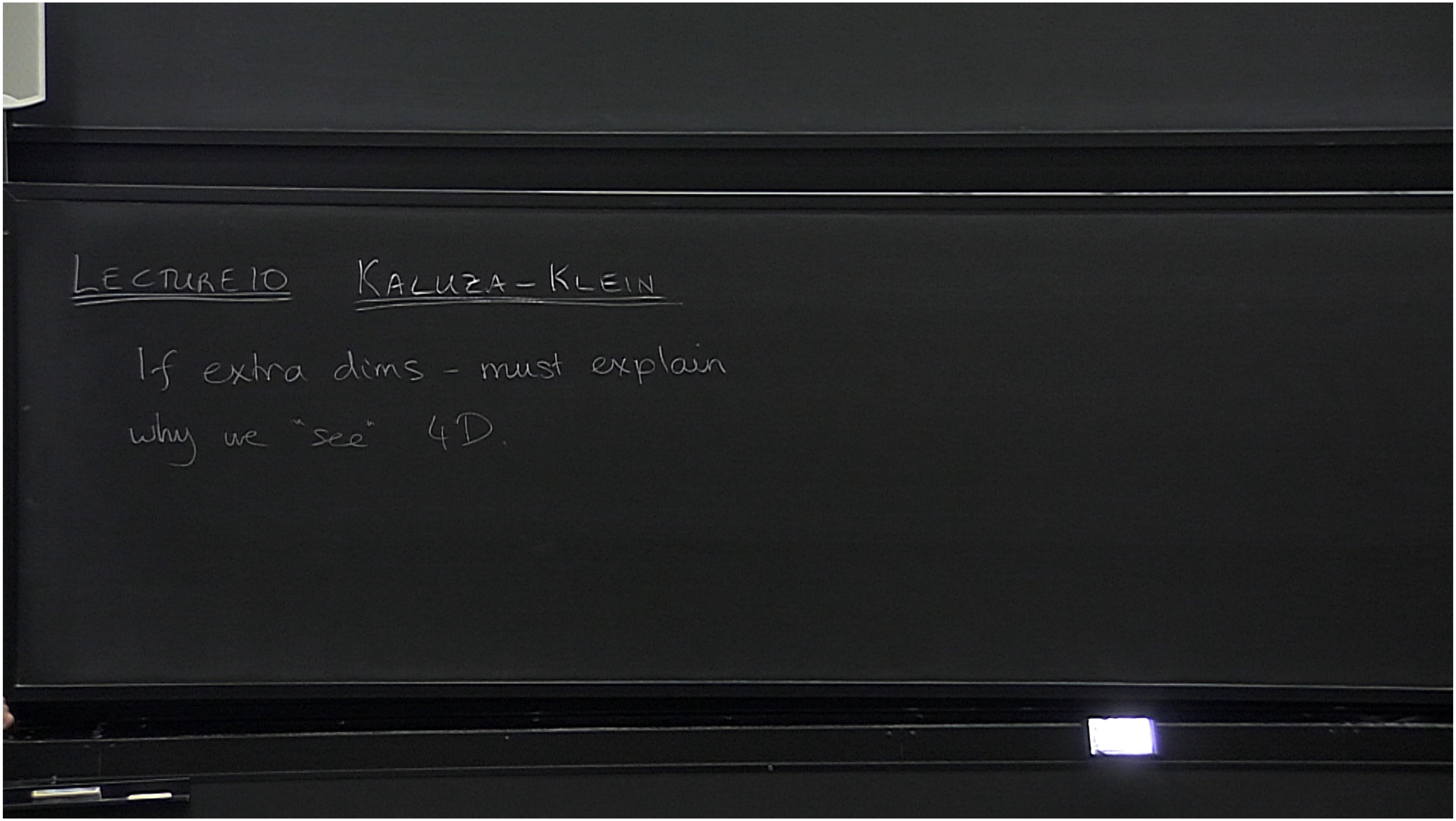
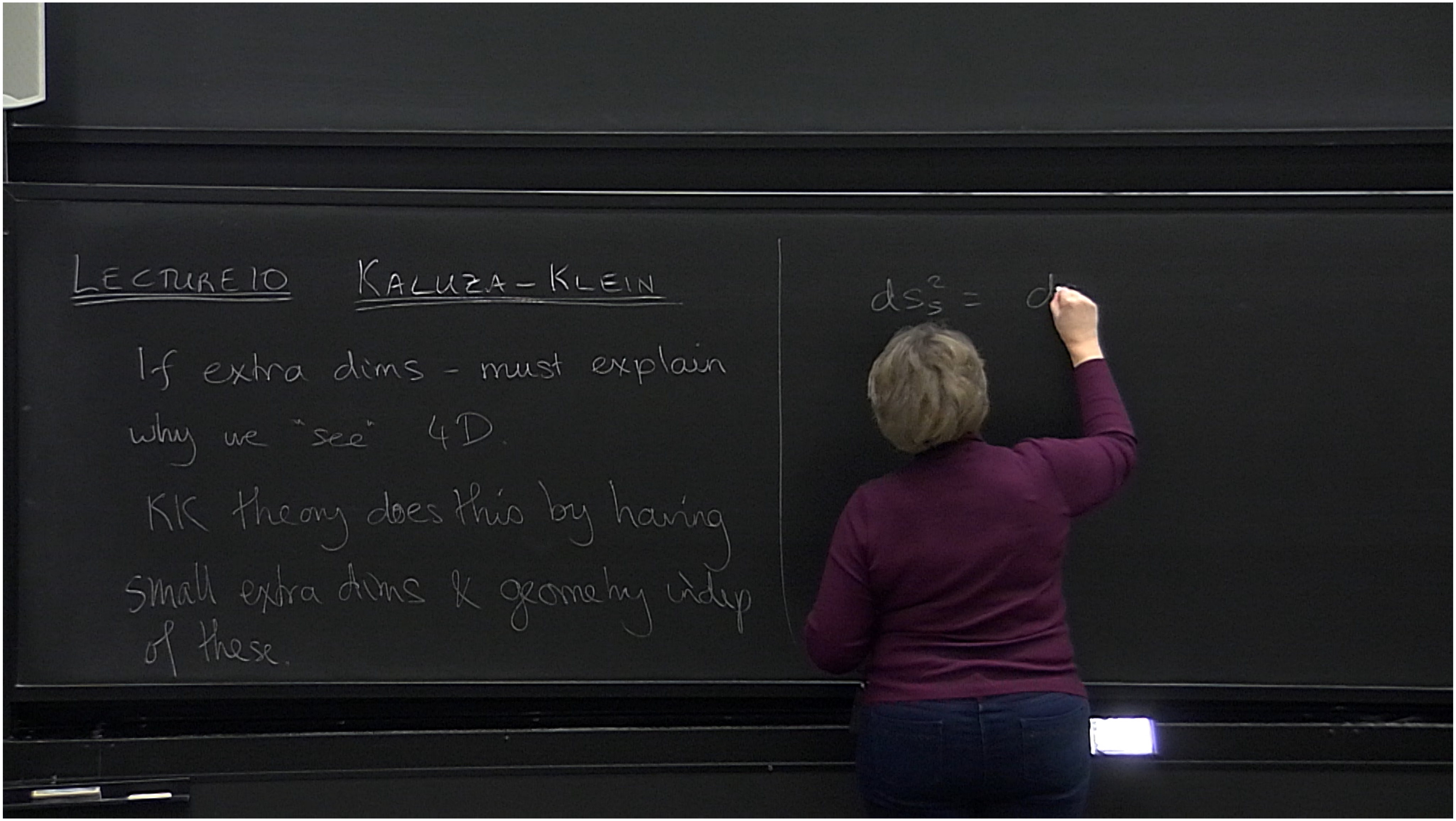
Title: PSI 2018/2019 - Gravitational Physics - Lecture 10

Date: Feb 11, 2019 10:15 AM

URL: http://pirsa.org/19020018

Abstract:


LECTURE 10
KALUZA-KLEIN
If extra dims - must explain
why we "see" 4D.


LECTURE 10
KALUZA-KLEIN
If extra dims - must explain
why we "see" 4D.
KK theory does this by having
small extra dims & geometry indep
of these.
$ds_5^2 = d$

N

plain

aving

y indep

$$ds_5^2 = ds_4^2 - \underbrace{L^2 d\chi^2}_{\text{size } L}$$

Wave fns that depend on χ have large masses.

$$\Psi(x,\chi) \sim \psi(x)\, e^{i\chi}$$

$$\nabla_5^2 \Psi \sim \left[\Box\psi + L^{-2}\psi\right] e^{i\chi}$$

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What about gravity?

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$$ds_5^2 = \hat{g}^{(4)}_{\mu\nu} dx^\mu dx^\nu - e^{2\sigma(x)}[dz + A^{(x)}_\mu dx^\mu]^2$$

$$\rightarrow \sqrt{g_5} = e^\sigma \sqrt{\hat{g}_4}$$

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$$\to \sqrt{g_5} = e^\sigma \sqrt{\hat{g}_4}$$

$\hat{g}_{\mu\nu}$ – tensor

A_μ – vector

σ – scalar

example with off-diagonal terms

$$\underline{A}$$

$$\underset{\sim}{\omega}^a = e^a{}_\mu \, \underline{d} x^\mu \qquad \underline{\omega}^5 = e^\sigma \left[\underline{d} z + A_\mu \underline{d} x^\mu \right]$$

example with off-diagonal terms

$$\underset{\sim}{\omega}^a = e^a{}_\mu \underset{\sim}{d} x^\mu \qquad \underset{\sim}{\omega}^5 = e^\sigma [\underset{\sim}{d} z + A_\mu \underset{\sim}{d} x^\mu] \quad \swarrow \underset{\sim}{A}$$

$$\left.\begin{aligned} \underset{\sim}{d}\underset{\sim}{\omega}^a &= -\underset{\sim}{\Theta}_0{}^a{}_b \wedge \underset{\sim}{\omega}^b \\ \underset{\sim}{d}\underset{\sim}{\omega}^5 &= \sigma_{,a}\, \underset{\sim}{\omega}^a \wedge \underset{\sim}{\omega}^5 + e^\sigma \underset{\sim}{F} \end{aligned}\right\} \text{ALGEBRAIC RELNS}$$

$$\rightarrow \quad \underset{\sim}{\Theta}^5{}_a = \sigma_{,a} \underset{\sim}{\omega}^5 + \tfrac{1}{2} e^\sigma F_{ab} \underset{\sim}{\omega}^b$$

$$\underset{\sim}{\Theta}^a{}_b = \underset{\sim}{\Theta}_0{}^a{}_b + \tfrac{1}{2} e^\sigma F^a{}_b \underset{\sim}{\omega}^5$$

$\underline{A}$

$A_\mu dx^\mu$

GEBRAIC

RELNS

Computation of Riemann straightforward

$$R^a{}_{bcd} = \hat{R}_0{}^a{}_{bcd} + \frac{1}{4} e^{2\sigma} \left(F^a{}_c F_{bd} - F^a{}_d F_{bc} + 2 F^a{}_b F_{cd} \right)$$

$$R^5{}_{a5b} = - \nabla_a \nabla_b \sigma - \nabla_b \sigma \nabla_a \sigma - \frac{1}{4} e^{2\sigma} F_{ac} F_b{}^c$$

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giving
$$S_5 = -\frac{1}{16\pi G_5} \int d^4x \, dz \sqrt{g^4} \left[e^{\sigma} \hat{R}_4 + \frac{1}{4} e^{3\sigma} F^2 - 2 \Box e^{\sigma} \right]$$

$\nabla_5 \Psi \sim \left(\Box\psi + n^2 L^{-2}\psi\right) e^{\frac{1}{2}\sigma}$ $\quad m^2.$

Use Cartan (handout).

Computation of Riemann straightforward

$$R^a{}_{bcd} = \hat{R}_0{}^a{}_{bcd} + \frac{1}{4} e^{2\sigma}\left(F^a{}_c F_{bd} - F^a{}_d F_{bc} + 2 F^a{}_b F_{cd}\right)$$

$$R^5{}_{a5b} = -D_a D_b \sigma - D_b\sigma D_a \sigma - \frac{1}{4} e^{2\sigma} F_{ac} F_b{}^c$$

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Total deriv - so no kinetic terms for scalar?

$$\text{giving} \quad S_5 = -\frac{1}{16\pi G_5}\int d^4x \, \underset{L}{dz} \sqrt{g^4}\left[e^{\sigma}\hat{R}_4 + \frac{1}{4} e^{3\sigma} F^2 - 2\Box e^{\sigma}\right]$$

$$\underset{\sim}{\Theta}{}^a{}_b = \underset{\sim}{\Theta}{}^a{}_b + \tfrac{1}{2} e^{\sigma} F^a{}_b \underset{\sim}{\omega}^s$$

giving

This is an example of scalar-tensor gravity.

$$\underset{\sim}{\Theta}{}^{a}{}_{b} = \underset{\sim\partial}{\Theta}{}^{a}{}_{b} + \tfrac{1}{2} e^{\sigma} F^{a}{}_{b} \underset{\sim}{\omega}{}^{s}$$

giving

This is an example of scalar-tensor gravity.
The scalar multiplies the Ricci term, & derivatives appear in the "Einstein eqns" through integration by parts. $\hat{g}_{ab}$ is said to be the '<u>Jordan frame</u>'

We can conformally transform our metric

$$\hat{g}_{ab} = \Omega^2(x)\, g_{ab}.$$

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Choose $\Omega = e^{-\sigma/2}$

giving $S_5 = -\frac{1}{16\pi G_5}\int d^4x\, dz \sqrt{g}\left[e^{\sigma} R_4 + \frac{1}{4} e^{\sigma} F^2 - 2\Lambda e^{\sigma} \right]$

L

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$$= \sqrt{g}\left(R + 3\Box\sigma - \frac{3}{2}(\nabla\sigma)^2\right)$$

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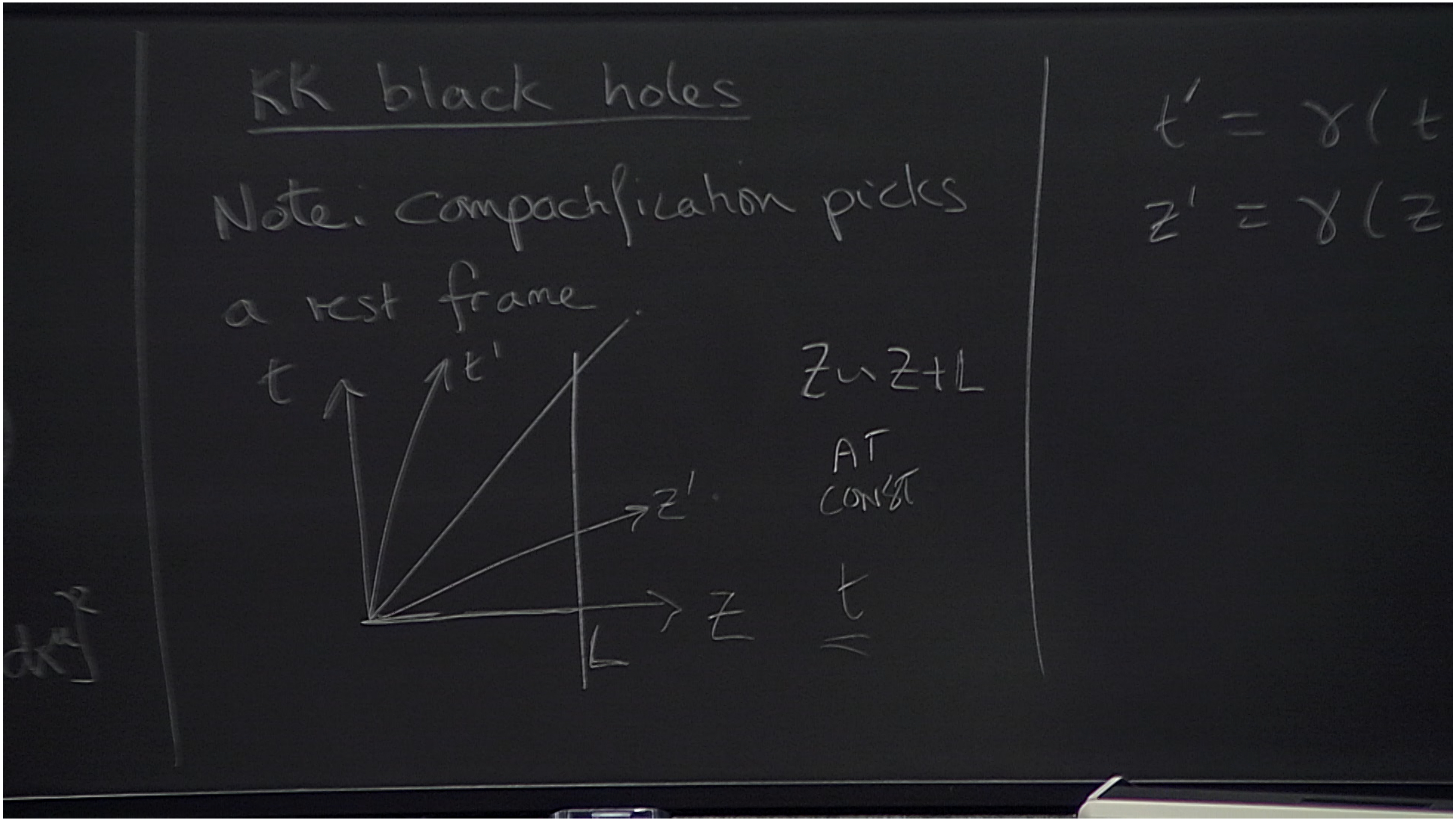
'Einstein frame'

Redefine $\varphi = \sigma/\sqrt{3}$ to canonically normalize kinetic terms:

$$S = \frac{1}{16\pi G_5/L} \int d^4x \sqrt{g} \left(-R + \frac{1}{2}(\partial\varphi)^2 - \frac{e^{\sqrt{3}\varphi}}{4} F^2 \right)$$

$$ds^2 = e^{-\sqrt{3}\varphi} g_{\mu\nu} dx^\mu dx^\nu - e^{2\sqrt{3}\varphi} \left[dz + A_\mu dx^\mu \right]^2$$


KK black holes
Note: compactification picks
a rest frame
t
t'
z'
z
L
z ~ z+L
AT
CONST
t
t' = γ (t
z' = γ (z

$$\left.\begin{aligned} t' &= \gamma(t - vz) \\ z' &= \gamma(z - vt) \end{aligned}\right\} \quad z \sim z + L \quad \leftrightarrow \quad \begin{aligned} z' &\sim z' + \gamma L \\ t' &\sim t - v\gamma L \end{aligned}$$

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What does this do to our black hole solns?

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SCH $\times \mathbb{R}$ is a soln of 5D gravity

$$ds^2 = \left(1 - \frac{2GM}{r}\right)dt^2 - \frac{dr^2}{1 - \frac{2GM}{r}} - r^2 d\Omega_{II}^2 - dz^2$$

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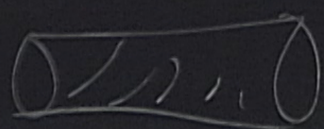 BLACK STRING.

Boost before identifying:

$$ds^2 = \left(1 - \frac{2GM}{r}\right) \frac{(dt + v\,dz)^2}{1 - v^2} - \frac{dr^2}{1 - \frac{2GM}{r}}$$

$$- \frac{(dz + v\,dt)^2}{1 - v^2} - r^2 d\Omega_{II}^2 .$$

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$$- \frac{(dz+v\,dt)^2}{1-v^2} - r^2\,d\Omega_{II}^2 .$$

$z \sim z + L$

rearrange to get

$$ds^2 = \left(1 - \frac{2GM}{(1-v^2)r} \right.$$

$$ds^2 = \left(1 - \frac{2GM}{(1-v^2)r + 2GMv^2}\right) dt^2 - r^2 d\Omega_{II}^2 - \frac{dr^2}{1-\frac{2GM}{r}}$$

$$- \left(1 + \frac{2GMv^2}{(1-v^2)r}\right)\left[dz + \frac{2GMv\,dt}{(1-v^2)r + 2GMv^2}\right]^2$$

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Velocity in extra dim $\leftrightarrow$ electric KK charge

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Velocity in extra dim $\leftrightarrow$ electric KK charge

Write $\hat{r} = r + \frac{2GMv^2}{1-v^2}$; $q = \frac{2GMv}{1-v^2}$; $\hat{M} = \frac{2GM}{1-v^2}$.

$$g_{\mu\nu}\,dx^\mu dx^\nu = \left(1-\frac{2G\hat{M}}{\hat{r}}\right)\left(1-\frac{vq}{\hat{r}}\right)^{-1/2} dt^2$$

$$-\left(1-\frac{vq}{\hat{r}}\right)^{3/2} \hat{r}^2\, d\Omega_{II}^2$$

$$-\left(1-\frac{2G\hat{M}}{\hat{r}}\right)^{-1}\left(1-\frac{vq}{\hat{r}}\right)^{1/2} d\hat{r}^2$$

KK electric b.h.

For a magnetic black hole

$$\underset{\sim}{F} = Q \sin\theta \, \underline{d}\theta \wedge \underline{d}\varphi$$

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For a magnetic black hole

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CLOSED but not EXACT

'Locally'

$$\underset{\sim}{A}_N = Q(1-\cos\theta)\,\underset{\sim}{d}\varphi$$

$$\underset{\sim}{A}_S = -Q\,(1+\cos\theta)\,\underset{\sim}{d}\varphi$$

$$\underset{\sim}{A}_S = \underset{\sim}{A}_N - 2Q\,\underset{\sim}{d}\varphi$$

With this Ansatz find the soln

$$ds^2 = \left(\frac{r-r_+}{r-r_-}\right) dt^2 - \frac{dr^2}{(1-r_+/r)} - r(r-r_-)\, d\Omega_{II}^2$$

$$- \left(1-\frac{r_-}{r}\right)\left[dz_N + Q\,(1-\cos\theta)\, d\varphi\right]^2$$

$$Q \downarrow \sqrt{r_+ r_-}$$

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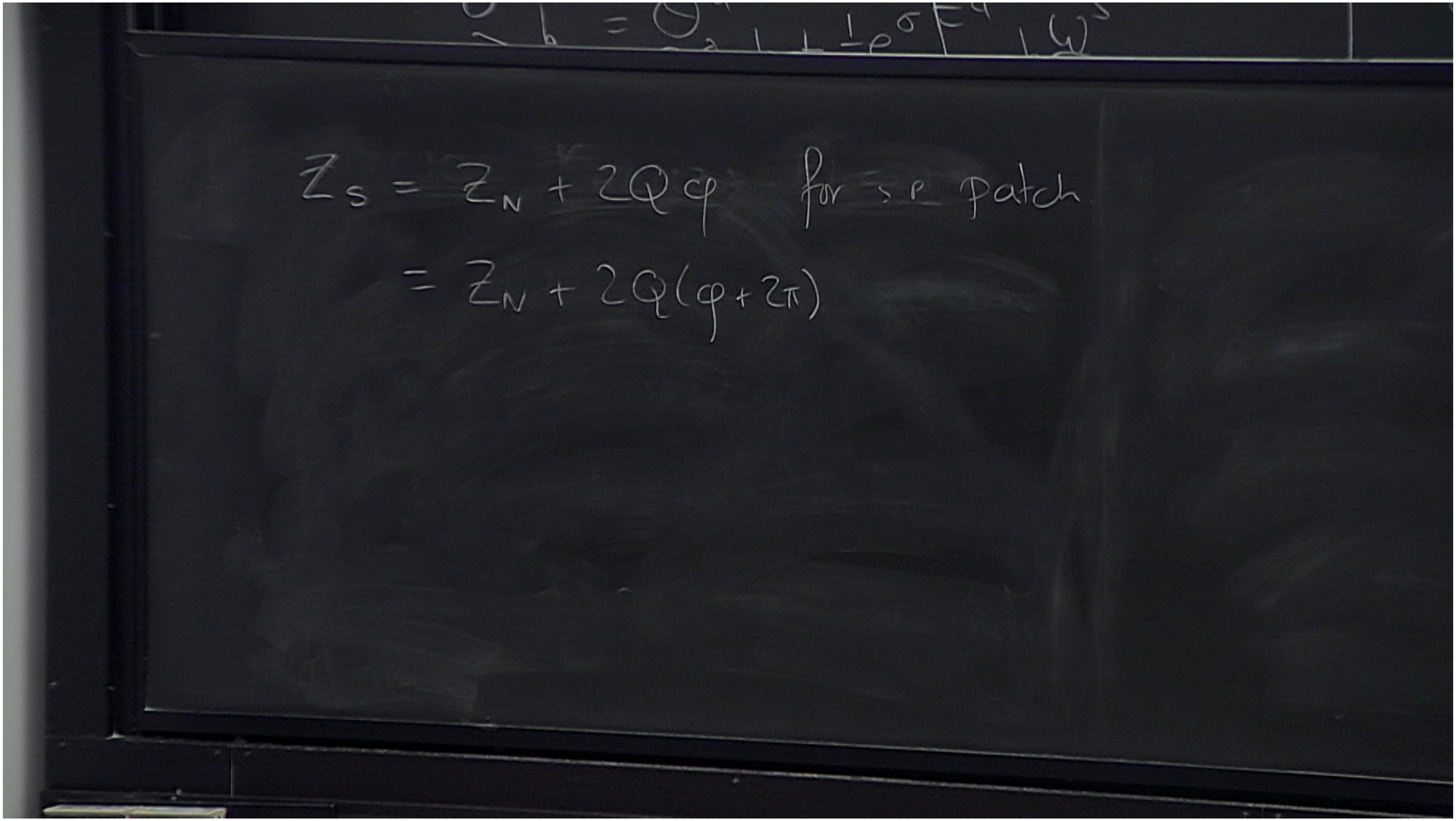
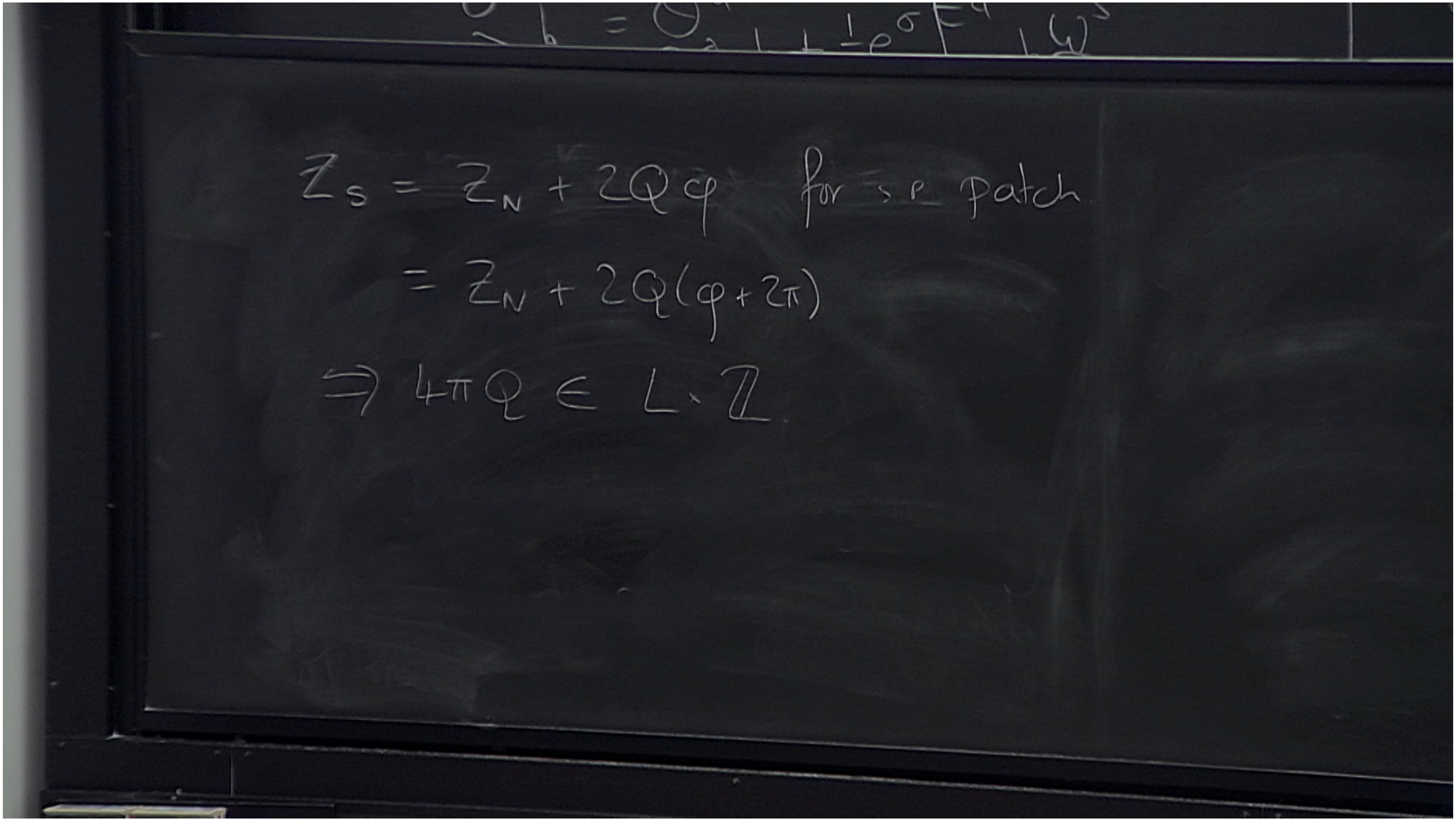
$Q \to \sqrt{r_+ r_-}$

$(1-\cos\theta)\,d\varphi \to A_N$


$Z_S = Z_N + 2Q\varphi$ for s.p. patch
$= Z_N + 2Q(\varphi + 2\pi)$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$$Z_S = Z_N + 2Q\varphi \quad \text{for s.p. patch}$$

$$= Z_N + 2Q(\varphi + 2\pi) \sim Z_S + L \times n$$

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$Z_S = Z_N + 2Q\varphi$ for S.P. patch

$= Z_N + 2Q(\varphi + 2\pi) \sim Z_S + L \times n$

$\Rightarrow 4\pi Q \in L \cdot \mathbb{Z}$

– Charge is quantized

Extremal limit $r_+ = r_-$ $\quad Q = r_+ = L/4\pi$

Let $\chi = 4\pi z / L \quad (\Delta\chi = 4\pi)$

$$ds^2 = dt^2 - \frac{dr^2}{1-Q/r} - \left(1-\frac{Q}{r}\right)\left[r^2 d\Omega_{II}^2 + Q^2 (d\chi_N + A_N)^2\right]$$

9) Causality
10) Wick
11) Fermionic rules
12) Yukawa diags
13) QED Maxwell

Extremal limit $r_+ = r_-$ $\quad Q = r_+ = L/4\pi$

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$r \to Q \quad \rho^2 = 4Q(r-Q) \quad$

$$ds^2 \sim dt^2 - d\rho^2 - \frac{\rho^2}{4}\left[d\theta^2 + \sin^2\theta \, d\varphi^2 + (d\chi + (1-\cos\theta)d\varphi)^2\right]$$

Extremal limit $r_+ = r_-$ $\quad Q = r_+ = L/4\pi$

Let $\chi = 4\pi z / L \quad (\Delta\chi = 4\pi)$

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$r \to Q$. $\quad \rho^2 = 4Q(r-Q)$.

$$ds^2 \sim dt^2 - d\rho^2 - \frac{\rho^2}{4}\underbrace{\left[d\theta^2 + \sin^2\theta\, d\varphi^2 + (d\chi + (1-\cos\theta)d\varphi)^2\right]}_{\text{ORIGIN OF } \mathbb{R}^4 \text{ in Euler angles}}$$

r gravity

& derivatives

integration

rdan frame'

$$\underset{\sim}{\Theta}^a{}_b = \underset{\sim}{\Theta}_0{}^a{}_b + \tfrac{1}{2} e^\sigma F^a{}_b \omega^s$$

$$x + iy = \rho \cos\theta/_2 \; e^{i\chi/_2}$$

$$z + iw = \rho \sin\theta/_2 \; e^{i(\varphi + \chi/_2)}$$

$$\Theta^a{}_b = \Theta_0{}^a{}_b + \tfrac{1}{2} e^{\sigma} F^a{}_b \,\omega^s$$

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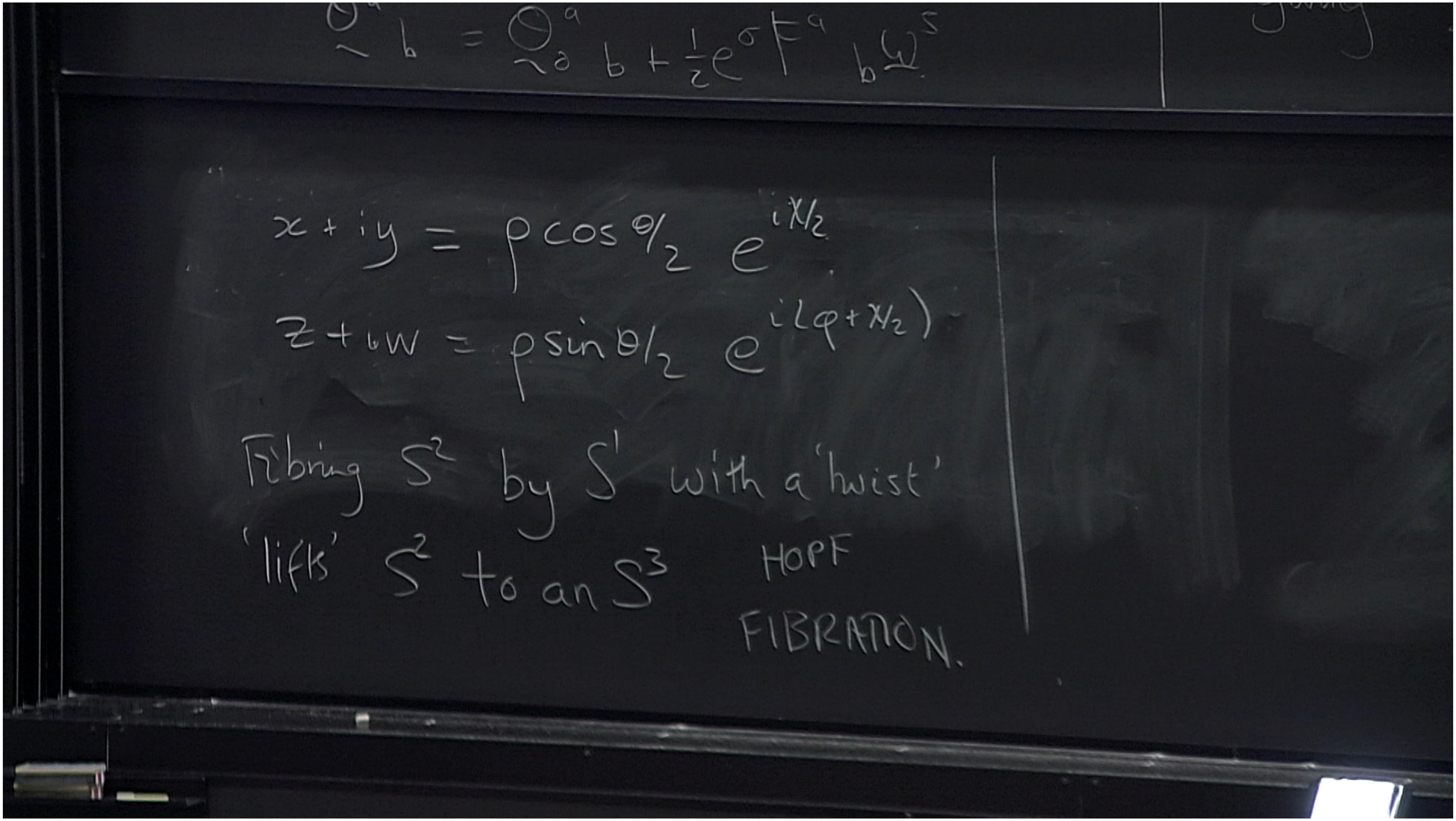
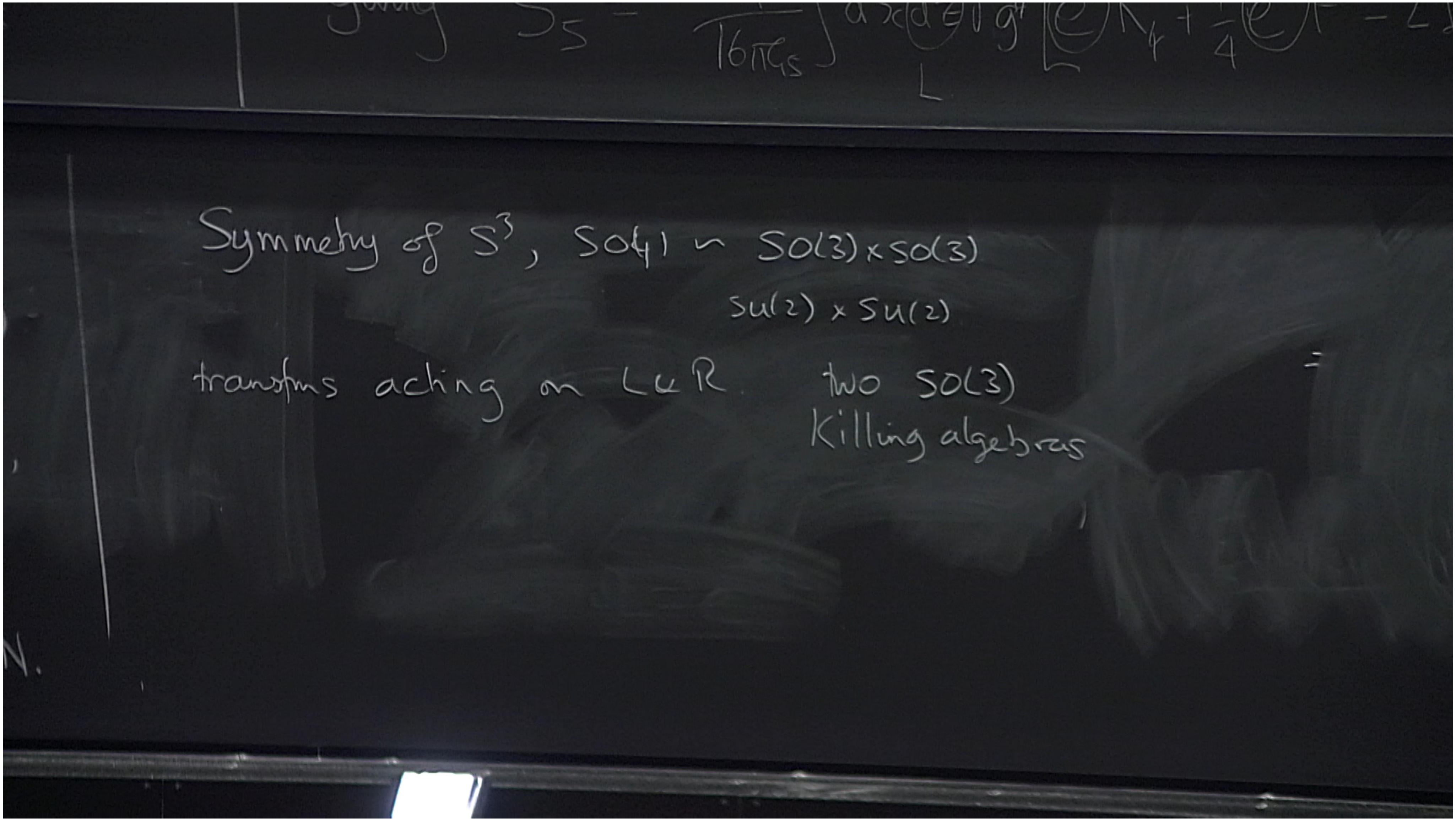
Fibring S^2 by S^1 with a 'twist'



$x + iy = \rho \cos \theta/_2 \, e^{i\chi/_2}$
$z + iw = \rho \sin \theta/_2 \, e^{i(\varphi + \chi/_2)}$
Fibring S^2 by S^1 with a 'twist'
'lifts' S^2 to an S^3
HOPF
FIBRATION.


Symmetry of S3, SO(4) ~ SO(3)×SO(3)
SU(2) × SU(2)
transfms acting on L & R
two SO(3)
Killing algebras