Title: PSI 2018/2019 - Standard Model Review - Lecture 12

Date: Feb 14, 2019 09:00 AM

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Abstract:

Anomalies

Classical symmetry of tree-level Lagrangian

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Classical symmetry of tree-level Lagrangian broken by quantum corrections (1-loop)

(1) Gauge anomalies

(2) Global anomalies

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(2) Global anomalies. Do exist in SM & have experimental (and cosmological) consequences
e.g. $U(1)_B$, $U(1)_L$

Anomalies

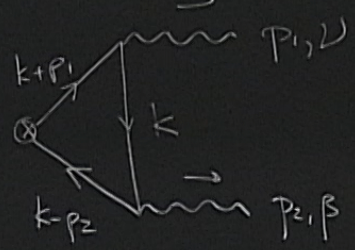
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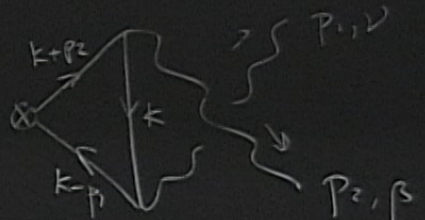
(2) Global anomalies: Do exist in SM & have experimental (and cosmological) consequences
e.g. $U(1)_B$, $U(1)_L$, chiral anomaly ($\pi^0 \to \gamma\gamma$)

Basic building block: famous triangle diagram

$$i\mathcal{M}_{\nu\beta} = iq\cdot$$

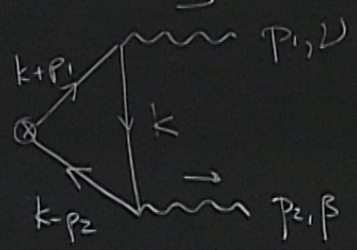


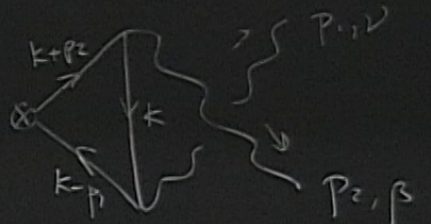
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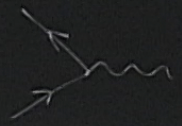
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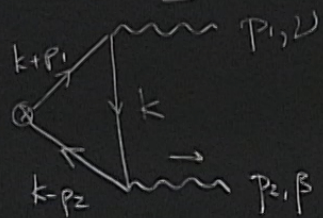
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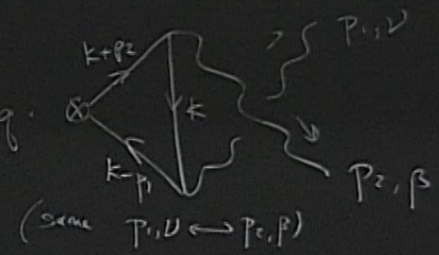
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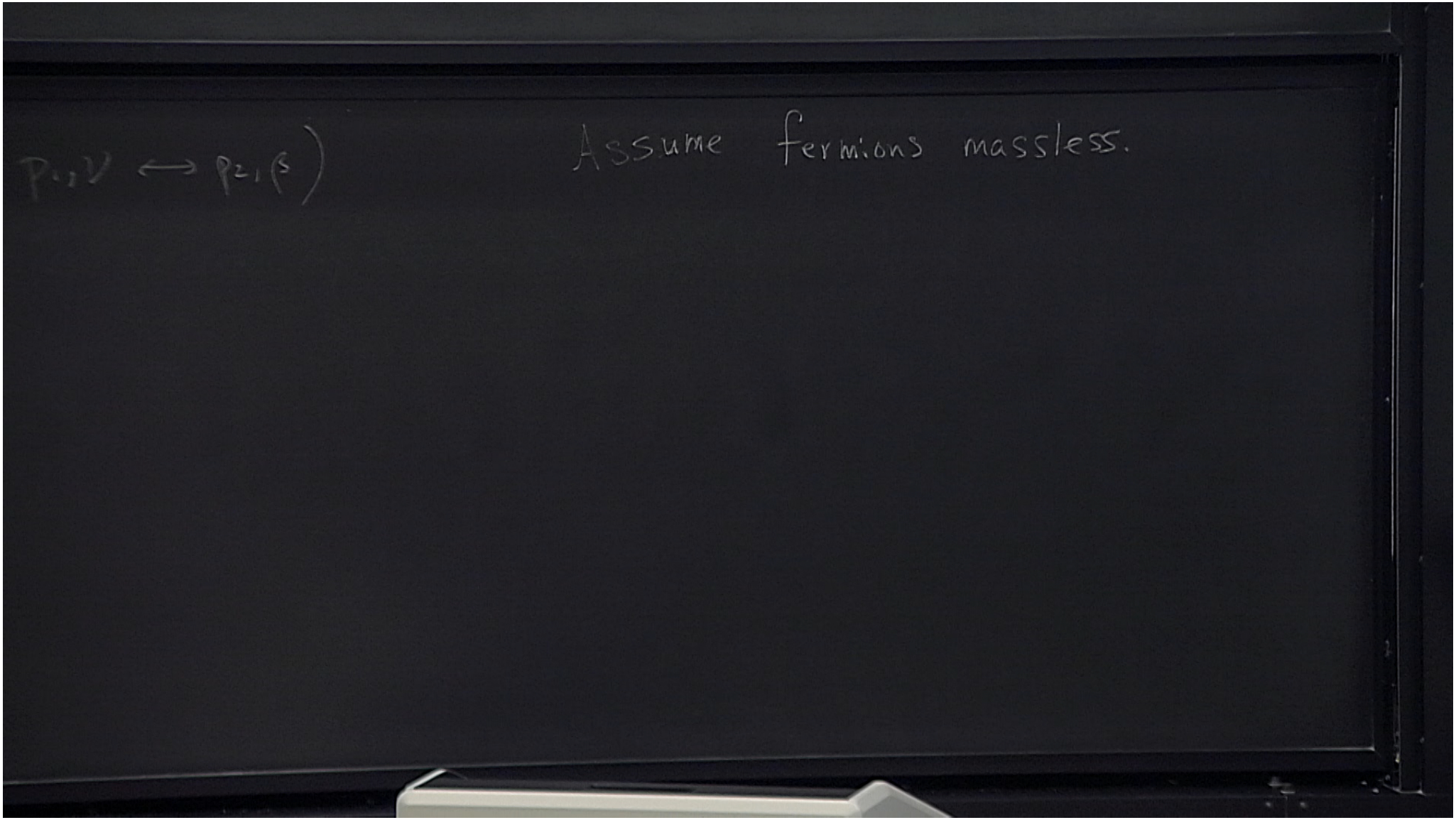
$$+\; iq\cdot$$ 

(same $p_1,\nu \longleftrightarrow p_2,\beta$)

⊗ is insertion of axial vector current $\gamma^\mu\gamma^5$, contracted into $q^\mu = (p_1+p_2)^\mu$

 = coupling to external gauge field via vector current = $i\gamma^\mu$ (factor out gauge coupling)

$$i\mathcal{M}_{\nu\beta} = (-1)\int \frac{d^4k}{(2\pi)^4}\,\mathrm{Tr}\left[i\not{q}\gamma^5 \frac{i}{\not{k}-\not{p}_2}\, i\gamma^\beta \frac{i}{\not{k}}\, i\gamma^\nu \frac{i}{\not{k}+\not{p}_1}\right] + \left(p_1,\nu \leftrightarrow p_2,\beta\right)$$


Assume fermions massless.

$$i\mathcal{M}_{\nu\beta} = (-1)\int \frac{d^4k}{(2\pi)^4}\,\mathrm{Tr}\left[i\not{p}\gamma^5 \frac{i}{\not{k}-\not{p}_2}\, i\gamma^\beta \frac{i}{\not{k}}\, i\gamma^\nu \frac{i}{\not{k}+\not{p}_1}\right] + \left(p_1,\nu \leftrightarrow p_2,\beta\right)$$

$$\text{Result} = \frac{1}{2\pi^2}\,\varepsilon_{\mu\nu\alpha\beta}\,p_1^\mu\,p_2^\alpha$$

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Put into context of $U(1)$ gauge theory:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}\, i\not{D}\,\psi$$

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Put into context of $U(1)$ gauge theory:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} + \bar\psi i\not D \psi, \qquad D_\mu = \partial_\mu + ig\gamma_\mu\left(Q_L P_L + Q_R P_R\right)$$

Chiral theory $Q_L \neq Q_R \Rightarrow$ no mass terms.

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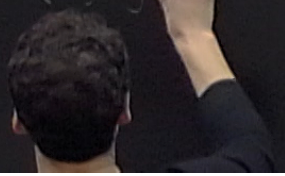
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Compute $\langle \partial_\mu j^\mu(x) \rangle$ at 2nd order in perturbation theory $\mathcal{O}(g^2)$

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$$\langle \partial_\mu j^\mu(x) \rangle = \langle A_{ext} | \, \partial_\mu j^\mu(x) \, \frac{1}{2} (-ig) \int d^4y \, \underbrace{A^\alpha(y) \, j_\beta(y)}_{\mathcal{L}_{int}} \, (-ig) \int d^4z \, A^\nu(z) \, j_\nu(z) \, | A_{ext} \rangle$$

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Compute $\langle \partial_\mu j^\mu(x) \rangle$ at 2nd order in perturbation theory $\mathcal{O}(g^2)$

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$$= -\frac{g^2}{2} \int d^4y \int d^4z \, A^\beta_{ext}(y) A^\nu_{ext}(z) \, \partial^x_\mu \langle j_\mu(x) j_\beta(y) j_\nu(z) \rangle$$

Evaluate 3-pt function:

$$\langle j^\mu(x) j^\beta(y) j^\nu(z) \rangle = \langle \bar\psi(x) \Gamma^\mu \psi(x) \, \bar\psi(y) \Gamma^\beta \psi(y) \, \bar\psi(z) \Gamma^\nu \psi(z) \rangle$$

$\Gamma^\mu = \gamma^\mu (Q_L P_L + Q_R P_R)$

$$\langle \partial_\mu j^\mu(x) \rangle = \langle A_{ext} | \; \partial_\mu j^\mu(x) \; \frac{1}{2} (-ig) \int d^4y \; \underbrace{A^\beta(y) \, j_\beta(y)}_{\mathcal{L}_{int}} \; (-ig) \int d^4z \; A^\nu$$

$$= -\frac{g^2}{2} \int d^4y \int d^4z \; A^\beta_{ext}(y) \, A^\nu_{ext}(z) \; \partial^\mu_x \langle \; j_\mu(x) \, j_\beta(y) \, j_\nu$$

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$$= -\frac{g^2}{2} \int d^4y \int d^4z \; A^\beta_{ext}(y) A^\nu_{ext}(z) \; \partial_x^\mu \langle J_\mu(x) j_\beta(y) j_\nu(z) \rangle$$

closed fermion loop

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$\Gamma^\mu = \gamma^\mu$

$$\langle \partial_\mu j^\mu(x) \rangle = \langle A_{ext} | \; \partial_\mu j^\mu(x) \; \frac{1}{2}(-ig)\int d^4y \; \underbrace{A^\beta(y)\, j_\beta(y)}_{\mathcal{L}_{int}} \; (-ig)\int d^4z \; A^\nu(z)\, j_\nu(z)$$

$$= -\frac{g^2}{2}\int d^4y \int d^4z \; A^\beta_{ext}(y)\, A^\nu_{ext}(z) \; \partial_x^\mu \langle j_\mu(x)\, j_\beta(y)\, j_\nu(z) \rangle$$

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+ other contraction $(y,\beta \leftrightarrow \nu,z)$

$\Gamma^\mu =$

Work in Momentum space:

$$A^{\nu}_{ext}(z) = \int \frac{d^4 p_1}{(2\pi)^4} e^{i p_1 \cdot z} \tilde{A}^{\nu}_{ext}(p_1)$$

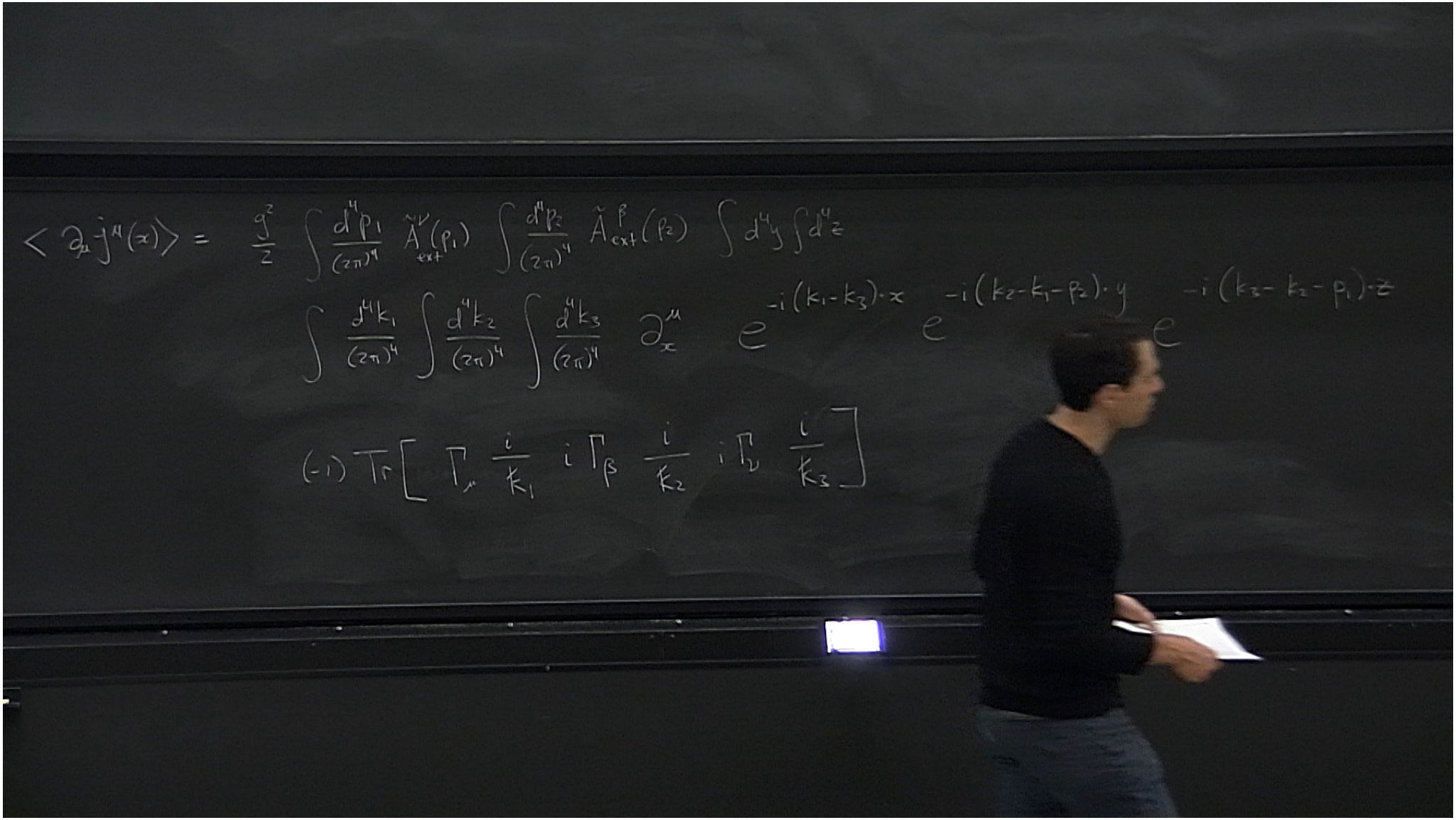
$$\overbrace{\psi(x)\bar{\psi}(y)} = \int \frac{d^4 k_1}{(2\pi)^4} \frac{i}{\not{k}_1} e^{-i k_1 \cdot (x-y)}$$

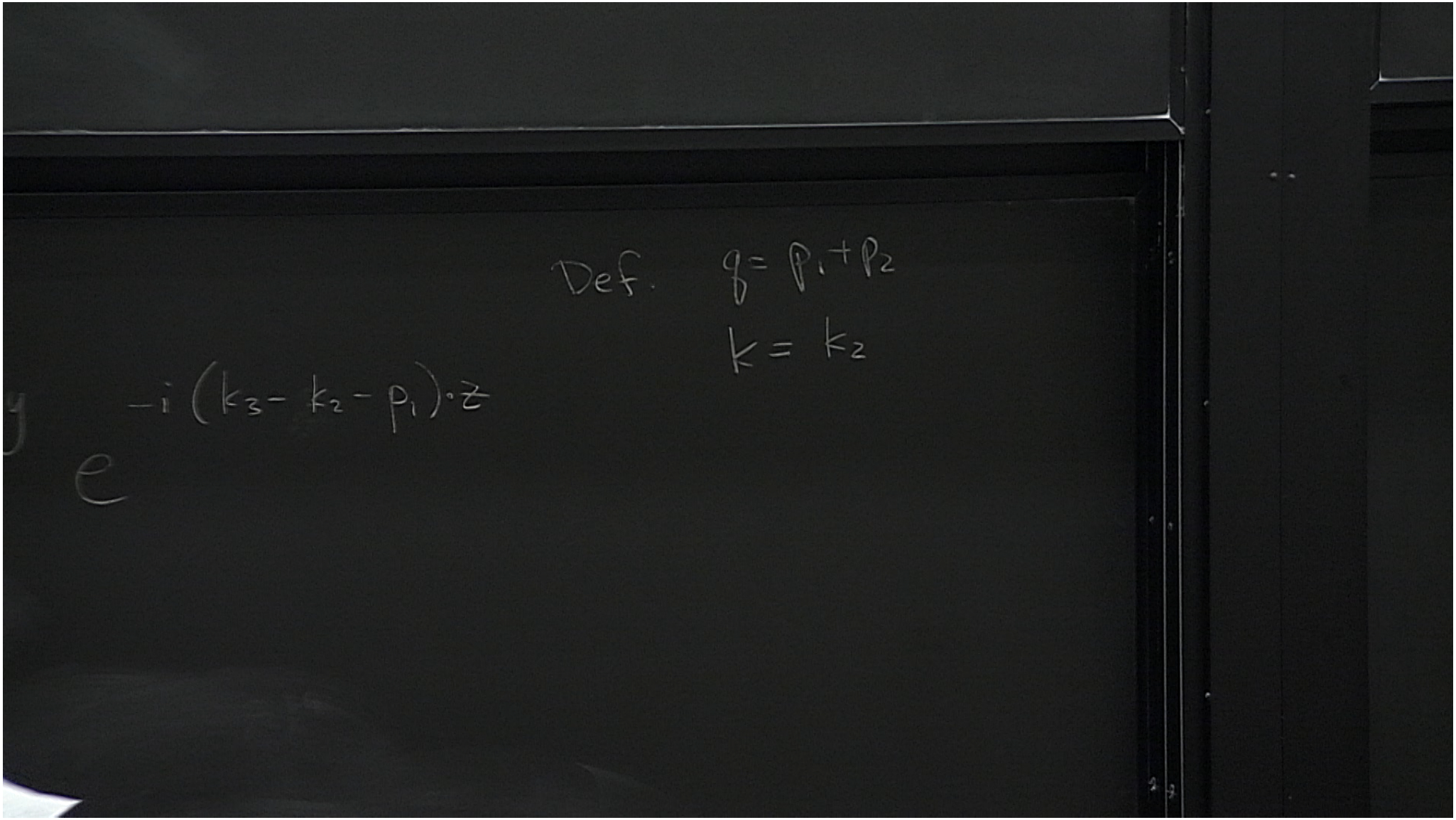
$$\langle \partial_\mu j^\mu(x) \rangle = \frac{g^2}{2} \int \frac{d^4p_1}{(2\pi)^4} \tilde{A}^\nu_{ext}(p_1) \int \frac{d^4p_2}{(2\pi)^4} \tilde{A}^\beta_{ext}(p_2) \int d^4y \int d^4z$$

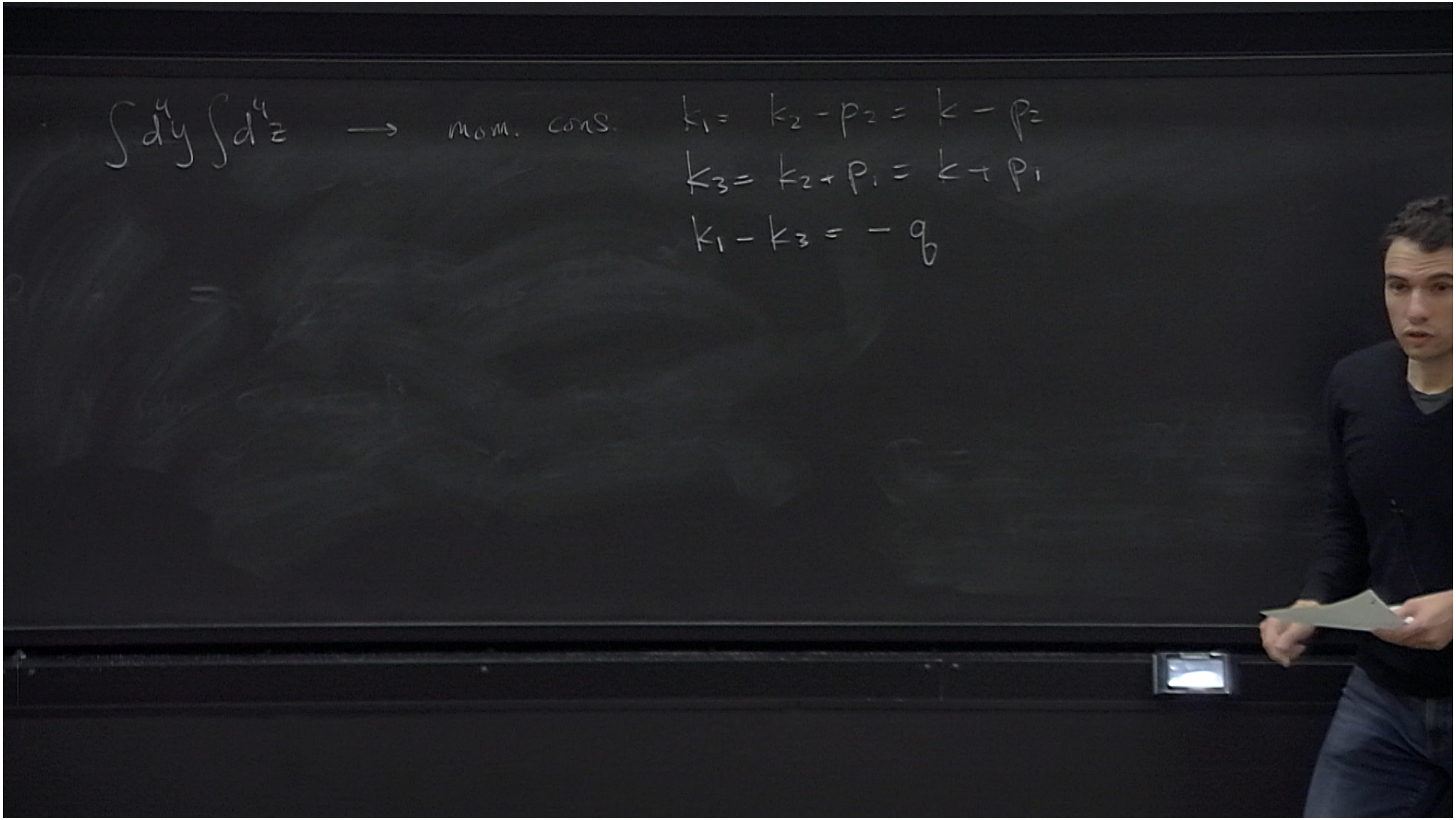
$$\int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \int \frac{d^4k_3}{(2\pi)^4}$$

$$\langle \partial_\mu j^\mu(x) \rangle = \frac{g^2}{2} \int \frac{d^4p_1}{(2\pi)^4} \tilde{A}^\nu_{ext}(p_1) \int \frac{d^4p_2}{(2\pi)^4} \tilde{A}^\beta_{ext}(p_2) \int d^4y \int d^4z$$

$$\int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \int \frac{d^4k_3}{(2\pi)^4} \partial_x^\mu \; e^{-i(k_1-k_3)\cdot x} \; e^{-i(k_2-k_1-p_2)\cdot y} \; e^{-i(k_3-k_2-p_1)\cdot z}$$


$\langle \partial_\mu j^\mu(x) \rangle = \frac{g^2}{2} \int \frac{d^4p_1}{(2\pi)^4} \tilde{A}^\nu_{ext}(p_1) \int \frac{d^4p_2}{(2\pi)^4} \tilde{A}^\beta_{ext}(p_2) \int d^4y \int d^4z$
$\int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \int \frac{d^4k_3}{(2\pi)^4} \partial_x^\mu e^{-i(k_1-k_3)\cdot x} e^{-i(k_2-k_1-p_2)\cdot y} e^{-i(k_3-k_2-p_1)\cdot z}$
$(-1)\,\mathrm{Tr}\left[\Gamma_\mu \frac{i}{\not{k}_1} i\Gamma_\beta \frac{i}{\not{k}_2} i\Gamma_\nu \frac{i}{\not{k}_3} \right]$


Def. $q = p_1 + p_2$
$k = k_2$
$e^{-i(k_3 - k_2 - p_1)\cdot z}$


$\int d^4y \int d^4z$ → mom. cons.
$k_1 = k_2 - p_2 = k - p_2$
$k_3 = k_2 + p_1 = k + p_1$
$k_1 - k_3 = -q$

$\int d^4y \int d^4z \longrightarrow$ mom. cons.

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$\partial_x^\mu \longrightarrow i q^\mu$

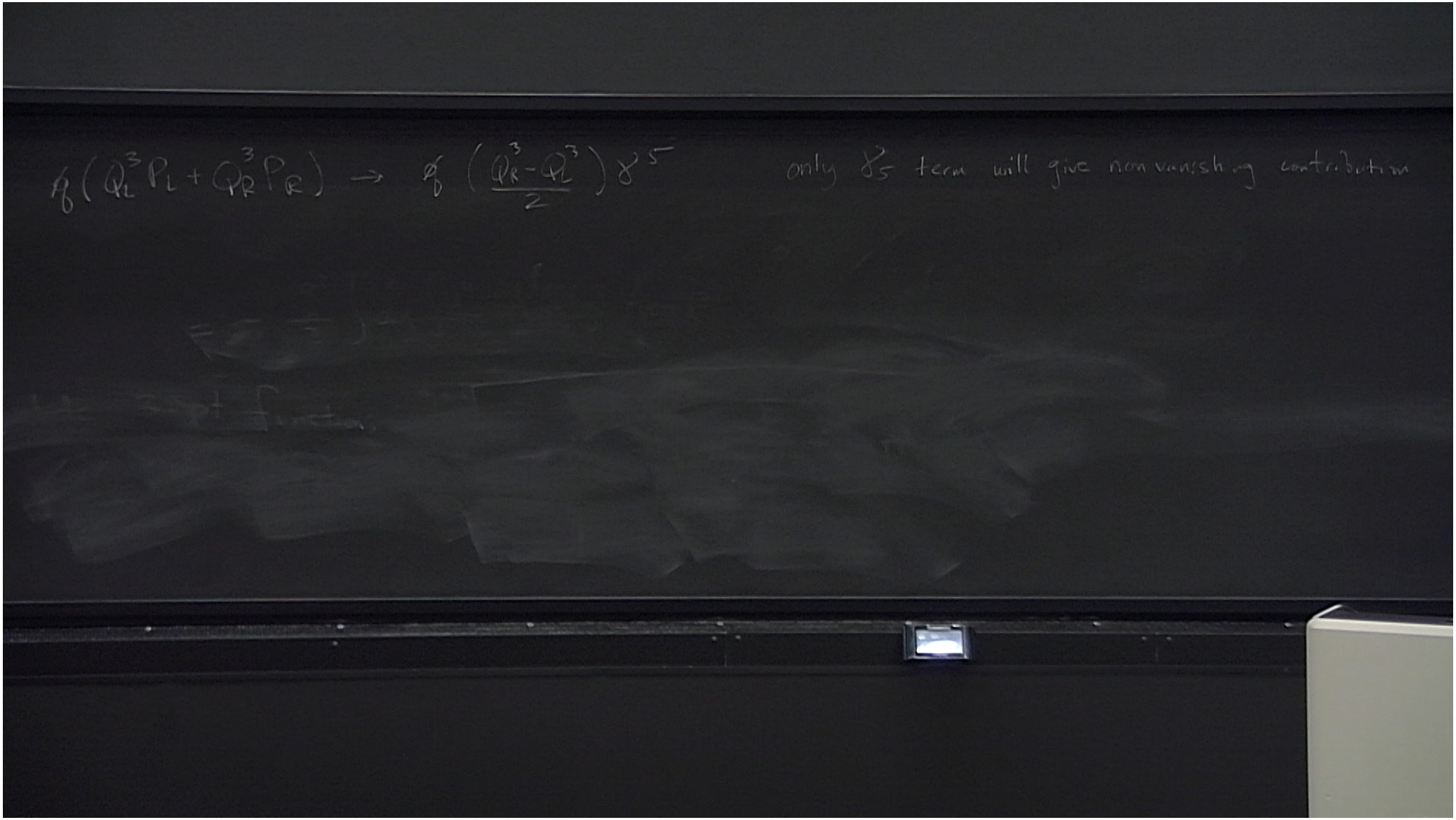
$$\langle \partial_\mu j^\mu(x) \rangle = \frac{q^2}{2} \int \frac{d^4p_1}{(2\pi)^4} \int \frac{d^4p_2}{(2\pi)^4} \tilde{A}^\nu_{ext}(p_1)\, \tilde{A}^\beta_{ext}(p_2)\, e^{iq\cdot x}$$

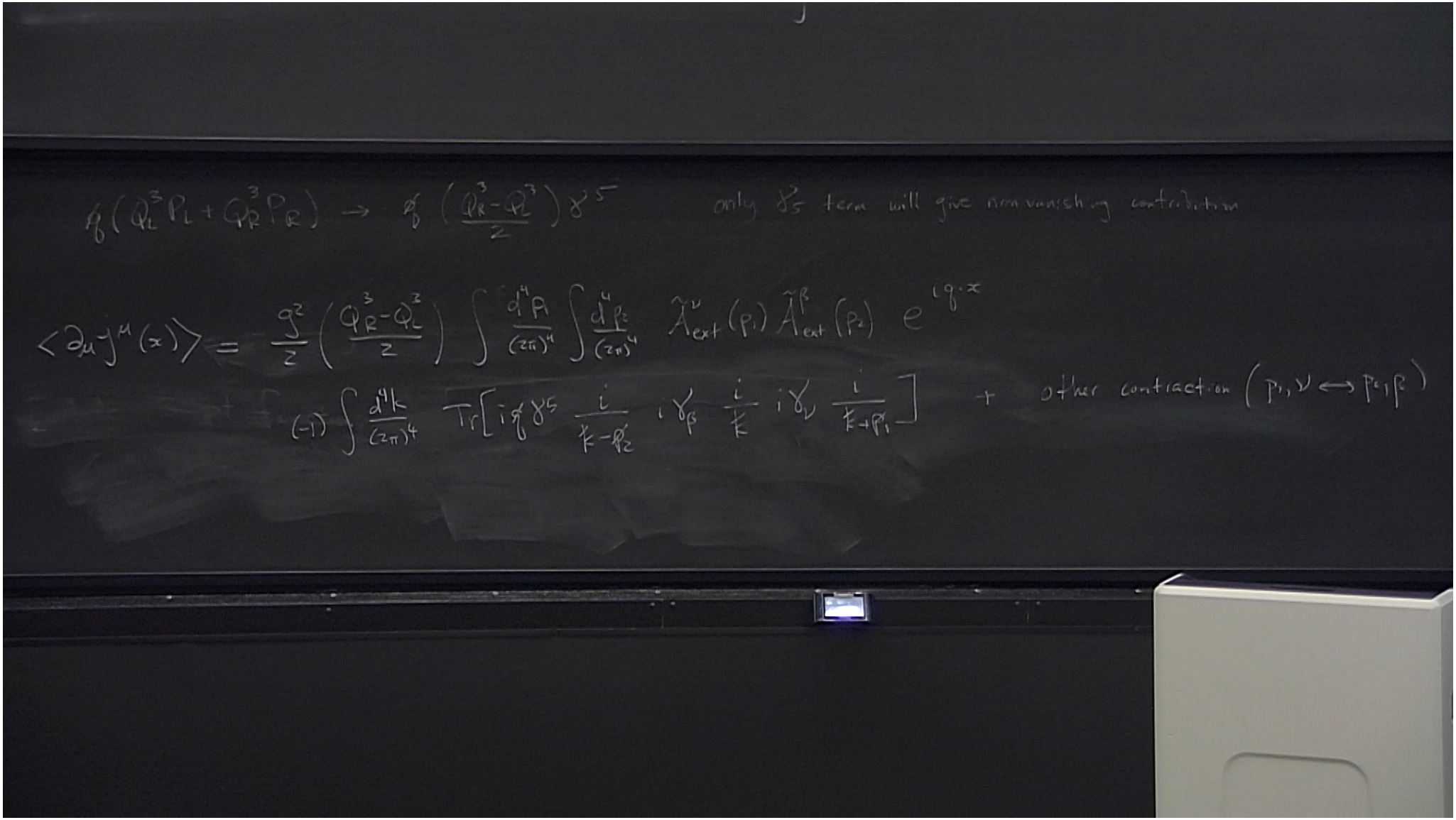
$$(-1) \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[i\not{q}\,(Q_L P_L + Q_R P_R)\, \frac{i}{\not{k}-\not{p}_2}\, i\gamma_\beta\,(Q_L P_L + Q_R P_R)\, \frac{i}{\not{k}}\, i\gamma_\nu\,(Q_L P_L + Q_R P_R)\, \frac{i}{\not{k}+\not{p}_1} \right]$$

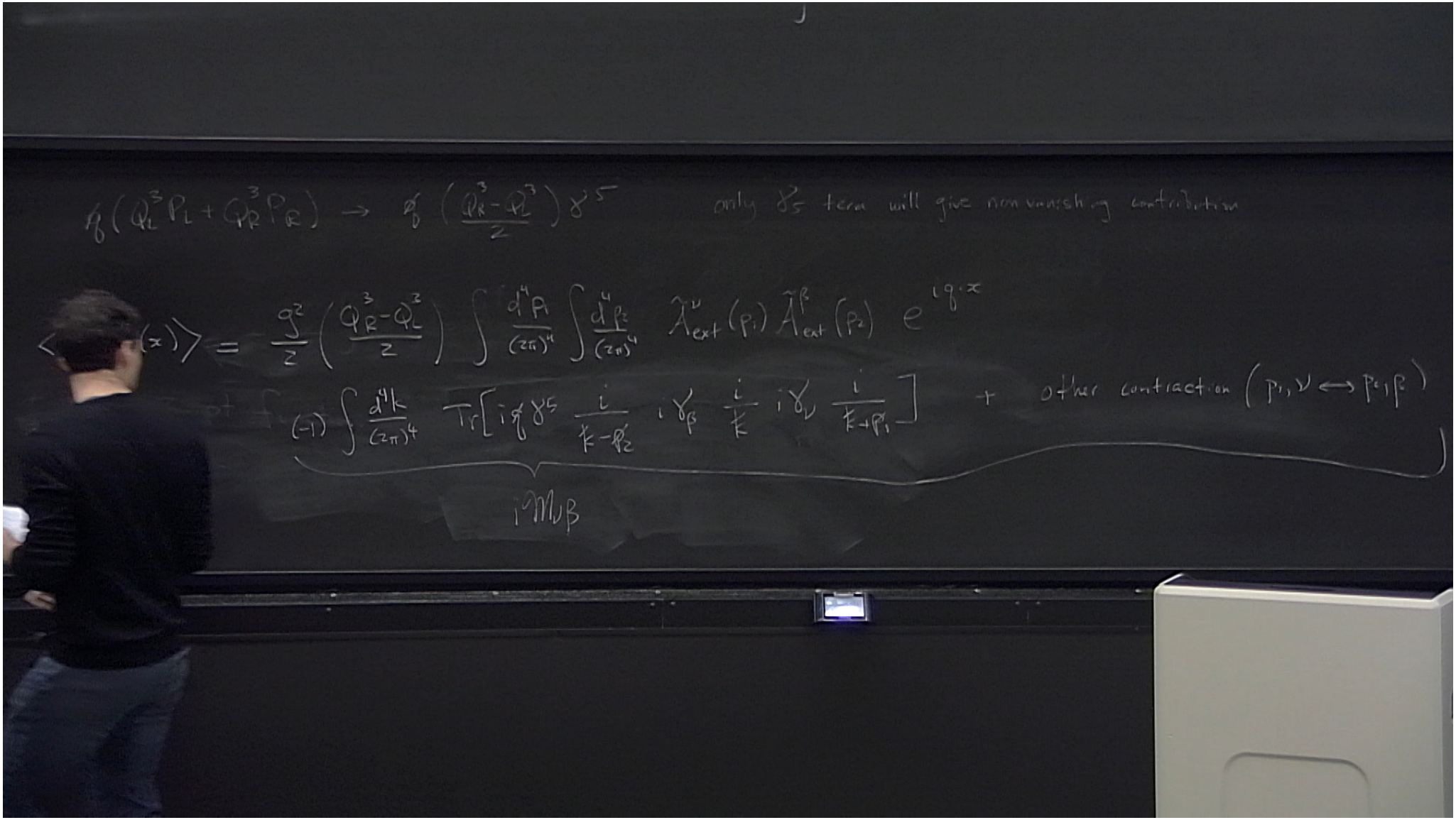
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$$(-1) \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[i\not{q}\,(Q_L P_L + Q_R P_R)\, \frac{i}{\not{k}-\not{p}_2}\, i\gamma_\beta\,(Q_L P_L + Q_R P_R)\, \frac{i}{\not{k}}\, i\gamma_\nu\,(Q_L P_L + Q_R P_R)\, \frac{i}{\not{k}+\not{p}_1} \right]$$

+ other contraction $(p_1,\nu \leftrightarrow p_2,\beta)$


$\bar{q}(Q_L^3 P_L + Q_R^3 P_R) \rightarrow \bar{q}\left(\frac{Q_R^3 - Q_L^3}{2}\right)\gamma^5$
only γ^5 term will give non vanishing contribution


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$\langle \partial_\mu j^\mu(x) \rangle = \frac{q^2}{2}\left(\frac{Q_R^3 - Q_L^3}{2}\right) \int \frac{d^4p_1}{(2\pi)^4} \int \frac{d^4p_2}{(2\pi)^4} \tilde{A}^\nu_{ext}(p_1) \tilde{A}^\beta_{ext}(p_2)\, e^{iq\cdot x}$
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$\bar{q}_0(Q_L^3 P_L + Q_R^3 P_R) \to \bar{q}_0\left(\frac{Q_R^3 - Q_L^3}{2}\right)\gamma^5$
only γ^5 term will give non-vanishing contribution
$\langle J^\mu(x)\rangle = \frac{g^2}{2}\left(\frac{Q_R^3 - Q_L^3}{2}\right)\int \frac{d^4p_1}{(2\pi)^4}\int \frac{d^4p_2}{(2\pi)^4}\, \tilde{A}^\nu_{ext}(p_1)\,\tilde{A}^\beta_{ext}(p_2)\, e^{iq\cdot x}$
$(-1)\int \frac{d^4k}{(2\pi)^4}\, Tr\left[i\bar{q}_0\gamma^5 \frac{i}{\not k - \not p_2} i\gamma_\beta \frac{i}{\not k} i\gamma_\nu \frac{i}{\not k + \not p_1}\right]$ + other contraction $(p_1,\nu \leftrightarrow p_2,\beta)$
$i\mathcal{M}_{\nu\beta}$

$$\bar{q}\left(Q_L^3 P_L + Q_R^3 P_R\right) \rightarrow \bar{q}\left(\frac{Q_R^3 - Q_L^3}{2}\right)\gamma^5 \qquad \text{only } \gamma_5 \text{ term will give non-vanishing contribution}$$

$$\langle \partial_\mu J^\mu(x) \rangle = \frac{g^2}{2}\left(\frac{Q_R^3 - Q_L^3}{2}\right) \int \frac{d^4p_1}{(2\pi)^4} \int \frac{d^4p_2}{(2\pi)^4} \tilde{A}^\nu_{ext}(p_1)\, \tilde{A}^\beta_{ext}(p_2)\, e^{iq\cdot x}$$

$$(-1)\int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[i\bar{q}\gamma^5 \frac{i}{\not{k} - \not{p}_2} i\gamma_\beta \frac{i}{\not{k}} i\gamma_\nu \frac{i}{\not{k} + \not{p}_1} \right] + \text{other contraction } (p_1, \nu \leftrightarrow p_2, \beta)$$

$$i\mathcal{M}_{\nu\beta} = \frac{1}{2\pi^2}\varepsilon_{\mu\nu\alpha\beta} p_1^\mu p_2^\alpha$$

$$p_1^\mu p_2^\alpha \; e^{i p_1 \cdot x} \; e^{i p_2 \cdot x} = (-i)\, \partial_x^\mu$$

$$\int \frac{d^4p_1}{(2\pi)^4}\, p_1^\mu\, \tilde{A}^\nu_{ext}(p_1)\, e^{ip_1\cdot x} = -i\partial_x^\mu \underbrace{\int \frac{d^4p_1}{(2\pi)^4}\, \tilde{A}^\nu_{ext}(p_1)\, e^{ip_1\cdot x}}_{A^\nu_{ext}(x)} = -i\partial^\mu A^\nu_{ext}(x)$$

$$\int \frac{d^4p_1}{(2\pi)^4}\, p_1^\mu\, \tilde{A}^\nu_{ext}(p_1)\, e^{ip_1\cdot x} = -i\partial_x^\mu \underbrace{\int \frac{d^4p_1}{(2\pi)^4}\, \tilde{A}^\nu_{ext}(p_1)\, e^{ip_1\cdot x}}_{A^\nu_{ext}(x)} = -i\partial^\mu A^\nu_{ext}(x)$$

$$\langle \partial_\mu j^\mu(x) \rangle = \frac{g^2}{2}\left(\frac{Q_L^3 - Q_R^3}{2}\right)\frac{1}{2\pi^2}$$

$$\int \frac{d^4p_1}{(2\pi)^4}\, p_1^\mu\, \tilde{A}^\nu_{ext}(p_1)\, e^{ip_1\cdot x} = -i\partial_x^\mu \underbrace{\int \frac{d^4p_1}{(2\pi)^4}\, \tilde{A}^\nu_{ext}(p_1)\, e^{ip_1\cdot x}}_{A^\nu_{ext}(x)} = -i\partial^\mu A^\nu_{ext}(x)$$

$$\langle \partial_\mu j^\mu(x)\rangle = \frac{q}{2}\left(\frac{Q_L^3 - Q_R^3}{2}\right)\frac{1}{2\pi^2}\underbrace{\varepsilon_{\mu\nu\alpha\beta}\,\partial^\mu A^\nu_{ext}\,\partial^\alpha A^\beta_{ext}}_{\frac{1}{4}\varepsilon_{\mu\nu\alpha\beta}F^{\mu\nu}_{ext}F^{\alpha\beta}_{ext}} = (Q_L^3 - Q_R^3)\,\frac{q^2}{32\pi^2}\,\varepsilon_{\mu\nu\alpha\beta}\,F^{\mu\nu}_{ext}\,F^{\alpha\beta}_{ext}$$

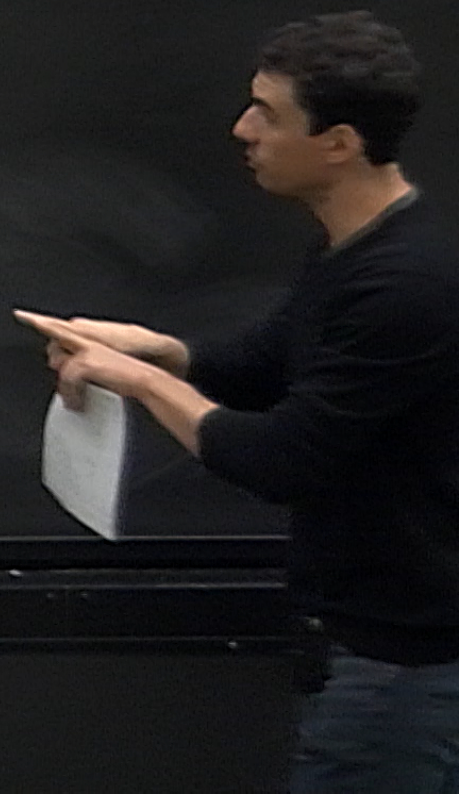
Abbreviated as

$$\partial_\mu j^\mu = \left(Q_L^3 - Q_R^3\right) \frac{g^2}{32\pi^2} \varepsilon_{\mu\nu\alpha\beta} F^{\mu\nu} F^{\alpha\beta}$$

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External field $\quad \varepsilon_{\mu\nu\alpha\beta} F^{\mu\nu} F^{\alpha\beta} = -8\, \vec{E} \cdot \vec{B}$



$$\frac{1}{4}\,\varepsilon_{\mu\nu\alpha\beta}\,F^{\mu\nu}_{ext}\,F^{\alpha\beta}_{ext}$$

Abbreviated as

$$\partial_\mu j^\mu = \left(Q_L^3 - Q_R^3\right)\frac{g^2}{32\pi^2}\,\varepsilon_{\mu\nu\alpha\beta}\,F^{\mu\nu}F^{\alpha\beta}$$

External field $\varepsilon_{\mu\nu\alpha\beta}F^{\mu\nu}F^{\alpha\beta} = -8\,\vec{E}\cdot\vec{B}$ act as a source of charge $q = \int_{\text{all space}} d^3x\, j^0(x)$

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$$\dot{q} = \frac{dq}{dt} = \int_{\text{all space}} d^3x \frac{\partial j^0}{\partial t} = \int d^3x\, \partial_\mu j^\mu = -\frac{g^2}{4\pi^2}\left(Q_L^3 - Q_R^3\right) \int_{\text{all space}} d^3x\, \vec{E} \cdot \vec{B}$$

Spatial part vanishes at ∞

Ward identity is violated $\Leftrightarrow$ gauge symmetry is violated.

Nonzero gauge boson mass  $\rightarrow$ m_A

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Want gauge anomalies to vanish in renormalizable theory.

One flavor $Q_L = Q_R \Rightarrow$ QED is anomaly free

Gauge anomalies in SM: Generalize

(1) Multiple fermion species $\psi_{L,R}^{(i)}$ i = flavor / type of fermion

$Q_{L,R}^{(i)}$

Gauge anomalies in SM. Generalize

(1) Multiple fermion species $\psi_{L,R}^{(i)}$ i = flavor / type of fermion.

$Q_{L,R}^{(i)}$

Sum over fermions i in loop.

$$\partial_\mu j^\mu = \frac{g^2}{32\pi^2}\left(\sum_i (Q_L^{(i)})^3 - (Q_R^{(i)})^3\right) \varepsilon_{\mu\nu\alpha\beta} F^{\mu\nu} F^{\alpha\beta}$$

Apply to hyper charge $U(1)_Y$

$$\partial_\mu j^\mu_Y = \frac{g'^2}{32\pi^2} \left(\sum_{i \in}

Apply to hyper charge $U(1)_Y$

$$\partial_\mu j^\mu_Y = \frac{g'^2}{32\pi^2}\left(\sum_{i\in \text{LH fermions}} (Y^i_L)^3 - \sum_{i\in \text{RH fermions}} (Y^i_R)^3\right) \varepsilon_{\mu\nu\alpha\beta} B^{\mu\nu} B^{\alpha\beta}$$

Apply to hypercharge $U(1)_Y$

$$\partial_\mu j^\mu_Y = \frac{g'^2}{32\pi^2}\left(\sum_{i\in LH\ \text{fermions}} (Y^i_L)^3 - \sum_{i\in RH\ \text{fermions}} (Y^i_R)^3\right)\varepsilon_{\mu\nu\alpha\beta}\, B^{\mu\nu} B^{\alpha\beta}$$

Must have $\sum_{i\in LH} (Y^i_L)^3 - \sum_{i\in RH} (Y^i_R)^3 = 0$

ex. $Y_{Q_L} = \frac{1}{6}$

$Y_{u_R} = \frac{2}{3}$

$B^{\mu\nu} B^{\alpha\beta}$

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$Y_{u_R} = \frac{2}{3}$

Sum over $SU(3)_C \times SU(2)_L$ d.o.f.

3×2 d.o.f

3 d.o.f.

Apply to hyper charge $U(1)_Y$

$$\partial_\mu j^\mu_Y = \frac{g'^2}{32\pi^2} \left(\sum_{i \in \text{LH fermions}} \right.$$

$U(1)_Y^3$: Must have $\sum_{i \in LH} \left(Y_L^i\right)^3 -$