 1. Primary (transform simply under CFT)
- 2. Descendants (fully determined by a primary operator)

$M_{\mu\nu}$: spin
 D : Δ (scaling dimension)

$$[D, \theta] = \Delta \theta$$

$$[D, [P_\lambda, \theta]] = (\Delta + 1) \theta$$

$$[D, [K_\lambda, \theta]] = (\Delta - 1) \theta$$

$$[D, \theta] = \Delta \theta$$

$$[D, [P_\mu, \theta]] = (\Delta + 1) \theta$$

$$[D, [K_\mu, \theta]] = (\Delta - 1) \theta$$

Primary operator $[K_\mu, \theta] = 0$

lowest weight vector

$$P_\mu \uparrow$$

$$K_\mu \downarrow$$

$$(J_- |s\rangle = 0)$$

$$[D, \theta] = \Delta \theta$$

$$[D, [P_\mu, \theta]] = (\Delta + 1) [P_\mu, \theta] \quad P_\mu \uparrow$$

$$[D, [K_\mu, \theta]] = (\Delta - 1) [K_\mu, \theta] \quad K_\mu \downarrow$$

Primary operator $[K_\mu, \theta] = 0$

lowest weight vector

$$(J_- |s\rangle = 0)$$

Representation

θ

$P_\mu \theta$

$P_{\mu_1} P_{\mu_2} \theta$

$\vdots$

$\vdots$

$\vdots$

$\vdots$

$$[D, \theta] = \Delta \theta$$

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Representation

θ

primis

$P_\mu \theta$

$P_{\mu_1} P_{\mu_2} \theta$

descendants

$\vdots$

$\vdots$

$\vdots$

$\vdots$

$\vdots$

$$[D, \theta] = \Delta \theta$$

$$[D, [P_\mu, \theta]] = (\Delta + 1) [P_\mu, \theta] \quad P_\mu \uparrow$$

$$[D, [K_\mu, \theta]] = (\Delta - 1) [K_\mu, \theta] \quad K_\mu \downarrow$$

$$\text{Primary operator } [K_\mu, \theta] = 0$$

lowest weight vector

$$[J_-, |s\rangle] = 0$$

Representation

$$[P_\mu, \theta] = \partial_\mu \theta$$

$P_{\mu_1} P_{\mu_2} \theta$

primaries

descendants

Transformation of Primary Operators under conformal transformations

$$x^\mu \rightarrow \hat{x}^\mu = (g \cdot x)^\mu \quad g(a, w, \lambda, b)$$

$$\frac{\partial \hat{x}}{\partial x} \frac{\partial \hat{x}}{\partial x} \eta = e^{2w} \eta$$

$$\theta(x) \longrightarrow$$

Transformation of Primary Operators under conformal transformations

$$x^\mu \rightarrow \tilde{x}^\mu = (g \cdot x)^\mu \quad g(a, w, \lambda, b)$$

$$\frac{\partial \tilde{x}}{\partial x} \frac{\partial \tilde{x}}{\partial x} \eta = e^{2w} \eta$$

$$\theta(x) \longrightarrow \hat{\theta}_{\Delta, \Delta}(x) =$$

Δ : Lorentz spin label
 Δ : scaling dimension

$$L_A^B(R(x)) \theta_B(\tilde{g}^{-1}x)$$

$$R_\mu^\nu(x) = \frac{\partial \tilde{x}^\mu}{\partial x^\nu} e^{f(w(x))}$$

spin quantum.

orbital

$$R^T \eta R = \eta$$

Transformation of Primary Operators under conformal transformations

$$x^M \rightarrow \tilde{x}^M = g(x)^M$$

$$\frac{\partial \tilde{x}}{\partial x} \frac{\partial \tilde{x}}{\partial x} \eta = e^{2\omega} \eta$$

$$\theta(x) \longrightarrow \hat{\theta}_{A,\Delta}(x) = \left| \frac{\partial x}{\partial \tilde{x}} \right|^{\Delta/D} L_A^B(R(x)) \theta_B(\tilde{g}^{-1}x)$$

A : Lorentz spin label
 Δ : scaling dimension

$$R_{\mu}^{\nu}(x) = \frac{\partial \tilde{x}^{\mu}}{\partial x^{\nu}} e^{\omega(x)}$$

spin quantum.

orbital

$$R^T \eta R = \eta$$

Transformation of Primary Operators under conformal transformations

$$x^M \rightarrow \tilde{x}^M = (g \cdot x)^M$$

$$g(a, w, \lambda, b)$$

$$L(g_1)L(g_2) = L(g_1 g_2)$$

$$\frac{\partial \tilde{x}}{\partial x} \frac{\partial \tilde{x}}{\partial x} \eta = e^{2w} \eta$$

$\Delta \neq 0$

$$\theta(x) \longrightarrow \hat{\theta}_{\Delta}^A(x) = \left| \frac{\partial x}{\partial \tilde{x}} \right| L_A^B(R(x)) \theta_B(\tilde{g}^{-1}x)$$

A : Lorentz spin label
 Δ : scaling dimension

$$R_{\mu}^{\nu}(x) = \frac{\partial \tilde{x}^{\nu}}{\partial x^{\mu}} e^{\int w(x)}$$

spin quantum.

orbital

$$R^T \eta R = \eta$$

Dilatation: $\tilde{x}^M = \lambda x^M$ (θ to be a scalar)

$$\hat{\theta}(x) = \tilde{\lambda}^{\Delta} \theta(\tilde{\lambda}^{-1} x)$$

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$$\tilde{\theta}(x) = \lambda^\Delta \theta(\lambda^{-1} x)$$

Under infinitesimal transf $\tilde{x} = x + \xi$

$$\delta \theta_A(x) = \tilde{\theta}_A(x) - \theta_A(x) = - \xi^\mu \underbrace{\partial_\mu \theta_A(x)}_{\uparrow \text{orbital}} + \frac{i}{2} \underbrace{\Omega_{\mu\nu}(x)}_{\uparrow \text{spin}} \left(M^{\mu\nu}_R \right)_A^B \theta_B(x) - \Delta \omega(x) \theta_A(x)$$

where

$$\Omega_{\mu\nu}(x) =$$

Dilatation: $\tilde{x}^M = \lambda x^M$ (θ to be a scalar)

$$\tilde{\theta}(x) = \lambda^\Delta \theta(\lambda^{-1} x)$$

Under infinitesimal transf $\tilde{x} = x + \xi$

$$\delta \theta_A(x) = \tilde{\theta}_A(x) - \theta_A(x) = - \xi^\mu \underbrace{\partial_\mu \theta_A(x)}_{\text{orbital}} + \frac{1}{2} \underbrace{\Omega_{\mu\nu}(x)}_{\text{spin}} \left(M^{\mu\nu}_R \right)_A^B \theta_B(x) - \Delta \omega(x) \theta_A(x)$$

where

$$\Omega_{\mu\nu}(x) = \omega_{\mu\nu} - 2(x_\mu b_\nu - x_\nu b_\mu)$$

$$\omega(x) = \lambda - 2x \cdot b$$

Dilatation: $\tilde{x}^M = \lambda x^M$ (θ to be a scalar)

$$\tilde{\theta}(x) = \lambda^\Delta \theta(\lambda^{-1} x)$$

Under infinitesimal transf $\tilde{x} = x + \xi$

$$\delta \theta_A(x) = \tilde{\theta}_A(x) - \theta_A(x) = - \xi^\mu \underbrace{\partial_\mu \theta_A(x)}_{\text{orbital}} + \frac{1}{2} \Omega_{\mu\nu}(x) \underbrace{\left(M^{\mu\nu}_R \right)_A^B}_{\text{spin}} \theta_B(x) - \Delta \omega(x) \theta_A(x)$$

where

$$\left\{ \begin{array}{l} \Omega_{\mu\nu}(x) = \omega_{\mu\nu} - 2(x_\mu b_\nu - x_\nu b_\mu) \\ \omega(x) = \lambda - 2x \cdot b \end{array} \right.$$

Infinitesimal conformal transform

$$\xi^M = a^M + \omega^M_{\nu} x^{\nu} + \lambda x^M + \left(\frac{1}{2} b^{\mu\nu} x^{\mu} x^{\nu} - 2x^M b \cdot x \right)$$

$\uparrow$
 P_{μ}

$\uparrow$
 $M_{\mu\nu}$

$\uparrow$
 D

$\uparrow$
 K_{μ}

$\mathbb{R}_2$ conformal transformation

$$I: x^{\mu} \rightarrow \frac{x^{\mu}}{x^2}$$

$$\begin{cases} I P_{\mu} I = K_{\mu} \\ I K_{\mu} I = P_{\mu} \\ I M_{\mu\nu} I = M_{\mu\nu} \\ I D I = -D \end{cases}$$

kinematics

$SO(2,D)$ conformal algebra

$ISO(1,D)$ Poincaré algebra

Label by quantum numbers under

1. Primary (transform simply under CGL) D Δ (scaling dimension)
2. Descendants (fully determined by a primary operator)

$$[\xi_{(1)}, \xi_{(2)}] = \xi_{(3)} \quad \xi_{(2)} \text{ is a CKV}$$

$$[\xi_{(1)}, \xi_{(2)}] = \xi_{(3)} \quad \xi_{(2)} \text{ is a CKV}$$

$$[\delta_{\xi_{(1)}}, \delta_{\xi_{(2)}}]\theta_A = \delta_{\xi_{(3)}}\theta_A$$

$$[\xi_{(1)}, \xi_{(2)}] = \xi_{(3)} \quad \xi_{(2)} \text{ is a CKV}$$

$$[\delta_{\xi_{(1)}}, \delta_{\xi_{(2)}}] \theta_A = \delta_{\xi_{(3)}} \theta_A \Rightarrow \text{confined algebra is realized on the operator}$$

Constraints observables

Constraints observables

- Position dependence of 2 and 3pt. functions are completely fixed!

Constraints observables.

- Position dependence of 2 and 3pt. functions are completely fixed ?

$$\langle \mathcal{O}_1(x_1) \mathcal{O}_2(x_2) \mathcal{O}_3(x_3) \rangle$$

Constraints observables

- Position dependence of 2 and 3pt. functions are completely fixed!

$$\langle \mathcal{O}_1(x_1) \mathcal{O}_2(x_2) \mathcal{O}_3(x_3) \rangle = f_{123}(x_1, x_2, x_3)$$

Constraints observables

- Position dependence of 2 and 3pt. functions are completely fixed!

$$\langle \mathcal{O}_1(x_1) \mathcal{O}_2(x_2) \mathcal{O}_3(x_3) \rangle = f_{123}(x_1, x_2, x_3)$$

x_1

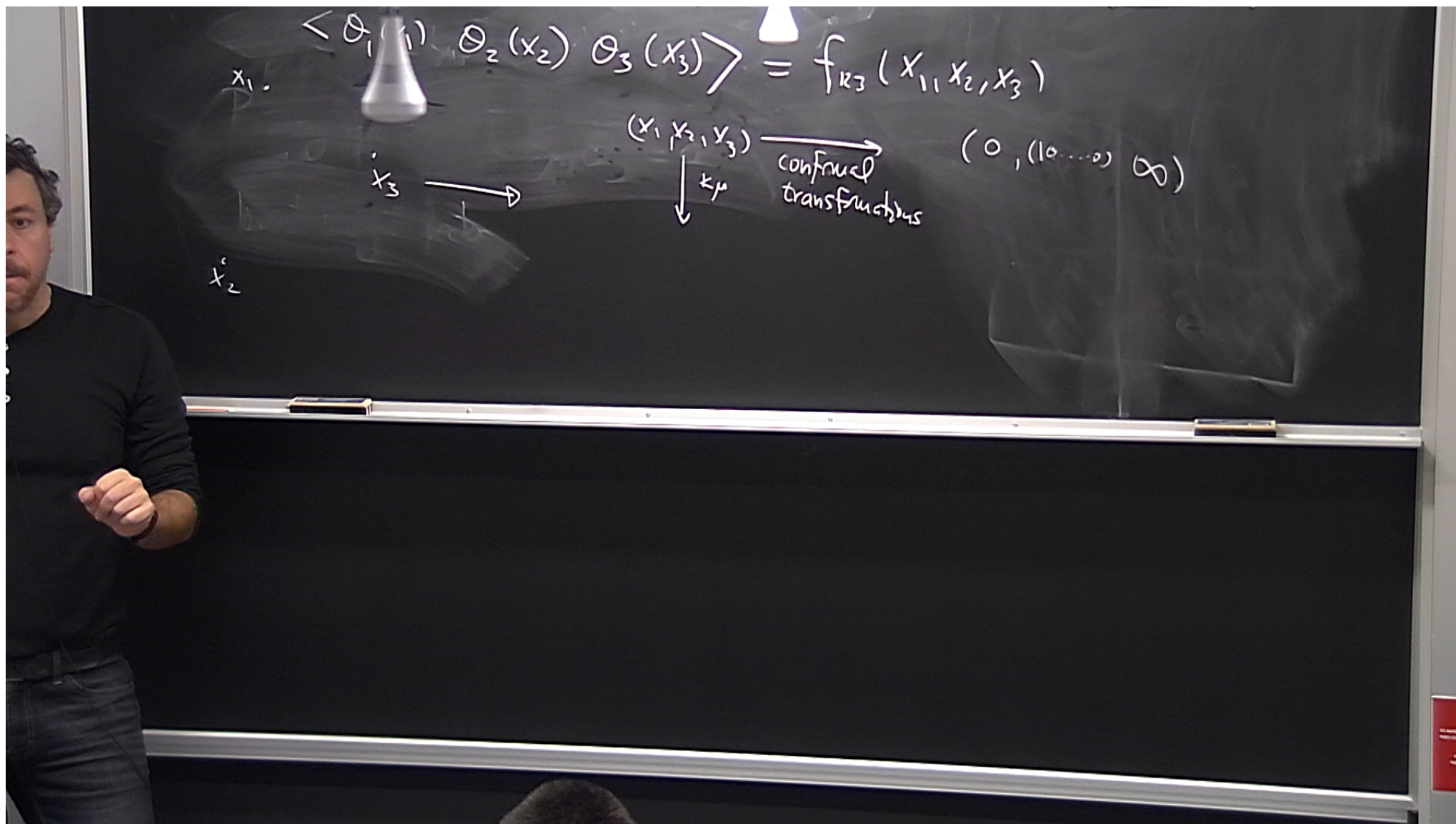
x_3

x_2

(x_1, x_2, x_3)

conformal
transformations

$(0, (1, 0, \dots, 0), \infty)$



$$[\xi_{(1)}, \xi_{(2)}] = \xi_{(3)} \quad \xi_{(2)} \text{ is a CKV}$$

$$[\delta_{\xi_{(1)}}, \delta_{\xi_{(2)}}] \theta_A = \delta_{\xi_{(3)}} \theta_A \Rightarrow \text{confined algebra is realized on the operator}$$

$$\tilde{X}^M = \frac{x^M + b^M x^2}{1 + 2b \cdot x + b^2 x^2}$$

$$\langle \theta_1(x_1) \theta_2(x_2) \theta_3(x_3) \rangle = f_{k_3}(x_1, x_2, x_3)$$

x_1
 $x_3 \rightarrow$
 x_2

$(x_1, x_2, x_3) \xrightarrow{\text{conformal transformations}} (0, (10 \dots 0), \infty)$
 $\downarrow k_\mu$
 (x'_1, x'_2, ∞)

$$\langle \theta_1(x_1) \theta_2(x_2) \theta_3(x_3) \rangle = f_{K_3}(x_1, x_2, x_3)$$

x_1
 $x_3 \rightarrow$
 x_2

$(x_1, x_2, x_3) \xrightarrow{\text{conformal transformations}} (0, (10 \dots 0), \infty)$
 $\downarrow K_\mu$
 (x'_1, x'_2, ∞)
 $\downarrow P_\mu$
 $(0, x''_2, \infty)$

$$C \perp D I = -D$$

algebra

$$[\xi_{(1)}, \xi_{(2)}] = \xi_{(3)} \quad \xi_{(2)} \text{ is a CKV}$$

$$[\delta_{\xi_{(1)}}, \delta_{\xi_{(2)}}] \theta_A = \delta_{\xi_{(3)}} \theta_A \Rightarrow \text{confined algebra is realized on the operators}$$

$$\tilde{X}^4 = \frac{x^4 + b^4 x^2}{1 + 2b \cdot x + b^2 x^2} \quad I P_A I$$

$$\langle \theta_1(x_1) \theta_2(x_2) \theta_3(x_3) \rangle = f_{K3}(x_1, x_2, x_3)$$

x_1
 $x_3 \rightarrow$
 x_2

$(x_1, x_2, x_3) \xrightarrow{\text{conformal transformations}} (0, (10 \dots 0) \infty)$
 $\downarrow k_\mu$
 (x'_1, x'_2, ∞)
 $\downarrow P_\mu$
 $(0, \frac{z''}{z''}, \infty)$
 $(x''_1, x''_2, \dots, x''_D)$

$j=2, \dots, D$
 M_{1j}
 $\rightarrow (0, (x''_1, 0, \dots, 0) \infty)$

$$\langle \theta_1(x_1) \theta_2(x_2) \theta_3(x_3) \rangle = f_{123}(x_1, x_2, x_3)$$

x_1
 x_3
 x_2

$(x_1, x_2, x_3) \xrightarrow{\text{conformal transformations}} (0, (1, 0, \dots, 0), \infty)$
 $\downarrow k_\mu$
 $(x'_1, x'_2, \infty) \xrightarrow{j=2, \dots, D} (0, (1, 0, \dots, 0), \infty)$
 $\downarrow P_\mu$
 $(0, \frac{z''}{z''}, \infty) \xrightarrow{M_{1j}} (0, (x''_1, 0, \dots, 0), \infty)$
 $(x''_1, x''_2, \dots, x''_D)$

D

CAUTION

DO NOT STAND ON THE BOARD OR ON THE TABLE

IF AN EMERGENCY OCCURS, PLEASE CALL 911

DO NOT TOUCH THE BOARD

x_2

$$\begin{array}{ccc} \downarrow P_A & & M_{1j} \\ (0, \bar{z}''_1, \infty) & \longrightarrow & (0, (x''_1, \dots, x''_n), \infty) \\ (x''_1, x''_2, \dots, x''_n) & & \end{array}$$

Constraints imposed on correlations (scalar)

$$\langle \hat{\theta}_1(\vec{x}_1) \dots \hat{\theta}_n(\vec{x}_n) \rangle = \langle \theta_1(\vec{x}_1) \dots \theta_n(\vec{x}_n) \rangle$$

CAUTION
DO NOT TOUCH THE BOARD
IF IT IS NECESSARY TO Wipe
PLEASE ASK THE TEACHER

$$(0, \vec{x}''_1, \infty) \xrightarrow{M_{1j}} (0, (x''_1, 0), \infty)$$

$\downarrow P_n$ M_{1j}
 $(x''_1, x''_2, \dots, x''_n)$

Constraints imposed on correlations (scalar)

$$\langle \hat{\theta}_1(\vec{x}_1) \dots \hat{\theta}_n(\vec{x}_n) \rangle = \langle \theta_1(\vec{x}_1) \dots \theta_n(\vec{x}_n) \rangle$$

or cl fm. $\sum_{I=1}^n \langle \theta_1(x_1) \dots \oint \theta_I(x_I) \dots \theta_n \rangle = 0$

x_2

$$(0, \vec{z}''', \infty) \xrightarrow{M_{1j}} (0, (x_1'', \dots, x_n''), \infty)$$

$(x_1'', x_2'', \dots, x_n'')$

Constraints imposed on correlations (scalar)

$$\langle \hat{\theta}_1(\vec{x}_1) \dots \hat{\theta}_n(\vec{x}_n) \rangle = \langle \theta_1(\vec{x}_1) \dots \theta_n(\vec{x}_n) \rangle$$

and f.m. $\sum_{I=1}^n \langle \theta_1(x_1) \dots \int \theta_2(x_2) \dots \theta_n \rangle = 0$

1. $\langle \theta(x) \rangle = 0$ unless $\theta = 1$

CAUTION
DO NOT TOUCH THE BOARD
IF IT IS DAMAGED BY YOU
YOU WILL BE RESPONSIBLE

$$(0, \frac{1}{2}, \infty) \xrightarrow{M_{1,2}} (0, (x_1'')_0, 0, \infty)$$

$(x_1'', x_2'', \dots, x_n'')$

Constraints imposed on correlations (scalon)

$$\langle \hat{\theta}_1(\tilde{x}_1) \dots \hat{\theta}_n(\tilde{x}_n) \rangle = \langle \theta_1(\tilde{x}_1) \dots \theta_n(\tilde{x}_n) \rangle$$

and from $\sum_{i=1}^n \langle \theta_1(x_1) \dots \theta_i(x_i) \dots \theta_n(x_n) \rangle = 0$

1. $\langle \theta(x) \rangle = 0$ unless $\theta = 1$

2. $\langle \theta(x_1) \theta(x_2) \rangle = \frac{1}{|x_1 - x_2|^{2\Delta}}$

$$(0, \frac{1}{2}, \infty, \dots) \rightarrow (0, (x_1'', 0) \infty) -$$

$$(x_1'', x_2'', \dots, x_n'')$$

Constraints imposed on correlations (scalar)

$$\langle \hat{\theta}_1(\vec{x}_1) \dots \hat{\theta}_n(\vec{x}_n) \rangle = \langle \theta_1(\vec{x}_1) \dots \theta_n(\vec{x}_n) \rangle$$

$$\text{or cl fm. } \sum_{I=1}^n \langle \theta_1(x_1) \dots \theta_I(x_I) \dots \theta_n(x_n) \rangle = 0$$

$$1. \langle \theta(x) \rangle = 0 \text{ unless } \theta = \mathbb{1}$$

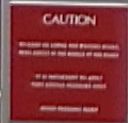
$$2. \langle \theta(x_1) \theta(x_2) \rangle = \frac{1}{|x_1 - x_2|^{2\Delta}}$$

$$C_{123}$$

$$|x_{12}|^{\Delta_1 + \Delta_2 - \Delta_3}$$

$$|x_{23}|^{\Delta_2 + \Delta_3 - \Delta_1}$$

$$|x_{13}|^{\Delta_1 + \Delta_3 - \Delta_2}$$



$$(0, \frac{1}{2}, \infty, \dots) \rightarrow (0, (x_1'', \dots, x_n'') \infty)$$

$$(x_1'', x_2'', \dots, x_n'')$$

Constraints imposed on correlations (scalar)

CFT data:
 $\{\Delta_I\}, \{C_{IJK}\}$

$$\langle \hat{\theta}_1(\hat{x}_1) \dots \hat{\theta}_n(\hat{x}_n) \rangle = \langle \theta_1(\vec{x}_1) \dots \theta_n(\vec{x}_n) \rangle$$

or eqn. $\sum_{I=1}^n \langle \theta_1(x_1) \dots \theta_I(x_I) \dots \theta_n(x_n) \rangle = 0$

$$\langle \theta_1(x_1) \theta_2(x_2) \theta_3(x_3) \rangle =$$

$$C_{123}$$

1. $\langle \theta(x) \rangle = 0$ unless $\theta = \mathbb{1}$

2. $\langle \theta(x_1) \theta(x_2) \rangle = \frac{1}{|x_1 - x_2|^{2\Delta}}$

$$|x_{12}|^{\Delta_1 + \Delta_2 - \Delta_3}$$

$$|x_{23}|^{\Delta_2 + \Delta_3 - \Delta_1}$$

$$|x_{13}|^{\Delta_1 + \Delta_3 - \Delta_2}$$

