

Title: PSI 2018/2019 - Standard Model Review - Lecture 10

Date: Feb 11, 2019 09:00 AM

URL: <http://pirsa.org/19020007>

Abstract:

$$\underline{h \rightarrow gg}$$

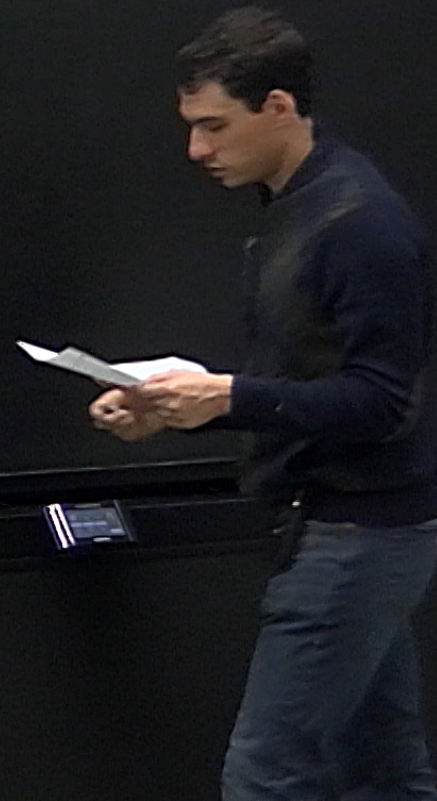
$$i\mathcal{M}_1 = h \text{ --- } g \rightarrow \begin{array}{c} \text{--- } k+p \text{ ---} \\ \text{--- } t \text{ ---} \\ \text{--- } k \text{ ---} \end{array} \begin{array}{c} \text{--- } p \text{ ---} \\ \text{--- } k+p' \text{ ---} \\ \text{--- } p' \text{ ---} \end{array} \begin{array}{c} g^A_\mu \\ g^B_\nu \end{array} =$$

$$\begin{aligned}
&= - g_s^2 \text{Tr}(T^A T^B) \frac{m_t^2}{V} \varepsilon_\mu(p) \varepsilon_\nu(p') \int dx dy dz \, 2 \delta(1-x-y-z) \\
&\quad \int \frac{d^d l}{(2\pi)^d} \frac{1}{(l^2 - M^2)^3} \left(\left(\frac{16}{d} - 4 \right) l^2 \gamma^{\mu\nu} + 2 \left(2m_t^2 - m_h^2 (1-2xz) \right) \gamma^{\mu\nu} + 4(1-4xz) p^\nu p'^\mu \right)
\end{aligned}$$

$$(1) = \int \frac{d^d l}{(2\pi)^d} \frac{l^2}{(l^2 - M^2)^3} = \frac{-i}{(4\pi)^{d/2}} \frac{d}{2} \frac{\Gamma(3 - \frac{d}{2} - 1)}{\Gamma(3)} \left(\frac{1}{M^2}\right)^{3 - \frac{d}{2} - 1}$$

$$(2) = \int \frac{d^d l}{(2\pi)^d} \frac{1}{(l^2 - M^2)^3} = \frac{i}{(4\pi)^{d/2}} \frac{\Gamma(3 - \frac{d}{2})}{\Gamma(3)} \left(\frac{1}{M^2}\right)^{3 - \frac{d}{2}}$$

Integral (1) is divergent $\sim \frac{1}{4-d}$, but $\left(\frac{16}{d}-4\right)\left(\frac{1}{4-d}\right) = 4\left(\frac{4-d}{d}\right)\frac{1}{4-d} = \frac{4}{d} \rightarrow 1$



$$\begin{aligned}
 \left(\frac{1}{M^2}\right)^{3-\frac{d}{2}-1} &= -\frac{i}{16\pi^2} \frac{d}{2} \frac{\Gamma\left(\frac{4-d}{2}\right)}{2} + \text{finite terms} \\
 &= -\frac{i}{16\pi^2} \frac{2}{4-d} + \dots
 \end{aligned}$$

Integral (i) is divergent $\sim \frac{1}{4-d}$, but $(\frac{16}{d}-4)(\frac{1}{4-d}) = 4(\frac{4-d}{d})\frac{1}{4-d} = \frac{4}{d} \rightarrow 1$

$$i\mathcal{M}_1 = -\frac{i}{16\pi^2} g_s^2 \text{Tr}(T^A T^B) \frac{m_t^2}{V} \varepsilon_\mu(p) \varepsilon_\nu(p') \int dx dz (4p^\nu p'^\mu - 2m_h^2 \eta^{\mu\nu}) \frac{1-4xz}{m_t^2 - xz m_h^2}$$

$\alpha m h$

$$\text{Tr}(T^A T^B) = \frac{1}{2} \delta^{AB}$$

$m_1 = m_2 \rightarrow \text{factor of 2.}$

$$i\mathcal{M}(h \rightarrow gg) = \frac{-i}{16\pi^2} \frac{g_s^2}{v} \delta^{AB} \varepsilon_\mu(p) \varepsilon_\nu(p') (4p^\nu p'^\mu - 2m_h^2 \eta^{\mu\nu}) \int_0^1 dx \int_0^{1-x} dy \frac{1-4xz}{1-xz(m_h^2/4m_t^2)}$$

$\propto m_h$

$$\text{Tr}(T^A T^B) = \frac{1}{2} \delta^{AB}$$

$m_1 = m_2 \rightarrow \text{factor of 2.}$

$$i\mathcal{M}(h \rightarrow gg) = \frac{-i}{16\pi^2} \frac{g_s^2}{v} \delta^{AB} \varepsilon_\mu(p) \varepsilon_\nu(p') (4p^\nu p'^\mu - 2m_h^2 \eta^{\mu\nu}) \underbrace{\int_0^1 dx \int_0^{1-x} dy \frac{1-4xz}{1-xz \left(\frac{m_h^2}{4m_t^2} \right)}}$$

$$\text{Tr}(T^A T^B) = \frac{1}{2} \delta^{AB}$$

$m_1 = m_2 \rightarrow \text{factor of 2.}$

$$i\mathcal{M}(h \rightarrow gg) = \frac{-i}{16\pi^2} \frac{g_s^2}{V} \delta^{AB} \varepsilon_\mu(p) \varepsilon_\nu(p') (4p^\nu p'^\mu - 2m_h^2 \eta^{\mu\nu}) \underbrace{\int_0^1 dx \int_0^{1-x} dy \frac{1-4xz}{1-xz(m_h^2/4m_t^2)}}$$

Take limit $\frac{m_h^2}{4m_t^2} \ll 1$: $\int_0^1 dx \int_0^{1-x} dz \frac{1-4xz}{1-xz} = \frac{1}{3}$

Take limit $\frac{m_h}{4m_t^2} \ll 1$: $\int_0^1 dx \int_0^1 dz \frac{1-4xz}{z} = 3$

$$i\mathcal{M} = -i \frac{\alpha_s}{3\pi v} \epsilon_\mu(p) \epsilon_\nu(p') \left(p^\nu p'^\mu - \frac{m_h^2}{2} \eta^{\mu\nu} \right) \delta^{AB}$$

$$\sum_{\text{spins colors}} |\mathcal{M}|^2 = \left(\frac{\alpha_s}{3\pi v} \right)^2 \underbrace{\sum_{AB} \delta^{AB} \delta_{AB}}_8 \left(p^\nu p'^\mu - \frac{m_h^2}{2} \eta^{\mu\nu} \right) \left(p_\nu p'_\mu - \frac{m_h^2}{2} \eta_{\mu\nu} \right)$$

$$= \frac{4\alpha_s^2 m_h^4}{q_\pi^2 v^2}$$

$$\Gamma(h \rightarrow gg) = \frac{\alpha_s^2 m_h^3}{64\pi^3 v^2} \quad \left(\text{also divided by 2 for identical gluons} \right)$$

$$= 0.23 \text{ MeV} \times \left[\text{QCD corrections enhance by } O(60\%) \right]$$

M_t -dependence cancels only in heavy quark limit.

$$\int_0^1 dx \int_0^{1-x} dz \frac{1-4xz}{1-xz \left(\frac{m_h^2}{m_q^2} z \right)} = \begin{cases} 1/3 & \text{if } m_q^2 \gg m_h^2 \\ \frac{1}{2} \frac{m_q^2}{m_h^2} \log^2 \left(\frac{m_h^2}{m_q^2} \right) & m_q^2 \ll m_h^2 \end{cases}$$

any quark limit.

$$\left\{ \begin{array}{l} 1/3 \quad \text{if } m_q^2 \gg m_h^2 \end{array} \right.$$

$$\left\{ \begin{array}{l} \frac{1}{2} \frac{m_q^2}{m_h^2} \log^2 \left(\frac{m_h^2}{m_q^2} \right) \quad m_q^2 \ll m_h^2 \end{array} \right.$$

$$\rightarrow 0 \text{ for } m_q \rightarrow 0$$

light quarks u, d, s, c, b have small contribution

$\rightarrow 0$ for $m_g \rightarrow 0$

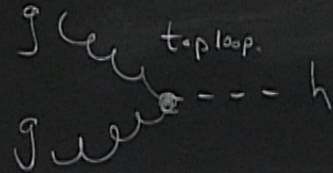
Higgs production at hadron colliders

LHC: pp collider at $\sqrt{s} = 7, 8, 13$ TeV

Cross section for higgs production?

Several production channels (parton level): in order of importance

(1) gluon-gluon fusion (ggF) $gg \rightarrow h$



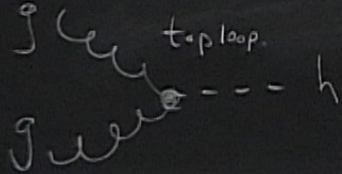
$\rightarrow 0$ for $m_g \rightarrow 0$

on at hadron colliders

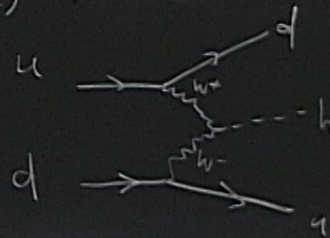
collider at $\sqrt{s} = 7, 8, 13$ TeV
for higgs production?

production channels (parton level): in order of importance

non-gluon fusion (ggF) $gg \rightarrow h$



(2) vector-boson fusion (VBF)



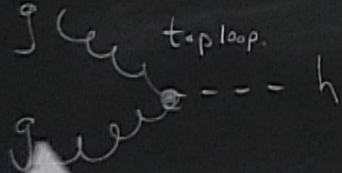
$\rightarrow 0$ for $m_g \rightarrow 0$

on at hadron colliders

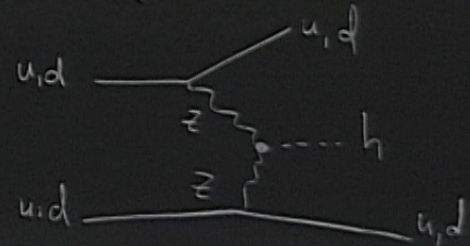
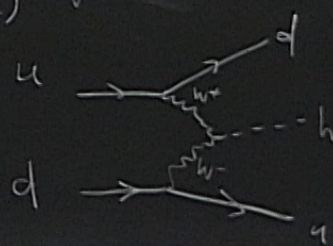
collider at $\sqrt{s} = 7, 8, 13$ TeV
for higgs production?

production channels (parton level): in order of importance

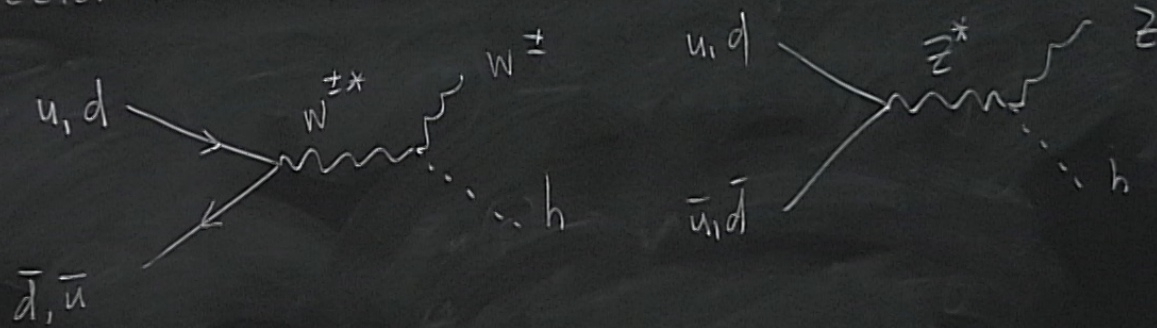
gluon-gluon fusion (ggF) $gg \rightarrow h$



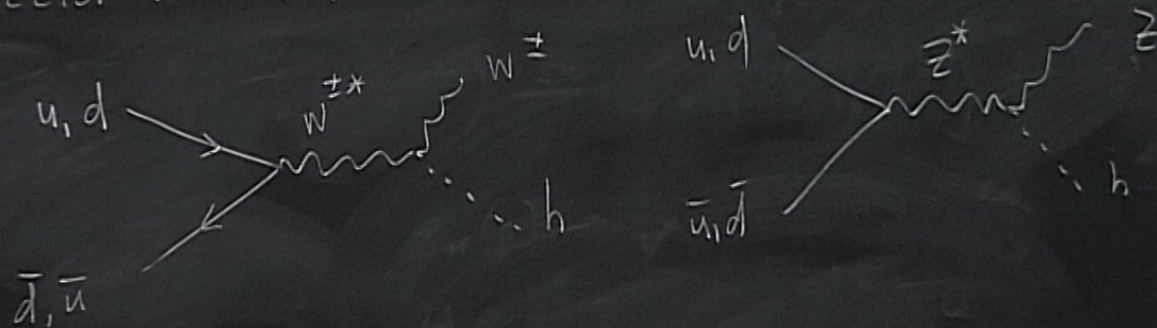
(2) vector-boson fusion (VBF)



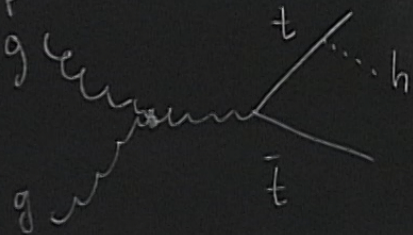
(3) Vector boson (W, Z) associated production (WH, ZH)



(3) Vector boson (W, Z) associated production (WH, ZH)



(4) top associated production ($t\bar{t}H$)



ggF cross section:

Parton-level cross section:

$$\hat{\sigma}(gg \rightarrow h) = \frac{1}{2E_1 2E_2 |v_{rel}|} \int \frac{d^3 q}{(2\pi)^3} \frac{1}{2E_q} (2\pi)^4 \delta^4(q - p_1 - p_2) \left(\frac{1}{8 \cdot 2} \right)^2 \sum |\mathcal{M}(gg \rightarrow h)|^2$$

$8 = \text{colors}$
 $2 = \text{spins}$

CM frame: $|v_{rel}| = 2$
 $4E_1 E_2 = \hat{s}$
 $E_q = m_h$

$$i\mathcal{M} = -i \frac{\alpha_s}{3\pi v} \epsilon_\mu(p) \epsilon_\nu(p') \left(p^\nu p'^\mu - \frac{m_h^2}{2} \eta^{\mu\nu} \right) \delta_{AB}$$

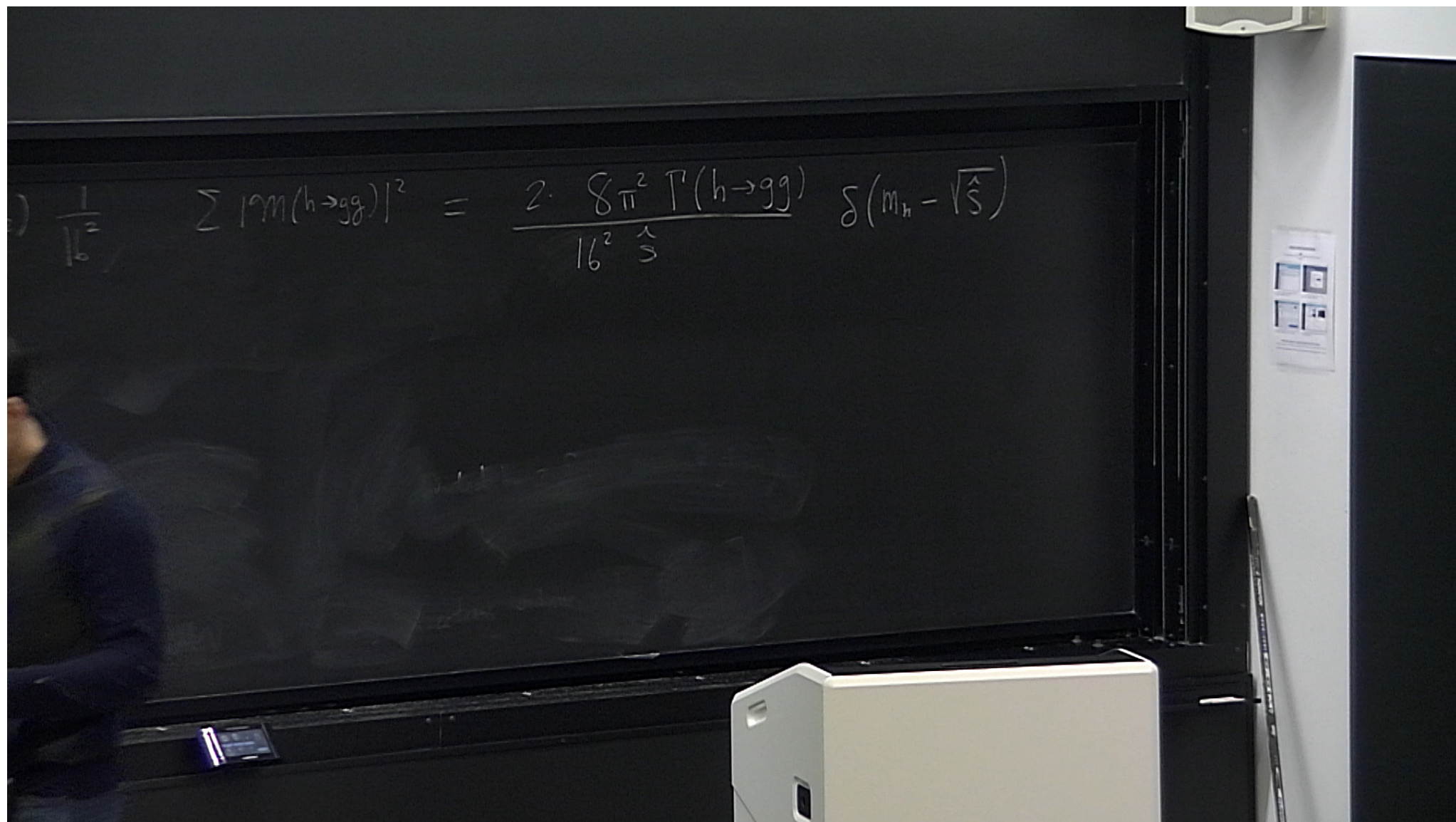
$$\left(\frac{\alpha_s}{3\pi v} \right)^2 \sum_{AB} \delta_{AB} \delta_{AB} \left(p^\nu p'^\mu - \frac{m_h^2}{2} \eta^{\mu\nu} \right) \left(p_\nu p'_\mu - \frac{m_h^2}{2} \eta_{\mu\nu} \right)$$

$$\hat{\sigma}(gg \rightarrow h) = \frac{1}{2\hat{s}} \frac{1}{2m_h} (2\pi) \delta(m_h - E_1 - E_2) \frac{1}{16} \sum |\mathcal{M}(h \rightarrow gg)|^2$$

$$\hat{\sigma}(gg \rightarrow h) = \frac{1}{2\hat{s}} \frac{1}{2m_h} (2\pi) \delta(m_h - E_1 - E_2) \frac{1}{16} \sum |\mathcal{M}(h \rightarrow gg)|^2$$

$$\Gamma(h \rightarrow gg) = \frac{1}{16\pi m_h} \frac{1}{2} \sum |\mathcal{M}(h \rightarrow gg)|^2$$

↑ identical gluons in final state.



$$\begin{aligned}
 \frac{1}{16} \sum |M(h \rightarrow gg)|^2 &= \frac{2 \cdot 8\pi^2 \Gamma(h \rightarrow gg)}{16^2 \hat{s}} \underbrace{\delta(m_h - \sqrt{\hat{s}})}_{\delta(\hat{s} - m_h^2) 2\sqrt{\hat{s}}} \\
 &= \delta(\hat{s} - m_h^2) 2\sqrt{\hat{s}}
 \end{aligned}$$

$$\hat{\sigma}(gg \rightarrow h) = \frac{1}{2\hat{s}} \frac{1}{2m_h} (2\pi) \delta(m_h - E_1 - E_2) \frac{1}{16} \sum |\mathcal{M}(h \rightarrow gg)|^2 =$$

$$\Gamma(h \rightarrow gg) = \frac{1}{16\pi m_h} \frac{1}{2} \sum |\mathcal{M}(h \rightarrow gg)|^2$$

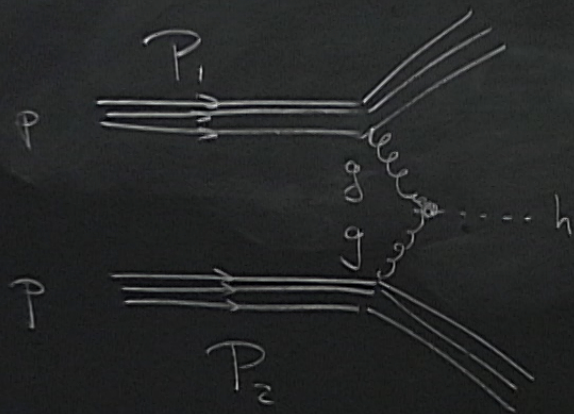
↑ identical gluons in final state

$$\hat{\sigma}(gg \rightarrow h) = \frac{\pi^2 \Gamma(h \rightarrow gg)}{16\hat{s}} \delta(\hat{s} - m_h^2)$$

Hadron-level cross section:

$$p(P_1) p(P_2) \rightarrow h + X$$

$X = \text{all}$



$h + X$

$X =$ all possible hadronic final states (inclusive higgs production)

$$p(P_1) p(P_2) \rightarrow h + X$$

$X =$ all possible hadronic final states (incl.)

$$\sigma(pp \rightarrow h X) = \int_0^1 dx_1 \int_0^1 dx_2 f_g(x_1) f_g(x_2) \hat{\sigma}(gg \rightarrow h)$$

X

X = all possible hadronic final states (inclusive higgs production)

$$\int_0^1 dx_1 \int_0^1 dx_2 f_g(x_1) f_g(x_2) \hat{\sigma}(gg \rightarrow h)$$

$x_{1,2}$ = momentum fractions
carried by "striking" gluons.

X

X = all possible hadronic final states (inclusive higgs production)

$$\int_0^1 dx_1 f_g(x_1) f_g(x_2) \hat{\sigma}(gg \rightarrow h)$$

$x_{1,2}$ = momentum fractions
carried by "struck" gluons.

$P_{1,2}$ = momenta of initial protons

$p_{1,2}$ = momenta of initial gluons

$$x_1 P_1 = p_1 \quad x_2 P_2 = p_2$$

$$p(P_1) p(P_2) \rightarrow h + X$$

$X =$ all possible hadronic final states

$$\sigma(pp \rightarrow h X) = \int_0^1 dx_1 \int_0^1 dx_2 f_g(x_1) f_g(x_2) \hat{\sigma}(gg \rightarrow h)$$

$$\hat{s} = (P_1 + P_2)^2 = 2 P_1 \cdot P_2 = 2 x_1 x_2 P_1 \cdot P_2 = x_1 x_2 s$$

($m_p \ll \sqrt{s}$)

$$(m_p \ll \sqrt{s})$$

$$\sigma(pp \rightarrow hX) = \int_0^1 dx_1 \int_0^1 dx_2 f_g(x_1) f_g(x_2) \frac{\pi^2 \Gamma(h \rightarrow gg)}{16 m_h} \delta(x_1 x_2 S - m_h^2)$$

$$(m_p \ll \sqrt{s})$$

$$\sigma(pp \rightarrow hX) = \int_0^1 dx_1 \int_0^1 dx_2 f_g(x_1) f_g(x_2) \frac{\pi^2 \Gamma(h \rightarrow gg)}{16 m_h} \delta(x_1 x_2 s - m_h^2)$$

Define $\tau = \frac{m_h^2}{s}$

$$\delta(x_1 x_2 s - m_h^2) = \frac{1}{s x_1} \delta(x_2 - \frac{\tau}{x_1}) \rightarrow x_2 = \frac{\tau}{x_1}$$

$$(m_p \ll \sqrt{s})$$

$$\sigma(pp \rightarrow hX) = \int_0^1 dx_1 \int_0^1 dx_2 f_g(x_1) f_g(x_2) \frac{\pi^2 \Gamma(h \rightarrow gg)}{16 m_h} \delta(x_1 x_2 s - m_h^2)$$

Define $\tau = \frac{m_h^2}{s}$

$$\delta(x_1 x_2 s - m_h^2) = \frac{1}{s x_1} \delta\left(x_2 - \frac{\tau}{x_1}\right) \rightarrow x_2 = \frac{\tau}{x_1}$$

We must have $0 < x_2 < 1 \Rightarrow 0 < \frac{\tau}{x_1} < 1 \Rightarrow x_1 > \tau$

$$\sigma(pp \rightarrow hX) = \int_0^1 \frac{dx_1}{x_1} f_g(x_1) f_g\left(\frac{\tau}{x_1}\right) \frac{\pi^2 \Gamma(h \rightarrow gg)}{8 m_h S}$$

$$\sigma(pp \rightarrow hX) = \int_{\tau}^1 \frac{dx_1}{x_1} f_g(x_1) f_g\left(\frac{\tau}{x_1}\right) \frac{\pi^2 \Gamma(h \rightarrow gg)}{8 m_h S}$$

$$\sigma(pp \rightarrow hX) = \int_0^1 \frac{dx_1}{x_1} f_g(x_1) f_g\left(\frac{\tau}{x_1}\right) \frac{\pi^2 \Gamma(h \rightarrow gg)}{8 m_h S}$$

PS: $f_g(x) = \frac{8}{x} (1-x)^7$

At LHC:

$\sqrt{s} = 7 \text{ TeV}$

8 TeV

13 TeV

Actual $f_g(x)$ (LO in α_s)

10.5 pb

11.4 pb

14.8 pb

PS ansatz

5.3 pb

6.8 pb

15.7 pb

s)

PS ansatz

5.3 pb

6.8 pb

15.7 pb

NNLO [including $\mathcal{O}(\alpha_s^2)$ corrections]

15.1 pb

19.3 pb

43.9 pb

How many h produced?

Simplified collider



Introduce luminosity

$$\frac{\text{Rate of collisions}}{\text{Volume}} = n_A$$

luminosity

$$\frac{\text{Rate of collisions}}{\text{Volume}} = \underbrace{n_A n_B}_{\text{number densities}} |\underline{v_{\text{rel}}}| \underbrace{\sigma}_{\text{cross section}}$$

$$\text{Rate} = \frac{\text{number of collisions}}{\text{time}} = \underbrace{(n_A n_B |\underline{v_{\text{rel}}}| \text{Volume})}_{\mathcal{L} = \text{luminosity}} \cdot \underbrace{\sigma}_{\frac{1}{\text{area} \cdot \text{time}}}$$

Integrated Luminosity $\int dt \mathcal{L} = \frac{\text{number of collisions}}{\sigma}$

Number of events = $\sigma \underbrace{\int dt \mathcal{L}}_{\text{depends on expt.}}$

Integrated Luminosity

$$\int dt \mathcal{L} = \frac{\text{number of collisions}}{\sigma}$$

Higgs

Number of events =

$$\sigma \underbrace{\int dt \mathcal{L}}_{\text{depends on expt.}}$$

Integrated Luminosity $\int dt \mathcal{L} = \frac{\text{number of collisions}}{\sigma}$

Number of events = $\sigma \underbrace{\int dt \mathcal{L}}_{\text{depends on expt.}}$

Dominant decays: $h \rightarrow b\bar{b}, WW^*, gg$ challenging
cleanest $h \rightarrow \gamma\gamma, \tau\tau^*$

$BR(h \rightarrow \gamma\gamma) = 0.15\% \rightarrow 250 \text{ events}$