

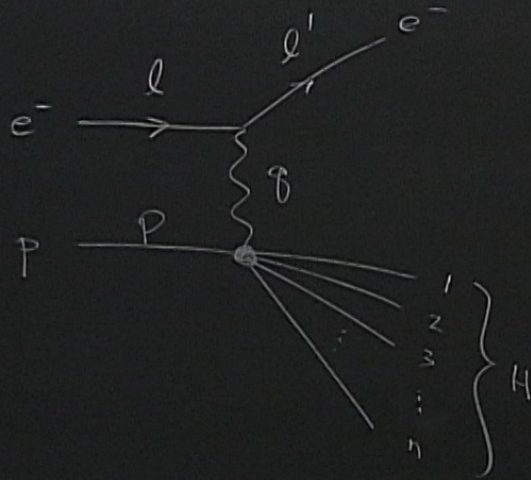
Title: PSI 2018/2019 - Standard Model Review - Lecture 8

Date: Feb 07, 2019 09:00 AM

URL: <http://pirsa.org/19020005>

Abstract:

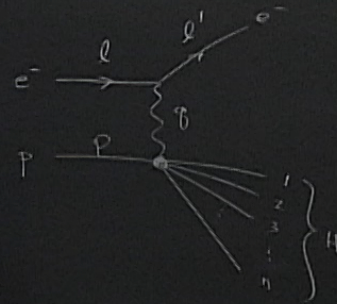
Deep inelastic scattering



Inclusive:

$$\sigma(e^- p \rightarrow e^- X) = \sum_H \sigma(e^- p \rightarrow e^- H)$$

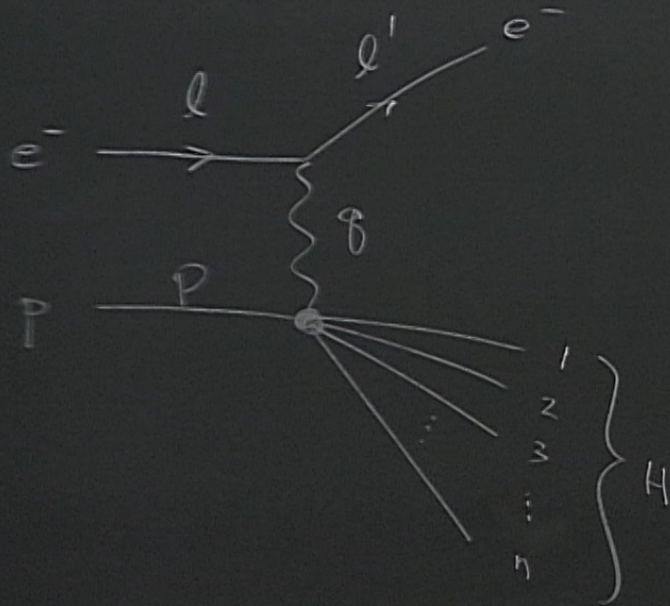
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Kinematic variables:

$$\text{Inelastic: } x + \{y, \nu, Q^2\}$$

$$\text{Elastic: } x=1 + \{y, \nu, Q^2\}$$

$$\sigma(e^- p \rightarrow e^- X) = \frac{1}{2E_e |\nu_{rel}|} (2\pi) \int \frac{d^3 \ell'}{(2\pi)^3} \frac{1}{2E_{\ell'}} \frac{e^4}{q^4} \underbrace{\frac{1}{2} \text{Tr}[\ell' \gamma^\mu \ell \gamma^\nu]}_{L^{\mu\nu} \text{ (leptonic tensor)}} W_{\mu\nu}$$

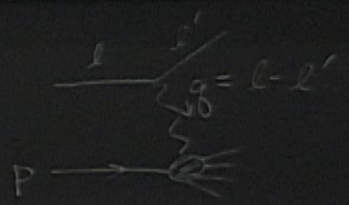
Hadronic tensor $W_{\mu\nu}$: use Lorentz invariance + Parity conservation.

$$\eta_{\mu\nu}, \quad 1$$

$$\int_0^4$$

Elastic: $\omega=1 + \{q, \nu, q^2\}$

$$\sigma(e^- p \rightarrow e^- X) = \frac{1}{2E_e |v_{rel}|} (2\pi) \int \frac{d^3 l'}{(2\pi)^3 2E_{l'}} \frac{e^4}{q^4} \underbrace{\frac{1}{2} \text{Tr}[\not{\epsilon}' \gamma^\mu \not{\epsilon} \gamma^\nu]}_{L^{\mu\nu} \text{ (leptonic tensor)}} W_{\mu\nu}$$



Hadronic tensor $W_{\mu\nu}$: use Lorentz invariance + Parity conservation.

$$\underbrace{\eta_{\mu\nu}, P^\mu P^\nu, \cancel{q^\mu q^\nu}, \cancel{P^\mu q^\nu}, \cancel{P^\nu q^\mu}}_{\text{Vanish contracted with } L^{\mu\nu}}, \quad \underbrace{\epsilon_{\mu\nu\alpha\beta} P^\alpha q^\beta}_{\text{P-violating}}$$

$$W_{\mu\nu} = -W_1 \eta_{\mu\nu} + W_2 \frac{P_\mu P_\nu}{m_p^2}$$

$W_{1,2}$ Lorentz scalars, only depend on other Lorentz scalars $(q^2, P \cdot q)$, or $(x, Q^2) \dots$
 By convention: define two other functions

$$F_1^e(x, Q^2) = 2mp W_1(x, Q^2)$$

$$F_2^e(x, Q^2) = \nu W_2(x, Q^2)$$

$W_{1,2}$ Lorentz scalars, only depend on other Lorentz scalars ($q^2, P \cdot q$), or $(x, Q^2) \dots$

By convention: define two other functions $F_1^e(x, Q^2) = 2m_p W_1(x, Q^2)$

$$F_2^e(x, Q^2) = \nu W_2(x, Q^2)$$

Work in "lab frame": $P = (m_p, 0, 0, 0)$ proton at rest $|v_{rel}| = 1$

High energy: $E, E', \nu \gg m_p$

$$S = (P + l)^2 = m_p^2 + 2m_p E_l \approx 2m_p E_l$$

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High energy: $E_0, E_0', \nu \gg m_p$

$$S = (P + l)^2 = m_p^2 + 2m_p E_l \approx 2m_p E_l$$

$$L_{\mu\nu} W^{\mu\nu} = 2(l^\mu l'^\nu + l^\nu l'^\mu - \eta^{\mu\nu} l \cdot l') \left(-W_1 \eta^{\mu\nu} + W_2 \frac{P^\mu P^\nu}{m_p^2} \right)$$

$$l^\mu = (E_l, 0, 0, E_l)$$

$$l'^\mu = (E_{l'}, \sin\theta E_{l'}, 0, \cos\theta E_{l'})$$

$\theta = \text{scattering angle}$

$$L_{\mu\nu} W^{\mu\nu} = 2 (l^\mu l'^\nu + l'^\mu l^\nu - \eta^{\mu\nu} l \cdot l') \left(-W_1 \eta^{\mu\nu} + W_2 \frac{p^\mu p^\nu}{m_p^2} \right)$$

$$= 2 E_e E_{e'} \left(\underbrace{2 W_1 (1 - \cos \theta)}_{\substack{\theta \text{ large} \\ W_1 \text{ term dominates}}} + \underbrace{W_2 (1 + \cos \theta)}_{\substack{\theta \text{ small} \\ W_2 \text{ term dominates}}} \right)$$

$$= 4 m_p E_e y x W_1 + 4 E_e^2 (1-y) W_2$$

$$l^\mu = (E_l, 0, 0, E_l)$$

$$l'^\mu = (E_{l'}, \sin\theta E_{l'}, 0, \cos\theta E_{l'})$$

$\theta = \text{scattering angle.}$

$$\int \frac{d^3 \ell'}{(2\pi)^3} \frac{1}{2E_{\ell'}} = \underbrace{\frac{1}{(2\pi)^2} \frac{1}{2} \int E_{\ell'} dE_{\ell'} d\cos\theta}_{\int dx dy m_p \nu} = \frac{1}{8\pi^2} m_p \int dx dy \nu$$

$$e^2 = 4\pi\alpha$$

$$\sigma(\bar{e}p \rightarrow \bar{e}X) = \int dx dy \frac{2\pi\alpha^2}{Q^4} \left[s x y^2 (2m_p W_1) + 2s(1-y)(\nu W_2) \right]$$

$$\frac{d^2\sigma(\bar{e}p \rightarrow \bar{e}\lambda)}{dx dy} = \frac{2\pi\alpha^2}{Q^4} \left[F_1^e(x, Q^2) x y^2 + 2 F_2^e(x, Q^2) (1-y) \right]$$

Inelastic scattering depends on unknown functions $F_{1,2}^e$
of 2 kinematic variables x, Q^2 .

$$\frac{d^2\sigma(e^-p \rightarrow e^-X)}{dx dy} = \frac{2\pi\alpha^2}{Q^4} \left[F_1^e(x, Q^2) x y^2 + 2 F_2^e(x, Q^2) (1-y) \right]$$

Inelastic scattering depends on unknown functions $F_{1,2}^e$ of 2 kinematic variables x, Q^2 .

Repeat calculation using quarks & factorization

$$\frac{d^2\sigma(e^-p \rightarrow e^-X)}{dx dy} = \frac{2\pi\alpha^2 S}{Q^4} \left[F_1^e(x, Q^2) xy^2 + 2 F_2^e(x, Q^2) (1-y) \right]$$

Incl

Repeat calculation using quarks & factorization

$$\sigma(e^-p \rightarrow e^-H) = \underbrace{\text{Prob}(q, p)}_{\substack{\text{Probability of finding} \\ \text{quark } q \text{ in proton} \\ \text{with momentum } p}} \times \underbrace{\hat{\sigma}(e^-q \rightarrow e^-q)}_{\substack{\text{cross section for} \\ \text{scattering off a} \\ \text{quark with mom. } p}} \times \underbrace{\text{Prob}(H)}_{\substack{\text{prob to hadronize} \\ \text{into final state } H}}$$

$$\frac{d^2\sigma(e^-p \rightarrow e^-X)}{dx dy} = \frac{2\pi\alpha^2}{Q^4} \left[F_1^e(x, Q^2) xy^2 + 2 F_2^e(x, Q^2) (1-y) \right]$$

Repeat calculation using quarks & factorization

$$\sigma(e^-p \rightarrow e^-H) = \underbrace{\text{Prob}(q, p)}_{\substack{\text{Probability of finding} \\ \text{quark } q \text{ in proton} \\ \text{with momentum } p \\ \text{large scales } \sim \text{fm}}} \times \underbrace{\hat{\sigma}(e^-q \rightarrow e^-q)}_{\substack{\text{cross section for} \\ \text{scattering off a} \\ \text{quark with mom } p \\ \text{small scale } \lambda \ll \text{fm}}} \times \underbrace{\text{Prob}(H)}_{\substack{\text{prob to hadronize} \\ \text{into final state } H \\ \text{large dist. } \sim \text{fm}}}$$

large scales $\sim \text{fm}$

small scale $\lambda \ll \text{fm}$

large dist. $\sim \text{fm}$

Inclusive cross section:

$$\sigma(e^- p \rightarrow e^- X) = \sum_H \sum_{q, P} \text{Prob}(q, P) \hat{\sigma}(e^- q \rightarrow e^- q) \text{Prob}(H)$$

$$= \sum_{q, P} \text{Prob}(q, P) \hat{\sigma}(e^- q \rightarrow e^- q) \underbrace{\sum_H \text{Prob}(H)}_{=1}$$

since must hadronize
by confinement.

$$\int \frac{d^3 \ell'}{(2\pi)^3} \frac{1}{2E_{\ell'}} = \frac{1}{(2\pi)^2} \frac{1}{2} \int E_{\ell'} dE_{\ell'} d\cos\theta = \frac{1}{8\pi^2} m_p \int dx dy \nu$$

$\int dx dy m_p \nu$

$$e^2 = 4\pi\alpha$$

$$\sigma(\bar{e} p \rightarrow \bar{e} X) = \int dx dy \frac{2\pi\alpha^2}{Q^4} \left[s x y^2 \underbrace{\left(2m_p W_1 \right)}_{F_1^e} + 2s(1-y) \underbrace{(\nu W_2)}_{F_2^e} \right]$$

Parton model

Predates QCD, but consistent with QCD

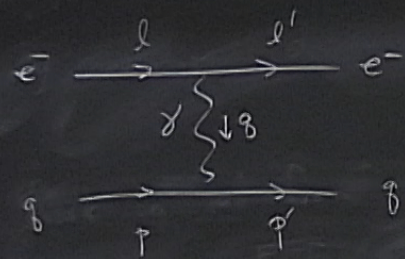
Partons are constituents of hadrons
i.e. quarks & gluons.

EM scattering: partons = quarks.

hats denote parton-level quantities e.g. $\frac{1}{G}$.

Parton-level cross section $\frac{1}{\sigma}$ Short-distance physics $\rightarrow$ perturbation theory (Zeroth order in α_s)

Assume quarks are massless.



$$i\mathcal{M}(\bar{e}q \rightarrow \bar{e}q) = \bar{u}(l')(ie\gamma^\mu)u(l) \frac{-i}{q^2} \bar{u}(p')(-ieQ_q\gamma_\mu)u(p)$$

$$\frac{1}{4} \sum |i\mathcal{M}(\bar{e}q \rightarrow \bar{e}q)|^2 = \frac{1}{4} \frac{e^4}{q^4} Q_q^2 \text{Tr}[\not{l}'\gamma^\mu\not{l}\gamma^\nu] \text{Tr}[\not{p}'\gamma_\mu\not{p}\gamma_\nu]$$

Mandelstam variables (parton)

$$\hat{s} = (l+p)^2 = (l'+p')^2 = 2l \cdot p = 2l' \cdot p'$$

$$\hat{t} = (l-l')^2 = (p-p')^2 = q^2$$

$$\hat{u} = (l-p')^2 = (l'-p)^2 = -2l \cdot p' = -2l' \cdot p$$

$$\hat{s} + \hat{t} + \hat{u} = 0$$

$$\frac{1}{4} \sum |\mathcal{M}(\bar{e}q \rightarrow \bar{e}q)|^2 = \frac{2e^4 Q_q^2}{\hat{t}^2} (\hat{s}^2 + \hat{u}^2)$$

Parton-level cross section. CM frame.

$$\hat{\sigma}(eq \rightarrow eq) = \frac{1}{2E_l 2E_p |\mathbf{v}_{rel}|} \int \frac{d^3 \mathbf{l}'}{(2\pi)^3} \frac{1}{2E_{l'}} \int \frac{d^3 \mathbf{p}'}{(2\pi)^3} \frac{1}{2E_{p'}} (2\pi)^4 \delta^4(l+p-l'-p') \frac{1}{4} \sum |M|^2$$

$$= \frac{1}{32\pi \hat{s}} \int d\cos\theta \frac{1}{4} \sum |M|^2$$

$$\frac{d\hat{\sigma}}{d\cos\theta} = \frac{\pi \alpha^2 Q_q^2}{\hat{s} \hat{t}} \left(\frac{\hat{s}^2 + \hat{u}^2}{\hat{s}^2} \right)$$

$$\hat{t} = (l - l')^2 = -2l \cdot l' = -2E_l E_{l'} (1 - \cos\theta)$$

$$d\hat{t} = 2E_l E_{l'} d\cos\theta = \frac{1}{2} \hat{s} d\cos\theta$$

$$\frac{d\hat{\sigma}}{d\hat{t}} = \frac{2\pi \alpha^2 Q_q^2}{\hat{s} \hat{t}} \left(\frac{\hat{s}^2 + \hat{t}^2}{\hat{s} + \hat{t}} \right)$$

Go parton-level $\rightarrow$ hadron-level

Photon momentum: $q^2 = \hat{t} = -Q^2$ (same)

p = 4-momentum of initial quark

P = 4-momentum of initial proton

"Struck" quark carries fraction x of total proton momentum

$$\hat{p} = x P \quad 0 < x < 1$$

Solve for x : $m_q = 0$, $p^2 = p'^2 = 0$

$$0 = p'^2 = (p + \underbrace{l - l'}_q)^2 = p \cdot q + q^2 = x P \cdot q + q^2$$

$$\Rightarrow x = -\frac{q^2}{2P \cdot q} \quad \text{Same as Bjorken } x.$$

Relate $\hat{s}$ (CM energy squared for parton level) to S (CM energy squared at hadron level)

$$\hat{s} = (p + l)^2 = 2p \cdot l = 2x P \cdot l = x S$$

$$y = \frac{P \cdot q}{P \cdot l} = \frac{x P \cdot q}{x P \cdot l} = - \frac{\hat{t}}{\hat{s}} \Rightarrow \hat{t} = -xy s$$

$$\hat{u} = -\hat{s} - \hat{t} = -s x(1-y)$$