Title: PSI 2018/2019 - Gravitational Physics - Lecture 6

Date: Feb 04, 2019 10:15 AM

URL: http://pirsa.org/19020013

Abstract:

LECTURE 6 MORE ON BLACK HOLES

Recall $\dot{r}^2 + V_{eff}(r) = 0$

$\theta = \pi/2$ $\quad \eta^2 - E^2 - \frac{2GM\eta}{r} + \frac{a^2(\eta - E^2) + h^2}{r^2} - \frac{2GM}{r^3}(h - aE)^2$

LECTURE 6   MORE ON BLACK HOLES

Recall $\dot{r}^2 + V_{eff}(r) = 0$

$\theta = \pi/2$ $\eta^2 - E^2 - \frac{2GM\eta}{r} + \frac{a^2(\eta - E^2) + h^2}{r^2} - \frac{2GM}{r^3}(h - aE)^2$

$g_{rr} \to \infty$ $\Delta = r^2 + a^2 - 2GMr \to 0$

Look at null geodesics

$\eta = 0 \qquad E = 1$

for algebraic simplicity

take $GM = a$

ie $\Delta = (r-a)^2$

Look at null geodesics

$\eta = 0 \qquad E = 1$

for algebraic simplicity

take $GM = a$

ie $\Delta = (r-a)^2$

EXTREMAL LIMIT

Look at null geodesics

$\eta = 0 \qquad E = 1$

for algebraic simplicity

take $GM = a$

ie $\Delta = (r-a)^2$

EXTREMAL LIMIT

$$1 + V_{eff} = \frac{h^2 - a^2}{r^2} - \frac{2a}{r^3}(h-a)^2$$

geodesics

1

implicity

a

MAL LIMIT.

$$1+V_{eff} = \frac{h^2-a^2}{r^2} - \frac{2a}{r^3}(h-a)^2$$

Initial conditions: $\dot{r} = 0$

$r = r_m$

$V=0$ has soln $r_m = h-a$.

Since $r_m \geq a$

geodesics

l

implicity

a

NAL LIMIT.

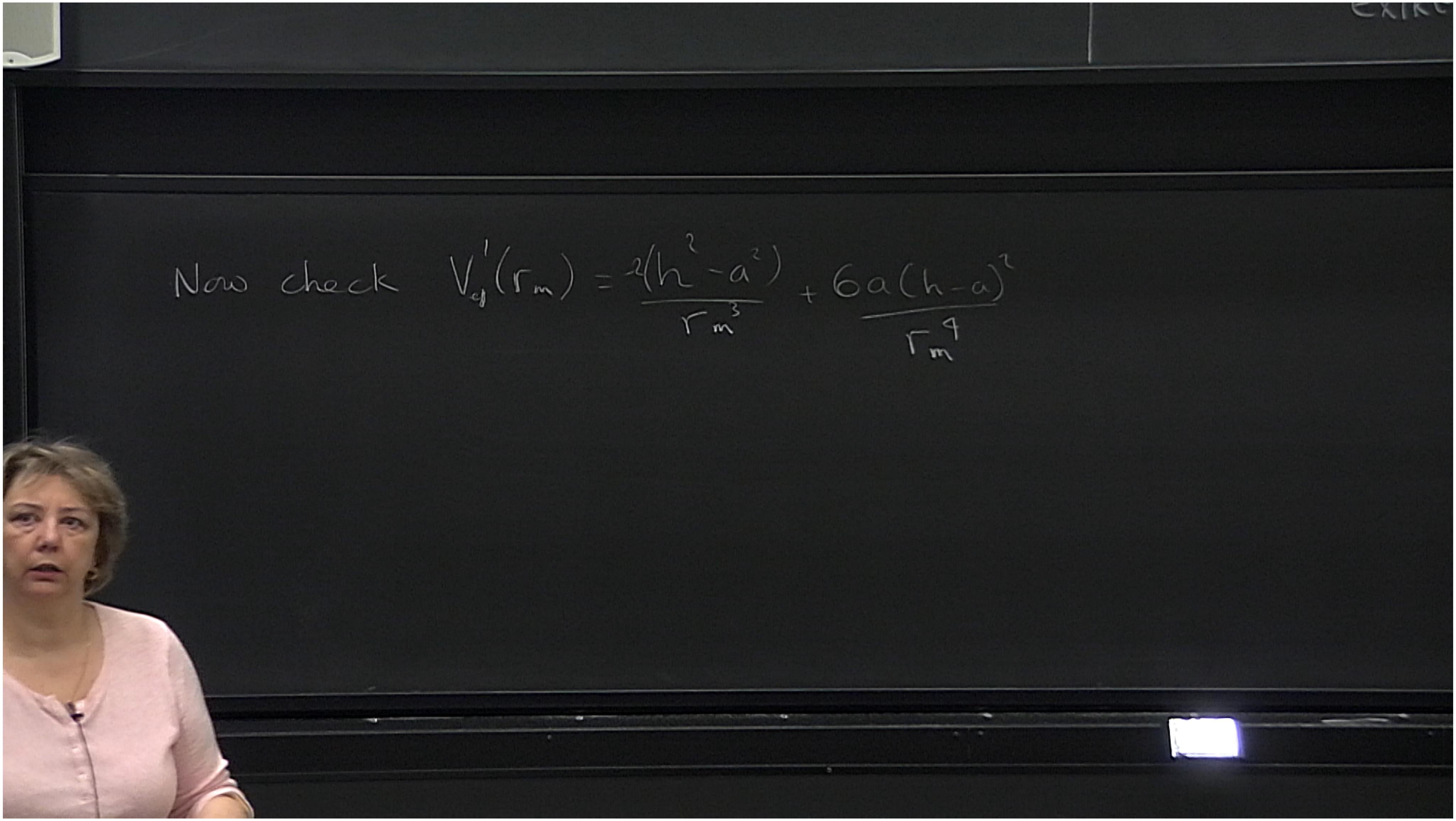
$$1+V_{eff} = \frac{h^2-a^2}{r^2} - \frac{2a}{r^3}(h-a)^2$$

Initial conditions: $\dot{r} = 0$
$r = r_m$

$V=0$ has soln $r_m = h-a$

Since $r_m \geqslant a$

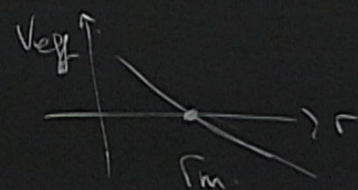
$$h \geqslant 2a.$$


Now check $V_{eff}'(r_m) = \frac{-2(h^2-a^2)}{r_m^3} + \frac{6a(h-a)^2}{r_m^4}$

Now check $V_{eff}'(r_m) = \dfrac{-2(h^2-a^2)}{r_m^3} + \dfrac{6a(h-a)^2}{r_m^4}$

$$= \frac{4a-2h}{r_m^2} \leq 0$$

hence r increases



Limit at $h = 2a$, $r_m = a$

Hence $\Delta = 0$ is event hor.

Limit at $h=2a$, $r_m=a$

Hence $\Delta=0$ is event hor.

Black holes do not exist

in vacuo.

Limit at $h=2a$, $r_m=a$

Hence $\Delta=0$ is event hor.

Black holes do not exist
in vacuo. Often have an
accretion disc - model by
concentric circular orbits.

t $h=2a$, $r_m = a$

$\Delta = 0$ is event hor.

oles do not exist

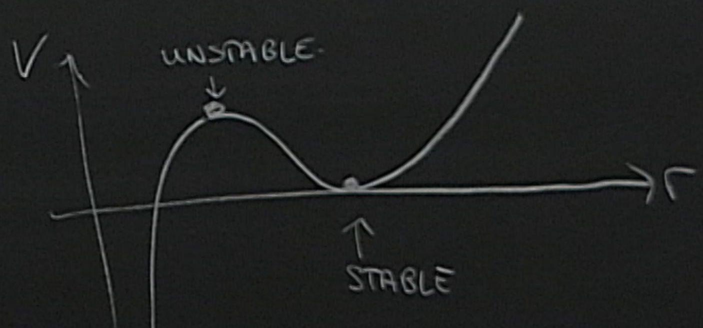
o. Often have an

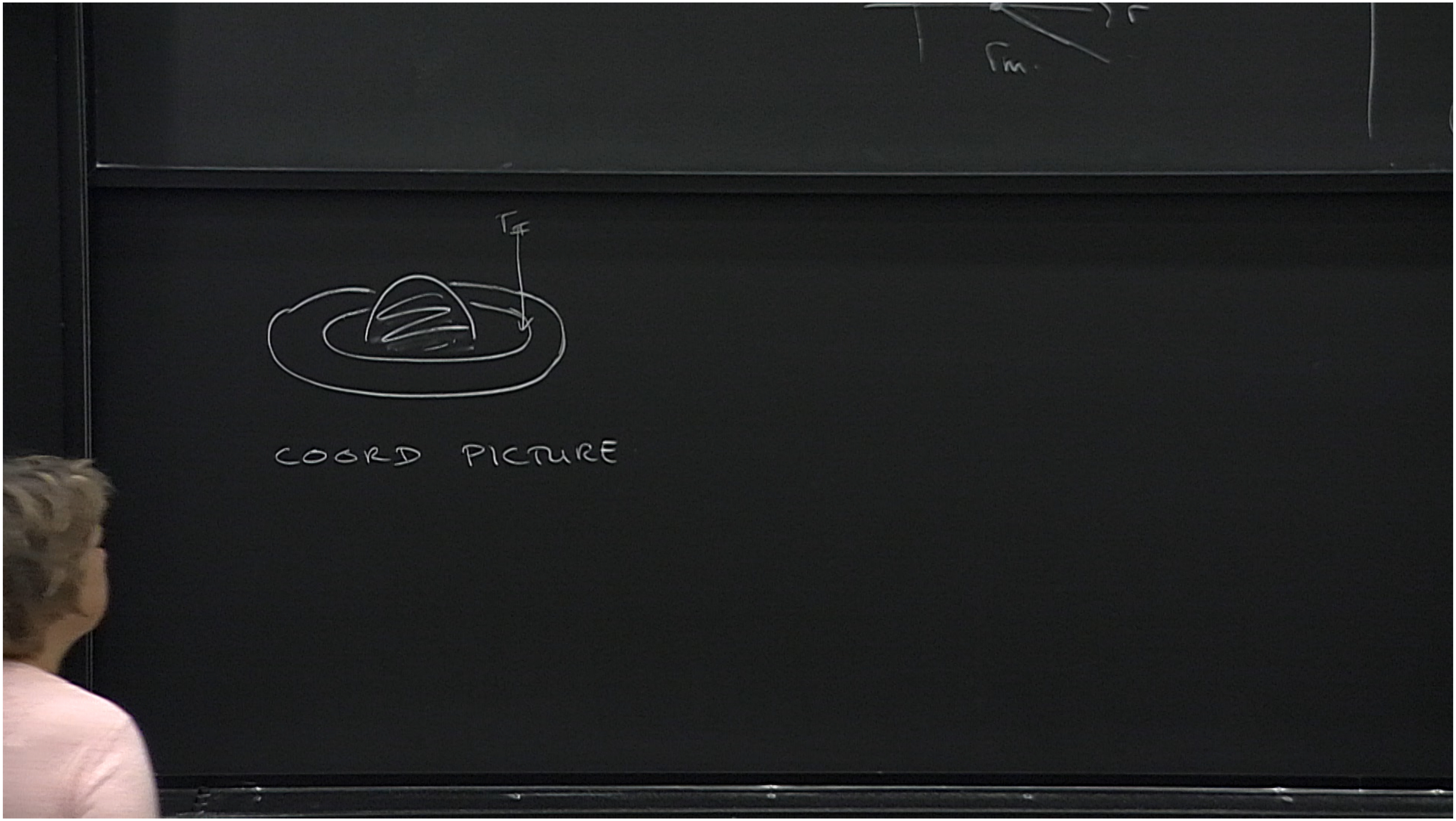
on disc - model by

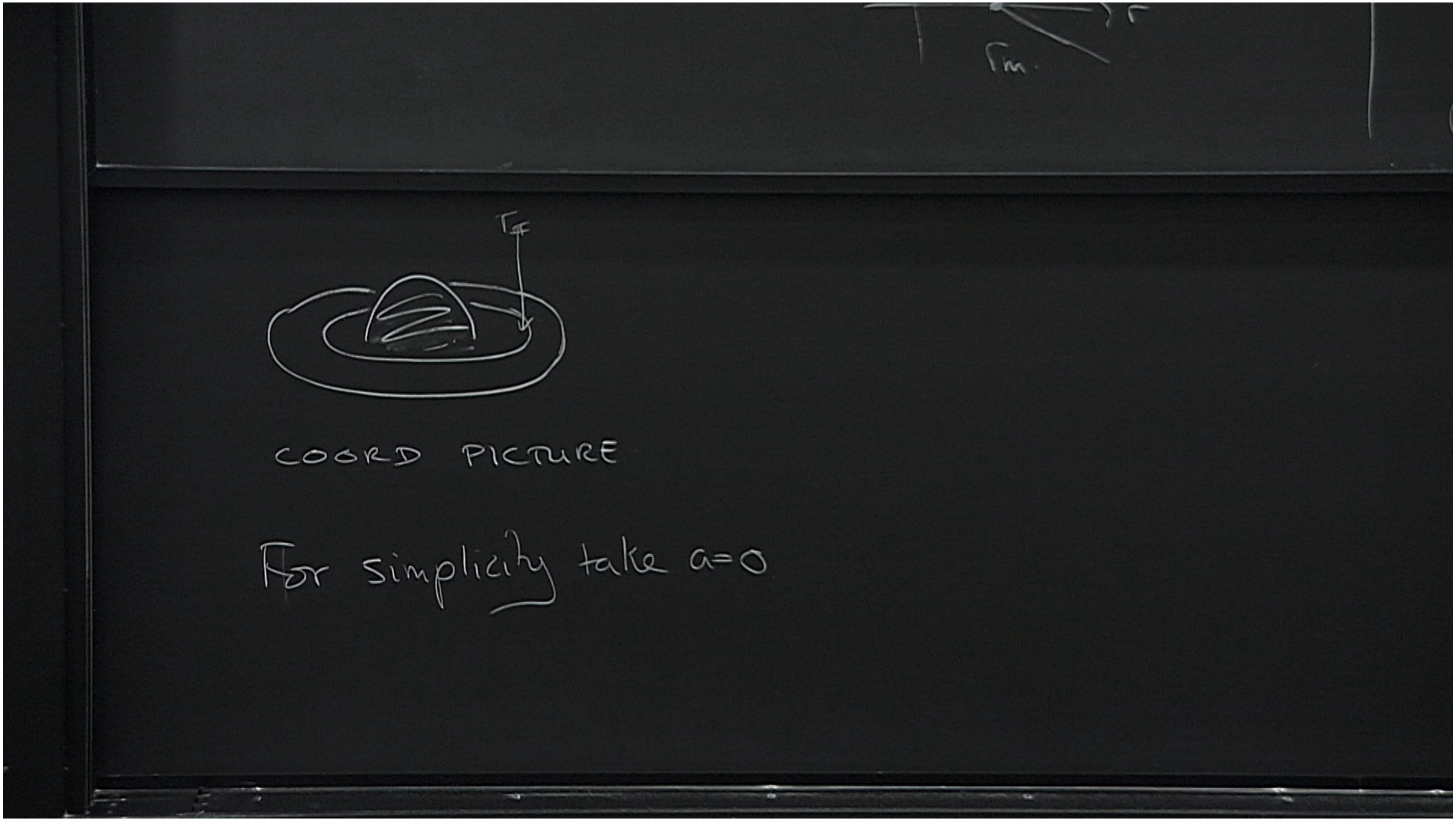
circular orbits.

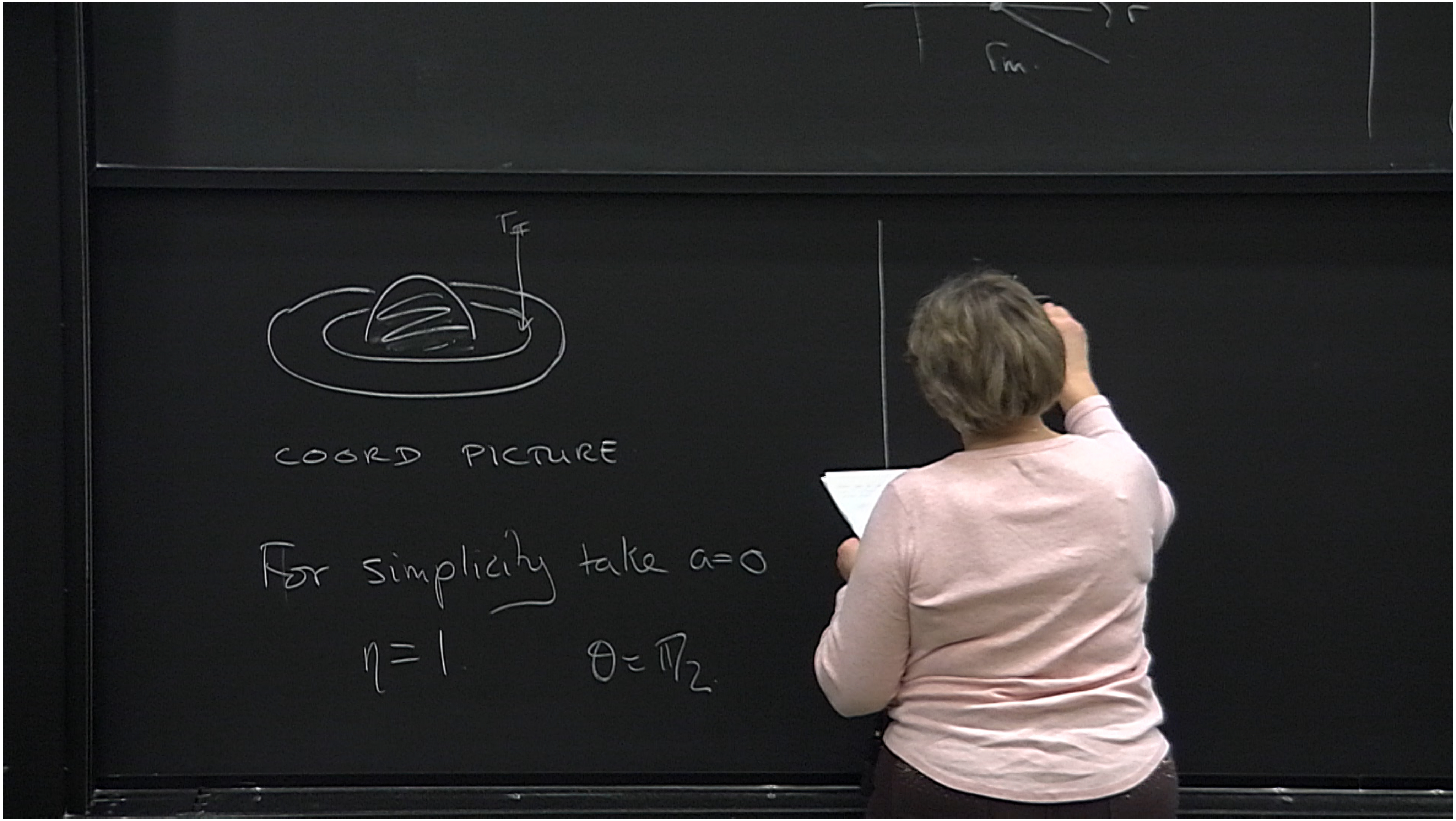
Circular orbit has $r = \text{const}$

$V_{eff} = V'_{eff} = 0$




COORD PICTURE


COORD PICTURE
For simplicity take a=0


COORD PICTURE
For simplicity take a=0
η=1 θ=π/2

accretion disc - model by concentric circular orbits.

r_m.

$$V = 1 - E^2 - \frac{2GM}{r} + \frac{h^2}{r^2} - \frac{2GMh^2}{r^3}$$

$$V' = \frac{2GM}{r^2} - \frac{2h^2}{r^3} + \frac{6GMh^2}{r^4}$$

$$V'' = -\frac{4GM}{r^3} + \frac{6h^2}{r^4} - \frac{24GMh^2}{r^5}$$

circular orbits.

$V = V' = 0$ determines h, E at given r.

circular orbits.

$V = V' = 0$ determines h, E at given r

$$h^2 = \frac{GMr^2}{r-3GM} \qquad E^2 = \frac{r-2GM}{r} + \frac{GM(r-2GM)}{r(r-3GM)}$$

circular orbits.

$V = V' = 0$ determines h, E at given r

$$h^2 = \frac{GMr^2}{r-3GM} \qquad E^2 = \frac{r-2GM}{r} + \frac{GM(r-2GM)}{r(r-3GM)}$$

$$V'' = \frac{2GM}{r^3}\,\frac{r-6GM}{r-3GM}$$

circular orbits.

$V = V' = 0$ determines h, E at given r.

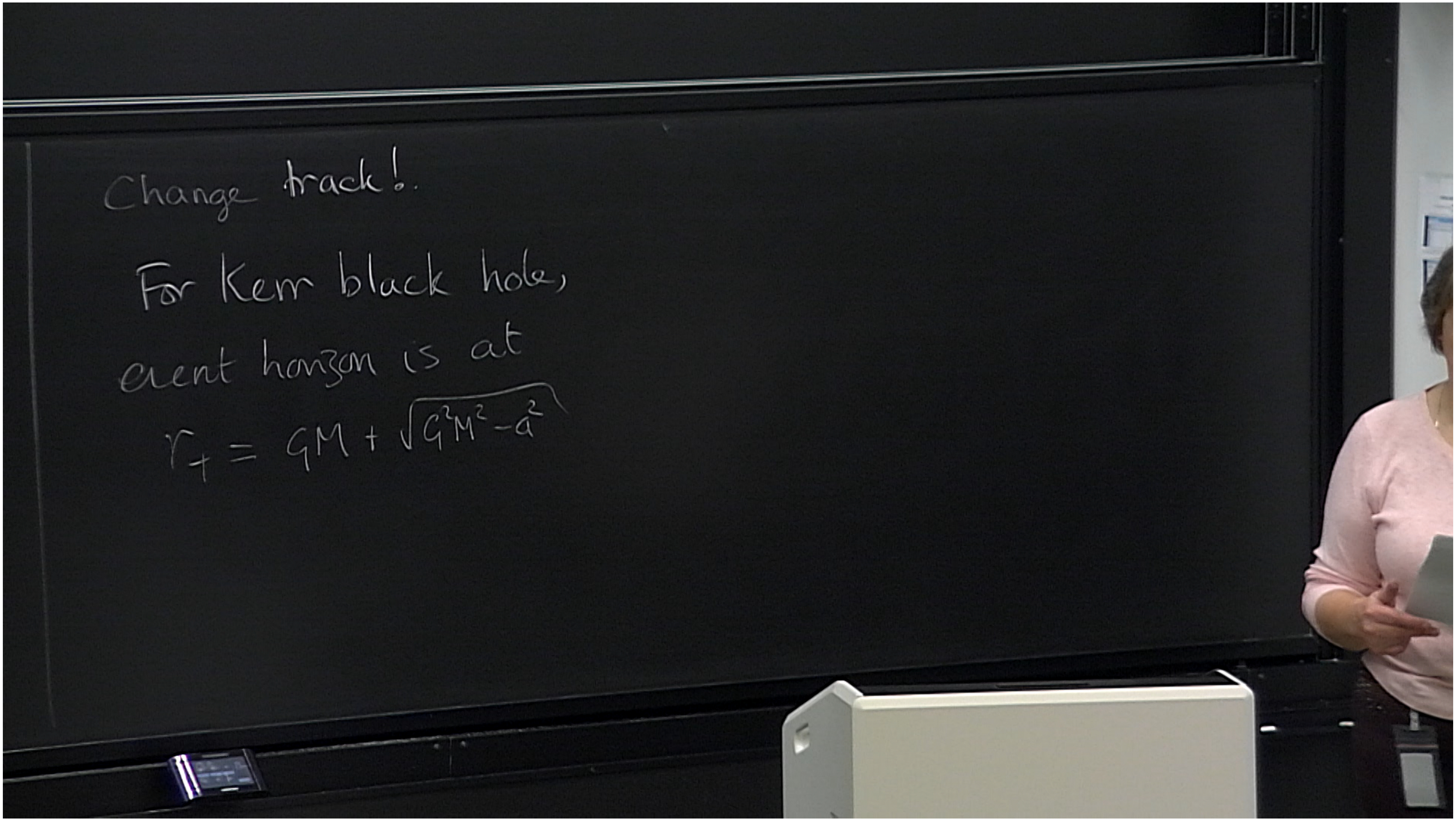
$$h^2 = \frac{GMr^2}{r-3GM} \qquad E^2 = \frac{r-2GM}{r} + \frac{GM(r-2GM)}{r(r-3GM)}$$

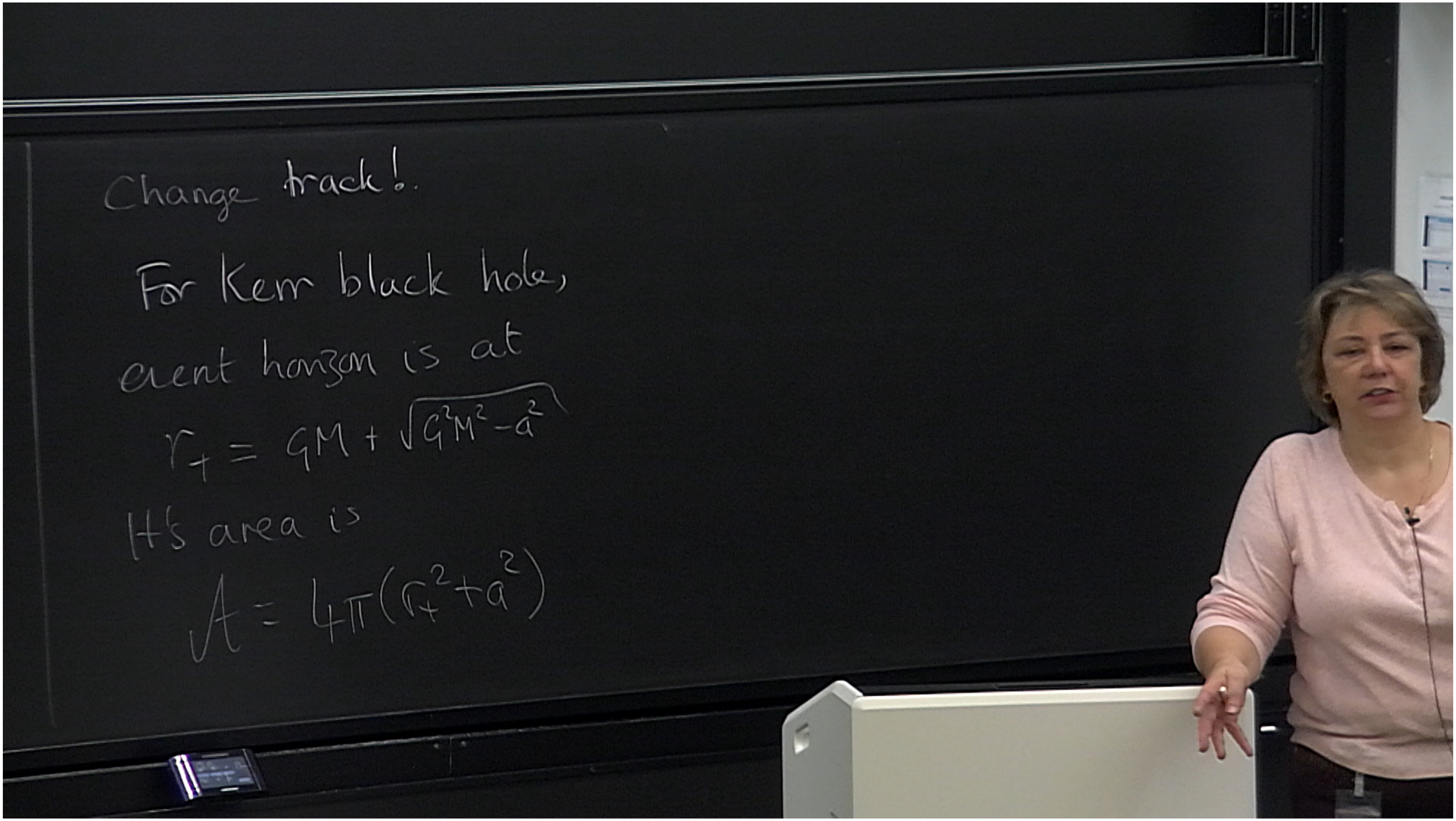
$$V'' = \frac{2GM}{r^3}\,\frac{r-6GM}{r-3GM} > 0 \text{ for } r > 6GM$$

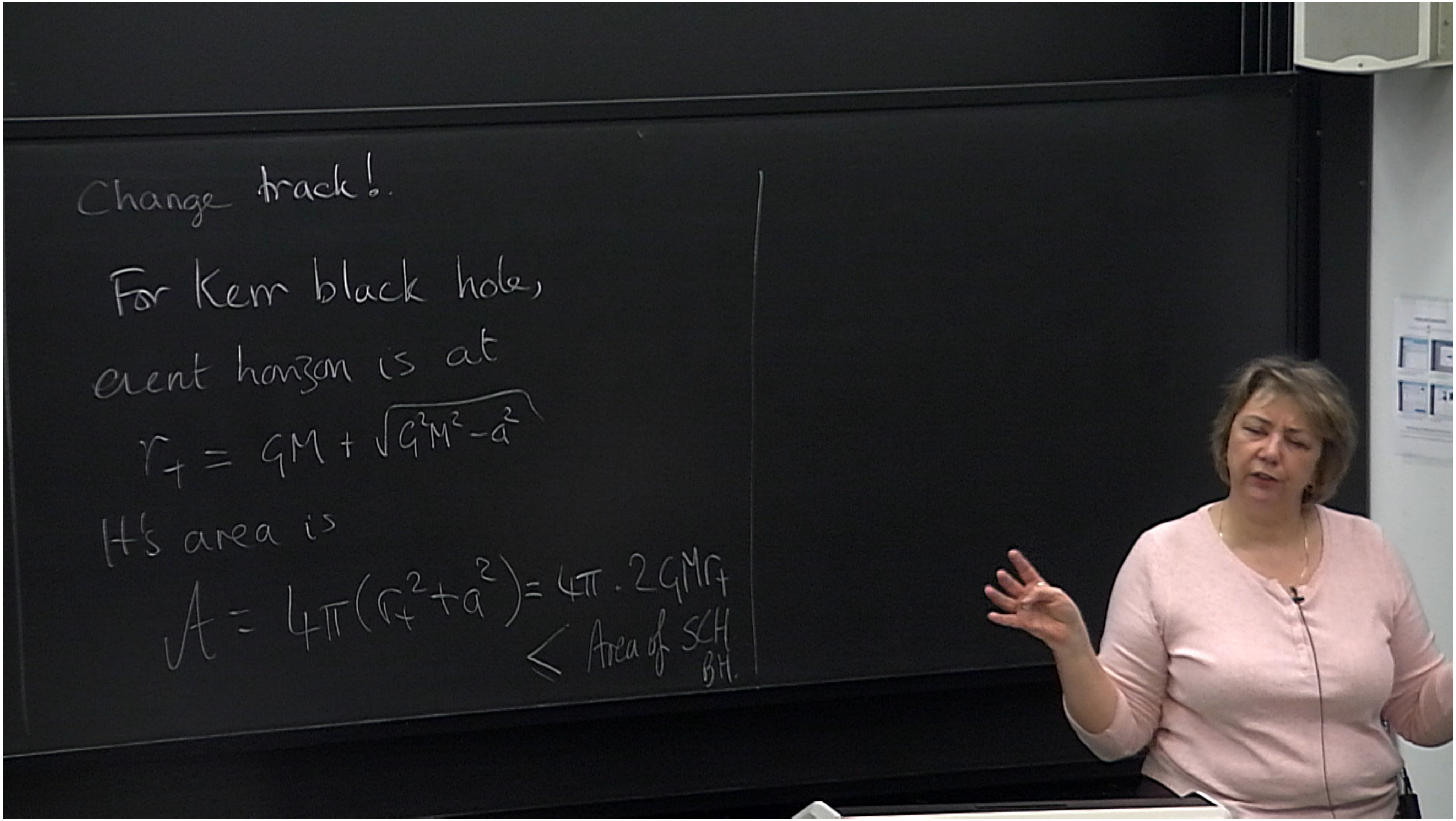
$r = 6GM$ is the limit of stable circular orbits

Innermost Stable Circular Orbit - ISCO.

For Kerr $r_{ISCO} < 6GM$, extremal: $r_{ISCO} = a = r_H$


Change track!.
For Kerr black hole,
event horizon is at
$r_+ = GM + \sqrt{G^2M^2 - a^2}$


Change track!
For Kerr black hole,
event horizon is at
$r_+ = GM + \sqrt{G^2M^2 - a^2}$
It's area is
$A = 4\pi(r_+^2 + a^2)$


Change track!.
For Kerr black hole,
event horizon is at
$r_+ = GM + \sqrt{G^2M^2 - a^2}$
It's area is
$A = 4\pi(r_+^2 + a^2) = 4\pi \cdot 2GMr_+$
$<$ Area of SCH BH.

Change track!.

For Kerr black hole,

event horizon is at

$$r_+ = GM + \sqrt{G^2M^2 - a^2}$$

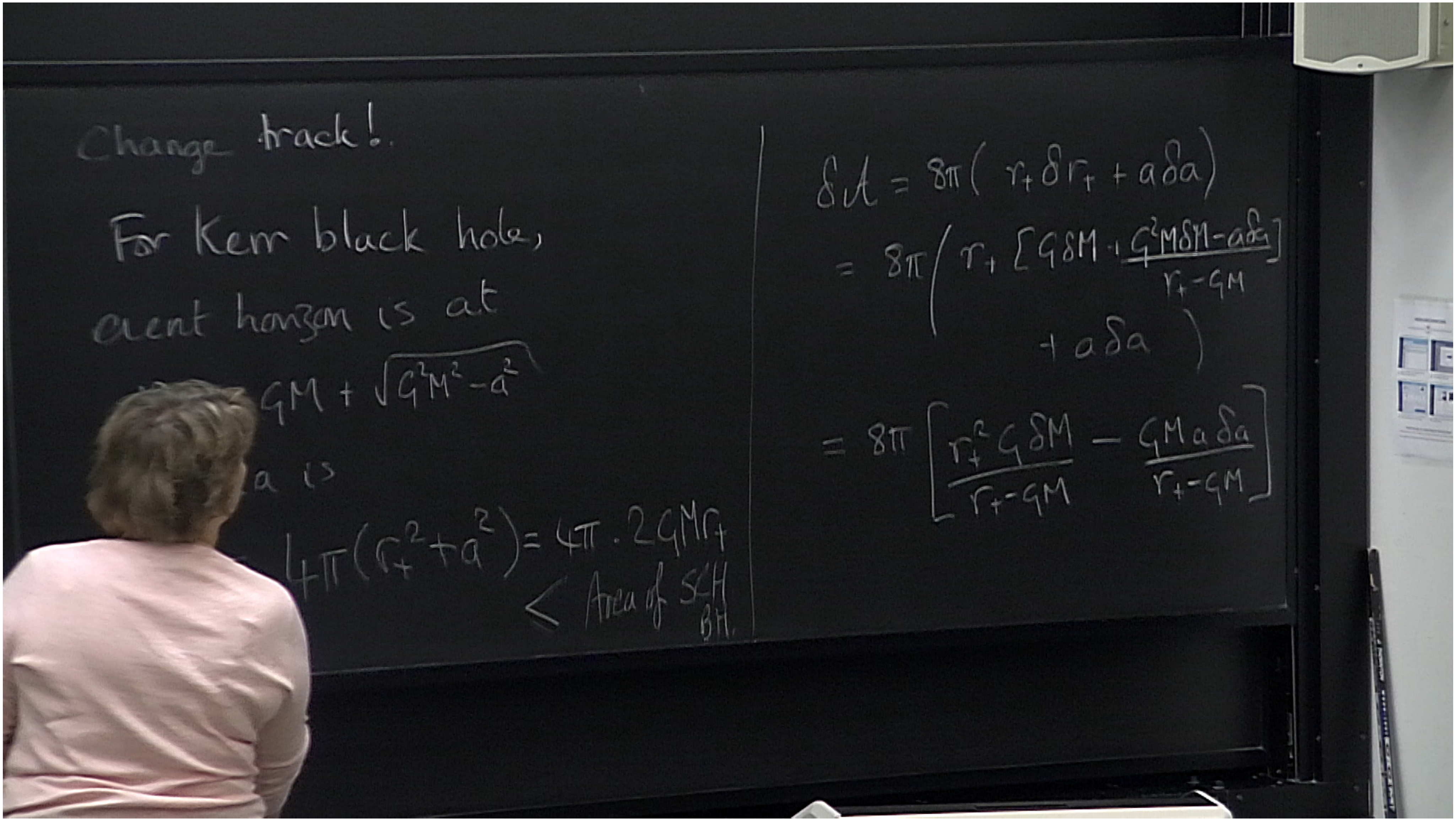
It's area is

$$A = 4\pi(r_+^2 + a^2) = 4\pi \cdot 2GMr_+$$

$<$ Area of SCH BH.

$$\delta A = 8\pi(r_+ \delta r_+ + a\delta a)$$

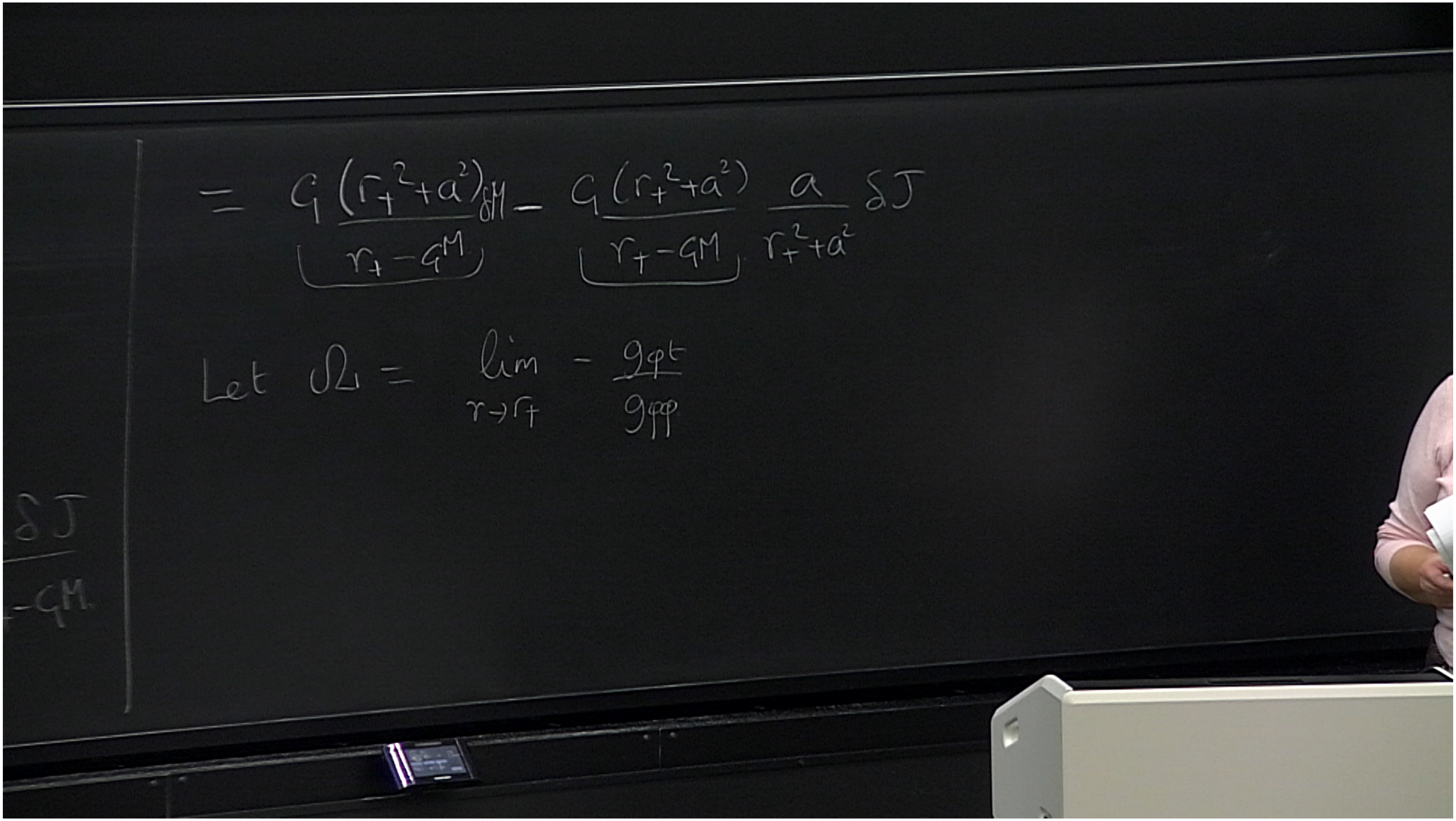
$$= 8\pi\left(r_+ \left[G\delta M + \frac{G^2 M \delta M - a\delta a}{r_+ - GM} \right] + a\delta a \right)$$


Change track!
For Kerr black hole,
event horizon is at
GM + √(G²M² − a²)
a is
4π(r+² + a²) = 4π . 2GMr+
< Area of SCH BH.
δA = 8π(r+ δr+ + aδa)
= 8π(r+ [GδM + (G²MδM − aδa)/(r+ − GM)]
+ aδa)
= 8π [r+² GδM/(r+ − GM) − GMaδa/(r+ − GM)]

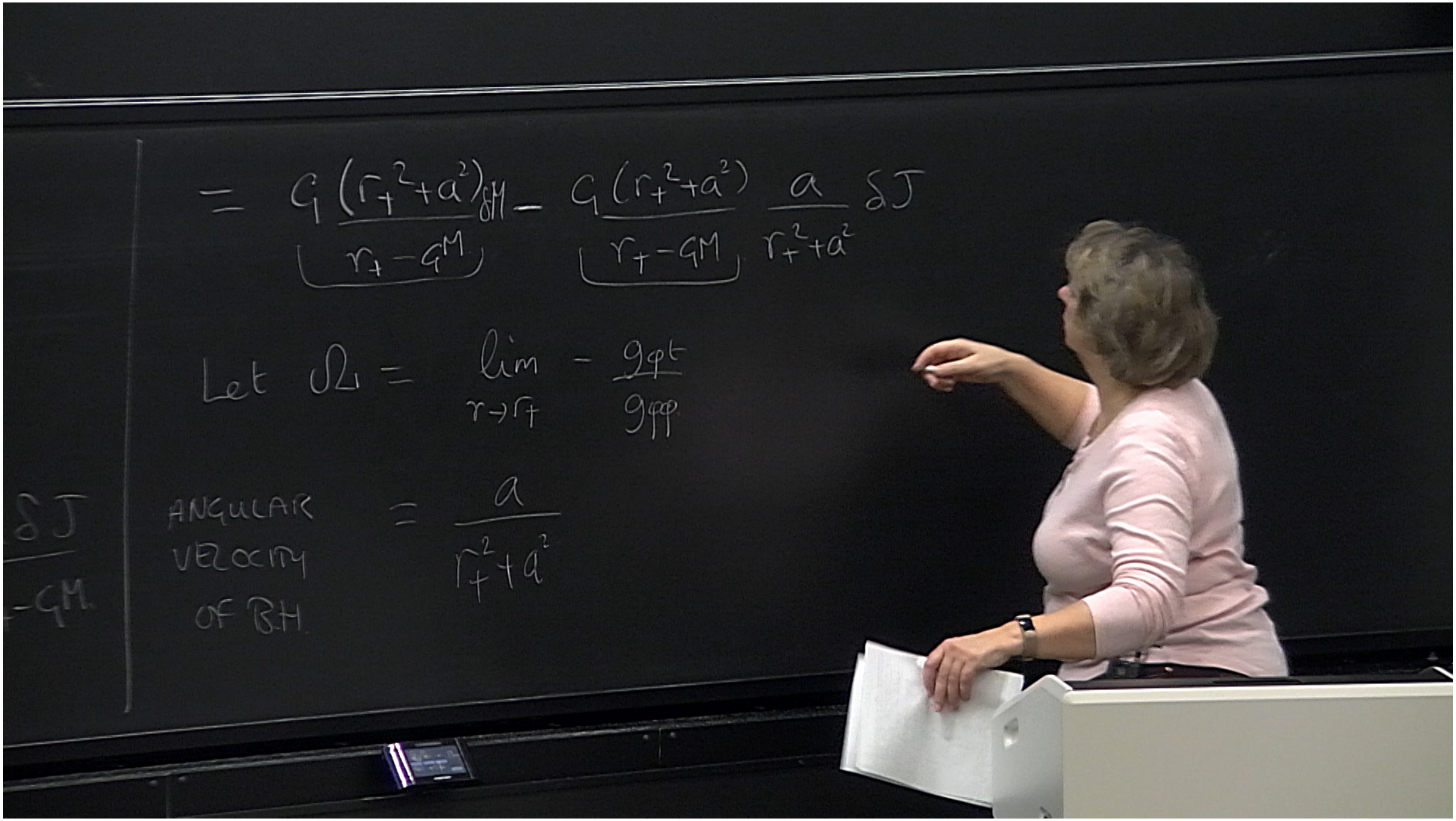
but $a = J/M$

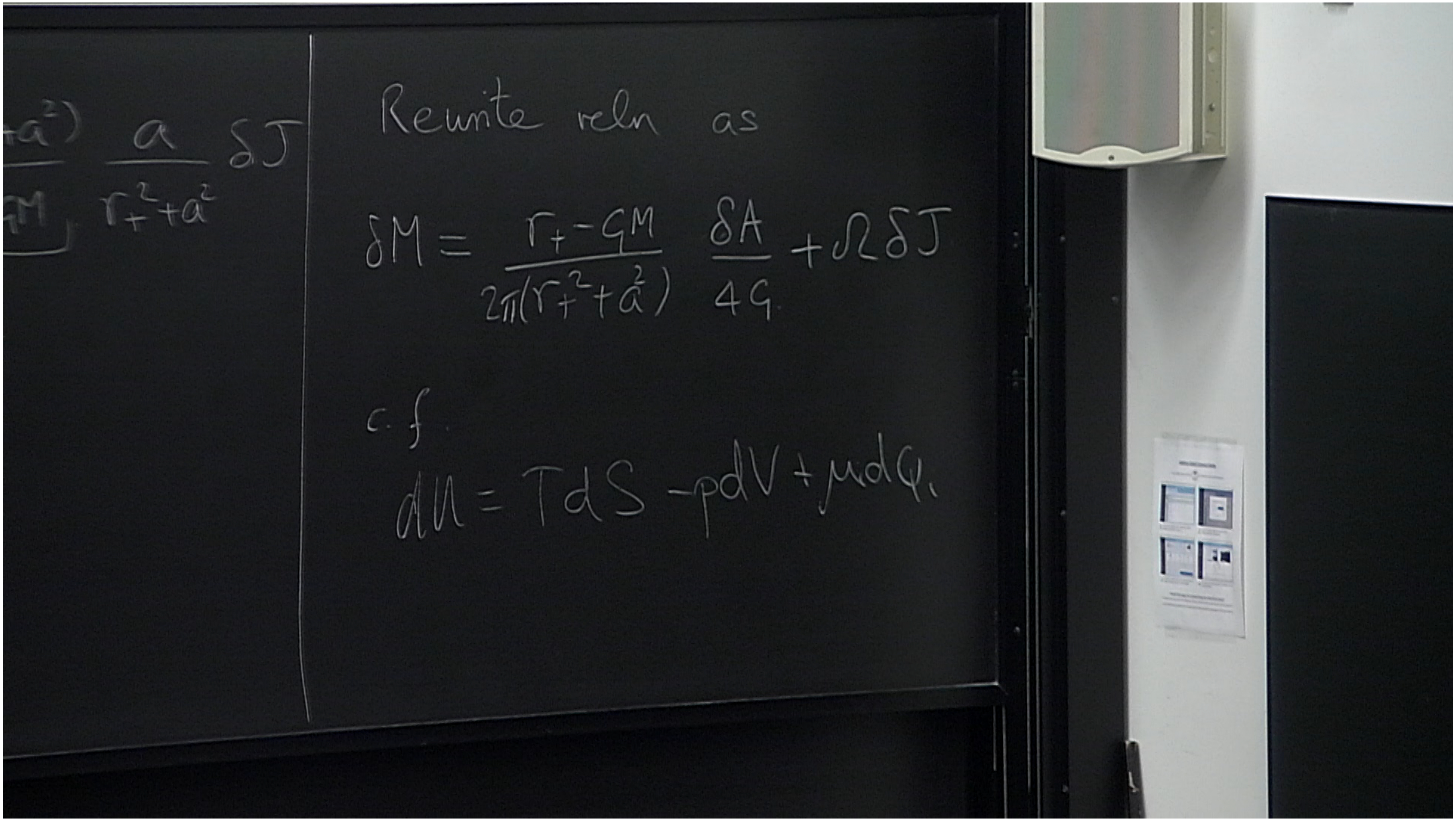
$$\delta a = \frac{\delta J}{M} - \overset{a}{\frac{J}{M}} \frac{\delta M}{M}$$

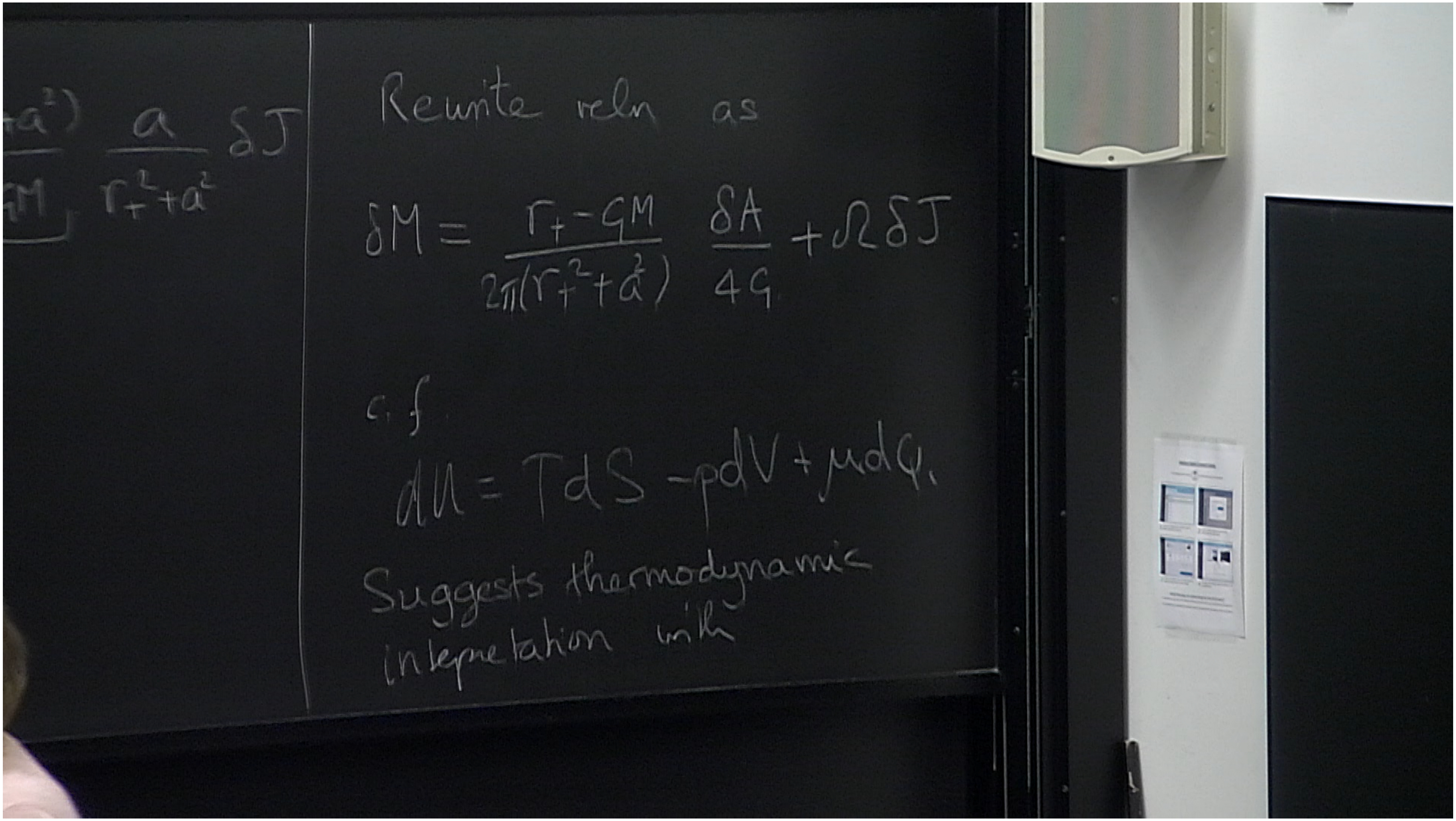
$$\Rightarrow \frac{\delta A}{8\pi} = \frac{r_+^2 G \delta M + G a^2 \delta M}{r_+ - GM} - \frac{G a \delta J}{r_+ - GM}$$



$= G\frac{(r_+^2+a^2)}{r_+-GM}\delta M - G\frac{(r_+^2+a^2)}{r_+-GM}\frac{a}{r_+^2+a^2}\delta J$
Let $\Omega = \lim_{r\to r_+} -\frac{g_{\varphi t}}{g_{\varphi\varphi}}$
δJ
$-GM.$


$= G\frac{(r_+^2+a^2)}{r_+ - GM}\delta M - G\frac{(r_+^2+a^2)}{r_+ - GM}\frac{a}{r_+^2+a^2}\delta J$
Let $\Omega = \lim_{r\to r_+} - \frac{g_{\varphi t}}{g_{\varphi\varphi}}$
ANGULAR VELOCITY OF B.H
$= \frac{a}{r_+^2+a^2}$
δJ
$-GM$


$+a^2)$ $\frac{a}{r_+^2+a^2}\delta J$
GM
Rewrite reln as
$\delta M = \frac{r_+ - GM}{2\pi(r_+^2+a^2)} \frac{\delta A}{4G} + \Omega\,\delta J$
c.f.
$dU = TdS - pdV + \mu dQ.$


$\frac{a}{r_+^2+a^2}\,\delta J$
Rewrite reln as
$\delta M = \frac{r_+ - GM}{2\pi(r_+^2+a^2)}\frac{\delta A}{4G} + \Omega\,\delta J$
c.f.
$dU = TdS - pdV + \mu dQ.$
Suggests thermodynamic
interpretation with

$M \Leftrightarrow$ energy *

$$T = \frac{r_+ - GM}{2\pi(r_+^2 + a^2)} \qquad S = \frac{A}{4G}.$$

$$\rightarrow \frac{1}{8\pi GM} \quad \text{for SCH}$$

$= \frac{A}{4G}$

CH

Explore a bit more ....

Take SCH, $t \to i\tau$

$$|ds^2| = \left(1 - \frac{2GM}{r}\right) d\tau^2 + \left(1 - \frac{2GM}{r}\right) dr^2 + r^2 d\Omega_{II}^2$$

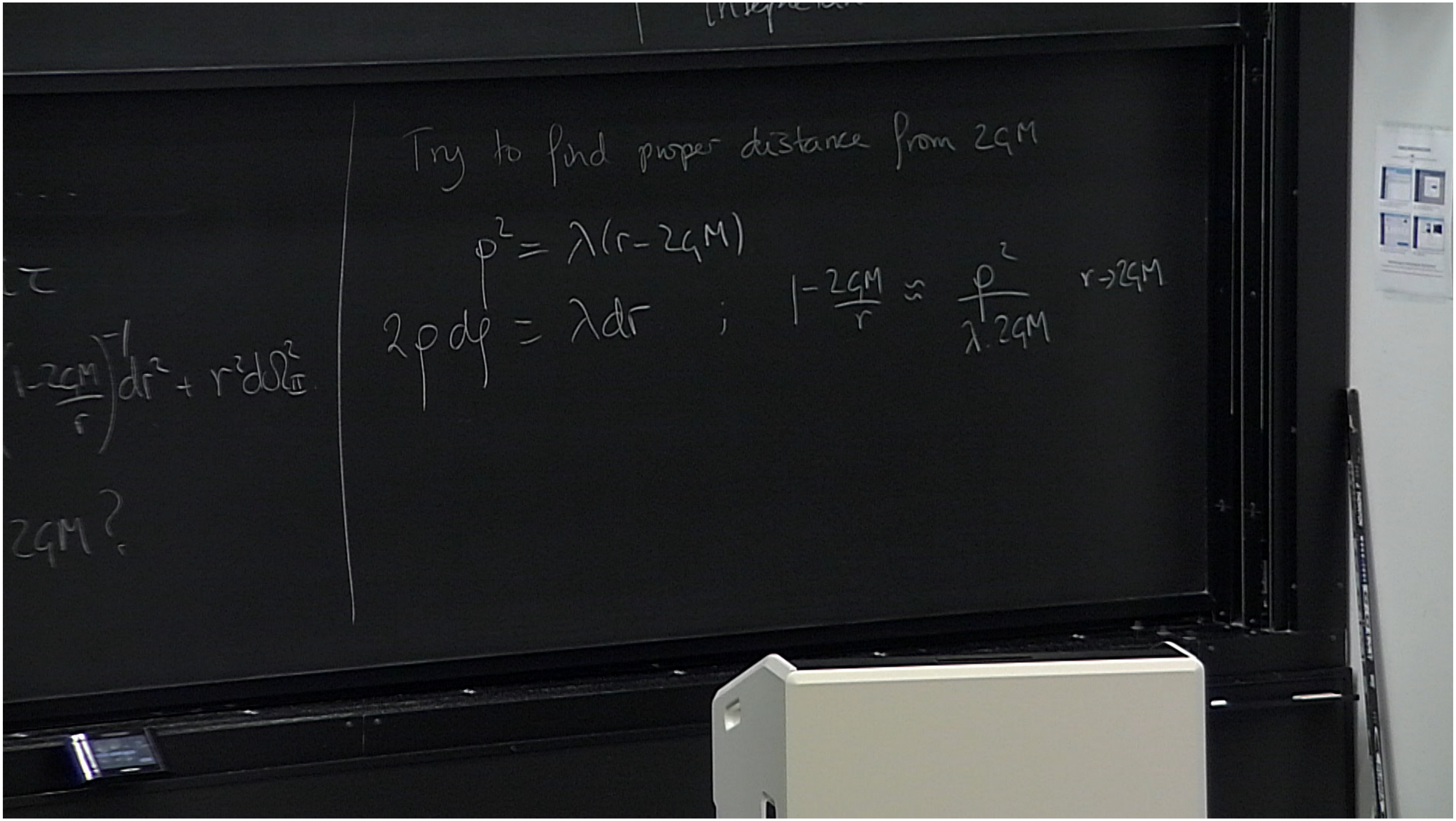
$= \frac{A}{4G}$

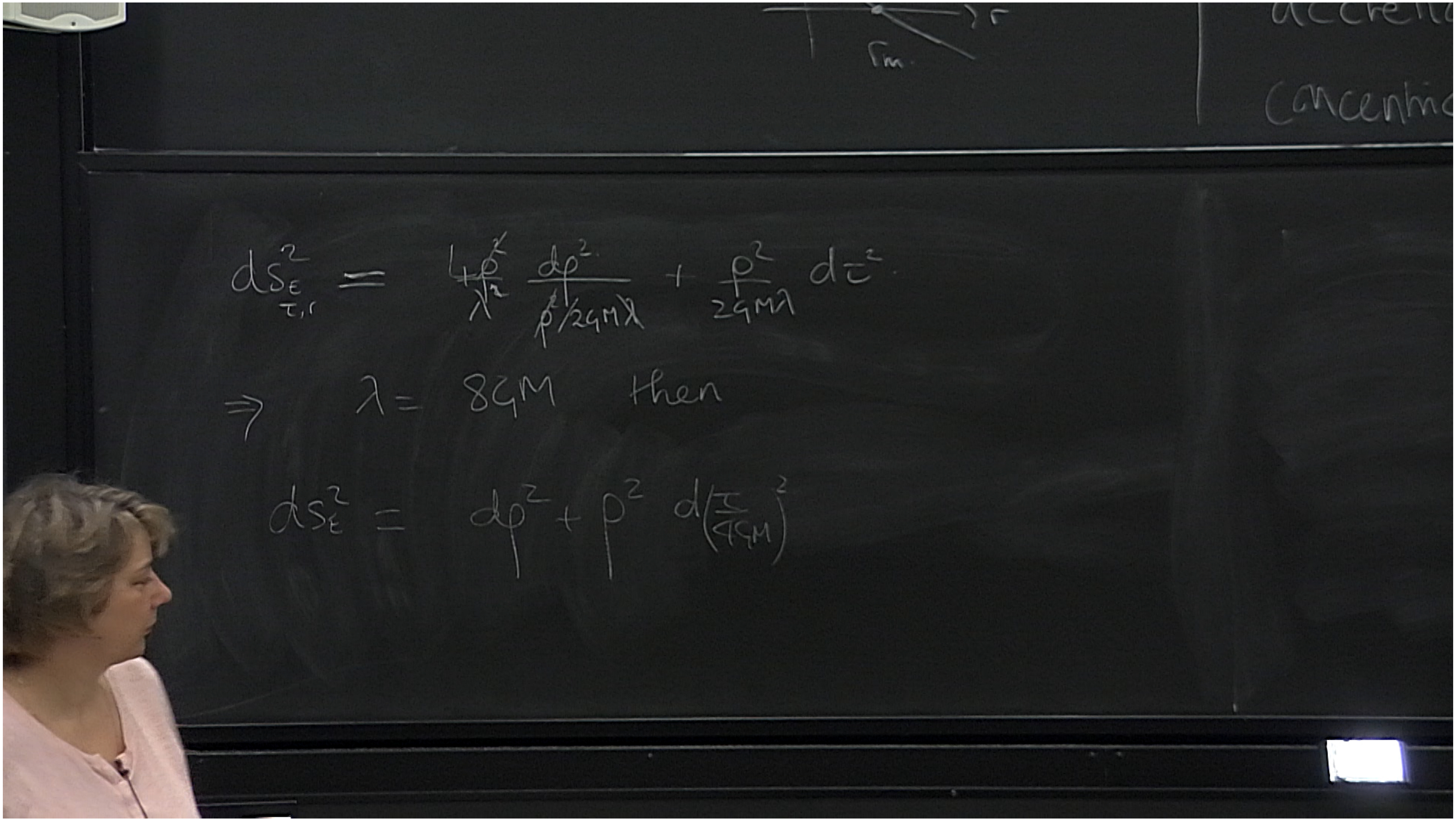
Explore a bit more ....

Take SCH, $t \to i\tau$

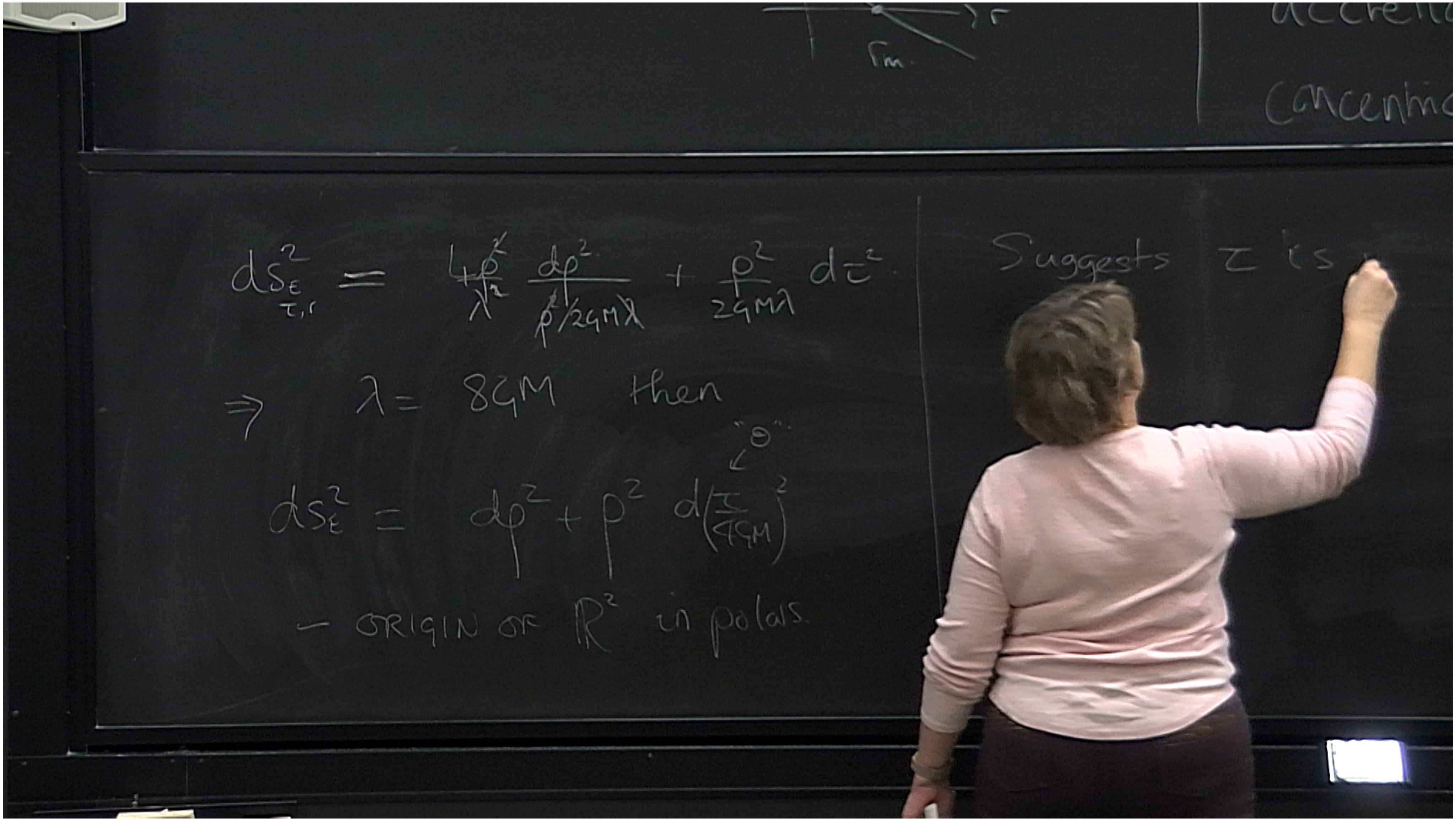
$$|ds^2| = \left(1-\frac{2GM}{r}\right)d\tau^2 + \left(1-\frac{2GM}{r}\right)dr^2 + r^2 d\Omega_{II}^2$$

? What happens at $2GM$?


Try to find proper distance from 2GM
$\rho^2 = \lambda(r - 2GM)$
$2\rho\, d\rho = \lambda\, dr \quad ; \quad 1 - \frac{2GM}{r} \simeq \frac{\rho^2}{\lambda \cdot 2GM} \quad r \to 2GM$
$\left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2 d\Omega_{II}^2$
$2GM?$



r
rm
concentric
ds²E τ,r = 4ρ²/λ² dρ²/(ρ²/2GMλ) + ρ²/2GMλ dτ²
⇒ λ = 8GM then
dsE² = dρ² + ρ² d(τ/4GM)²

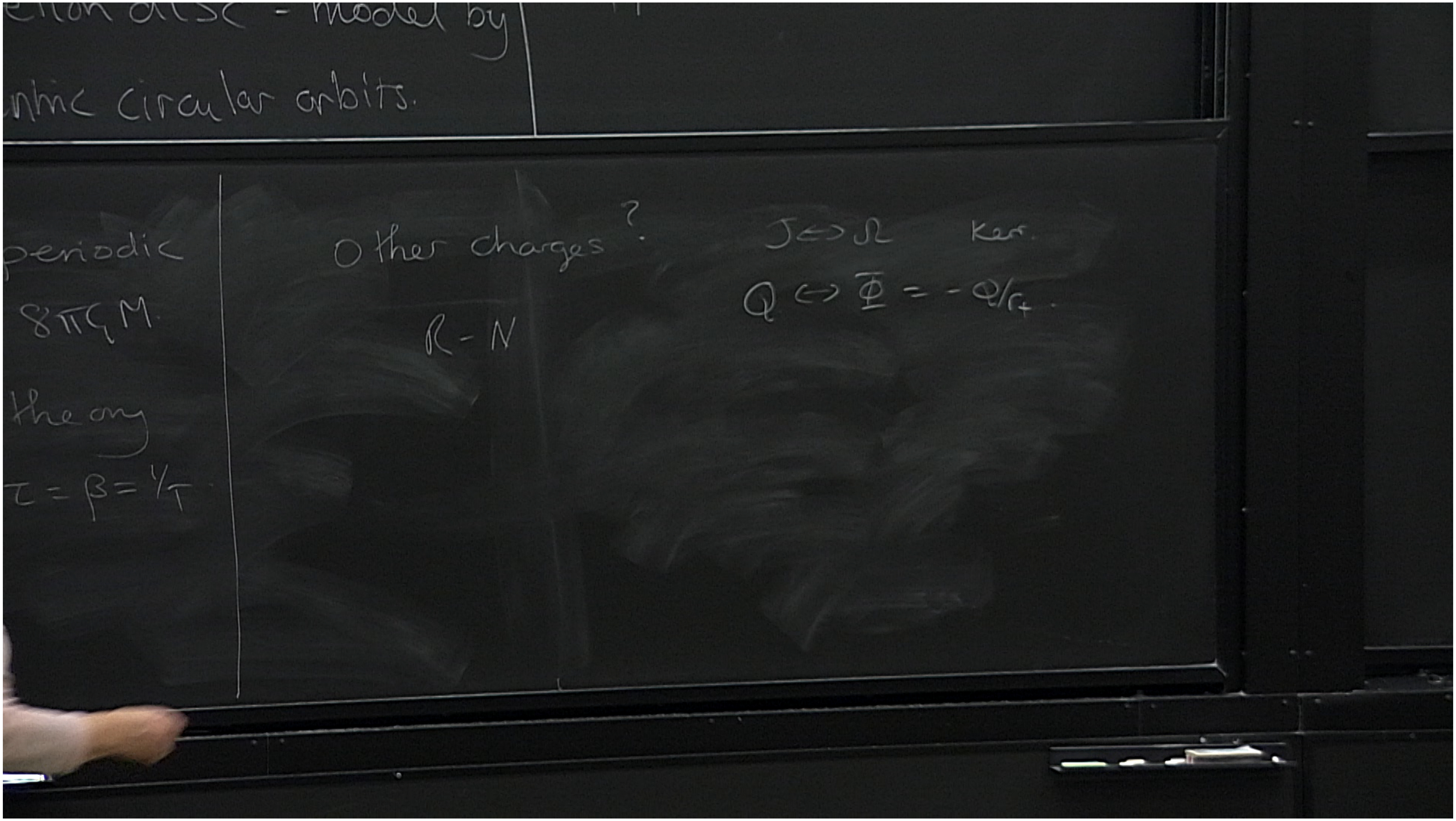


$ds^2_{E\,\tau,r} = \frac{4\rho^2}{\lambda^2}\frac{d\rho^2}{\rho^2/2GM\lambda} + \frac{\rho^2}{2GM\lambda}d\tau^2$
$\Rightarrow \lambda = 8GM$ then
"θ"
$ds^2_E = d\rho^2 + \rho^2\, d\left(\frac{\tau}{4GM}\right)^2$
– ORIGIN OF $\mathbb{R}^2$ in polars.
Suggests τ is
concentric

accretion disc - model by concentric circular orbits.

Suggests τ is periodic with periodicity $8\pi G M$.

- signal of field theory at finite T, $\Delta\tau = \beta = 1/T$


model by
circular orbits.
periodic
8πGM.
τ = β = 1/T
Other charges?
R - N
J ↔ Ω
Kerr.
Q ↔ Φ = -Q/r+

ection disc - model by

onic circular orbits.

periodic

$8\pi G M$.

theory

$\tau = \beta = 1/T$.

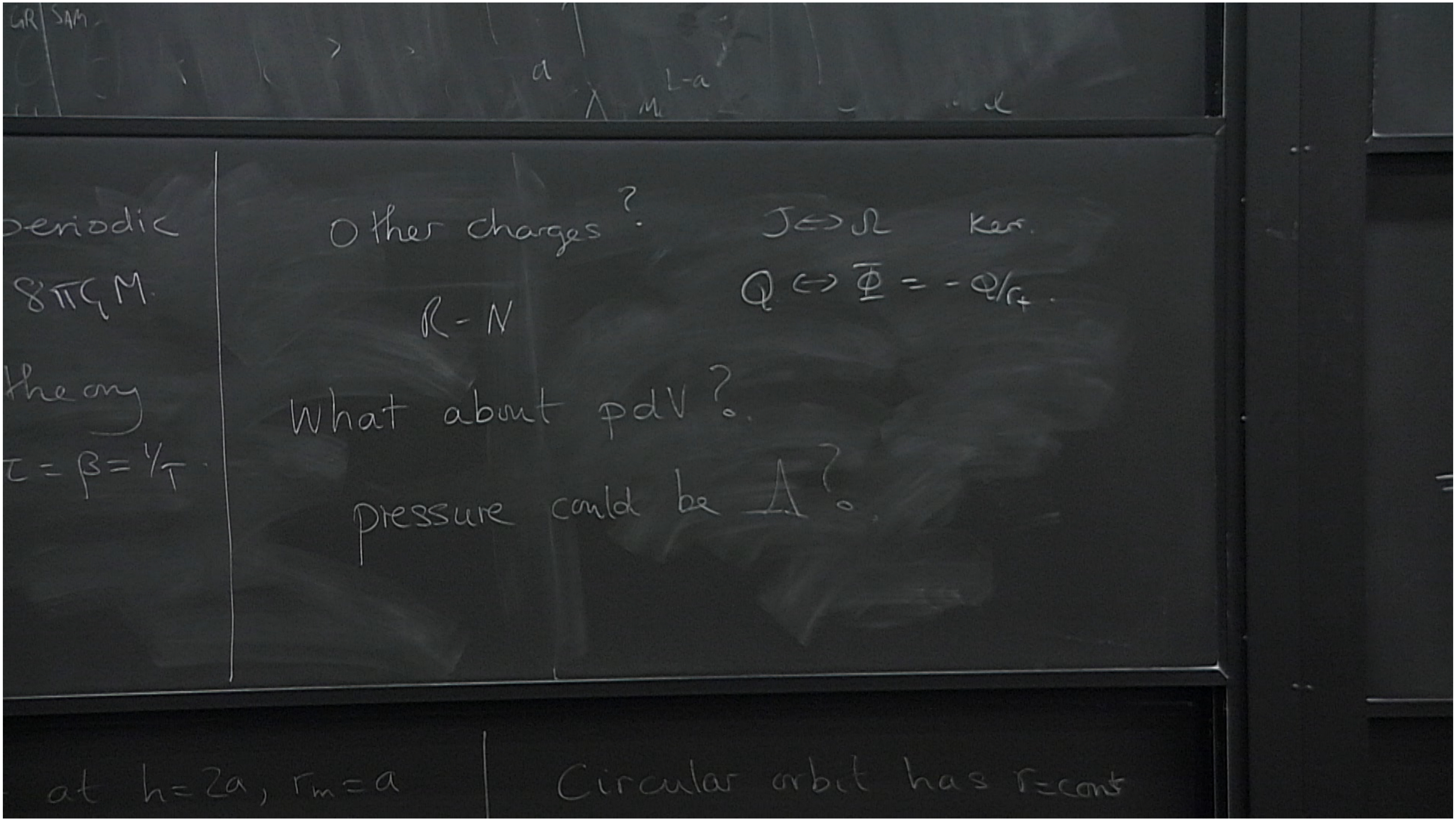
Other charges?

R - N

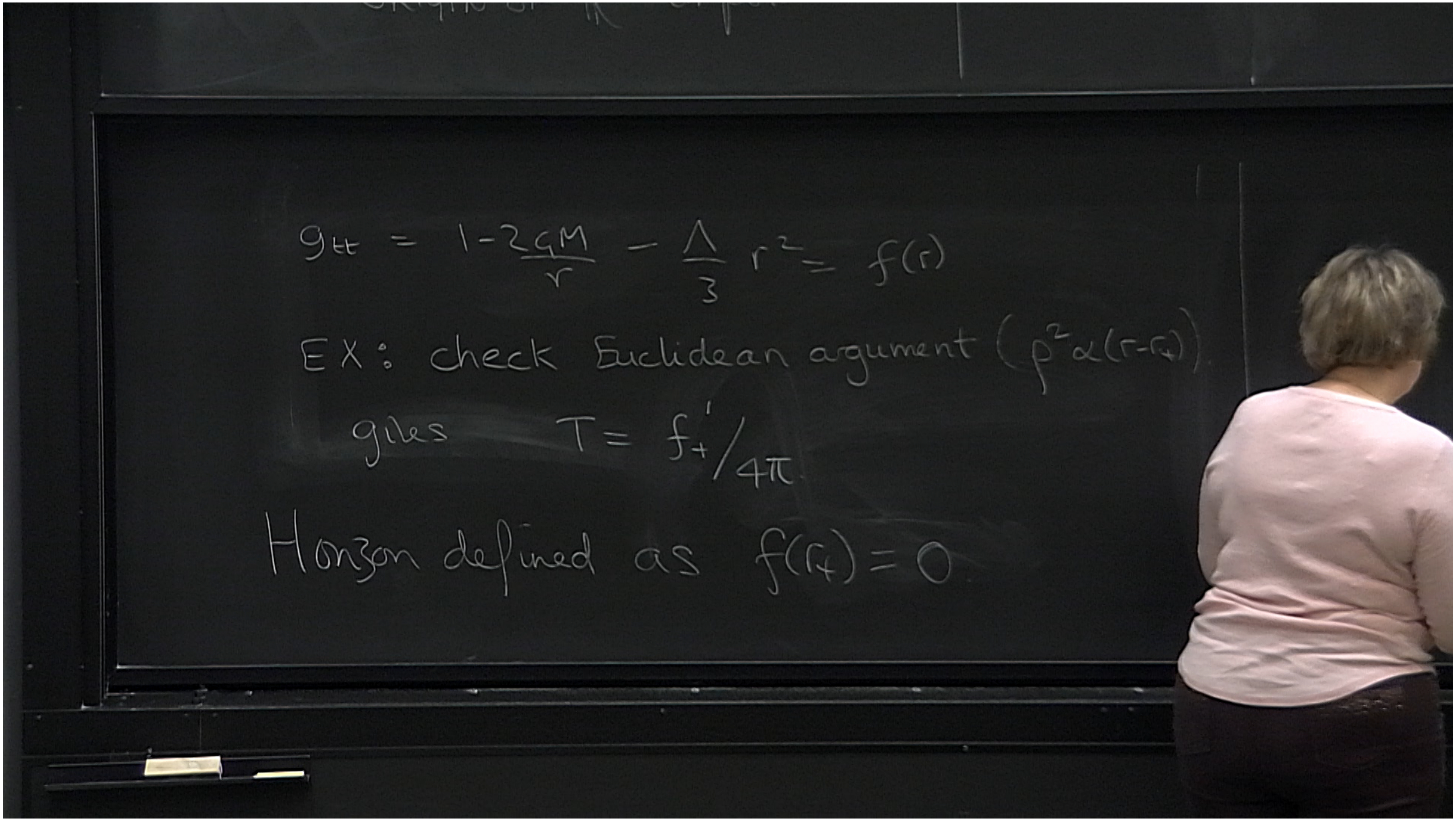
$J \leftrightarrow \Omega$ Kerr.

$Q \leftrightarrow \Phi = -Q/r_+$.

What about pdV?



periodic
8πGM.
theory
τ = β = 1/T.
Other charges?
J ↔ Ω Kerr.
Q ↔ Φ = -Q/r+.
R-N
What about pdV?
pressure could be Λ?
at h=2a, rm=a
Circular orbit has r=const


$g_{tt} = 1 - \frac{2GM}{r} - \frac{\Lambda}{3} r^2 = f(r)$
EX: check Euclidean argument ($\rho^2 \propto (r-r_+)$)
gives $T = f'_+/4\pi$
Horizon defined as $f(r_+) = 0$

$$P = \Lambda / 8\pi G$$

$$(r - r_+))$$

$$(f + \delta f)(r_+ + \delta r_+) = 0$$

$$\delta r_+ \underbrace{f'_+}_{4\pi T} - \frac{2G\,\delta M}{r_+} - \frac{\delta\Lambda}{3} r_+^2 = 0$$

$$\Rightarrow \delta M = T\frac{\delta A}{4G} + \frac{r_+^3}{6G}\delta\Lambda = T\delta S + \underbrace{\frac{4\pi}{3} r_+^3}_{V} \delta P$$

$$P = \Lambda / 8\pi G$$

$$(f+\delta f)(r_+ + \delta r_+) = 0$$

$$\delta r_+ \underbrace{f'_+}_{4\pi T} - \frac{2G\delta M}{r_+} - \frac{\delta \Lambda}{3} r_+^2 = 0$$

$$\Rightarrow \delta M = T \frac{\delta A}{4G} + \frac{r_+^3}{6G} \delta \Lambda = T\delta S + \underbrace{\frac{4\pi}{3} r_+^3}_{V} \delta P$$

Suggest M is enthalpy

$$P = \Lambda / 8\pi G.$$

$$(f+\delta f)(r_+ + \delta r_+) = 0$$

$$\delta r_+ \underbrace{f'_+}_{4\pi T} - \frac{2G\,\delta M}{r_+} - \frac{\delta\Lambda}{3} r_+^2 = 0$$

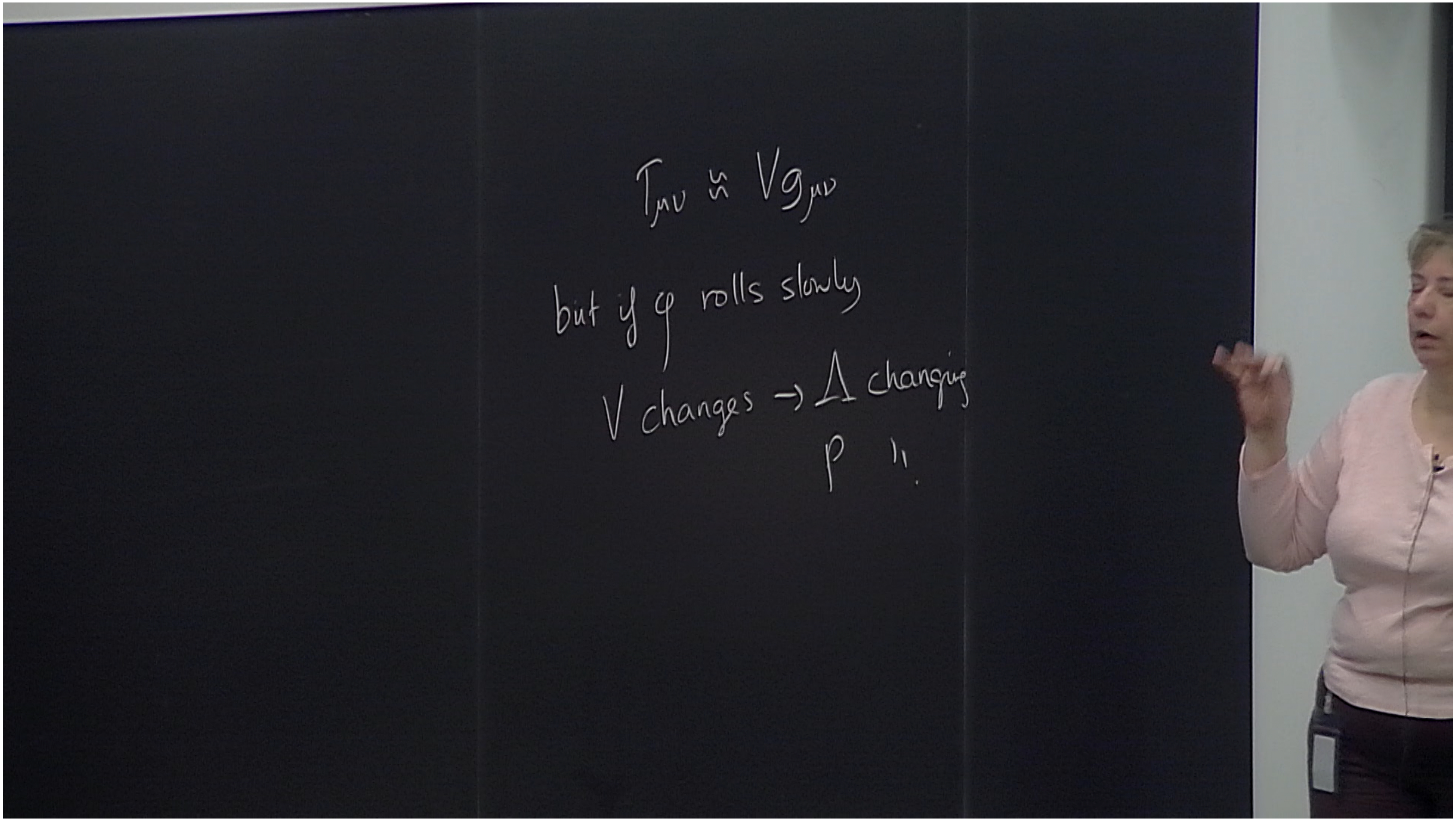
$$\Rightarrow \delta M = T\frac{\delta A}{4G} + \frac{r_+^3}{6G}\,\delta\Lambda$$

$$= T\delta S + \underbrace{\frac{4\pi}{3} r_+^3}_{V}\,\delta P$$

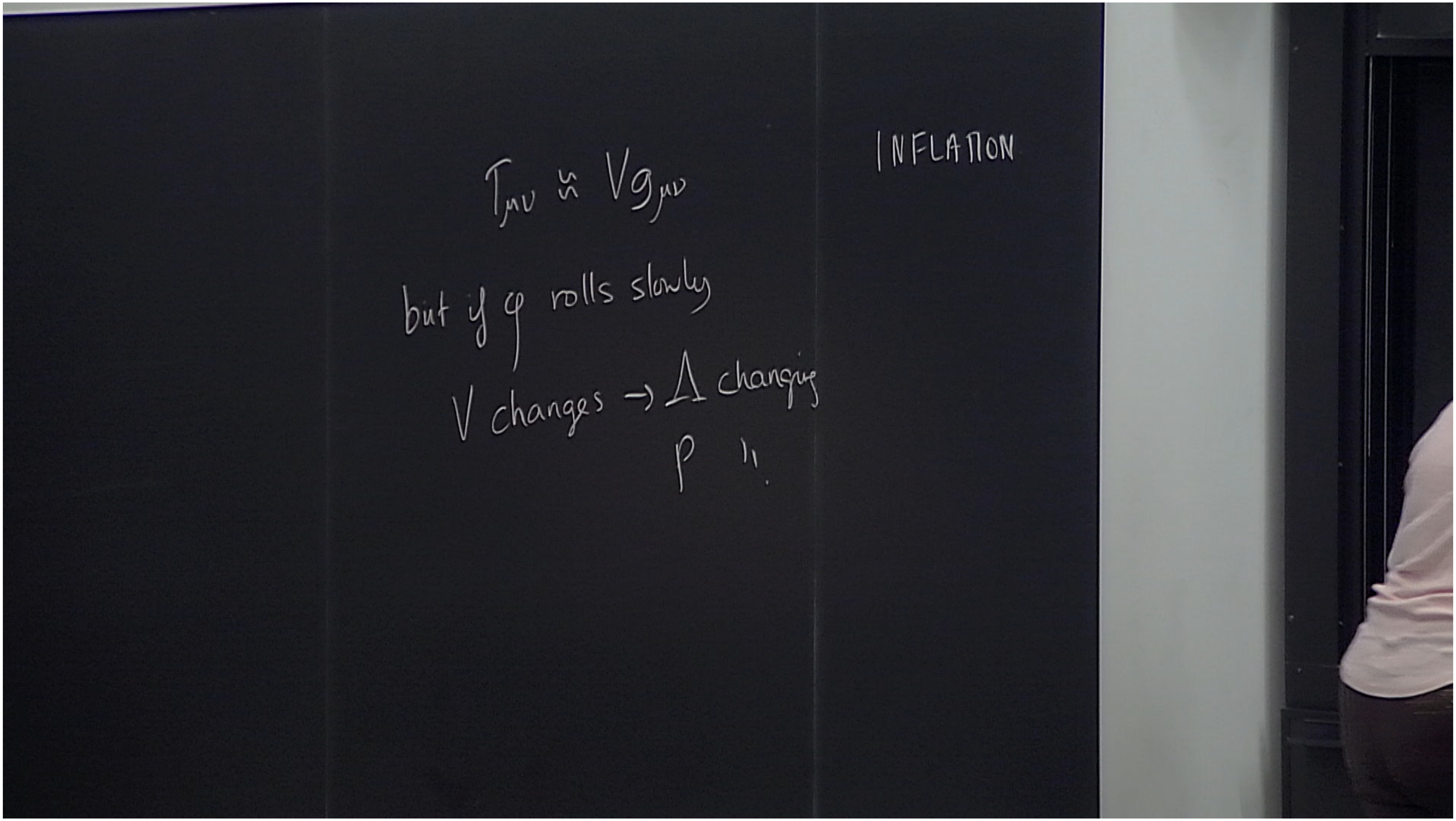
Suggest M is enthalpy.

$$T_{\phi\,\mu\nu} = \partial_\mu\phi\,\partial_\nu\phi - g_{\mu\nu}\left(\tfrac{1}{2}(\partial\phi)^2 - V\right)$$

SLOW ROLL.



$T_{\mu\nu} \simeq V g_{\mu\nu}$
but if φ rolls slowly
V changes $\rightarrow$ Λ changing
ρ "


INFLATION
$T_{\mu\nu} \simeq V g_{\mu\nu}$
but if φ rolls slowly
V changes → Λ changing
ρ "