Title: PSI 2018/2019 - Standard Model Review - Lecture 9

Date: Feb 08, 2019 09:00 AM

URL: http://pirsa.org/19020006

Abstract:

Deep inelastic scattering

$$e^- q \to e^- q$$

$$\frac{d\hat{\sigma}}{d\hat{t}} = \frac{2\pi \alpha^2 Q_q^2}{\hat{s}^2 \hat{t}^2} \left(\hat{s}^2 + \hat{u}^2 \right)$$

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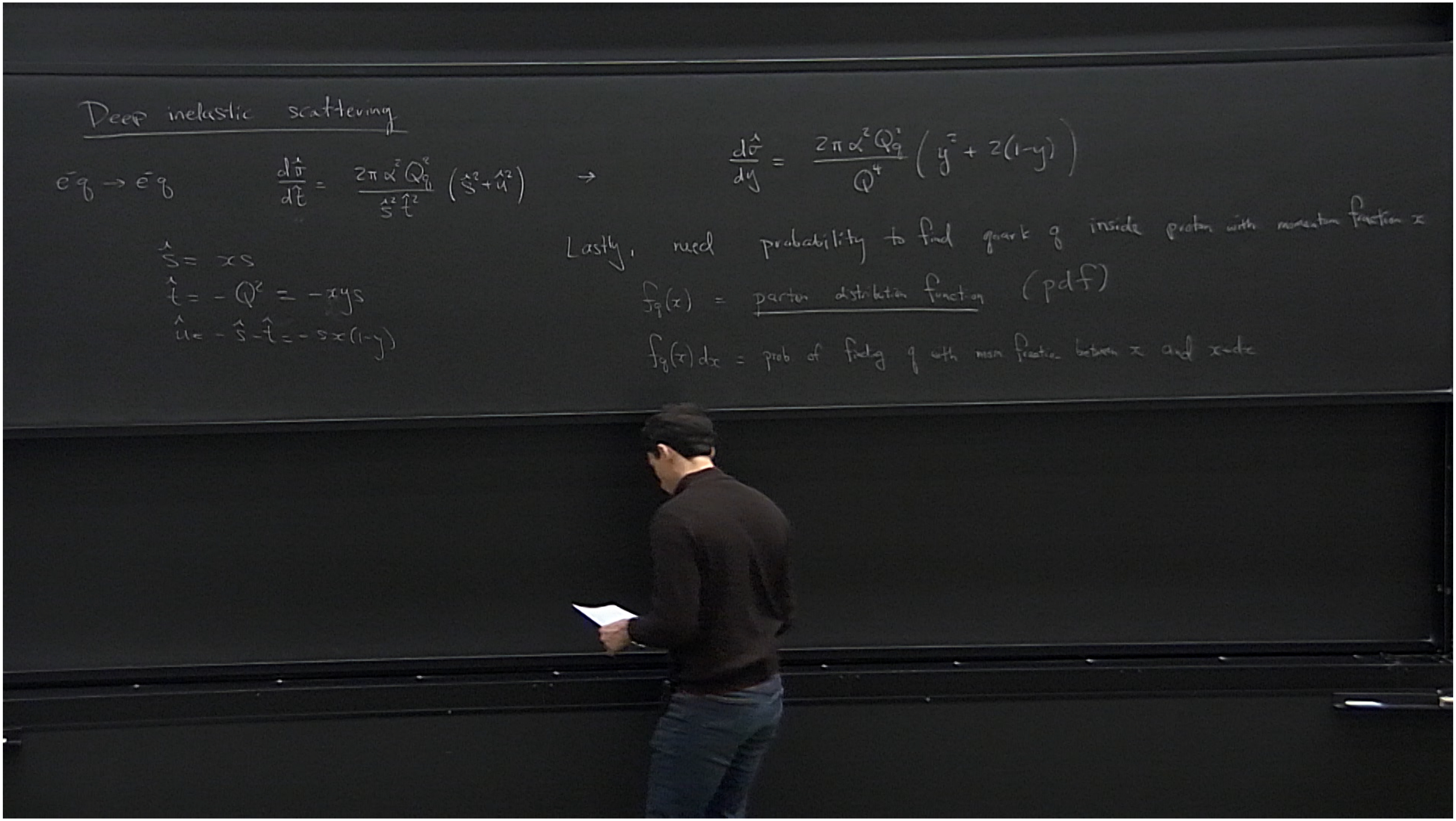
$$\hat{s} = xs$$

$$\hat{t} = -Q^2 = -xys$$

$$\hat{u} = -\hat{s} - \hat{t} = -sx(1-y)$$


Deep inelastic scattering
$e^- q \to e^- q$
$\frac{d\hat\sigma}{d\hat t} = \frac{2\pi\alpha^2 Q_q^2}{\hat s^2 \hat t^2}\left(\hat s^2+\hat u^2\right)$ $\rightarrow$ $\frac{d\hat\sigma}{dy} = \frac{2\pi\alpha^2 Q_q^2}{Q^4}\left(y^2+2(1-y)\right)$
$\hat s = xs$
$\hat t = -Q^2 = -xys$
$\hat u = -\hat s - \hat t = -sx(1-y)$
Lastly, need probability to find quark q inside proton with momentum fraction x
$f_q(x)$ = parton distribution function (pdf)
$f_q(x)\,dx$ = prob of finding q with mom fraction between x and x+dx

$f_q(x)\,dx$ = prob.

Total inclusive cross section (hadron level)

$$\sigma(e^- p \to e^- X) = \sum_q \int_0^1 f_q(x)\,dx\ \hat{\sigma}(e^- q \to e^- q)$$

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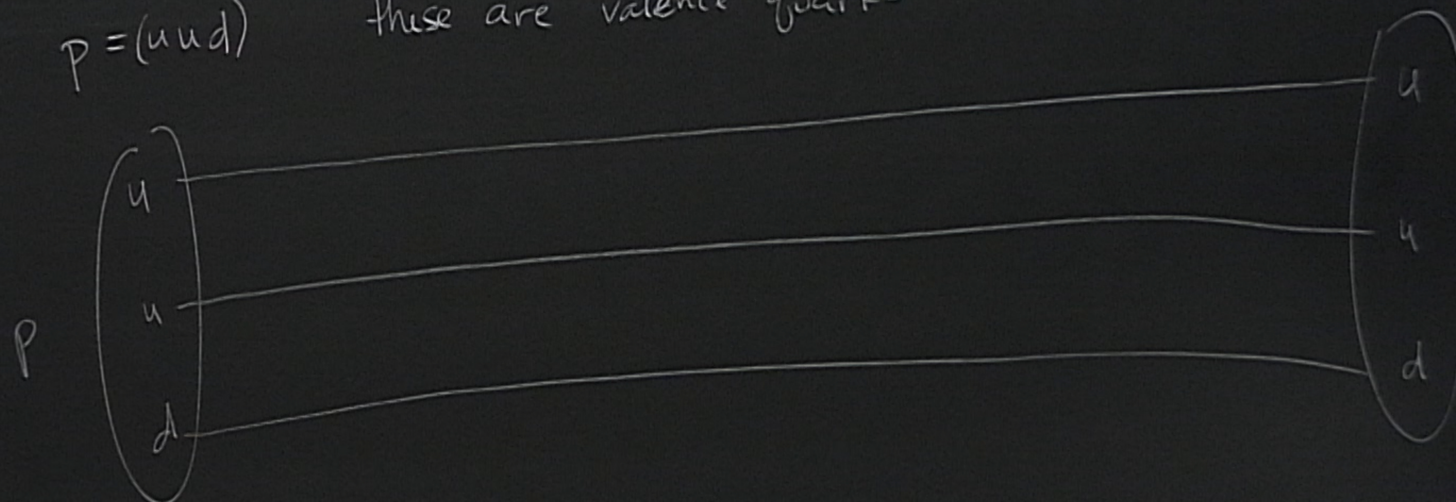
What are pdfs? What quarks do we sum over?

$p = (uud)$ these are valence quarks



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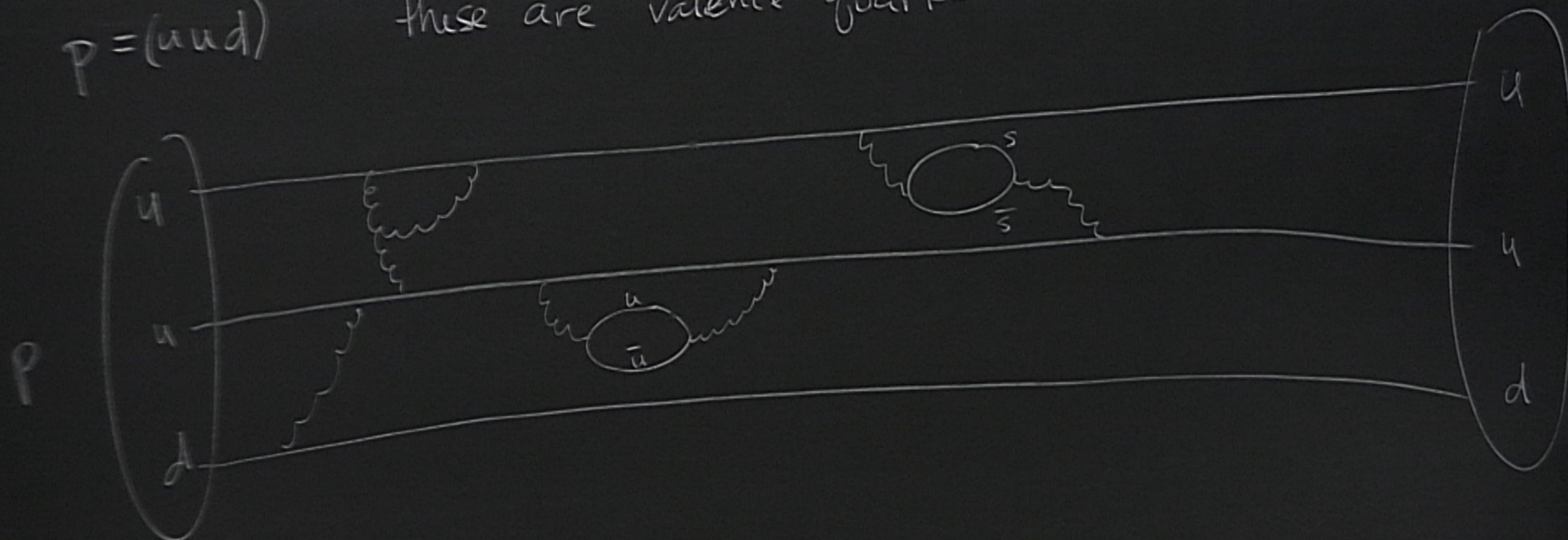
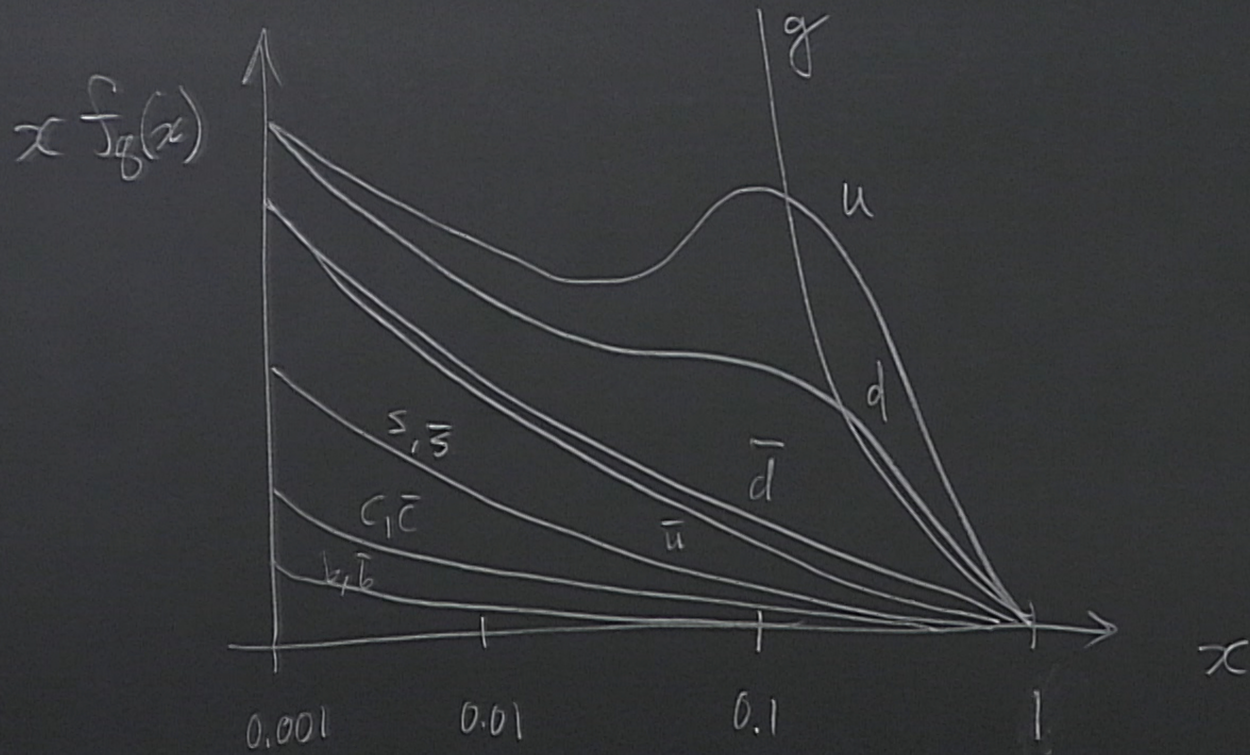


Sum over all quarks & antiquarks

u

u

d



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$$f_q(x) = \underline{\text{parton distribution function}} \quad (\text{pdf})$$

$f_q(x)\,dx$ = prob of finding q with mom fraction between x and $x+dx$

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Previously:

$$\frac{d^2\sigma(e^- p \to e^- X)}{dx\,dy} = \frac{2\pi\alpha^2 s}{Q^4}\left(F_1^e(x,Q^2)\, x\, y^2 + 2F_2^e(x,Q^2)(1-y) \right)$$

Parton calculation:

$$\frac{d^2\sigma(e^- p \to e^- X)}{dx\,dy} = \frac{2\pi\alpha^2 s}{Q^4}\left(\sum_q Q_q^2 f_q(x)\, x\, y^2 + 2(1-y) \sum_q Q_q f_q(x) \right)$$

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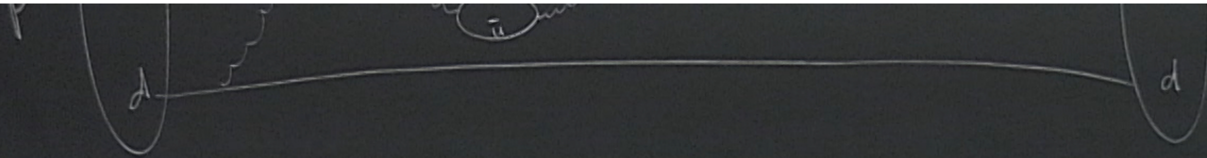


We have: $x F_1(x,Q^2) = F_2(x,Q^2) = x \sum_q Q_q^2 f_q(x) = x\left(\frac{2}{3}\right)^2 f_u(x) + x\left(-\frac{2}{3}\right)^2 f_{\bar{u}}(x) + x\left(\frac{-1}{3}\right)^2 f_d(x) + \dots$

(1) Prediction from parton model: F_1^e, F_2^e only depend on x, not Q^2. Bjorken Scaling

(Underlying scattering is elastic.) "Scaling violations" $F_{1,2}^e(x) \to F_{1,2}^e(x,Q^2)$ at order α_s.

(2) $x F_1^e(x) = F_2^e(x)$ Callan-Gross relation

(If quark had spin-0 $\Rightarrow$ $F_1^e(x) = 0$.)

Higgs decays & production (at hadron colliders)

Higgs has largest couplings to W, Z, t. But $m_h \simeq 125$ GeV.

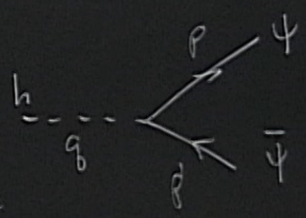
$h \to WW, ZZ, t\bar{t}$ all kinematically forbidden

Tree-level decays

- $h \to$ fermions

$$\mathcal{L}_{int} = -\left(\frac{m_\psi}{v}\right)\bar{\psi}\psi h$$

$i\mathcal{M} =$



$$= -i\,\frac{m_\psi}{v}\,\bar{u}(p)\,v(p')$$

$$\sum |\mathcal{M}|^2 = \frac{m_\psi^2}{v^2}\,4p\cdot p' = \frac{2m_\psi^2 m_h^2}{v^2}$$

$$\Gamma(h \to \psi\bar{\psi}) = \frac{1}{16\pi m_h} \sum |\mathcal{M}|^2$$

$$\Gamma(h \to \tau\bar{\tau}) = \frac{m_\tau^2 m_h}{8\pi v^2} \approx 0.26 \text{ MeV} \qquad (6.3\%)$$

$$\Gamma(h \to b\bar{b}) = \frac{3 m_b^2 m_h}{8\pi v^2} \approx 4.3 \text{ MeV} \qquad (58\%)$$

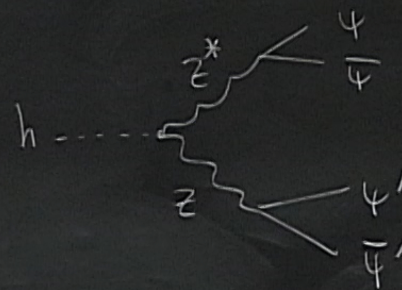
$$\Gamma(h \to c\bar{c}) = \frac{3 m_c^2 m_h}{8\pi v^2} \approx 0.4 \text{ MeV} \qquad (2.9\%)$$

Renormalization group running of m_c, m_b (due to QCD) from m_c, m_b to m_h reduces m_c, m_b by factors of 4 & 2 resp.

$$\frac{\quad}{8\pi v^2} \approx 4.3\ \text{MeV} \qquad (58\%)$$

$$\Gamma(h \to c\bar{c}) = \frac{3 m_c^2 m_h}{8\pi v^2} \approx 0.4\ \text{MeV} \qquad (2.9\%)^*$$

- $h \to W^+W^-, ZZ$ forbidden unless one (both) gauge bosons are of shell



"golden mode" $\psi, \psi' = e, \mu$

$h \to ZZ^* \to 4$ leptons

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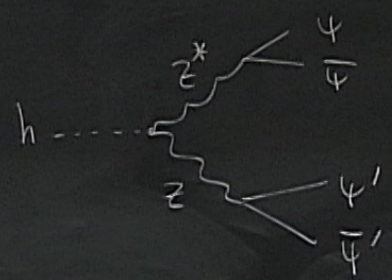
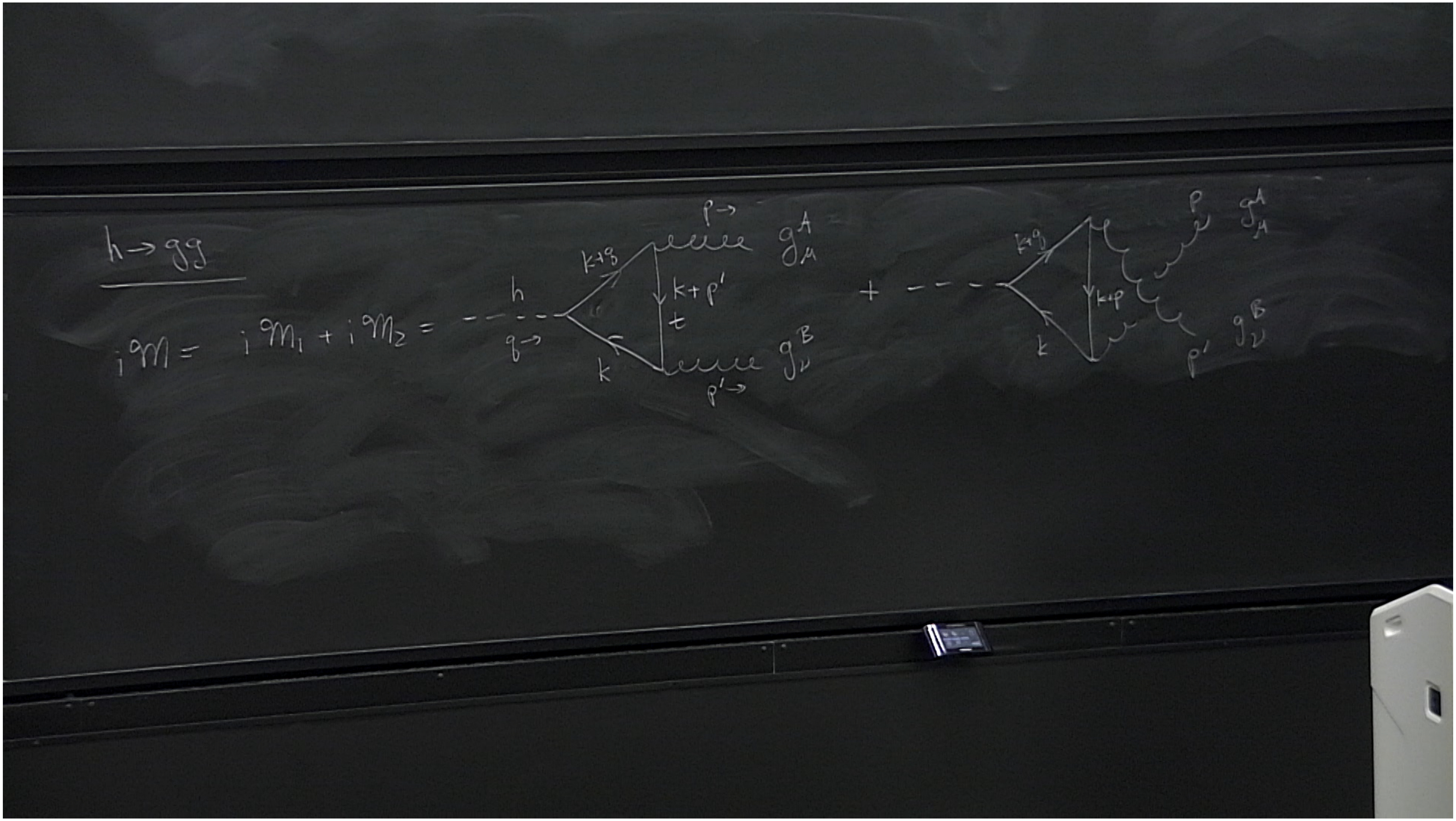


"golden mode" $\psi, \psi' = e, \mu$

$h \to ZZ^* \to 4$ leptons

$$\Gamma(h\to ZZ^*) \approx 0.11\ \text{MeV} \quad (2.6\%)$$

$$\Gamma(h\to WW^*) \approx 0.88\ \text{MeV} \quad (22\%)$$


h → gg
iM = iM₁ + iM₂ =
h
q →
k+q
k+p'
t
k
p →
p' →
g^A_μ
g^B_ν
+
k+q
k+p
k
p
p'
g^A_μ
g^B_ν

$$i\mathcal{M}_1 = \int \frac{d^dk}{(2\pi)^d} (-1)\, \mathrm{Tr}\left[\frac{i(\not{k}+m_t)}{k^2-m_t^2}\, i g_s T^B \gamma^\nu \frac{i(\not{k}+\not{p}'+m_t)}{(k+p')^2-m_t^2}\, i g_s T^A \gamma^\mu \frac{i(\not{k}+\not{q}+m_t)}{(k+q)^2-m_t^2} \left(-\frac{i m_t}{v}\right) \right] \varepsilon_\mu \varepsilon_\nu$$

$$= -g_s^2\, \mathrm{Tr}(T^A T^B) \left(\frac{m_t}{v}\right) \varepsilon_\mu \varepsilon_\nu \int \frac{d^dk}{(2\pi)^d} \frac{\mathrm{Tr}\left[(\not{k}+m_t)\gamma^\nu(\not{k}+\not{p}'+m_t)\gamma^\mu(\not{k}+\not{q}+m_t)\right]}{(k^2-m_t^2)\left((k+p')^2-m_t^2\right)\left((k+q)^2-m_t^2\right)}$$

Feynman parameters:

$$i\mathcal{M}_1 = -g_s^2 \, \mathrm{Tr}(T^A T^B) \frac{m_t}{v} \varepsilon_\mu \varepsilon_\nu \int dx\, dy\, dz \; 2\, \delta(1-x-y-z) \frac{\mathrm{Tr}[\dots]}{\left[(k^2-m_t^2)x + ((k+p')^2-m_t^2)y + ((k+q)^2-m_t^2)z\right]^3}$$

$$[\,\cdot\,] = (k^2-m_t^2)x + ((k+p')^2-m_t^2)y + ((k+q)^2-m_t^2)z$$

$$= \ell^2 - M^2 \qquad \text{where } \ell = k + y p' + z q, \quad M^2 = m_t^2 - xz\, m_h^2$$

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$$p' \cdot q = p' \cdot (p' + p) = p \cdot p' = \tfrac{1}{2}(p+p')^2 = \tfrac{1}{2} q^2 = \tfrac{1}{2} m_h^2$$

Shift $k = \ell - y p' - z q$ in numerator:

$$\mathrm{Tr}[\ldots] = 16\, m_t^2\, \ell^\mu \ell^\nu - 4\, \ell^2 m_t\, \eta^{\mu\nu} + 4\, m_t^3\, \eta^{\mu\nu} - 2 m_t m_h^2 (1 - 2xz)\, \eta^{\mu\nu} + 4 m_t (1 - 4xz)\, p_1^\nu p_2^\mu$$

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Neglected terms $\sim p^\mu$ or p'^ν since $p\cdot\varepsilon(p) = 0$, $p'\cdot\varepsilon(p') = 0$

Neglected terms odd in ℓ

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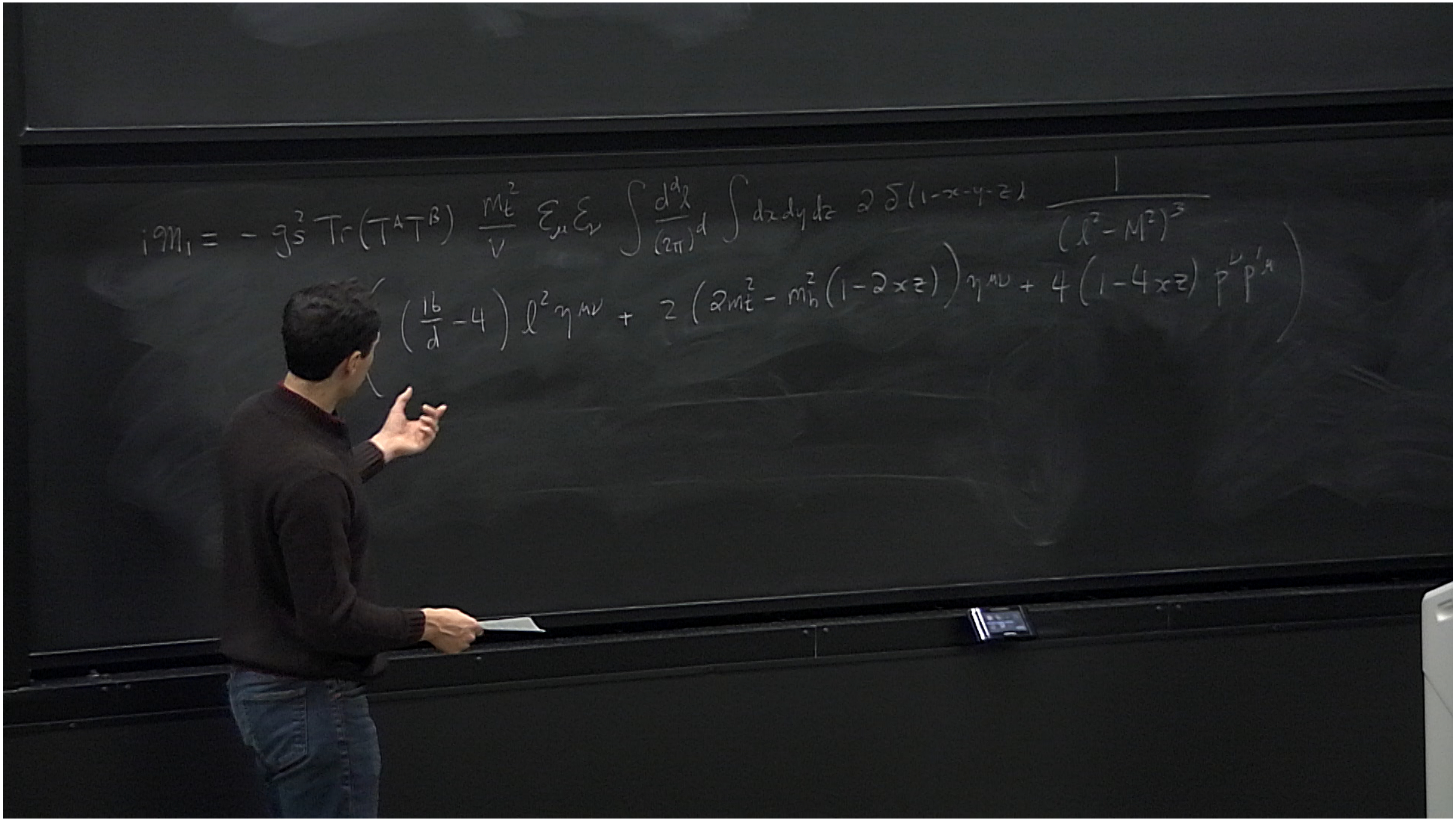
Neglected terms odd in ℓ

In d-dim: $\ell^\mu \ell^\nu \to \frac{1}{d}\ell^2\eta^{\mu\nu}$ Check. $\eta^{\mu\nu}\ell_\mu\ell_\nu \to \eta^{\mu\nu}\frac{1}{d}\eta_{\mu\nu}\ell^2 = \frac{1}{d}d\,\ell^2$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$\left(\left(\frac{16}{d}-4\right) l^2 \eta^{\mu\nu} + 2\left(2m_t^2 - m_h^2(1-2xz)\right)\eta^{\mu\nu} + 4(1-4xz)\,p^\nu p'^\mu\right)$