Title: The first law of general quantum resource theories

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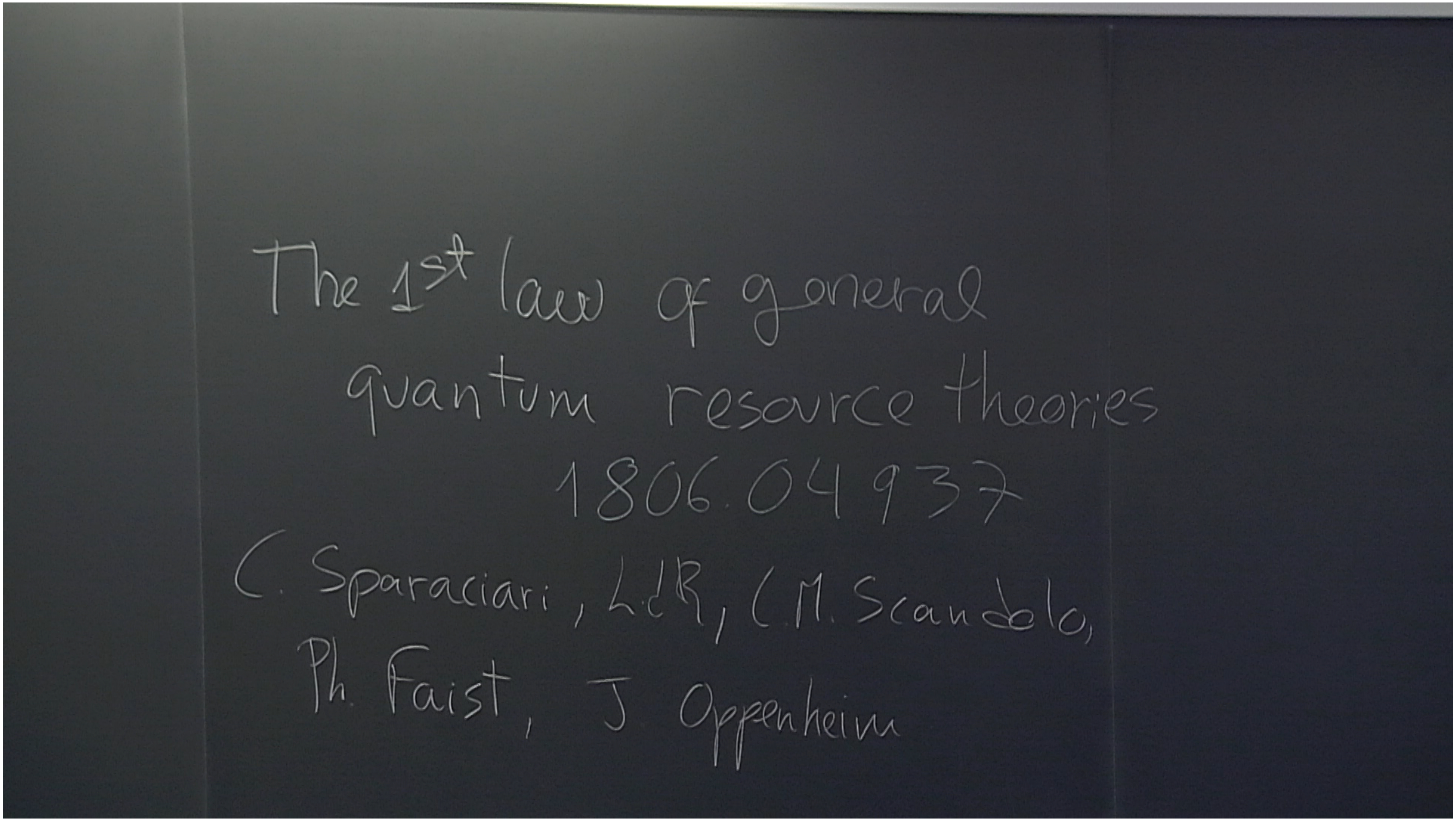
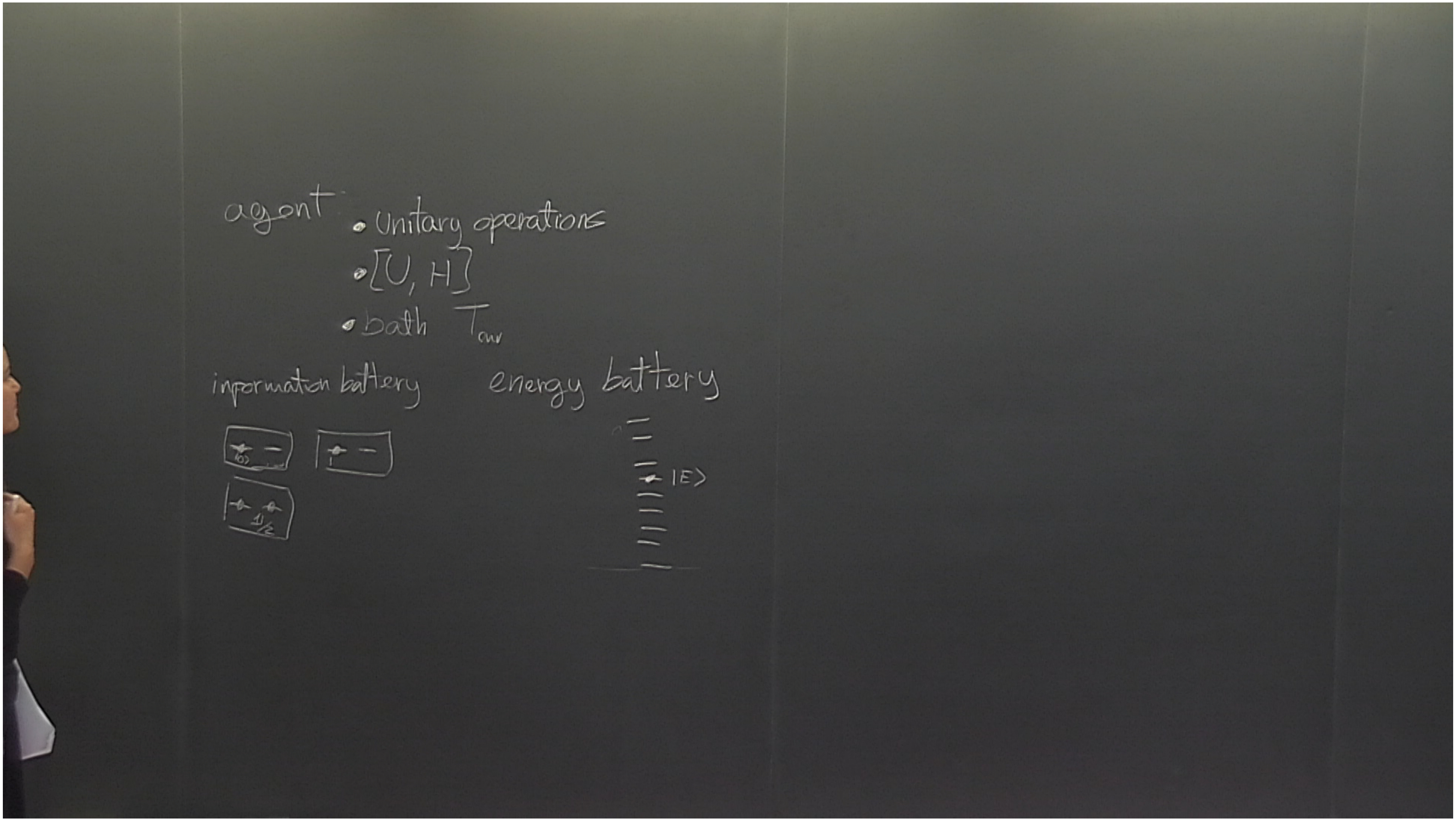
Abstract: <p>From arXiv: 1806.04937, with Carlo Sparaciari, Carlo Maria Scandolo, Philippe Faist and Jonathan Oppenheim</p>

<p>We extend the tools of quantum resource theories to scenarios in which multiple quantities (or resources) are present, and their interplay governs the evolution of the physical systems. We derive conditions for the interconversion of these resources, which generalise the first law of thermodynamics. We study reversibility conditions for multi-resource theories, and find that the relative entropy distances from the invariant sets of the theory play a fundamental role in the quantification of the resources. The first law for general multi-resource theories is a single relation which links the change in the properties of the system during a state transformation and the weighted sum of the resources exchanged. In fact, this law can be seen as relating the change in the relative entropy from different sets of states. In contrast to typical single-resource theories, the notion of free states and invariant sets of states become distinct in light of multiple constraints. Additionally, generalisations of the Helmholtz free energy, and of adiabatic and isothermal transformations, emerge. </p>


The 1st law of general
quantum resource theories
1806.04937
C. Sparaciari, L.d.R, C.M. Scandolo,
Ph. Faist, J. Oppenheim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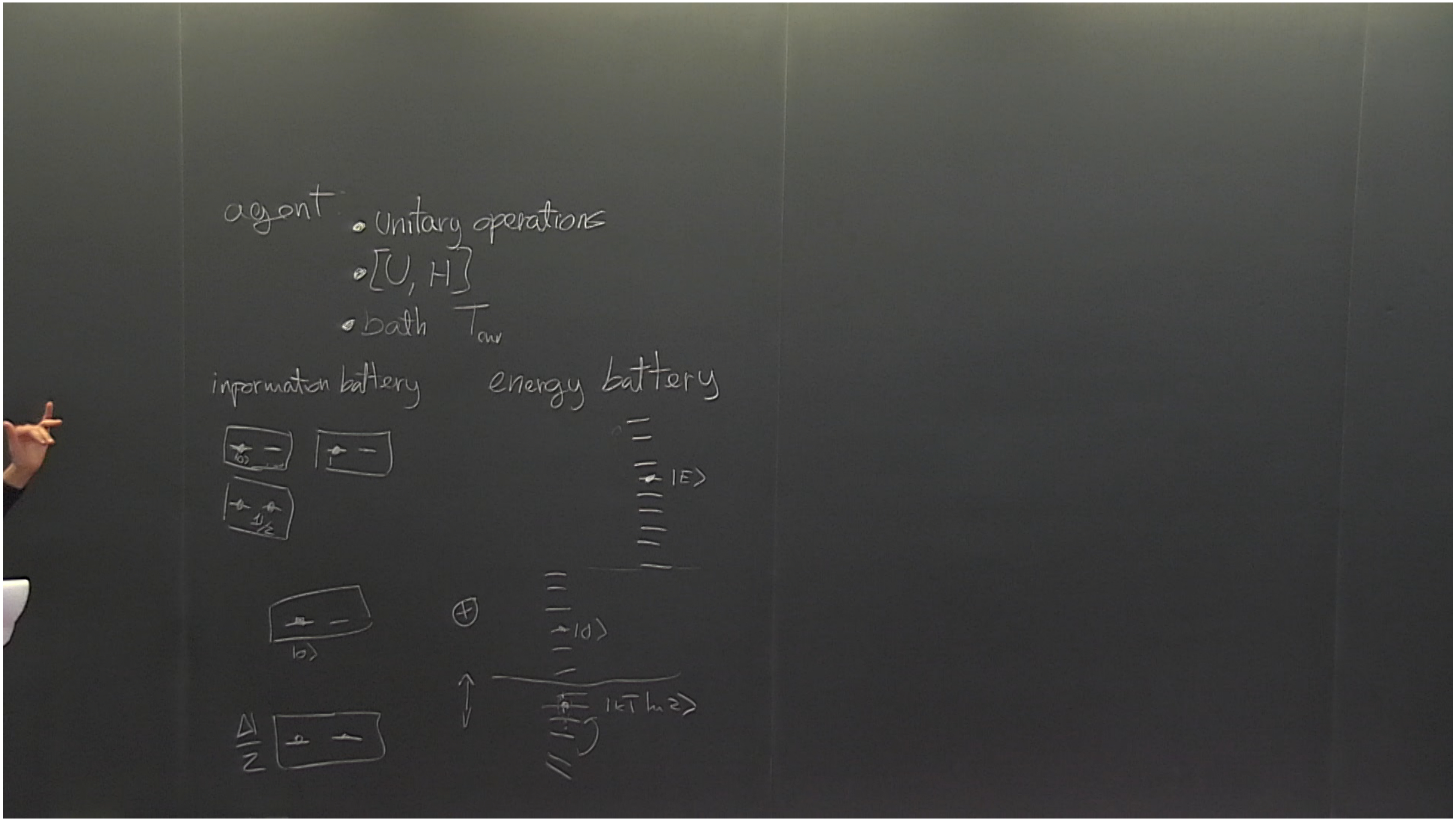
Unitary operations
[U, H]
bath T_env
information battery
energy battery
|0>
|E>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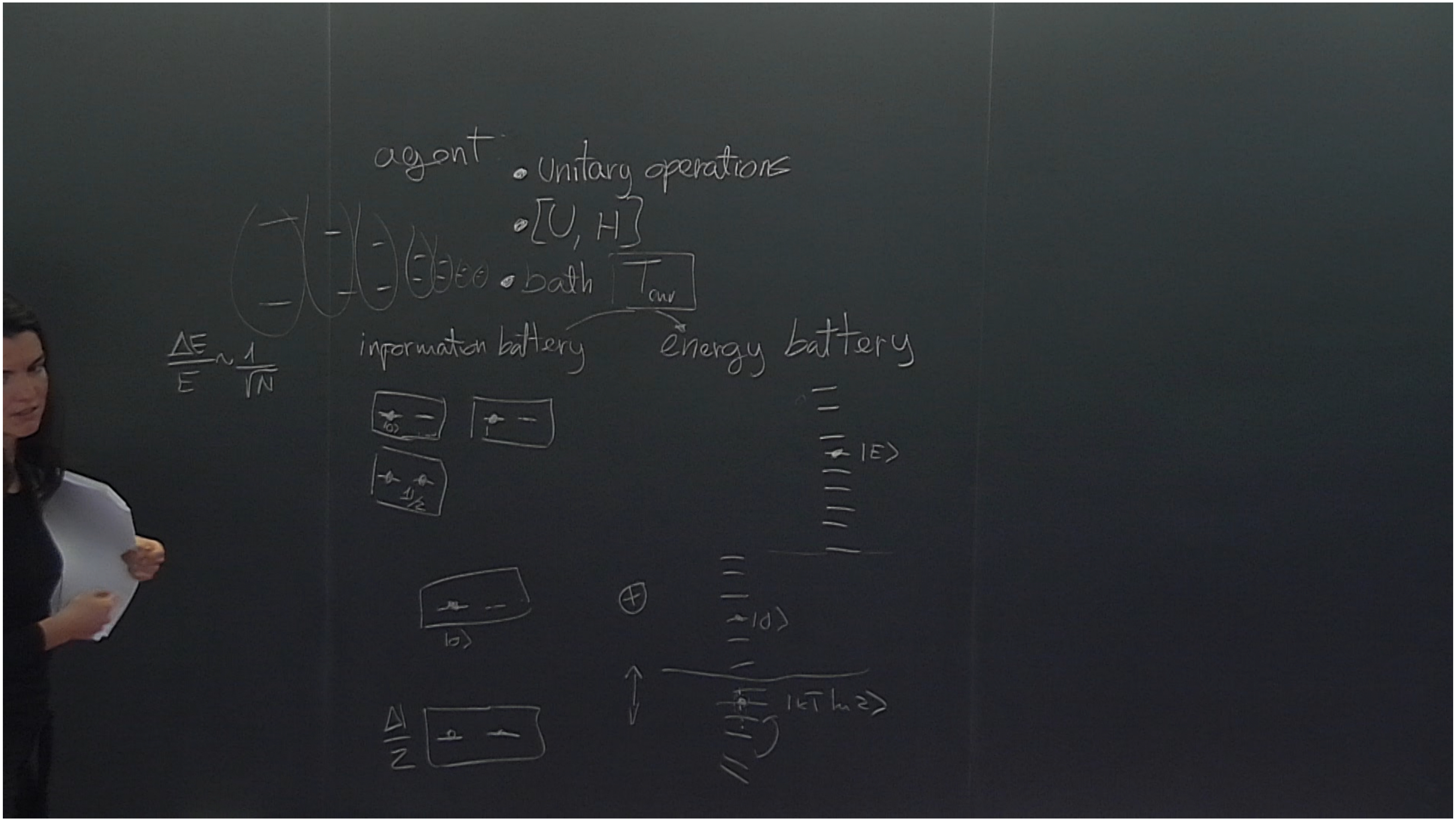

agent:
Unitary operations
[U, H]
bath
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ttery

$|E\rangle$

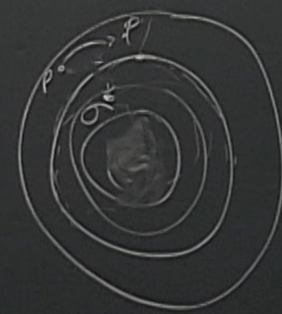
$|h_2\rangle$

Single resource theory R_i

allowed operations $\mathcal{A}_i$

States $S(\mathcal{H})$

- invariant sets: $\mathcal{I} = \{\sigma : \forall \mathcal{E} \in \mathcal{A}_i,\ \mathcal{E}(\sigma) \in \mathcal{I}\}$
- Free sets: $\mathcal{F}_i\ \{\sigma : \forall \rho, \exists \mathcal{E} \in \mathcal{A}_i,\ \mathcal{E}(\rho) = \sigma\}$



ttery

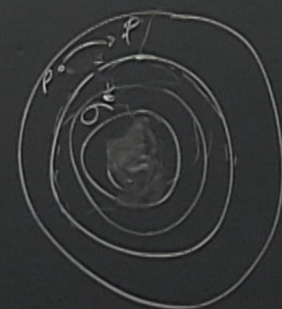
$|E\rangle$

Single resource theory R_i

allowed operations $\mathcal{A}_i$

States $S(\mathcal{H})$

- invariant sets: $\mathcal{I} = \{\sigma : \forall \mathcal{E} \in \mathcal{A}_i \;\; \mathcal{E}(\sigma) \in \mathcal{I}\}$
- Free sets. $\mathcal{F}_i$ $\{\sigma : \forall \rho, \exists \mathcal{E} \in \mathcal{A}_i, \;\; \mathcal{E}(\rho) = \sigma\}$



Monotones

$$F_i(\rho) \geq F_i(\mathcal{E}(\rho)), \; \forall \mathcal{E} \in \mathcal{A}$$

$$\rho \to \sigma \to F_i(\rho) \geq F_i(\sigma)$$

ory R_i

$\mathcal{A}_i$

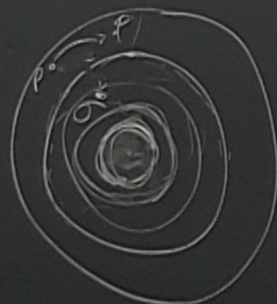
$S(\mathcal{H})$

$=\{\sigma : \forall \mathcal{E} \in \mathcal{A}_i,\ \mathcal{E}(\sigma) \in \mathcal{I}\}$

$\sigma : \forall \rho, \exists \mathcal{E} \in \mathcal{A}_i,\ \mathcal{E}(\rho) = \sigma\}$

$(\mathcal{E}(\rho)),\ \forall \mathcal{E} \in \mathcal{A}$

$f_i(\rho) \geq f_i(\sigma)$



$$d(\rho,\sigma) \geq d(\mathcal{E}(\rho), \mathcal{E}(\sigma)),\ \forall \mathcal{E} \in \mathcal{A}_i$$

$\hookrightarrow f_i(\rho) : \min_{\sigma \in F_i} d(\rho,\sigma)$ is a monotone

n copies $\mathcal{H}$, demand $F^{(n)} \supseteq F^{\otimes n}$

regularized monotone $f^{\infty}(\rho) = \lim_{n\to\infty} \frac{F(\rho^{\otimes n})}{n}$

eg: $f^{\infty}(\rho) = \min_{\sigma \in F_i} D(\rho \| \sigma)$

$\downarrow$

$-\mathrm{Tr}(\rho(\log \rho - \log \sigma))$

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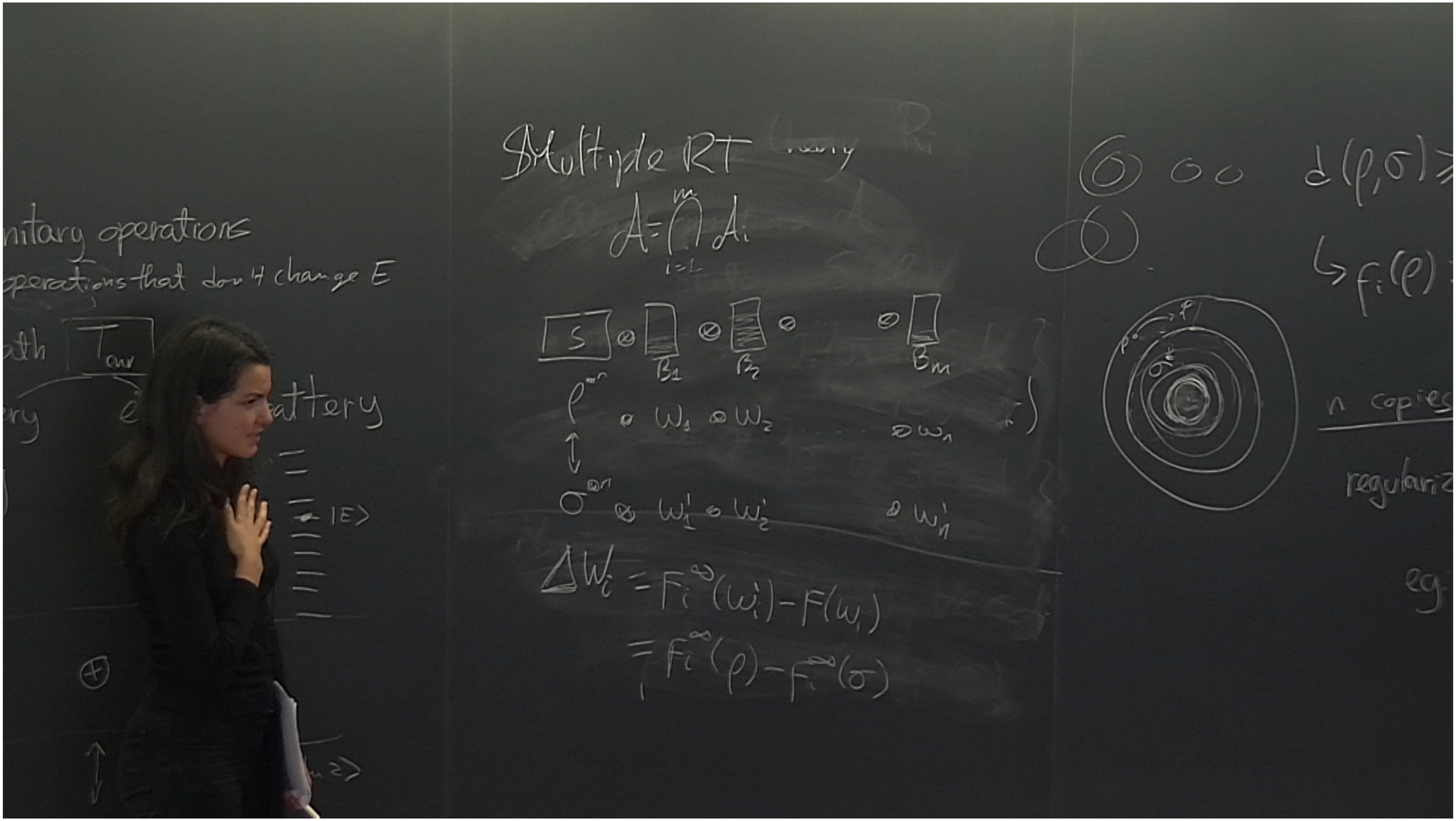
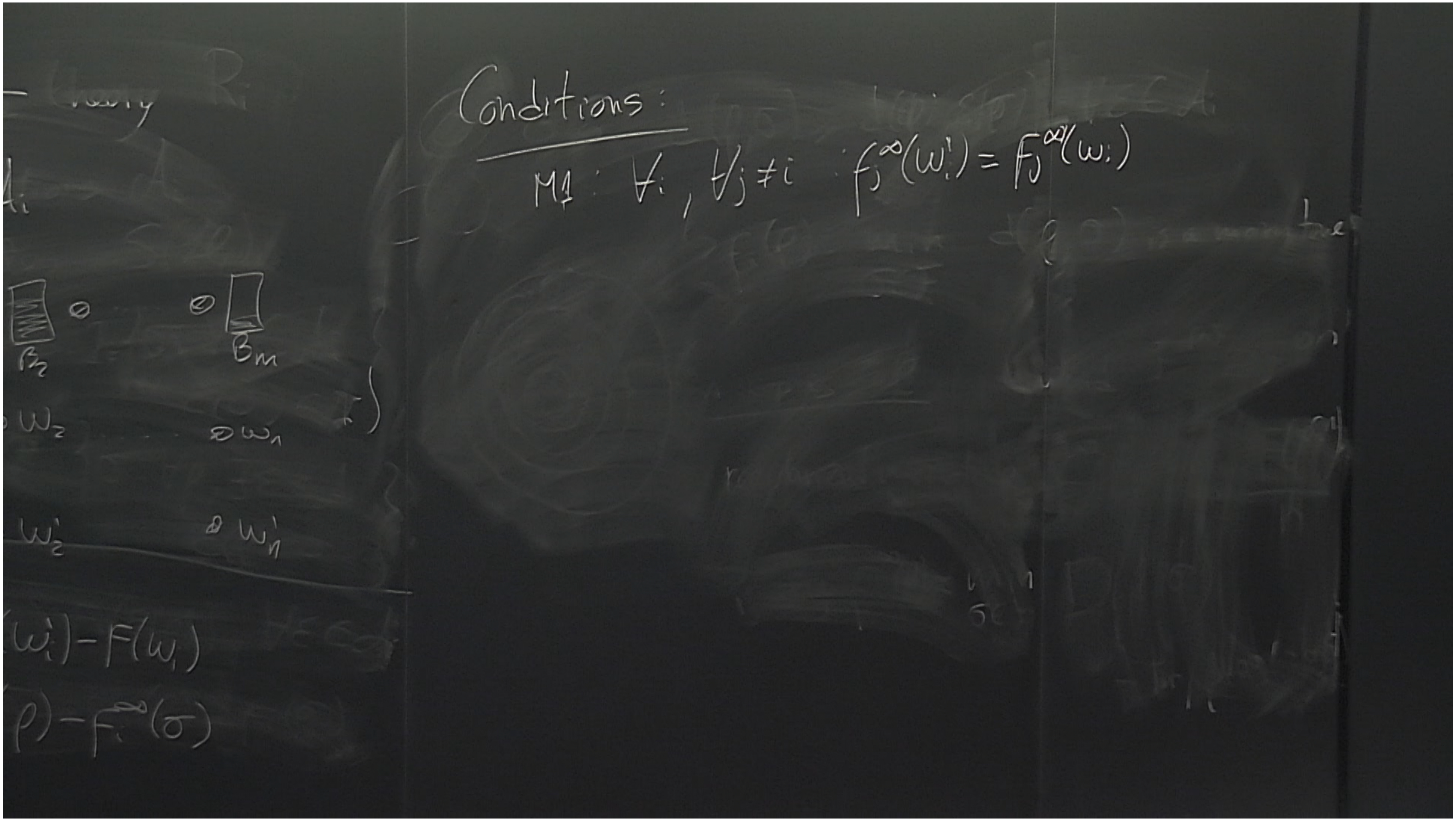
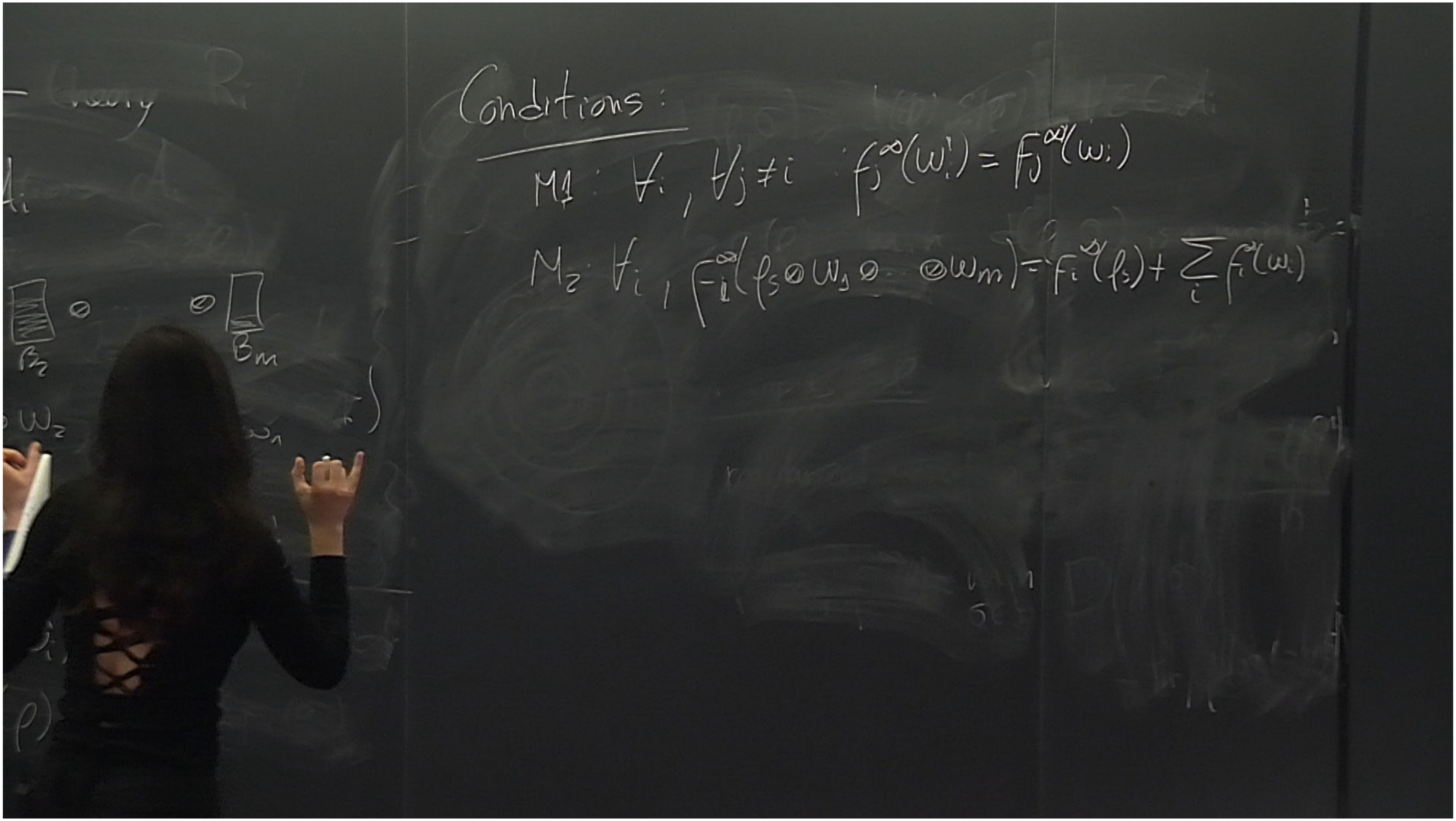
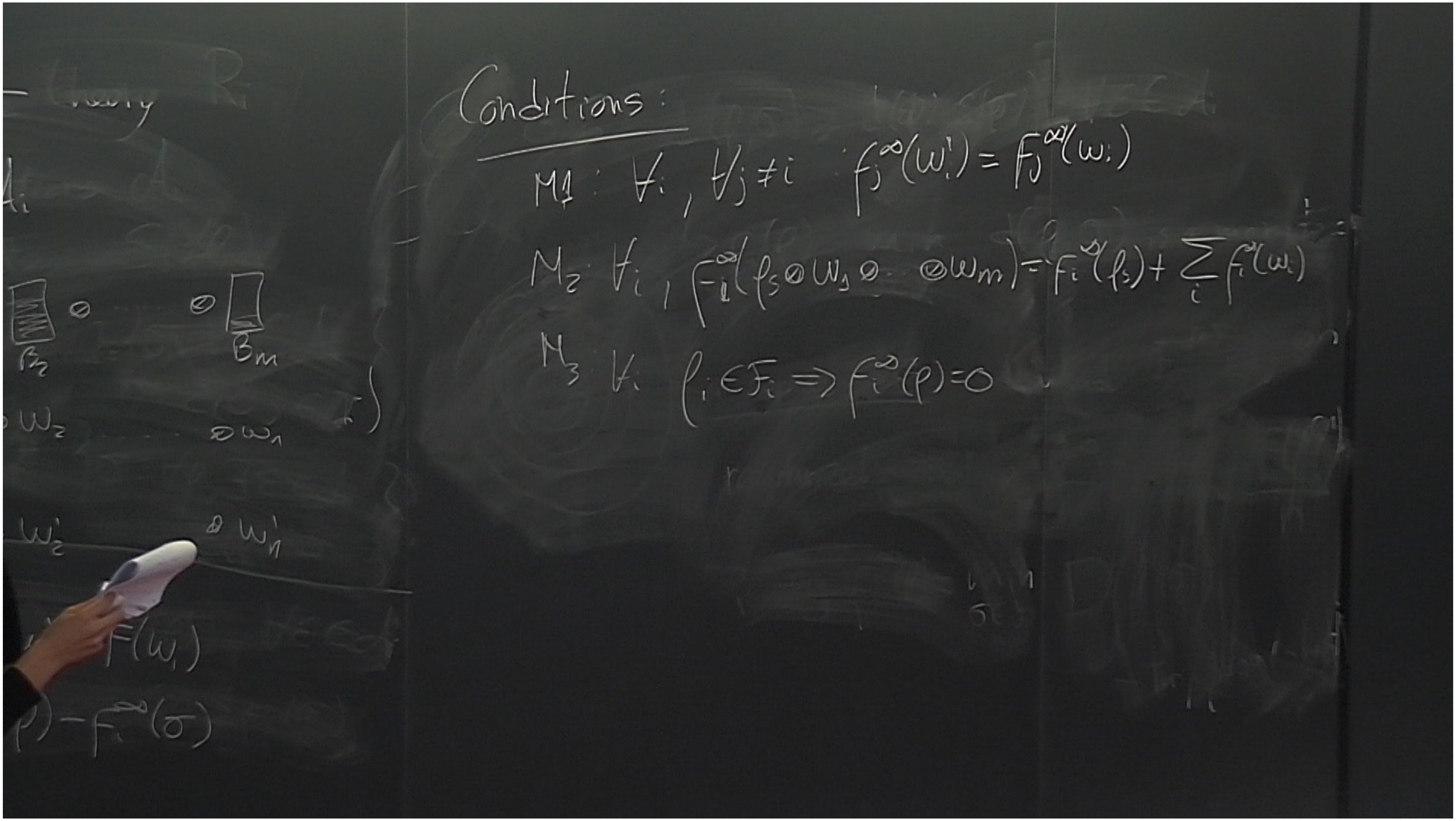
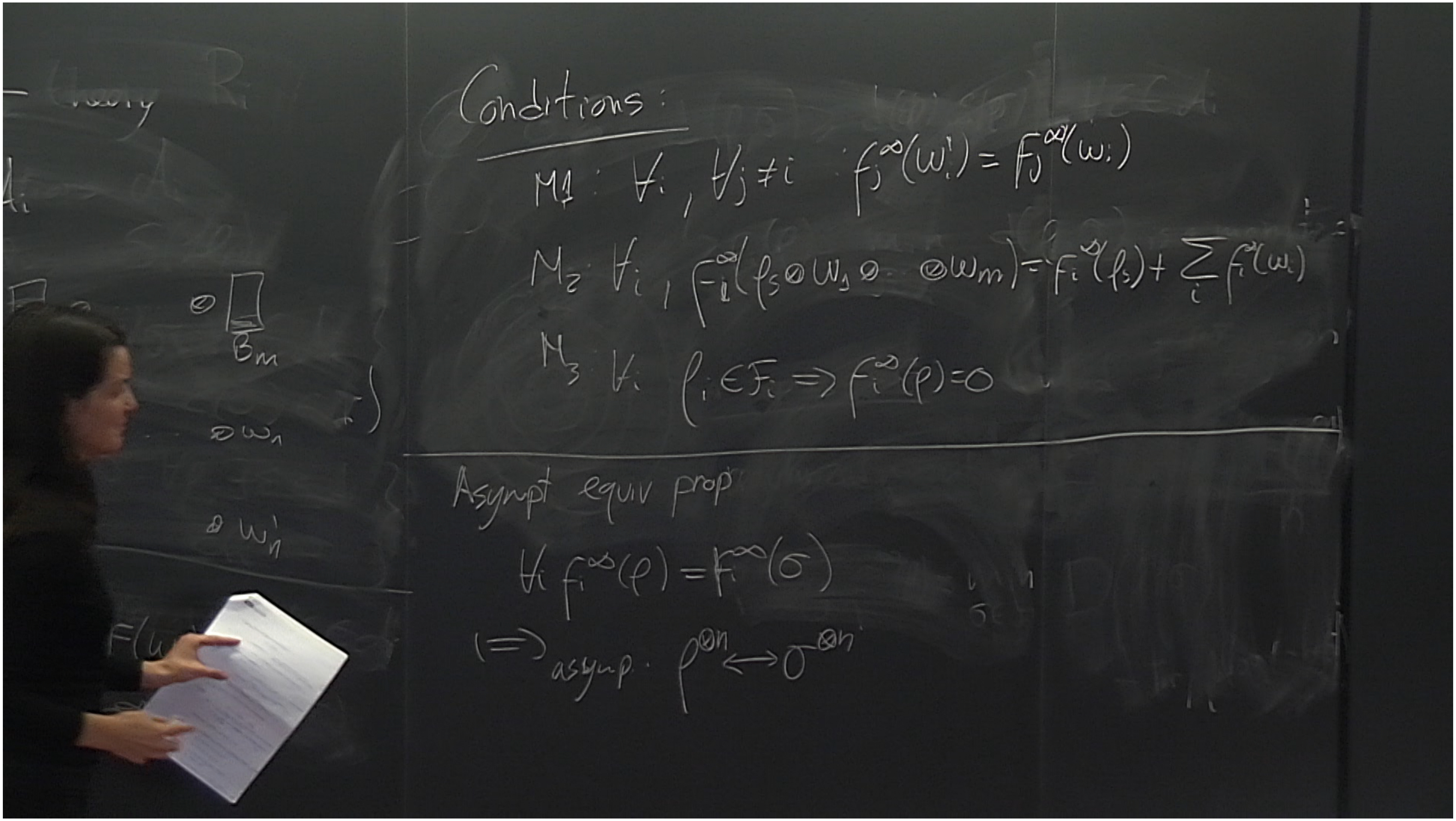
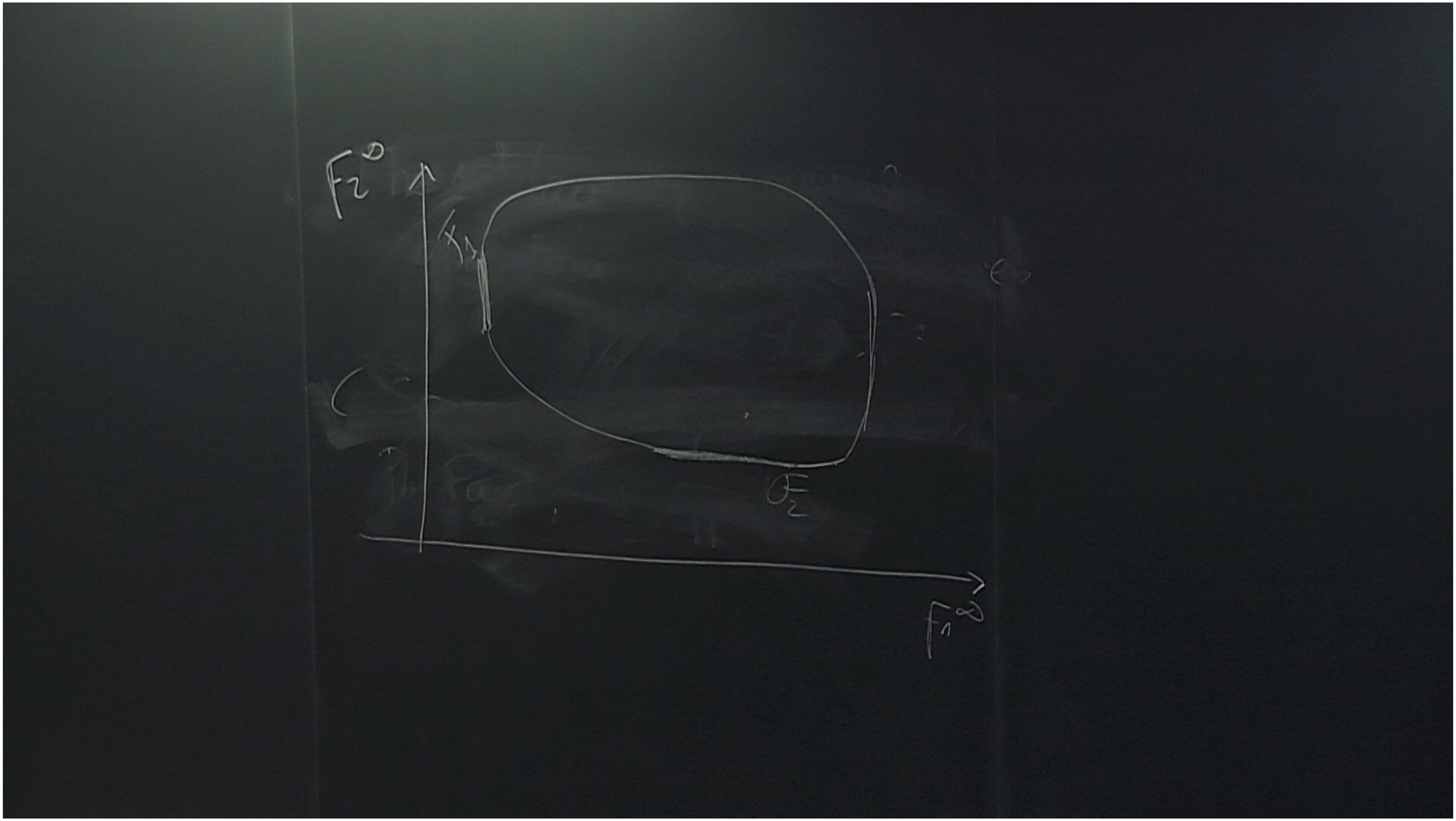

Multiple RT
nitary operations
operations that don't change E
attery
n copies
regulari


Conditions:
M1 : ∀i , ∀j≠i : F_j^∞(ω_i') = F_j^∞(ω_i)
B_2
B_m
ω_2
ω_n
ω_2'
ω_n'

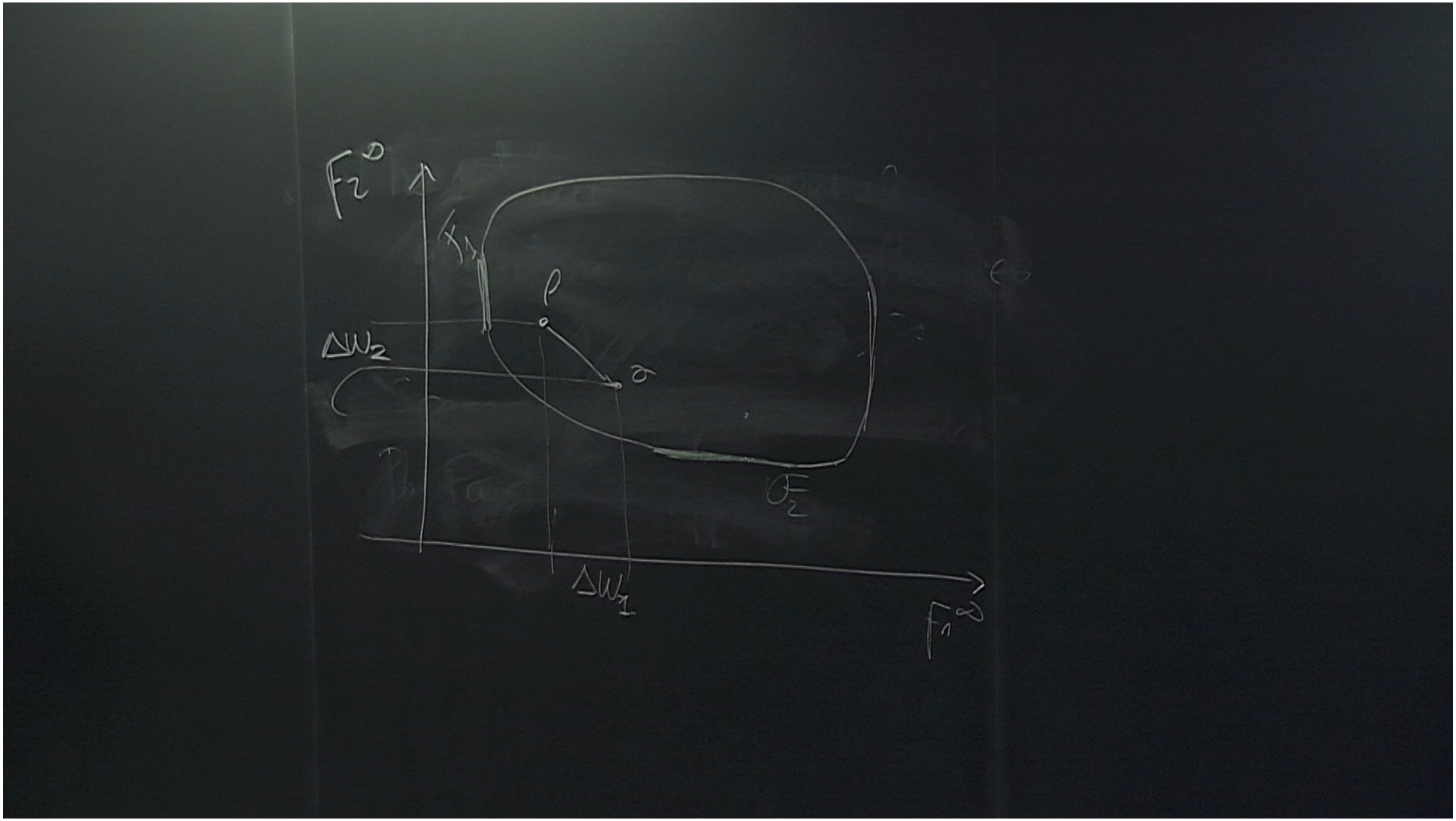

Conditions:
M1: $\forall i, \forall j \neq i$: $f_j^{\infty}(\omega_i') = f_j^{\infty}(\omega_i)$
M2: $\forall i$, $f_i^{\infty}(\rho_S \otimes \omega_1 \otimes \dots \otimes \omega_m) = f_i^{\infty}(\rho_S) + \sum_i f_i^{\infty}(\omega_i)$


Conditions:
M1: ∀i, ∀j≠i f_j^∞(ω_i') = f_j^∞(ω_i)
M2: ∀i, f_i^∞(ρ_S ⊗ ω_1 ⊗ … ⊗ ω_m) = f_i^∞(ρ_S) + Σ_i f_i^∞(ω_i)
M3: ∀i ρ_i ∈ F_i ⇒ f_i^∞(ρ) = 0

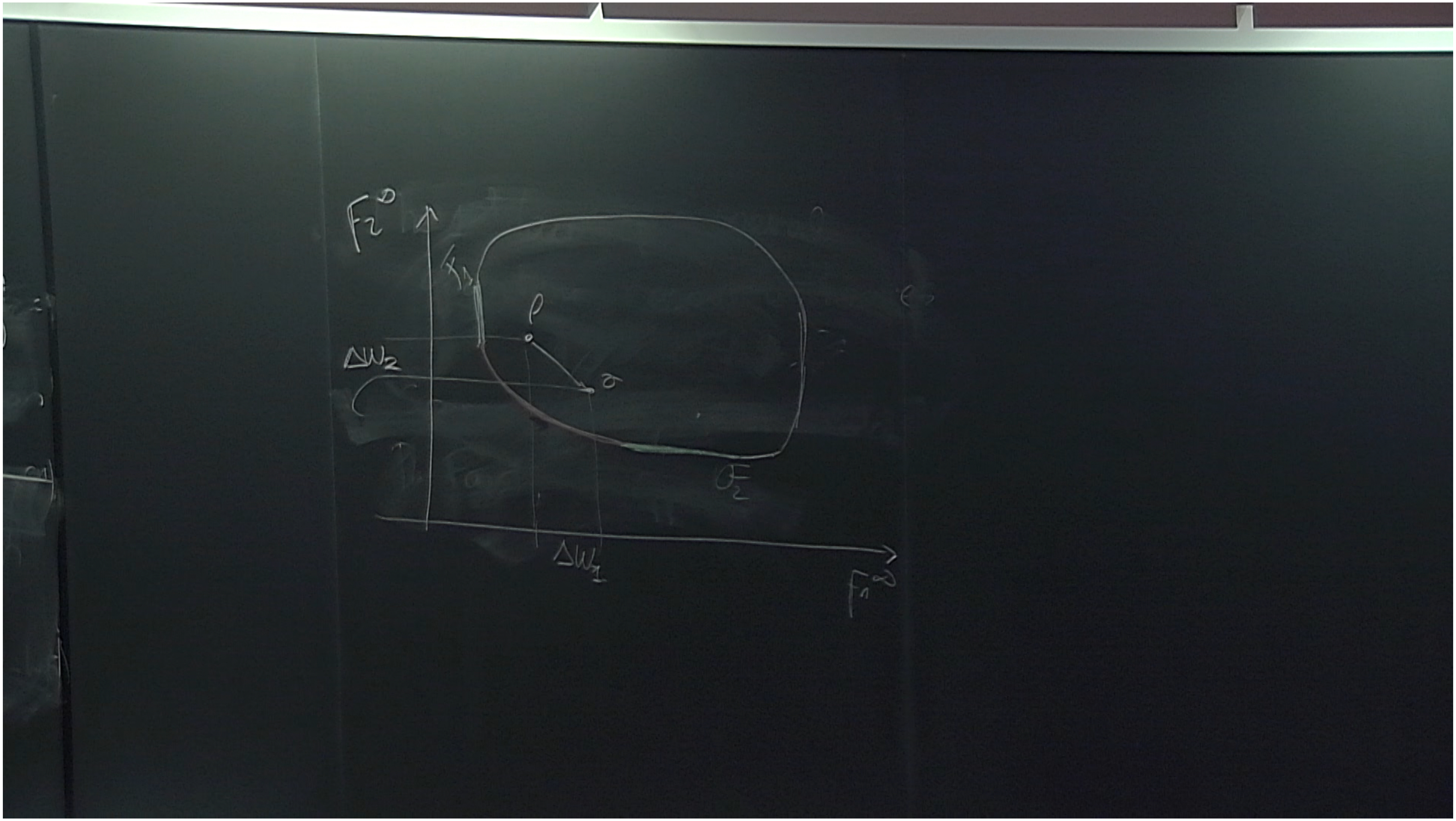

Conditions:
M1: $\forall i, \forall j \neq i \quad f_j^\infty(\omega_i') = f_j^\infty(\omega_i)$
M2: $\forall i, \quad f_i^\infty(\rho_S \otimes \omega_1 \otimes \dots \otimes \omega_m) = f_i^\infty(\rho_S) + \sum_i f_i^\infty(\omega_i)$
M3: $\forall i \quad \rho_i \in F_i \Rightarrow f_i^\infty(\rho) = 0$
Asympt equiv prop
$\forall i \; f_i^\infty(\rho) = f_i^\infty(\sigma)$
$\Longleftrightarrow_{\text{asymp.}} \rho^{\otimes n} \longleftrightarrow \sigma^{\otimes n}$
$\otimes B_m$
$\otimes \omega_n$
$\otimes \omega_n'$



F_2^∞
F_1^∞



F_2^∞
ρ
σ
ΔW_2
ΔW_1
F_1^∞



F_2^∞
ΔW_2
ρ
σ
ΔW_1
F_1^∞

operations

...ons that don't change E

T_{env}

energy battery

$|E\rangle$

$|0\rangle$

$|kT \ln 2\rangle$

$$F_i^{(n)} = F_i^{\otimes n}$$

$$\text{Bank} := \{ \rho_S : \forall \sigma_S \; F_1(\sigma) > F_1(\rho) \text{ or } F_2(\sigma) > F_2(\rho) \text{ or } F_{1,2}(\rho) = F_{1,2}(\sigma) \}$$

Conditions:

M1: $\forall i, \forall j \neq i$

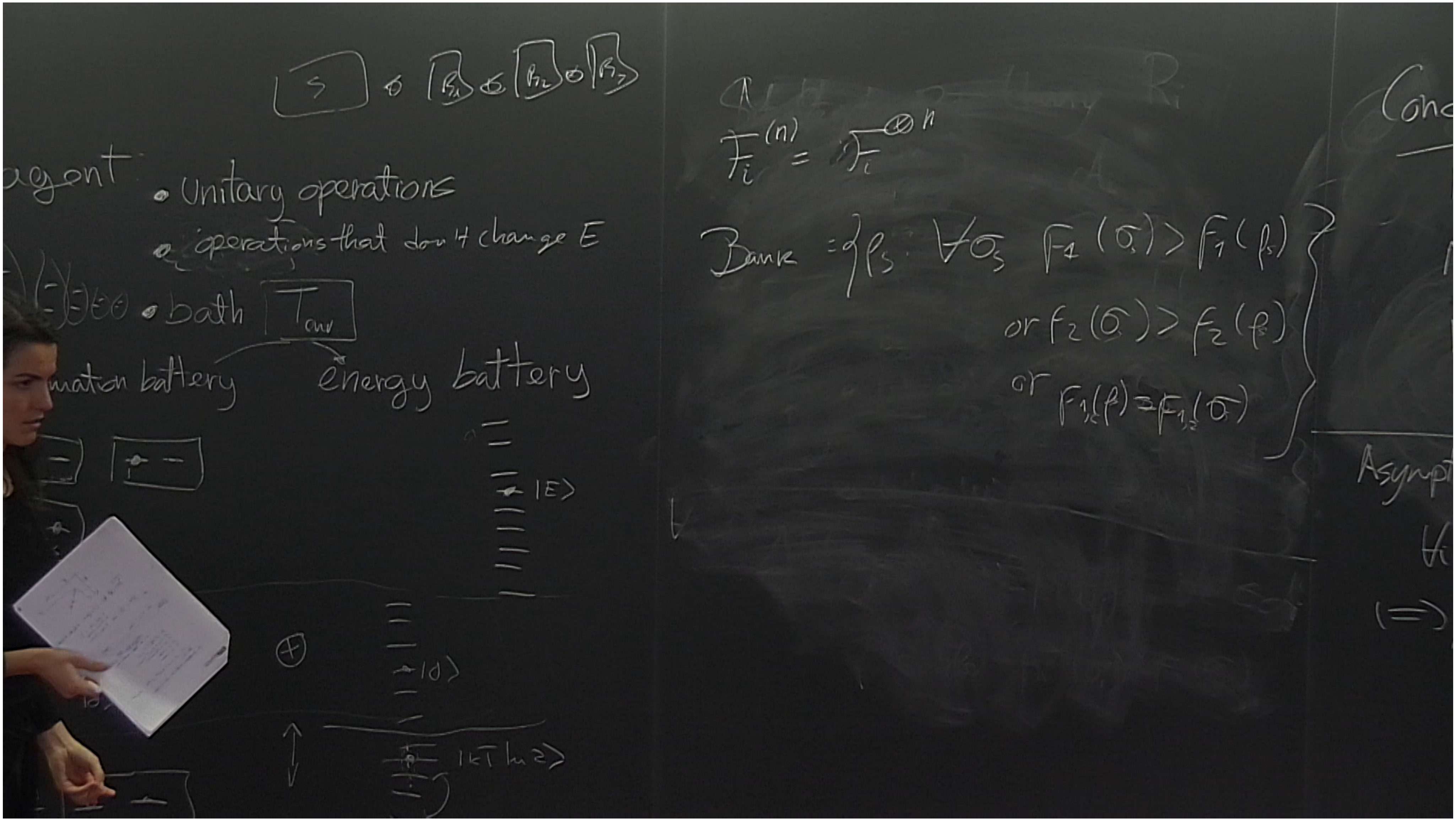
M2: $\forall i, F_i^{\infty}(\rho_S \otimes w$

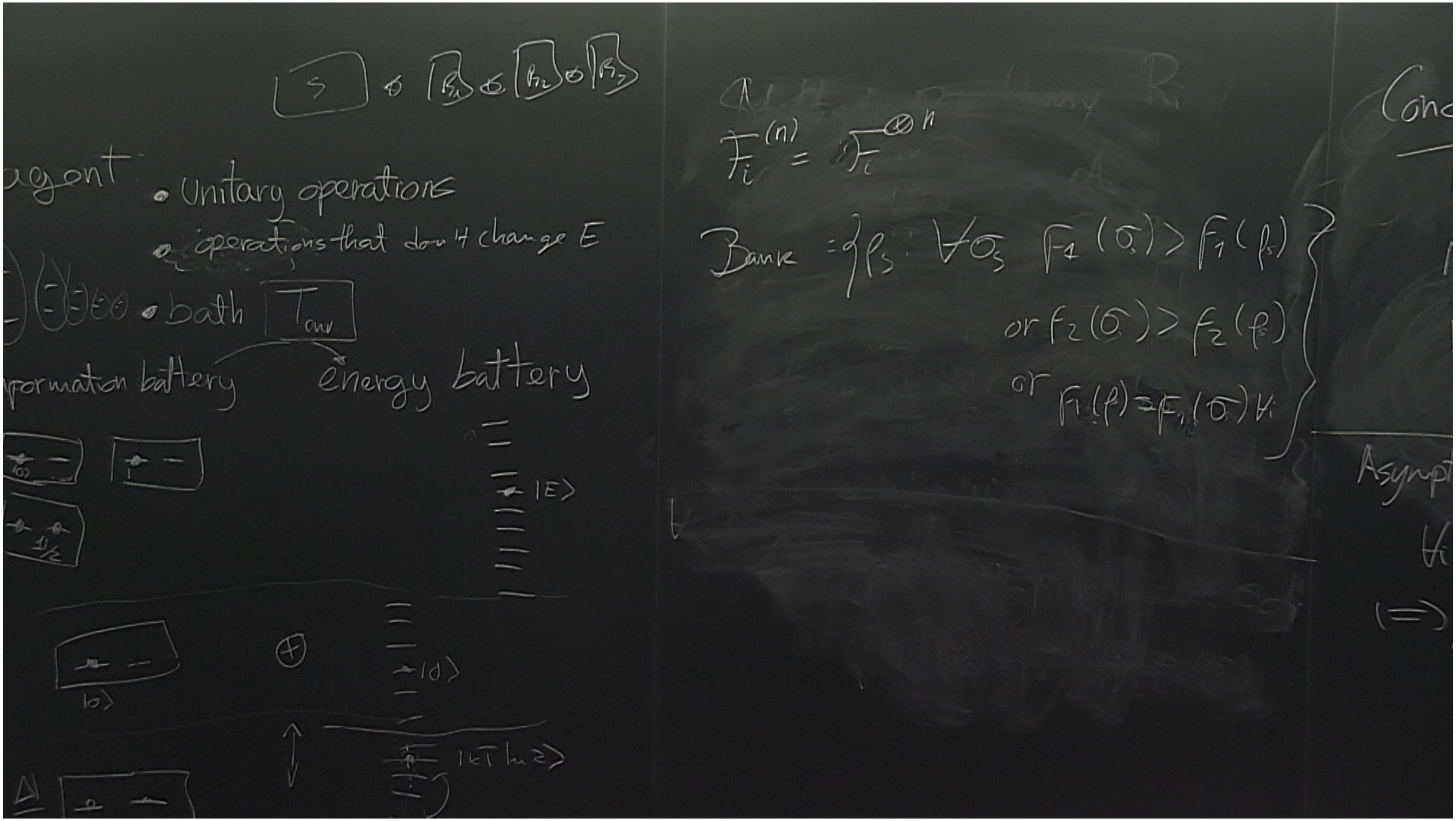
M3: $\forall i \quad \rho_i \in F_i \Rightarrow$

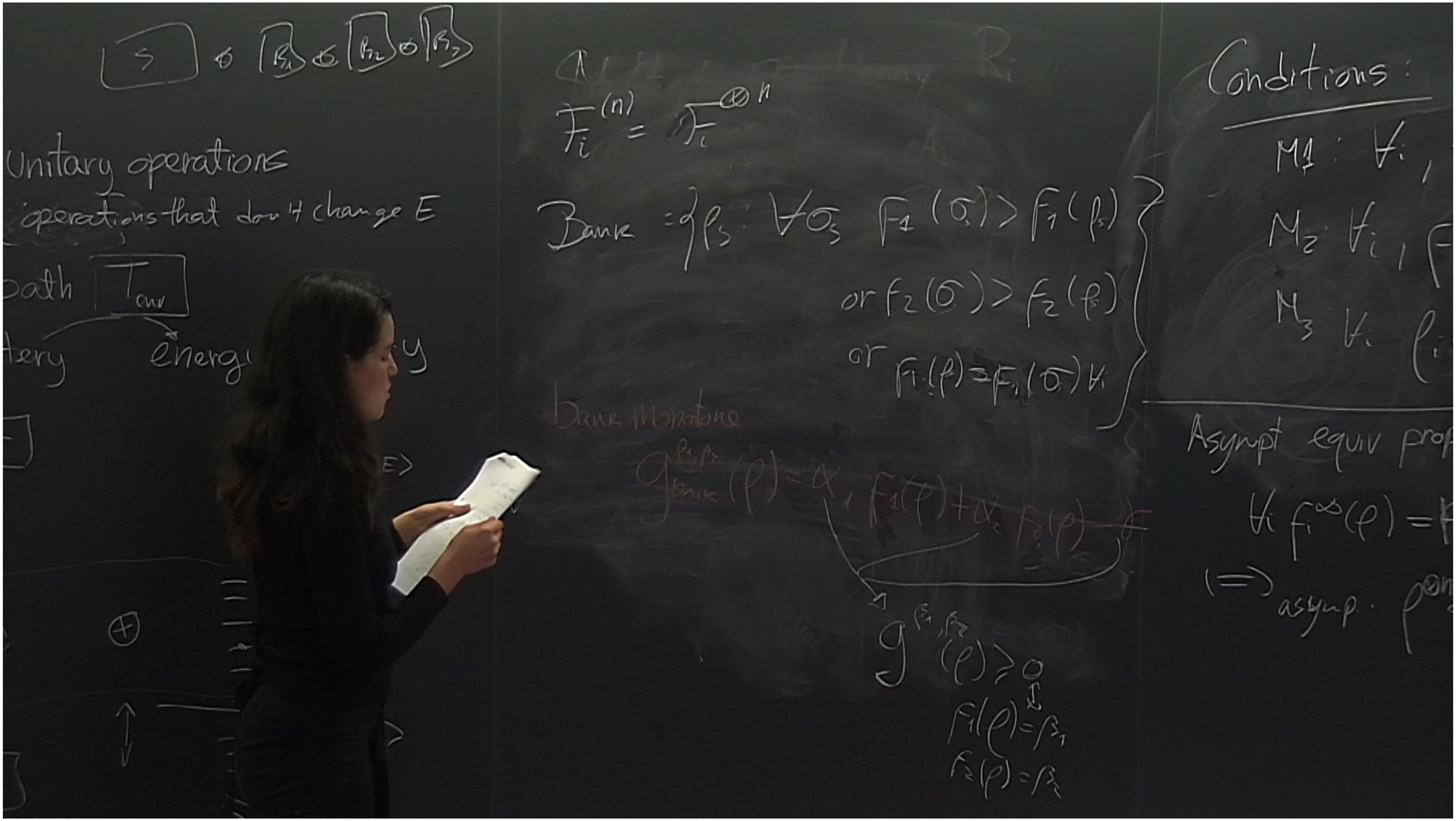
Asympt equiv prop

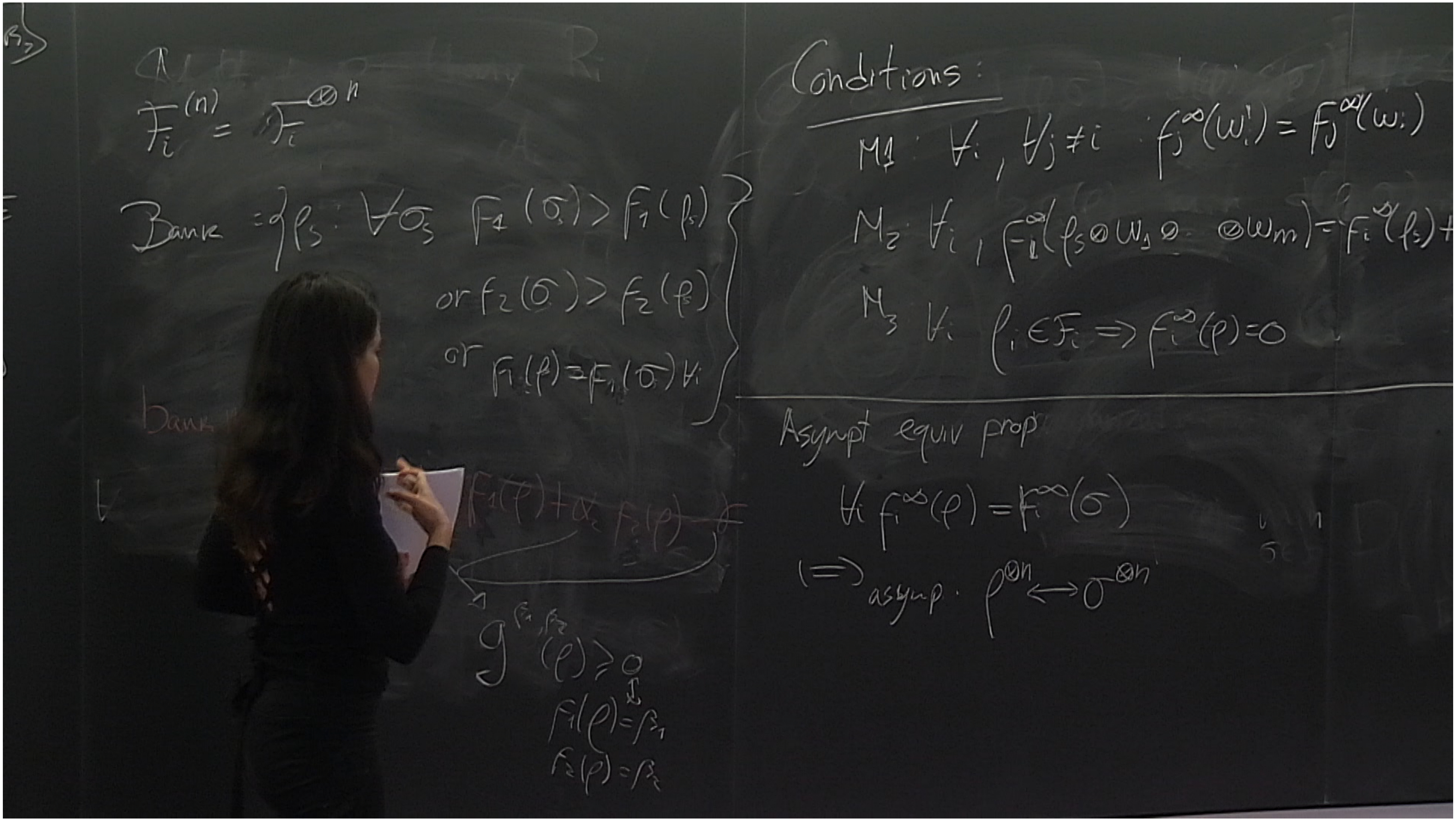
$$\forall i \; F_i^{\infty}(\rho) = F_i^{\infty}(\sigma)$$

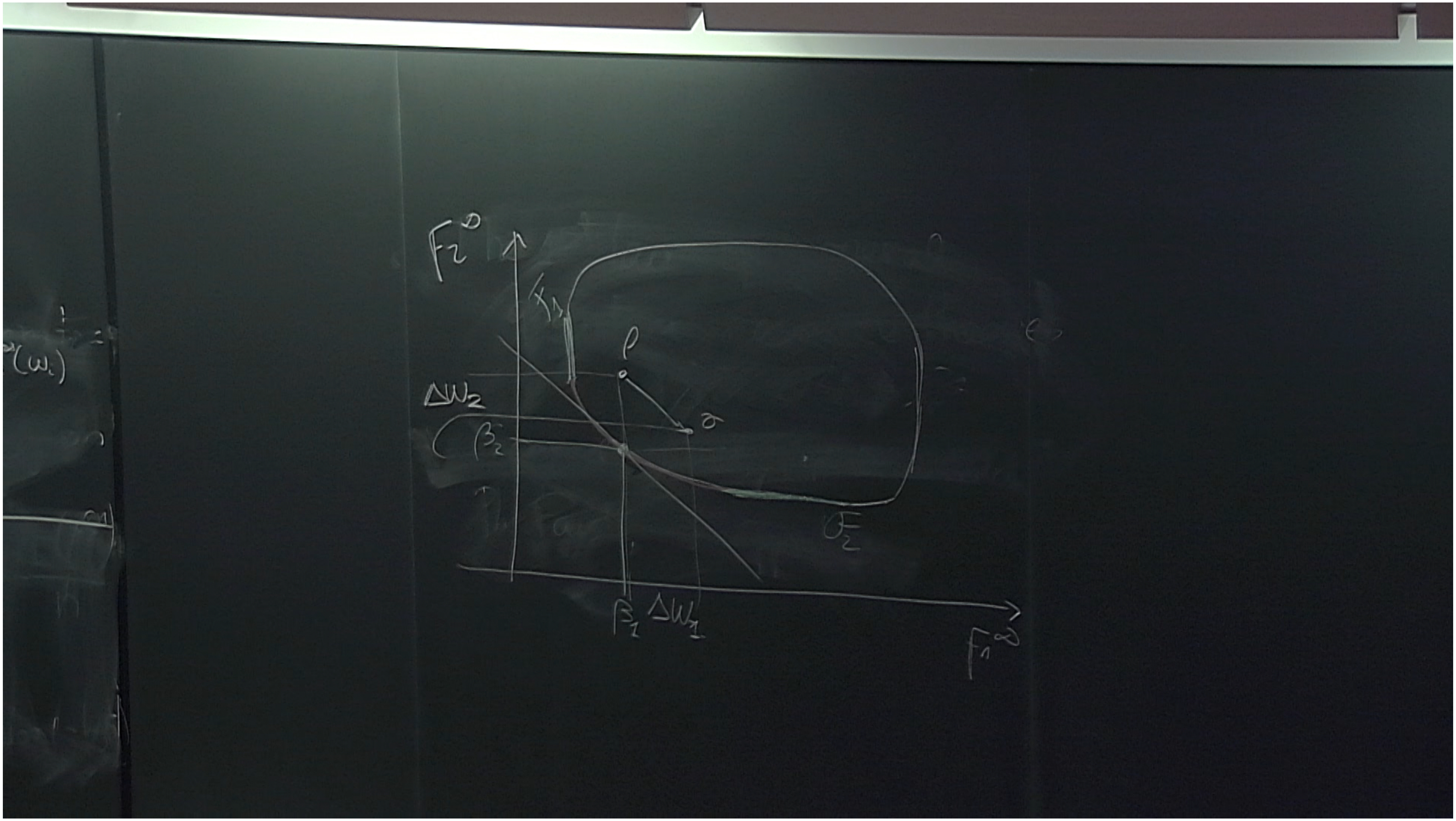
$$\Longleftrightarrow_{\text{asymp.}} \rho^{\otimes n} \longleftrightarrow \sigma^{\otimes n}$$

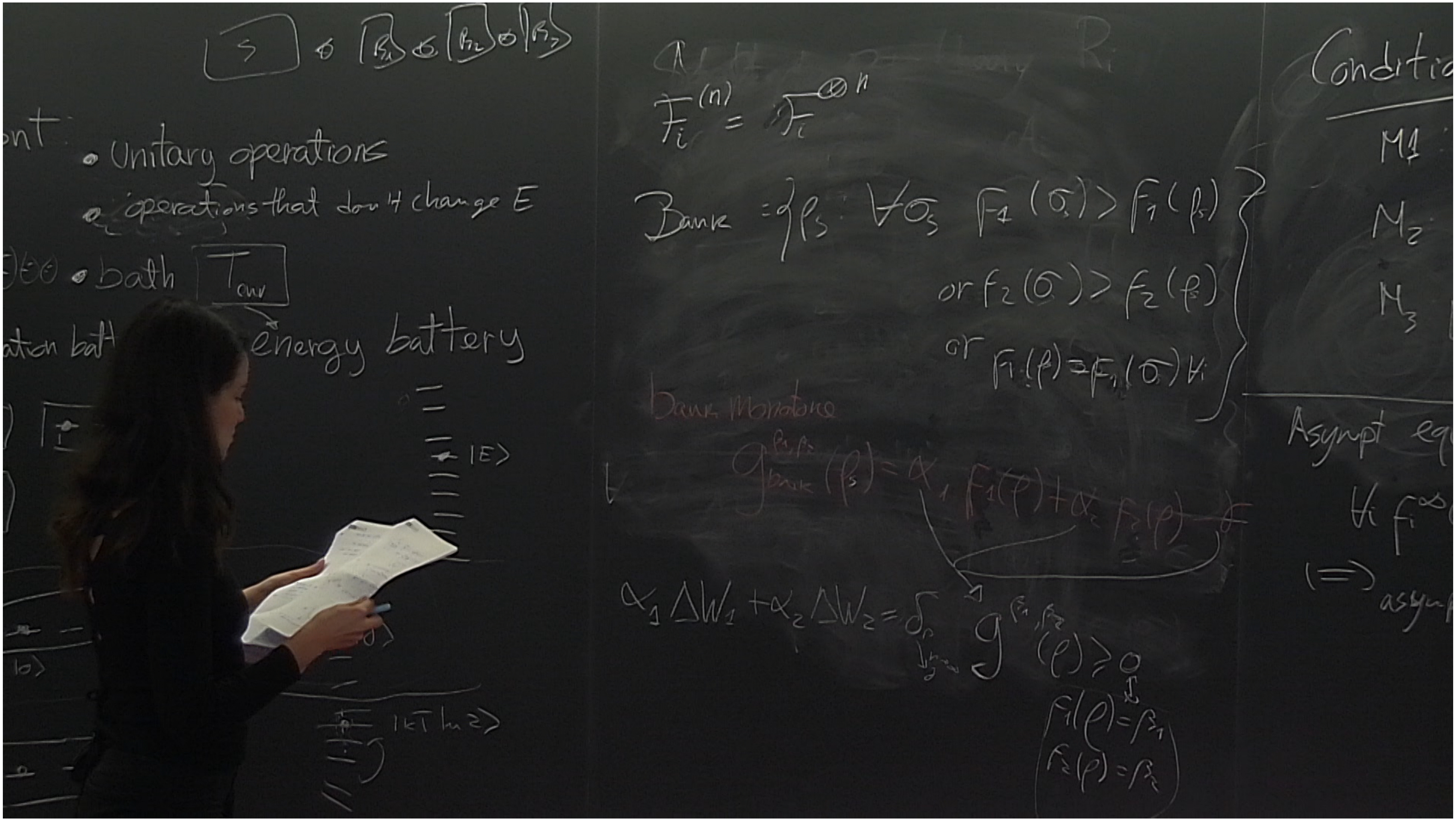

agent
Unitary operations
operations that don't change E
bath
energy battery
mation battery
|E>
|0>
|kT ln 2>
Bank
or
Asympt


Unitary operations
operations that don't change E
bath
energy battery
|E>
|0>
Bank
or
or
Asympt


Unitary operations
operations that don't change E
Conditions :
Bank monotone
Asympt equiv prop


$F_i^{(n)} = F_i^{\otimes n}$
Bank $= \{ \rho_S : \forall \sigma_S \; f_1(\sigma_S) > f_1(\rho_S)$
or $f_2(\sigma) > f_2(\rho)$
or $f_i(\rho) = f_i(\sigma) \; \forall i \}$
$g^{\beta_1, \beta_2}(\rho) \geq 0$
$f_1(\rho) = \beta_1$
$f_2(\rho) = \beta_2$
Conditions:
M1: $\forall i, \forall j \neq i \quad f_j^\infty(w_i) = f_j^\infty(w_i)$
M2: $\forall i, \; f_i^\infty(\rho_S \otimes w_1 \otimes \dots \otimes w_m) = f_i^\infty(\rho_S) +$
M3: $\forall i \quad \rho_i \in F_i \Rightarrow f_i^\infty(\rho) = 0$
Asympt equiv prop
$\forall i \; f_i^\infty(\rho) = f_i^\infty(\sigma)$
$\Longleftrightarrow_{asymp.} \rho^{\otimes n} \leftrightarrow \sigma^{\otimes n}$


F_2^∞
ρ
ΔW_2
σ
β_2
β_1
ΔW_1
F_1^∞



Unitary operations
operations that don't change E
bath
energy battery
Bank
Bank Monotone
Asympt eq

$$\ldots^{(n)} = \mathcal{F}_i^{\otimes n}$$

$$\ldots\text{ank} = \{\rho_S : \forall \sigma_S \; f_1(\sigma_S) > f_1(\rho_S) \text{ or } f_2(\sigma) > f_2(\rho) \text{ or } f_i(\rho) = f_i(\sigma)\ \forall i\}$$

...ank Monotone

$$g^{\beta_1,\beta_2}_{\text{bank}}(\rho_S) = \alpha_1 f_1(\rho) + \alpha_2 f_2(\rho) + \gamma$$

$$\Delta W_1 + \alpha_2 \Delta W_2 = \delta_n \qquad g^{\beta_1,\beta_2}(\rho) \geq 0$$

$$f_1(\rho) = \beta_1 \qquad f_2(\rho) = \beta_2$$

Conditions:

M1: $\forall i, \forall j \neq i \quad f_j^\infty(\omega_i) = f_j^\infty(\omega_i)$

M2: $\forall i, \; f_i^\infty(\rho_S \otimes \omega_1 \otimes \cdots \otimes \omega_m) = f_i^\infty(\rho_S) + \sum_i f_i^\infty(\omega_i)$

M3: $\forall i \quad \rho_i \in \mathcal{F}_i \Rightarrow f_i^\infty(\rho) = 0$

Asympt equiv prop

$$\forall i \; f_i^\infty(\rho) = f_i^\infty(\sigma)$$

$$\Longleftrightarrow_{\text{asymp.}} \rho^{\otimes n} \longleftrightarrow \sigma^{\otimes n}$$

$$\alpha_1 \beta_1 + \alpha_2 \beta_2 + \gamma = 0$$