Title: PSI 2018/2019 - Standard Model Review - Lecture 11

Date: Feb 13, 2019 09:00 AM

URL: http://pirsa.org/19020009

Abstract:

Parity of Higgs boson

(1) Couplings all predicted $\propto m$

(2) Verify that it is $J^P = 0^+$ state

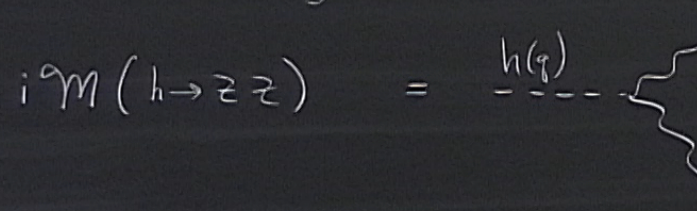
$h \to \gamma\gamma$ spin-1 excluded by Landau-Yang theorem

Four possibilities: $J = 0, 2$, $P = +, -$ $\Rightarrow$ $0^+, 0^-, 2^+$

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Scalar coupling $\mathcal{L}_{hZZ} = c_S\, h Z_\mu Z^\mu$

Pseudoscalar coupling $\mathcal{L}_{hZZ} = h\, \varepsilon_{\mu\nu\alpha\beta} Z^{\mu\nu} Z^{\alpha\beta}$ $\qquad Z_{\alpha\beta} = \partial_\alpha Z_\beta - \partial_\beta Z_\alpha$

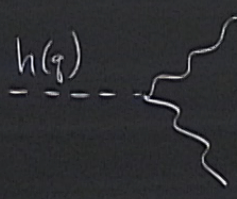
$i\mathcal{M}(h \to ZZ)$ = 

$= \begin{cases} 2ic_S\, \eta_{\mu\alpha}\, \varepsilon^\mu(p)\, \varepsilon^\alpha(p') & \text{scalar } 0^+ \\ 4ic_P\, \varepsilon_{\mu\nu\alpha\beta}\, p^\nu p'^\beta\, \varepsilon^\mu(p) \varepsilon^\alpha(p') & \text{pseudo. } 0^- \end{cases}$

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Scalar coupling: $\mathcal{L}_{hZZ} = c_S\, h\, Z_\mu Z^\mu$

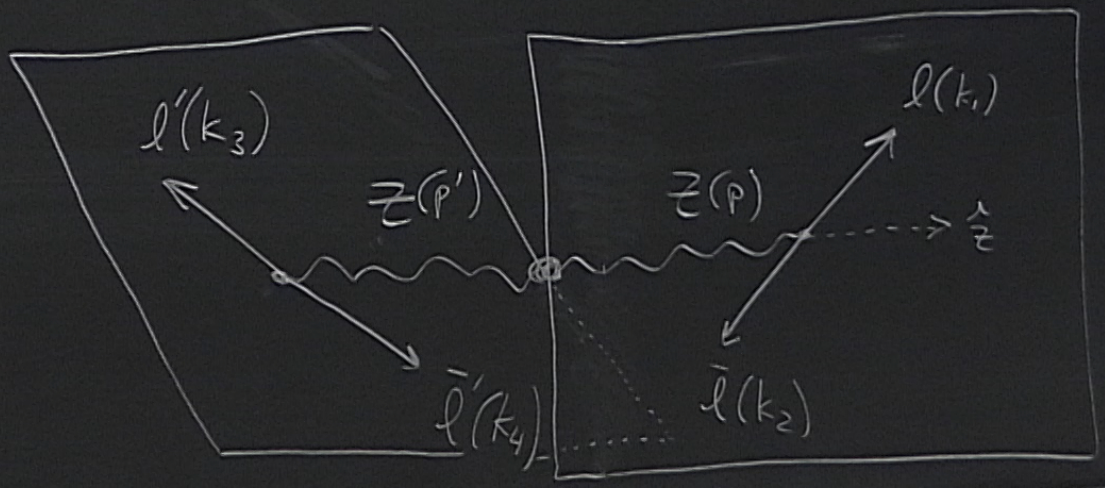
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$i\mathcal{M}(h \to ZZ)$ = 

$= \begin{cases} 2i c_S\, \eta_{\mu\alpha}\, \varepsilon^\mu(p)\, \varepsilon^\alpha(p') & \text{scalar } 0^+ \\ 4i c_P\, \varepsilon_{\mu\nu\alpha\beta}\, p^\nu p'^\beta\, \varepsilon^\mu(p) \varepsilon^\alpha(p') & \text{pseudo. } 0^- \end{cases}$

0^+ vs 0^-: couple differently to Z polarizations.
Use Z polarization → parity.

$h \to ZZ \to \ell\bar{\ell}\,\ell'\bar{\ell}'$, $\ell, \ell' = e, \mu$.

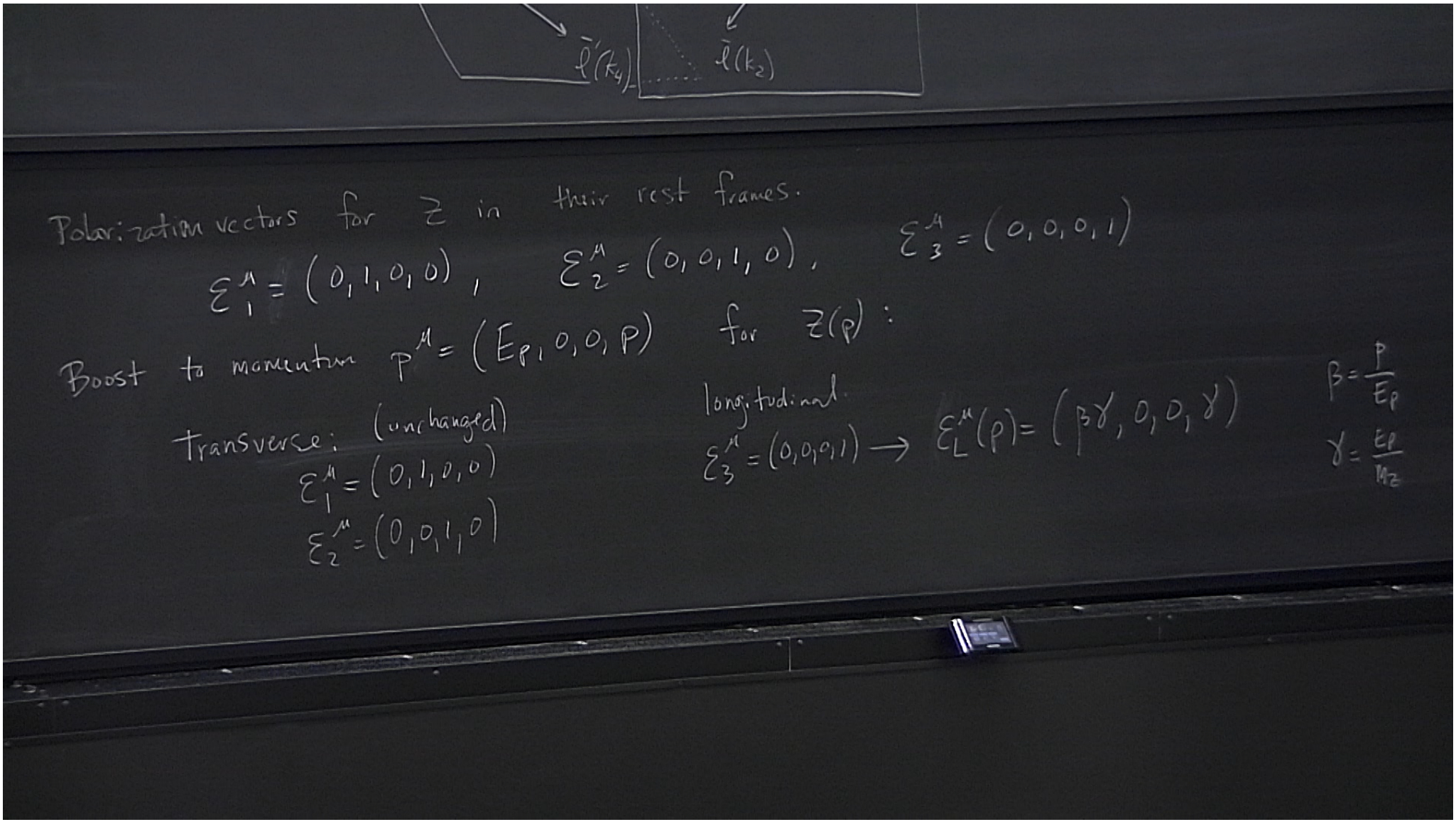


ϕ = relative angle between planes

= relative azimuthal angle between $l(k_1)$ & $\bar{l}(k_3)$

$= \phi_1 - \phi_3$

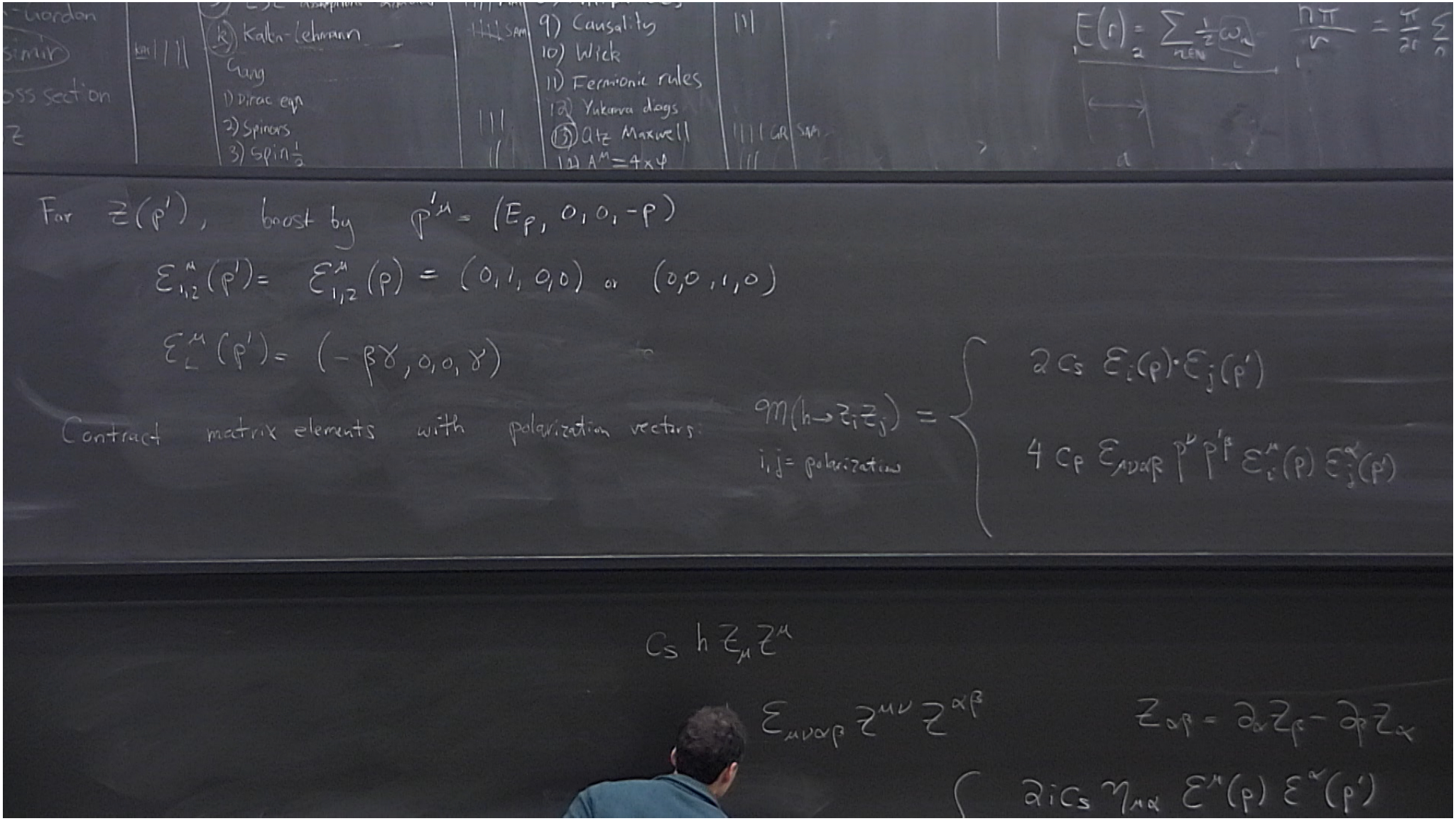
θ_1, θ_3 = polar angles in Z CM frames.


$\bar{f}'(k_4)$
$\bar{f}(k_2)$
Polarization vectors for Z in their rest frames.
$\varepsilon_1^\mu = (0,1,0,0)$, $\varepsilon_2^\mu = (0,0,1,0)$, $\varepsilon_3^\mu = (0,0,0,1)$
Boost to momentum $p^\mu = (E_p, 0, 0, p)$ for $Z(p)$:
transverse: (unchanged)
$\varepsilon_1^\mu = (0,1,0,0)$
$\varepsilon_2^\mu = (0,0,1,0)$
longitudinal.
$\varepsilon_3^\mu = (0,0,0,1) \rightarrow \varepsilon_L^\mu(p) = (\beta\gamma, 0, 0, \gamma)$
$\beta = \frac{p}{E_p}$
$\gamma = \frac{E_p}{m_Z}$

For $z(p')$, boost by $p'^{\mu} = (E_p, 0, 0, -p)$

$$\varepsilon^{\mu}_{1,2}(p') = \varepsilon^{\mu}_{1,2}(p) = (0,1,0,0) \text{ or } (0,0,1,0)$$

$$\varepsilon^{\mu}_{L}(p') = (-\beta\gamma, 0, 0, \gamma)$$


Kallen-Lehmann
Gang
1) Dirac eqn
2) Spinors
3) Spin $\frac{1}{2}$
9) Causality
10) Wick
11) Fermionic rules
12) Yukawa diags
13) Qtz Maxwell
14) $A^\mu = 4 \times 4$
$E(r) = \sum_n \frac{1}{2} \omega_n$
$\frac{n\pi}{r} = \frac{\pi}{2r} \sum_n$
For $\Xi(p')$, boost by $p'^\mu = (E_p, 0, 0, -p)$
$\varepsilon^\mu_{1,2}(p') = \varepsilon^\mu_{1,2}(p) = (0,1,0,0)$ or $(0,0,1,0)$
$\varepsilon^\mu_L(p') = (-\beta\gamma, 0, 0, \gamma)$
Contract matrix elements with polarization vectors:
i, j = polarization
$\mathcal{M}(h \to \Xi_i \Xi_j) =$
$2 C_S \, \varepsilon_i(p) \cdot \varepsilon_j(p')$
$4 C_P \, \varepsilon_{\mu\nu\alpha\beta} p^\nu p'^\beta \varepsilon_i^\mu(p) \varepsilon_j^\alpha(p')$
$C_S \, h \, \Xi_\mu \Xi^\mu$
$\varepsilon_{\mu\nu\alpha\beta} \Xi^{\mu\nu} \Xi^{\alpha\beta}$
$\Xi_{\alpha\beta} = \partial_\alpha \Xi_\beta - \partial_\beta \Xi_\alpha$
$2 i C_S \, \eta_{\mu\alpha} \varepsilon^\mu(p) \varepsilon^\alpha(p')$

$\varepsilon_L^\mu(p') = (-\beta\gamma, 0, 0, \gamma)$

Contract matrix elements with polarization vectors:

$\mathcal{M}(h \to Z_i Z_j) =$ i, j = polarizations

$$\begin{cases} 2 c_S \, \varepsilon_i(p) \cdot \varepsilon_j(p') \\ 4 c_P \, \varepsilon_{\mu\nu\alpha\beta} \, p^\mu p'^\beta \, \varepsilon_i^\nu(p) \, \varepsilon_j^\alpha(p) \end{cases}$$

Scalar case:

$$\varepsilon_i(p) \cdot \varepsilon_j(p') = \begin{cases} -1 & \text{if } i = j = 1,2 \\ -\gamma^2(1+\beta^2) & \text{if } i = j = L \\ 0 & i \neq j \end{cases}$$

$\longrightarrow$ long. polarizations dominate for $\gamma \gg 1$

Pseudoscalar case: Only transverse components $i=1, j=2$ or $i=2, j=1$

Scalar case: $i\mathcal{M}(h \to Z_L Z_L) = -2 C_S \gamma^2 (1+\beta^2)$

Pseudoscalar case: $i\mathcal{M}(h \to Z_1 Z_2) = -i\mathcal{M}(h \to Z_2 Z_1) = 4 C_P \varepsilon_{\mu\nu\alpha\beta} p^\nu p^\beta \varepsilon_1^\mu \varepsilon_2^\alpha$

Z Polarization determines angular distribution of leptons.

Illustrative limit: heavy Higgs limit: $m_h \gg m_Z \gg m_\ell$ (easier if both Z on shell)

Goal: Compute differential decay rate

Consider $Z_i(p) \longrightarrow \ell(k_1)\,\bar{\ell}(k_2)$ i = polarization of initial Z.

Work in CM frame of Z

$$\Gamma(Z_i \to \ell\bar{\ell}) = \frac{1}{64\pi^2 m_Z} \int d\cos\theta_1 \int d\cos\phi_1 \sum_{\substack{\text{ferm.}\\ \text{spins}}} |\mathcal{M}(Z_i \to \ell\bar{\ell})|^2$$

Feynman rule: Z_μ — ℓ, $\bar{\ell}$

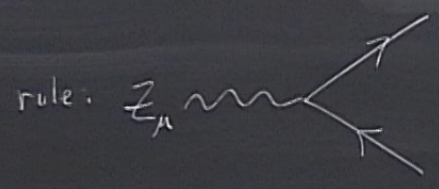
$$= \frac{ig}{4c_W}\,\gamma^\mu\left((1-4s_W^2) - \gamma^5\right)$$

Consider $Z_i(p) \longrightarrow \ell(k_1)\,\bar{\ell}(k_2)$ $\quad i$ = polarization of initial Z.

Work in CM frame of Z

$$\Gamma(Z_i \to \ell\bar{\ell}) = \frac{1}{64\pi^2 m_Z} \int d\cos\theta_1 \int d\cos\phi_1 \sum_{\substack{\text{fern.}\\ \text{spins}}} |\mathcal{M}(Z_i \to \ell\bar{\ell})|^2$$

Experimentally: $s_W^2 \approx 0.23$.

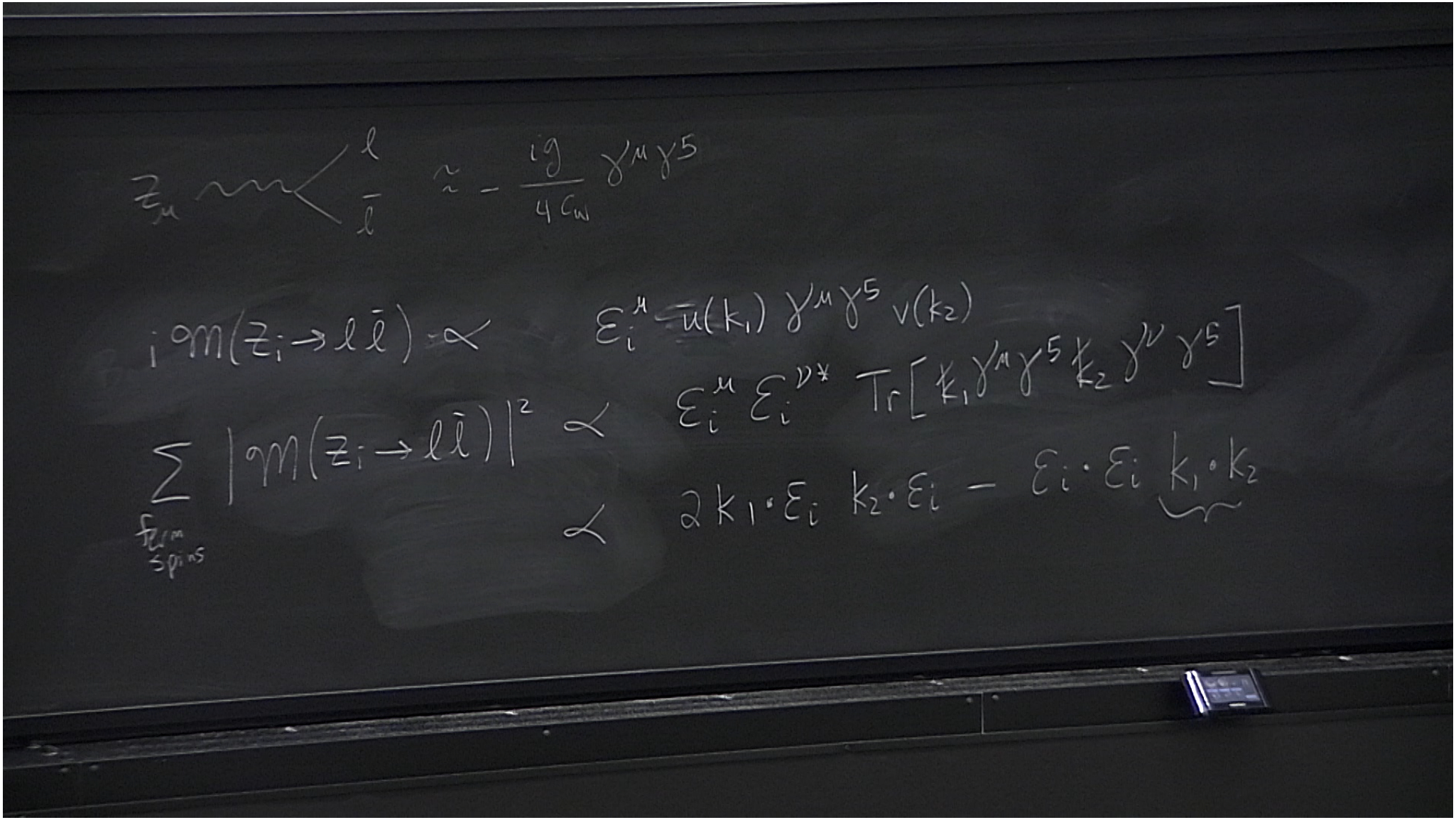
Feynman rule: 

$$= \frac{ig}{4c_W}\,\gamma^\mu\left((1-4s_W^2) - \gamma^5\right)$$

neglect

Weak charge of $\ell, \mu(p) \propto (1-4s_W^2) \ll 1$.

$$\varepsilon_3^\mu = (0,0,0,1)$$


Z_μ
ℓ
$\bar{\ell}$
$\simeq -\frac{ig}{4c_W}\gamma^\mu\gamma^5$
$i\mathcal{M}(Z_i \to \ell\bar{\ell}) \propto \varepsilon_i^\mu\, \bar{u}(k_1)\,\gamma^\mu\gamma^5\, v(k_2)$
$\sum_{\text{fermion spins}} |\mathcal{M}(Z_i \to \ell\bar{\ell})|^2 \propto \varepsilon_i^\mu \varepsilon_i^{\nu *}\, \mathrm{Tr}\left[\not{k}_1 \gamma^\mu \gamma^5 \not{k}_2 \gamma^\nu \gamma^5\right]$
$\propto 2\, k_1 \cdot \varepsilon_i\; k_2 \cdot \varepsilon_i - \varepsilon_i \cdot \varepsilon_i\, k_1 \cdot k_2$

Feynman rule: Z_μ … $\bar{l}$ $= \frac{ig}{4c_W}\gamma^\mu\left((1-4s_W^2)-\gamma^5\right)$

neglected

Weak charge of $e,\mu(l) \propto (1-4s_W^2) \ll 1$.

Z_μ … l, $\bar{l}$ $\simeq -\frac{ig}{4c_W}\gamma^\mu\gamma^5$

$i\mathcal{M}(Z_i \to l\bar{l}) \propto \varepsilon_i^\mu\, \bar{u}(k_1)\,\gamma^\mu\gamma^5\, v(k_2)$

$\sum_{\text{final spins}} |\mathcal{M}(Z_i \to l\bar{l})|^2 \propto \varepsilon_i^\mu \varepsilon_i^{\nu *}\, \mathrm{Tr}\left[\not{k}_1\gamma^\mu\gamma^5\not{k}_2\gamma^\nu\gamma^5\right]$

$\propto \underbrace{2k_1\cdot\varepsilon_i\; k_2\cdot\varepsilon_i}_{2k_{1i}k_{2i}} - \varepsilon_i\cdot\varepsilon_i\,\underbrace{k_1\cdot k_2}_{\frac{m_Z^2}{2}}$

$p^2 = m_Z^2 = (k_1+k_2)^2 = 2k_1\cdot k_2$

Write $\underline{k}_1 = (k_{1x}, k_{1y}, k_{1z})$

$= k_1(\sin\theta_1 \cos\phi_1, \sin\theta_1 \sin\phi_1, \cos\theta_1)$

CM frame $k_1 = \frac{m_Z}{2}$

$\underline{k}_2 = -\underline{k}_1$

$$\sum_{\text{spins}} |\mathcal{M}(Z_L \to l\bar{l})|^2 \propto \frac{m_Z^2}{2}(1 - \cos^2\theta_1)$$

$$\sum_{\text{spins}} |\mathcal{M}(Z_1 \to \ell\bar{\ell})|^2 \propto \frac{m_Z^2}{2}\left(1 - \sin^2\theta_1 \cos^2\phi_1\right)$$

$$\sum_{\text{spins}} |\mathcal{M}(Z_2 \to \ell\bar{\ell})|^2 \propto \frac{m_Z^2}{2}\left(1 - \sin^2\theta_1 \sin^2\phi_1\right)$$

For future reference

$$\sum_{\text{spins}} \mathcal{M}(Z_1 \to l\bar{l})\ \mathcal{M}(Z_2 \to l\bar{l})^* \propto 2\, k_1\cdot\varepsilon_1\, k_2\cdot\varepsilon_2 - \underbrace{\varepsilon_1\cdot\varepsilon_2}_{0}\, k_1\cdot k_2$$

$$\propto -\frac{m_Z^2}{2}\sin^2\theta_1 \sin\phi_1 \cos\phi_1$$

$$\mathcal{M}(h \to ZZ \to \ell\bar{\ell}\,\ell'\bar{\ell}') = \sum_{\text{pol } i,j} \mathcal{M}(h \to Z_i Z_j)\, \mathcal{M}(Z_i \to \ell\bar{\ell})\, \mathcal{M}(Z_j \to \ell'\bar{\ell}')$$

Scalar case: only need $i = j = L$

$$\left|\mathcal{M}(h \to ZZ \to \ell\bar{\ell}\,\ell'\bar{\ell}')\right|^2 \propto \left|\mathcal{M}(h \to Z_L Z_L)\right|^2 \left|\mathcal{M}(Z_L \to \ell\bar{\ell})\right|^2 \left|\mathcal{M}(Z_L \to \ell'\bar{\ell}')\right|^2$$

$$\propto (\text{constant})\ (1 - \cos^2\theta_1)(1 - \cos^2\theta_2)$$

$\mathcal{L}$ (constant)

No dependence on $\phi_1, \phi_3 \rightarrow \dfrac{d\Gamma}{d\phi} = \text{const.}$

Pseudoscalar case:

$$\sum |\mathcal{M}(h \to Z Z \to \ell\bar{\ell})|^2 = \sum_{\substack{\text{fermi} \\ \text{spins}}} \left| \sum_{i,j=1,2} \mathcal{M}(h \to Z_i Z_j)\, \mathcal{M}(Z_i \to \ell\bar{\ell})\, \mathcal{M}(Z_j \to \ell'\bar{\ell}') \right|^2$$

$$= |\mathcal{M}(h \to Z_1 Z_2)|^2 \sum_{\substack{\text{fermi} \\ \text{spins}}} \left| \mathcal{M}(Z_1 \to \ell\bar{\ell})\, \mathcal{M}(Z_2 \to \ell'\bar{\ell}') - \mathcal{M}(Z_2 \to \ell\bar{\ell})\, \mathcal{M}(Z_1 \to \ell'\bar{\ell}') \right|^2$$

$$= |\mathcal{M}(h \to ZZ)|^2 \left\{ \sum_{\substack{\text{ferm} \\ \text{spins}}} |\mathcal{M}(Z_1 \to \ell\bar{\ell})|^2 \, |\mathcal{M}(Z_2 \to \ell\bar{\ell})|^2 + |\mathcal{M}(Z_2 \to \ell\bar{\ell})|^2 \, |\mathcal{M}(Z_1 \to \ell'\bar{\ell}')|^2 \right.$$

$$\left. - 2\,\mathrm{Re} \sum \mathcal{M}(Z_1 \to \ell\bar{\ell}) \, \mathcal{M}(Z_2 \to \ell\bar{\ell})^* \, \mathcal{M}(Z_2 \to \ell'\bar{\ell}') \, \mathcal{M}(Z_1 \to \ell'\bar{\ell}')^* \right\}$$

$$= |\mathcal{M}(h \to ZZ)|^2 \left\{ \sum_{\text{ferm spins}} \underbrace{|\mathcal{M}(Z_1 \to \ell\bar{\ell})|^2}_{(1 - \sin^2\theta_1 \cos^2\phi_1)} \underbrace{|\mathcal{M}(Z_2 \to \ell\bar{\ell})|^2}_{(1 - \sin^2\theta_3 \sin^2\phi_3)} + \underbrace{|\mathcal{M}(Z_2 \to \ell\bar{\ell})|^2}_{(1 - \sin^2\theta_1 \sin^2\phi_1)} \underbrace{|\mathcal{M}(Z_1 \to \ell'\bar{\ell}')|^2}_{(1 - \sin^2\theta_3 \cos^2\phi_3)} \right.$$

$$\left. - 2\,\mathrm{Re} \sum \underbrace{\mathcal{M}(Z_1 \to \ell\bar{\ell})\, \mathcal{M}(Z_2 \to \ell\bar{\ell})^*}_{-\sin^2\theta_1 \sin\phi_1 \cos\phi_1} \underbrace{\mathcal{M}(Z_2 \to \ell'\bar{\ell}')\, \mathcal{M}(Z_1 \to \ell'\bar{\ell}')^*}_{-\sin^2\theta_3 \sin\phi_3 \cos\phi_3} \right\}$$

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$$\left. - 2\,\mathrm{Re} \sum \underbrace{\mathcal{M}(Z_1 \to \ell\bar{\ell})\, \mathcal{M}(Z_2 \to \ell\bar{\ell})^*}_{-\sin^2\theta_1 \sin\phi_1 \cos\phi_1} \underbrace{\mathcal{M}(Z_2 \to \ell'\bar{\ell}')\, \mathcal{M}(Z_1 \to \ell'\bar{\ell}')^*}_{-\sin^2\theta_3 \sin\phi_3 \cos\phi_3} \right\}$$

Integrate over $\int d\theta_1 \int d\theta_3$.

Define $\phi_1 - \phi_3 = \phi$. (No dependence on $\phi_1 + \phi_3$)

After integration $\sum_{\substack{\text{ferm} \\ \text{spins}}} |\mathcal{M}|^2 \propto \left(1 - \frac{1}{4}\cos 2\phi\right)$

$$\frac{d\Gamma}{d\phi} \propto \begin{cases} \text{const.} & \text{scalar} \\ 1 - \frac{1}{4}\cos 2\phi & \text{pseudoscalar} \end{cases}$$

Use total partial width

$$\Gamma = \Gamma(h \to ZZ \to \ell\bar{\ell}\,\ell'\bar{\ell}')$$

$$\frac{d\Gamma}{d\phi} = \begin{cases} \dfrac{\Gamma}{2\pi} & \text{scalar} \\ \dfrac{\Gamma}{2\pi}\left(1 - \dfrac{1}{4}\cos 2\phi\right) & \text{pseudoscalar} \end{cases}$$

$\frac{d\Gamma}{d\phi}$

