Title: PSI 2018/2019 - Standard Model Review - Lecture 6

Date: Feb 04, 2019 09:00 AM

URL: http://pirsa.org/19020002

Abstract:

2) Spinors
3) Spin 1

(3) $a_{1,2}$ Maxwell

III GR SM

CC interaction ($W^{\pm}$):

$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}}\left(\bar{u}_L^i V_{ij} \not{W}^+ d_L^j + \bar{\nu}_L^i U_{ij} \not{W}^+ \right.$$

Effective theory for weak interaction.

Weak processes with energy $E \ll M_{W,Z}$ we can integrate out

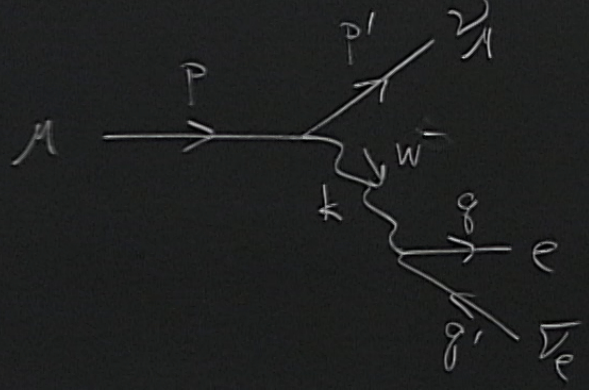
$\bar{\nu}_L^i U_{ij} \not{W}^+ e_L^j$) + h.c.

$U =$ PMNS

can integrate out heavy degrees of freedom to get an effective Lagrangian.

ex. μ-decay

$$i\mathcal{M}(\mu \to \nu_\mu e \bar{\nu}_e) = \bar{u}_e \left(\frac{-ig}{\sqrt{2}}\right) \gamma^\mu P_L V_{\nu e} \frac{-i}{k^2 - m}$$



Consider $k_\mu k_\nu$ term &

$$\bar{u}_e \not{k} P_L V_{\nu e} = \bar{u}_e (\not{q}$$

$$\bar{u}_e \left(\frac{-ig}{\sqrt{2}}\right) \gamma^\mu P_L V_{\nu_e} \quad \frac{-i}{k^2 - m_N^2} \left(\eta_{\mu\nu} - \frac{k_\mu k_\nu}{m_W^2}\right) \bar{u}_{\nu_\mu} \left(\frac{-ig}{\sqrt{2}}\right) \gamma^\nu P_L u_\mu$$

Consider $k_\mu k_\nu$ term & use Dirac eqn.

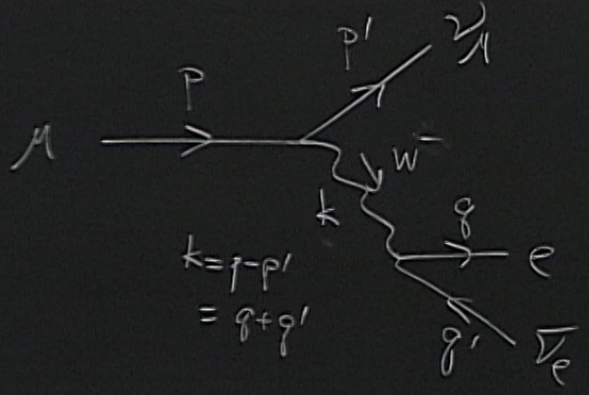
$$\bar{u}_e \not{k} P_L V_{\nu_e} = \bar{u}_e (\not{q} + \not{q}') P_L V_{\nu_e} = m_e \bar{u}_e P_L V_{\nu_e}$$

$$\bar{u}_{\nu_\mu} \not{k} P_L u_\mu = m_\mu \bar{u}_{\nu_\mu} P_R u_\mu$$

$$\left(\frac{m_e m_\mu}{m_W^2}\right) \sim 10^{-8}$$

ex. μ-decay

$$i\mathcal{M}(\mu \to \nu_\mu e \bar{\nu}_e) = \bar{u}_e \left(\frac{-ig}{\sqrt{2}}\right) \gamma^\mu P_L V_{\nu_e} \quad \frac{\overline{}}{k^2}$$



Consider $k_\mu k_\nu$ term &

$\bar{u}_e \not{k} P_L V_{\nu_e} = \bar{u}_e$

$\bar{u}_{\nu_\mu} \not{k} P_L u_\mu = m_\mu$

$\left(\frac{m_e m_\mu}{m_W^2}\right) \sim 10^{-8}$

$$\frac{-i}{k^2 - m_W^2}\left(\eta_{\mu\nu} - \frac{k_\mu k_\nu}{m_W^2}\right)\bar{u}_{\nu_\mu}\left(\frac{-ig}{\sqrt{2}}\right)\gamma^\nu P_L\, u_{\mu}$$

$\not{p}\, u(p) = m\, u(p)$

use Dirac eqn.

$$\bar{u}_{\nu_e}(\not{q} + \not{q}')P_L\, v_{\nu_e} = \bar{u}_{\nu_e}\not{q}\, P_L\, v_{\nu_e} + \bar{u}_{\nu_e} P_R \not{q}'\, v_{\nu_e}$$

$m_\mu\, \bar{u}_{\nu_\mu} P_R\, u_\mu$

$-\varepsilon$

0

$$i\mathcal{M}(\mu \to \nu_\mu e \bar{\nu}_e) = \frac{ig^2}{8m_W^2}\, \bar{u}_e \gamma^\alpha (1-\gamma^5) v_{\nu_e}\; \bar{u}_{\nu_\mu} \gamma_\alpha (1-\gamma^5) u_\mu$$

This can be expressed by operator:

$$\mathcal{L}_{eff} = \frac{G_F}{\sqrt{2}}\, \bar{e} \gamma^\alpha (1-\gamma^5) \nu_e\; \bar{\nu}_\mu \gamma_\alpha (1-\gamma^5) \mu + h.c.$$

G_F = Fermi's constant.

where $\frac{G_F}{\sqrt{2}} = \frac{g^2}{8 m_W^2}$

V_{ke} $\bar{u}_{\nu_e} \gamma_\alpha (1-\gamma^5) u_\mu$ ($m_W \gg$ all momentum scales i.e. k)

$\bar{\nu}_\mu \gamma_\alpha (1-\gamma^5) \mu + h.c.$

G_F = Fermi's constant. $\Rightarrow \frac{G_F}{\sqrt{2}} = \frac{1}{2v^2}$

$= \frac{gv}{2}$

vev $v = (\sqrt{2} G_F)^{-1/2} \simeq 246$ GeV fixed by Fermi's constant.

$H = \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix}$ $\xrightarrow{\text{alternative convention}}$ $H = \begin{pmatrix} 0 \\ v + \frac{h}{\sqrt{2}} \end{pmatrix}$, $v \simeq 174$ GeV.

Current interactions described by (at low-energy):

$$\mathcal{L}_{CC\text{-eff}} = \sum_{ij} \frac{G_F}{\sqrt{2}} \bar{e}^i \gamma^\mu (1-\gamma^5) \nu^i \, \bar{\nu}^j \gamma_\mu (1-\gamma^5) e^j + h.c. \quad \text{(leptonic)}$$

$$+ \frac{G_F}{\sqrt{2}} \sum_{ijk} V_{ij} \, \bar{u}^i \gamma^\mu (1-\gamma^5) d^j \, \bar{e}^k \gamma_\mu (1-\gamma^5) \nu^k + h.c. \quad \text{(semileptonic)}$$

$$+ \frac{G_F}{\sqrt{2}} \sum_{ijkl} V_{ij} V_{kl}^* \, \bar{u}^i \gamma^\mu (1-\gamma^5) d^j \, \bar{d}^l \gamma_\mu (1-\gamma^5) u^k + h.c. \quad \text{(hadronic)}$$

$\sqrt{2}\ i,j,k,l$

All couplings involve same G_F + CKM matrix.

Consequence of $SU(2)_L \times U(1)_Y$ gauge theory (g is universal gauge coupling)

3) spin $\frac{1}{2}$

4) $A^\mu = 4 \times 4$

Weak decay of hadrons e.g. Charged pions

$\pi^+ = (u\bar{d})$

$\pi^- = (d\bar{u})$

$J^P = 0^-$ pseudoscalar mesons

$m_{\pi^\pm} \approx 140$ MeV

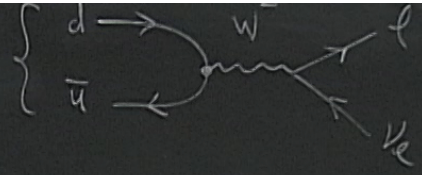
Parity $P = -$

Spin $J = 0$

Matrix element: $(l = e, \mu)$

$$i\mathcal{M}(\pi^- \to l^- \bar{\nu}_l) = \pi^- \{ d \to W^- \to l, \ \bar{u} \to \nu_l$$

$$(\pi^- \to \ell^- \bar{\nu}_\ell) = \pi^-$$



$$= \langle \ell(p_1)\, \bar{\nu}_\ell(p_2) \,|\, i \int d^4x \frac{G_F}{\sqrt{2}} V_{ud}\, \bar{\ell}\, \gamma_\mu (1-\gamma^5) \nu_\ell\, \bar{u}\, \gamma_\mu (1-\gamma^5) d \,|\, \pi^-(q) \rangle$$

$$= i \frac{G_F}{\sqrt{2}} V_{ud} \int d^4x\, e^{i(p_1+p_2)\cdot x}\, \bar{u}_\ell(p_1)\, \gamma_\mu (1-\gamma^5) v_{\nu_\ell}(p_2)\, \underbrace{\langle 0 \,|\, \bar{u}(x)\, \gamma_\mu (1-\gamma^5) d(x) \,|\, \pi^-(q) \rangle}$$

Unknown from 1st principles, can't solve nonperturbative dynamics of strong int at

$(1-\gamma^5)\nu_e \; \bar{u}\gamma_\mu(1-\gamma^5)d \; |\pi^-(q)\rangle$

$\gamma_\mu(1-\gamma^5)\nu_e(p_2) \; \underbrace{\langle 0|\bar{u}(x)\gamma_\mu(1-\gamma^5)d(x)|\pi^-(q)\rangle}$

Unknown from 1st principles, can't solve nonperturbative dynamics of strong int. at low energy

$(1-\gamma^5)\nu_\ell \; \bar{u}\gamma_\mu(1-\gamma^5)d \; |\pi^-(q)\rangle$

$\gamma_\mu(1-\gamma^5)\nu_\ell(p_2) \; \underbrace{\langle 0|\bar{u}(x)\gamma_\mu(1-\gamma^5)d(x)|\pi^-(q)\rangle}$

Unknown from 1st principles, can't solve nonperturbative dynamics of strong int. at low energy

Use symmetry

V $\langle 0 | \bar{u}(x) \gamma^\mu d(x) | \pi^-(q) \rangle = 0$ P invariance of

AV $\langle 0 | \bar{u}(x) \gamma^\mu \gamma^5 d(x) | \pi^-(q) \rangle = \underbrace{-i f_\pi}_{\text{overall coeff.}} q^\mu e^{-iq\cdot x}$

$\langle 0 | \bar{u}(x) \gamma^\mu \gamma^5 d(x) | \pi^-(q) \rangle = \langle 0 | e^{i\hat{P}\cdot x} \bar{u}(0) \gamma^\mu \gamma^5$

$e^{i\hat{P}\cdot x} e^{-i\hat{P}\cdot x}$ $\qquad$ $e^{i\hat{P}\cdot x} e^{-i\hat{P}\cdot x}$

...ance of strong interaction

$$\bar{u}(0)\gamma^{\mu}\gamma^{5}d(0)\, e^{-i\hat{P}\cdot x}\,|\pi^{-}(q)\rangle = \underbrace{\langle 0|\,\bar{u}(0)\gamma^{\mu}\gamma^{5}d(0)\,|\pi^{-}(q)\rangle}_{-if_{\pi}q^{\mu}}\, e^{-iq\cdot x}$$

$\hat{P} \downarrow q$

$$f_\pi(q^2) = f_\pi(m_\pi^2)$$

$$i\mathcal{M}(\pi^- \to \ell \bar{\nu}_\ell) = \frac{G_F V_{ud}}{\sqrt{2}} f_\pi \, \bar{u}_\ell(p_1) \not{q} (1-\gamma^5) V_{\nu_\ell}(p_2)$$

$$\Gamma(\pi^- \to \ell \bar{\nu}_\ell) = \frac{G_F^2 |V_{ud}|^2}{8\pi} f_\pi^2 \, m_\ell^2 \, m_{\pi^\pm} \left(1 - \frac{m_\ell^2}{m_{\pi^\pm}^2}\right)^2$$

Dirac eqn: $\bar{u}_\ell(p_1)(\not{p}_1 + \not{p}_2)(1-\gamma^5) V_{\nu_\ell}(p_2) = m_\ell \, \bar{u}_\ell(p_1)(1-\gamma^5) V_{\nu_\ell}(p_2)$

$$= \frac{G_F V_{ud}}{\sqrt{2}} f_\pi \, \bar{u}_\ell(p_1) \not{q} (1-\gamma^5) V_{\nu_\ell}(p_2) \qquad \left(\times (2\pi)^4 \delta^4(q - p_1 - p_2) \right)$$

$$\frac{G_F^2 |V_{ud}|^2}{8\pi} f_\pi^2 m_\ell^2 m_{\pi^\pm} \left(1 - \frac{m_\ell^2}{m_{\pi^\pm}^2}\right)^2 \qquad (m_\nu = 0)$$

$$(p_1 + p_2)(1-\gamma^5) V_{\nu_\ell}(p_2) = m_\ell \, \bar{u}_\ell(p_1)(1-\gamma^5) V_{\nu_\ell}(p_2)$$

Amplitude $\propto m_e \Rightarrow \Gamma(\pi^- \to \mu \bar{\nu}_\mu) >> \Gamma(\pi^- \to e^- \bar{\nu}_e)$

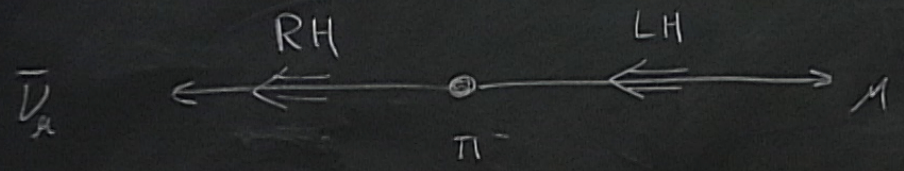
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Observed value of $\Gamma(\pi^- \to \mu \bar{\nu}_\mu)$ fixes $f_\pi \simeq 130$ MeV.

Experimental proof that weak CC interaction is maximally chiral.

$W^\pm$ only couples to LH particles & RH antiparticles.

Consider π^- decaying at rest.

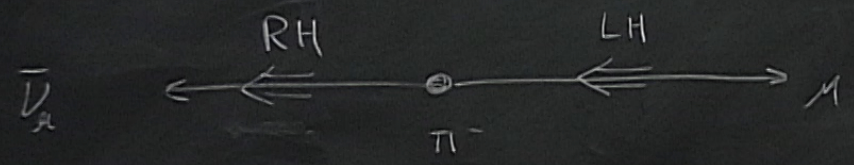


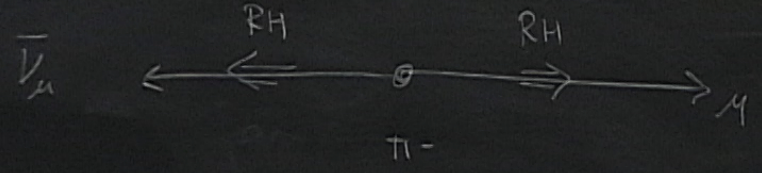
If all fermions were massless then chirality = helicity.

$\Rightarrow$ total spin = J=1

(initial state J=0)

Consider π^- decaying at rest.





If all fermions were massless then chirality = helicity.

$\Rightarrow$ total spin = J=1
(initial state J=0)

Allowed but must depend on mass for helicity-flip.

$\Rightarrow$ Amplitude $\propto m_\ell$

Additional parameter θ_{QCD} (expt. $\theta_{QCD} \lesssim 10^{-10}$)

Everything can be computed except

(1) strong interaction at low-energy

(2) Unsolved SM issues e.g. neutrino masses, dark matter etc.

Parameters of SM

3 gauge couplings g, g', g_s

2 Higgs parameters: μ^2, λ

} m_W, m_Z, v, m_h

9 Fermion mass: $m_e, m_\mu, m_\tau, m_u, m_c, m_t, m_d, m_s, m_b$

4 CKM matrix elements: V_{ij}

(19 parameters)

Addit

Every