Title: Quantum gravity and black hole spin in gravitational wave observations: a test of the Bekenstein-Hawking entropy

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Abstract: <p>Black hole entropy is a robust prediction of quantum gravity with no observational test to date. We use the Bekenstein-Hawking entropy formula to determine the probability distribution of the spin of black holes at equilibrium in the microcanonical ensemble. We argue that this ensemble is relevant for black holes formed in the early universe and predicts the existence of a population of black holes with zero spin. Observations of such a population at LIGO, Virgo, and future gravitational wave observatories would provide the first experimental test of the statistical nature of black hole entropy.</p>

Quantum Gravity and Black Hole Spin in

Gravitational Wave Observations:
a test of the Bekenstein-Hawking entropy

Hal M. Haggard, Bard College
Quantum Gravity Seminar
January 24th, 2019

Quantum Gravity and Black Hole Spin in GW Observations: a Test of the Bekenstein-Hawking Entropy

Joint work with Penn State collaborators:

Eugenio Bianchi, Anuradha Gupta & Sathya Sathyaprakash

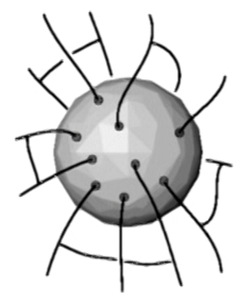
[E. Bianchi, A. Gupta, HMH, & B.S. Sathyaprakash, arXiv:1812.05127]

Are black holes simple or complex?

Pure classical solutions to GR:

Mass M, Spin J

vs.



Huge entropy:

$$S(M,J) = \frac{A(M,J)}{4\ell_P^2}$$

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At fixed mass, rotating black holes have a smaller entropy

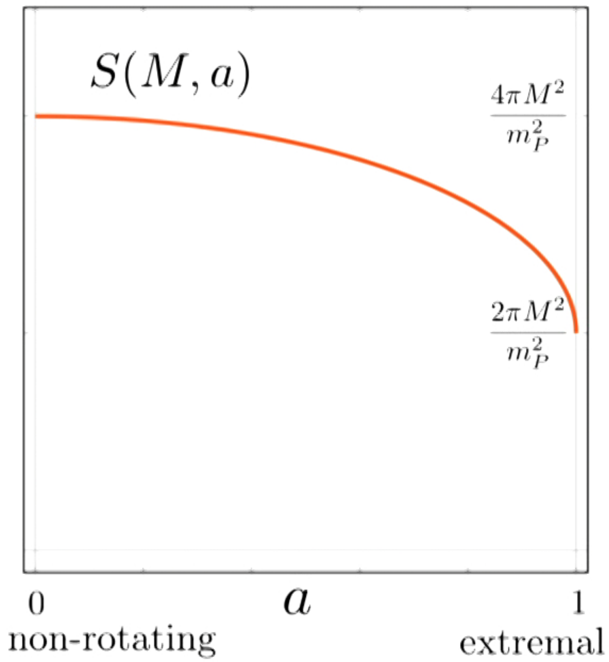
Dimensionless spin parameter

$$a = \frac{|\vec{J}|}{GM^2/c},$$

$a \in [0, 1]$.

Bekenstein-Hawking entropy

$$S(M,a) = (1+\sqrt{1-a^2}\,)\frac{2\pi M^2}{m_P^2}$$



Allows an in-principle **test** of the Bekenstein-Hawking entropy

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Outline

I. The Black Hole Spin Ensemble

II. Observation of BH Spins in Gravitational Waves

III. Microcanonical Equilibration

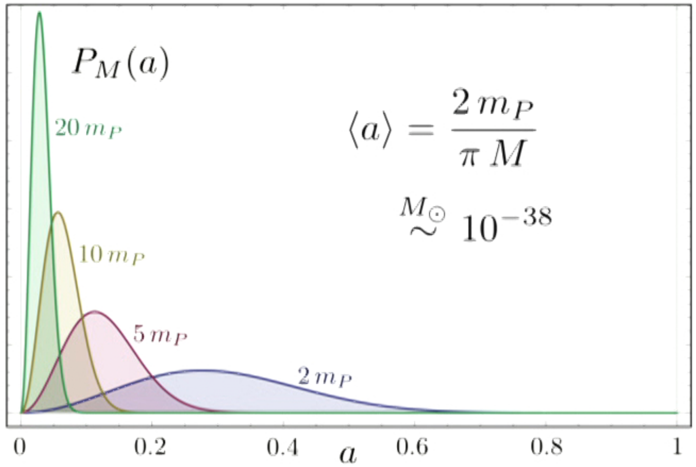
The BH spin ensemble

Number of microstates (M, a)

$$\mathcal{N} \sim e^{S(M,a)}$$

Probability of finding a BH of spin a in a population of BHs of mass M

$$P_M(a) = \frac{e^{A(M,a)/4\ell_P^2} a^2}{\int_0^1 e^{A(M,a')/4\ell_P^2} a'^2 da'}$$



Applies to an unbiased population. Small spins are far more likely.

[Bianchi 2010, Bianchi & HMH NJP 2018]

Rotating black holes can be represented as particular mixed states

Quantum BHs with asymptotically flat b.c.s are characterized by

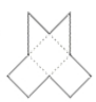
$$\hat{M}|M,j,\alpha\rangle = M|M,j,\alpha\rangle \quad \text{and} \quad \hat{J}|M,j,\alpha\rangle = \hbar\sqrt{j(j+1)}|M,j,\alpha\rangle$$

where α enumerates a basis of the Hilbert space $\mathcal{H}_{Mj}$.

This allows a decomposition of $\mathcal{H}$ into sectors

$$\mathcal{H} = \bigoplus_{M} \mathcal{H}_M, \quad \text{with} \quad \mathcal{H}_M = \bigoplus_{j=0,\frac{1}{2},1,\ldots} \mathcal{H}_{Mj}.$$

A rotating BH is maximally-mixed at fixed M and j

$\rho($  $) \equiv \rho_{Mj} = \dfrac{1}{\dim \mathcal{H}_{Mj}} \displaystyle\sum_{\alpha=1}^{\dim \mathcal{H}_{Mj}} |M,j;\alpha\rangle\langle M,j;\alpha| \,.$

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Semiclassical methods ideal for computing $\dim \mathcal{H}_{Mj}$ at $M \gg m_P$

Use the canonical partition function

$$Z(\beta, \vec{\omega}) = \mathrm{Tr}(e^{-\beta \hat{M} - \vec{\omega}\cdot\vec{J}/\hbar}) = \int [Dg_{\mu\nu}]\, e^{-S_E[g_{\mu\nu}]/\hbar},$$

where S_E is the Euclidean gravitational action. [Gibbons Hawking '77, Sen '12]

The result

$$\dim \mathcal{H}_{Mj} \sim \sqrt{S(M, a_j)^{\frac{212}{45} - \frac{3}{2}}}\; e^{\,S(M,a_j)}\; a_j^2$$

$$\sum_j \dim \mathcal{H}_{Mj} \sim \sqrt{S(M, 0)^{\frac{212}{45}}}\; e^{\,S(M,0)}$$

with $S(M, a)$ the BH entropy and $a_j = \sqrt{j(j+1)} m_P^2 / M^2$. Hence

$$p_M(j) \to P_M(a) \qquad \text{for} \quad M \gg m_P.$$

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BH mergers and GW observations at LIGO/Virgo

$$(M_1, \vec{J}_1) + (M_2, \vec{J}_2) + \vec{L} \to (M_f, \vec{J}_f) + GW$$

Final spin

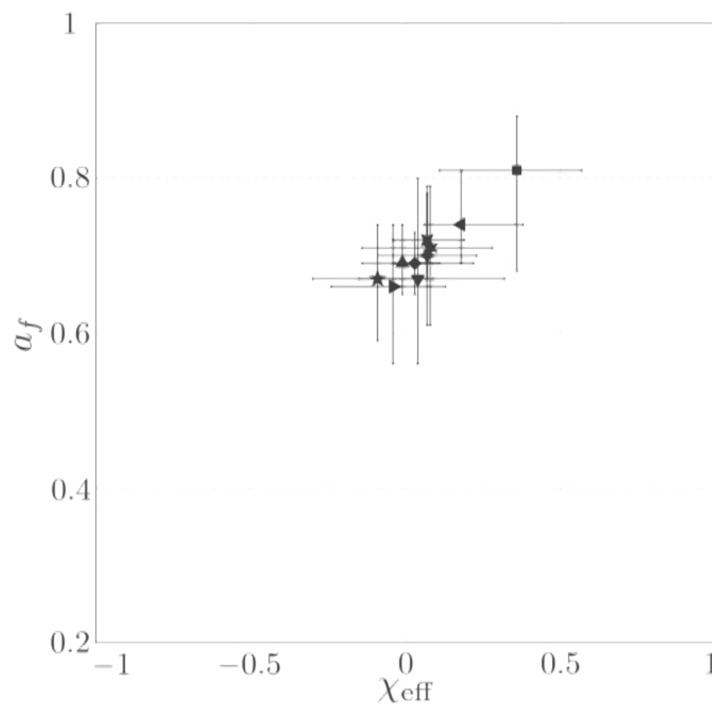
$$a_f = \frac{|\vec{J}_f|}{GM_f^2/c}$$

Effective initial spin

$$\chi_{\text{eff}} = \left(\frac{M_1\,\vec{a}_1 + M_2\,\vec{a}_2}{M_1 + M_2} \right) \cdot \hat{L},$$

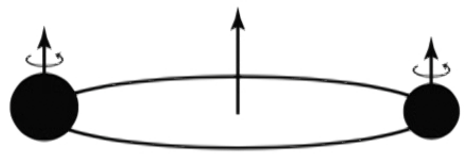
with $\chi_{\text{eff}} \in [-1, 1]$.

[Blanchet '14, Kesden et al '15, Gerosa et al '15]



▲GW150914 ▼GW151012 ◀GW151226 ▶GW170104 ◆GW170608 ■GW170729 ✦GW170809 ✖GW170814 ★GW170818 ✷GW170823 astrophysical spin model 1:

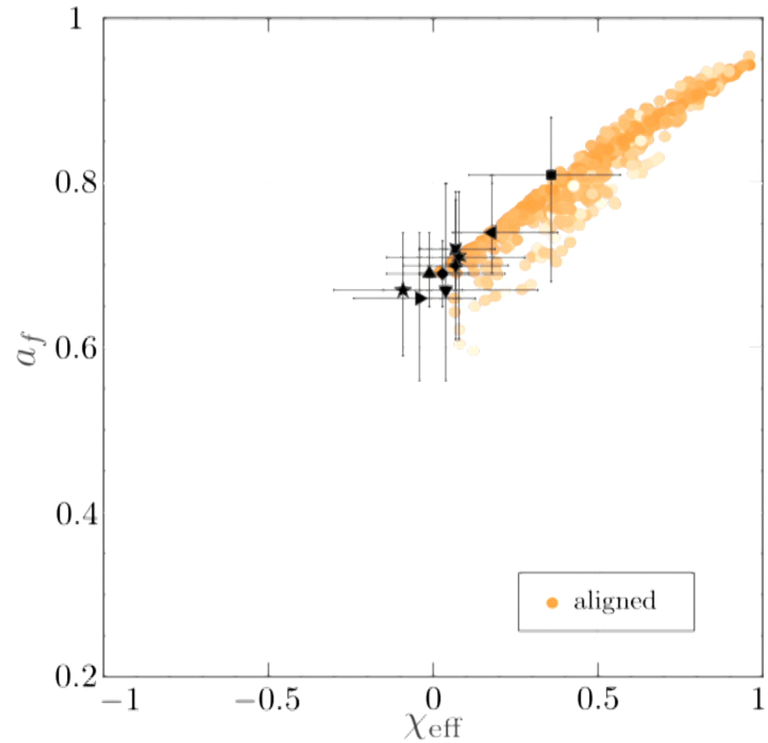
aligned BH spins



a uniform in $[0, 1]$

mass ratio

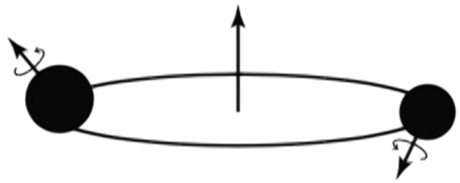
$$q = \frac{M_2}{M_1} \geq 1$$



Binaries formed through common envelope evolution in galactic fields are expected to have aligned spins [Postnov & Yungelson, 2014]

▲GW150914 ▼GW151012 ◀GW151226 ▶GW170104 ◆GW170608 ■GW170729 ✦GW170809 ✖GW170814 ★GW170818 ✶GW170823 astrophysical spin model 2:

isotropic BH spins



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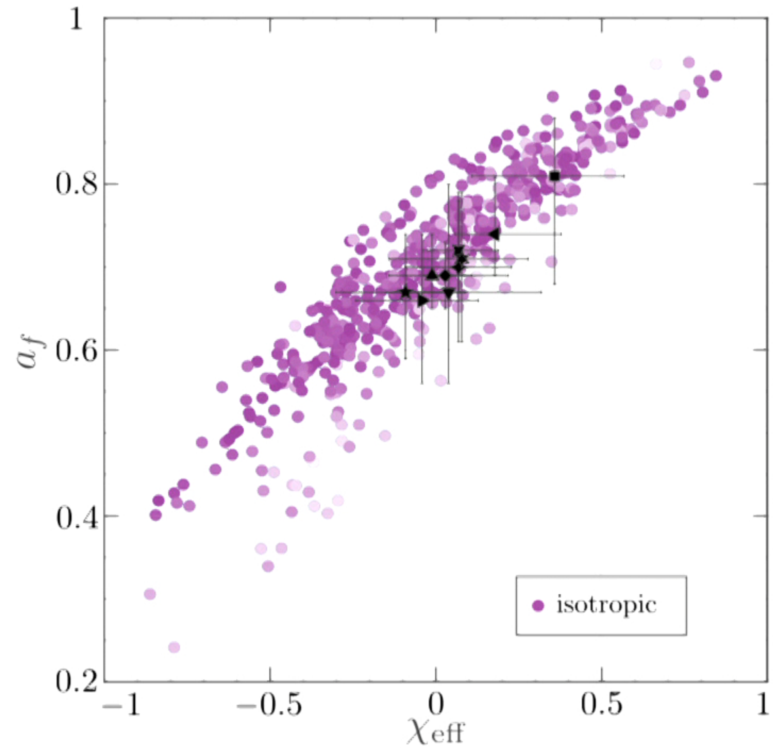


Binaries formed in globular clusters or stellar clusters near active galactic nuclei are expected to have isotropic spins

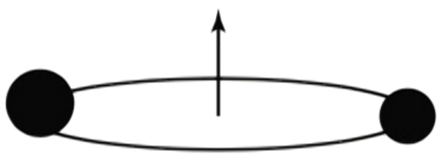
[Benacquista & Downing, 2013, Miller & Lauburg, 2009]

▲GW150914 ▼GW151012 ◀GW151226 ▶GW170104 ✦GW170809 ✖GW170814 ★GW170818 ✶GW170823

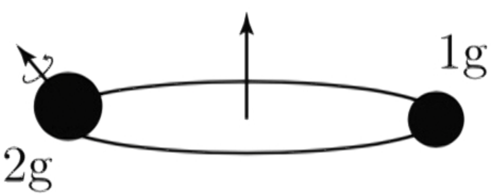
Microcanonical BHs:

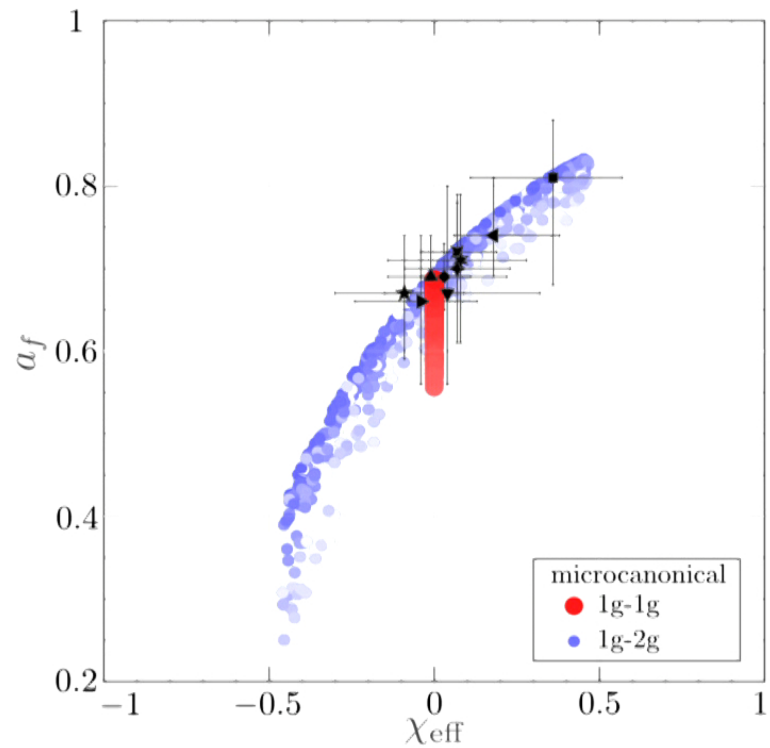
BH spins: $P_M(a)$ ensemble

1st generation mergers:
1g-1g $\rightsquigarrow a_f \approx 0.69$



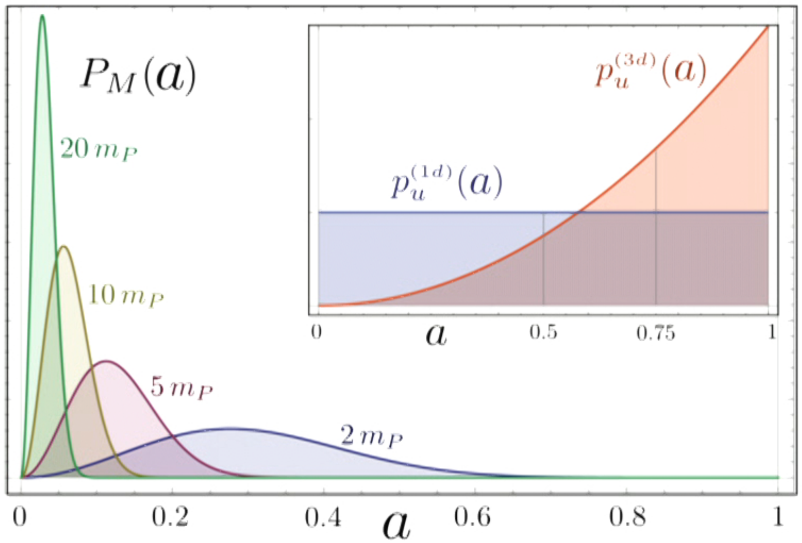
1g-2g mergers:





[E. Bianchi, A. Gupta, HMH, & B.S. Sathyaprakash, arXiv:1812.05127]

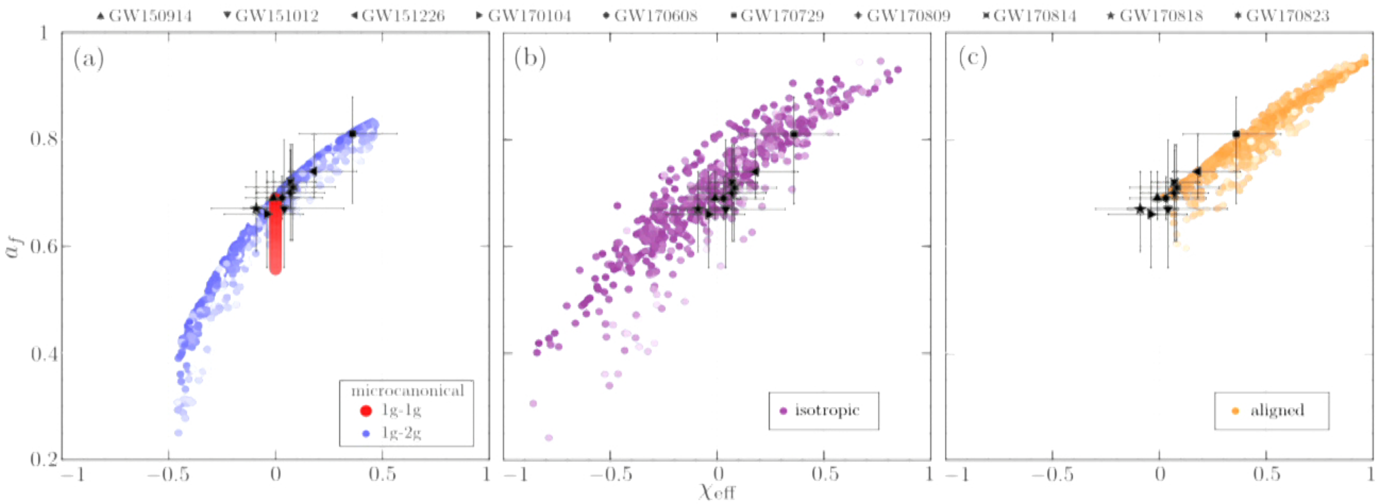
The BH microcanonical and astrophysical ensembles



Microcanonical BHs: small spins by far most likely

[E. Bianchi & HMH 2018, E. Bianchi, A. Gupta, HMH, & B.S. Sathyaprakash, arXiv:1812.05127]

Microcanonical vs. astrophysical ensembles



Imminent transition from single-event GW analysis to population statistics can provide the first observational test of BH entropy

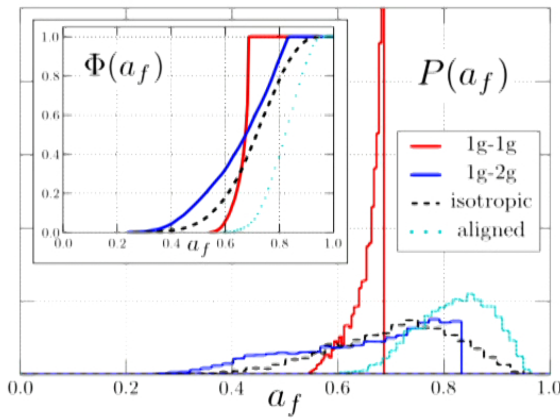
Cumulatives for final spin a_f

M1: 90% aligned + 10% 1g-1g

M2: 90% aligned + 10% 1g-2g

M3: 90% isotropic + 10% 1g-1g

M4: 90% isotropic + 10% 1g-2g



Anderson-Darling (AD) and Kolmogorov-Smirnov (KS) tests

population	p-value (AD test)	p-value (KS test)
A vs. M1	7.8×10^{-11}	1.6×10^{-8}
A vs. M2	1.3×10^{-7}	3.2×10^{-7}
I vs. M3	0.04	0.01
I vs. M4	0.81	0.99

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When is a population of black holes well-described by the microcanonical ensemble?

Not in stellar collapse:

(i) the initial matter distribution already has angular momentum and is far from microcanonical equilibrium.

(ii) angular momentum exchanges are not efficient and microcanonical equilibrium cannot be reached.

(iii) in a dense environment, accretion can drive spin up.

Hence, larger spins are expected.

For example, X-ray binaries typically have $a > 0.25$, with several having $a \gtrsim 0.9$.

Microcanonical BHs as primordial BHs

In QCD phase trans. pressure drops $\rightsquigarrow$ enhanced BH production

Mass estimate:

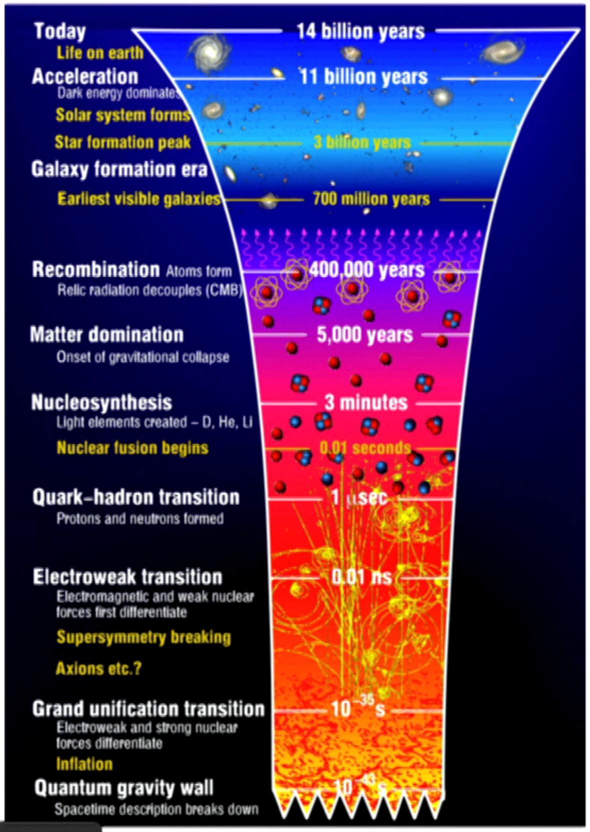
$$\rho_0 = \frac{M_0}{r_S^3} \quad \text{and} \quad r_S = \frac{2GM_0}{c^2}$$

$$\implies M_0 = \frac{c^3}{2\sqrt{2}G^{3/2}} \frac{1}{\sqrt{\rho_0}}$$

Then with

$$\rho_{\text{QCD}} \sim (150\text{MeV})^4/\hbar^3 c^5$$
$$\rightsquigarrow M_{\text{QCD}} \sim 25 M_\odot$$

Current GW observations probe same mass range as PBHs formed in QCD phase transition



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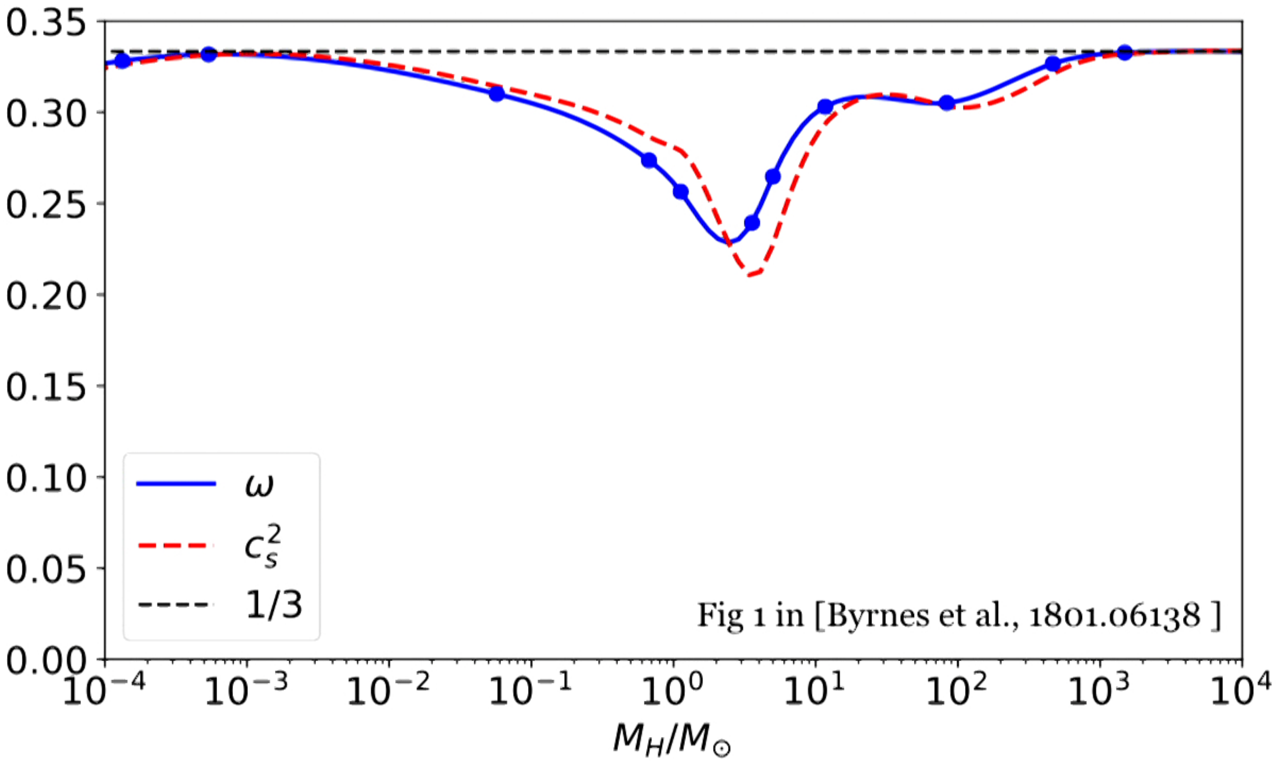

The equation of state parameter ω and the sound speed squared c2s for the Standard Model, plotted against horizon mass, in units of solar mass.

[Byrnes et al., "PBH with an accurate QCD equation of state" 1801.06138]

[Borsanyi et al, "Lattice QCD for cosmology"Nature 539 (2016)]

- Early universe: during the QCD phase transition, the pressure drops $\Longrightarrow$ enhanced BH production

- mass estimate: $\rho_0 = M_0/r_S^3$ and $r_S = 2GM_0/c^2$ $\Longrightarrow$ $M_0 = \frac{c^3}{2\sqrt{2}G^{3/2}}\frac{1}{\sqrt{\rho_0}}$

$\rho_0 \sim (150 \text{ MeV})^4/\hbar^3 c^5$ $\quad M_0 \sim 25\, M_\odot$

- lattice QCD simulations: BH mass range $0.1 - 100\ M_\odot$

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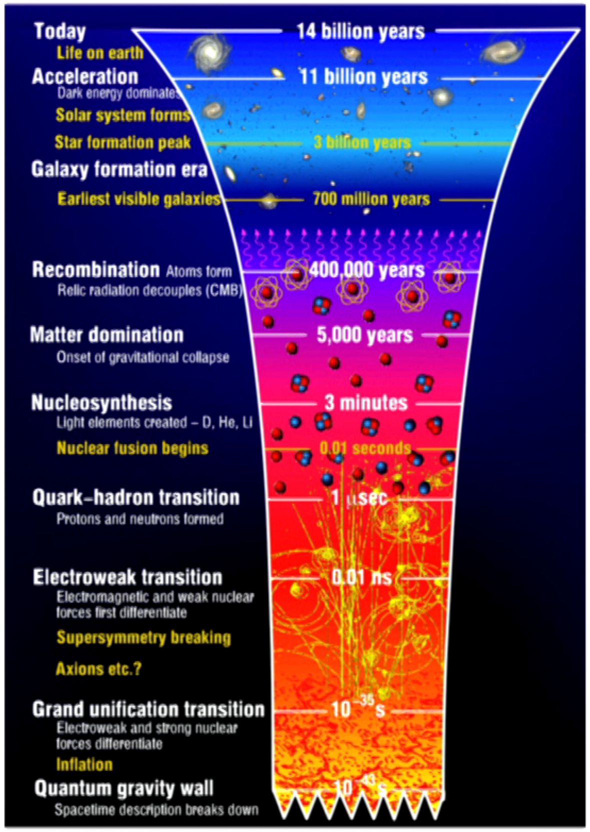
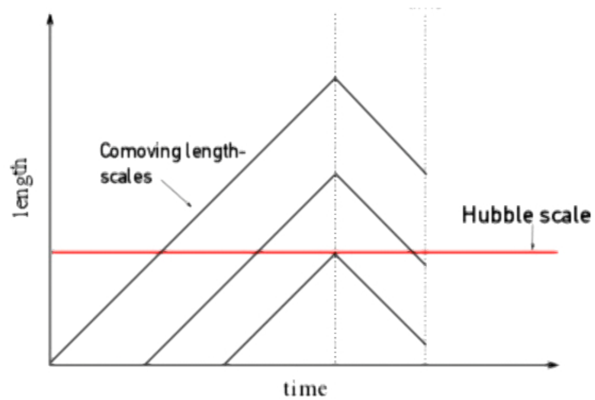


Primordial black holes formed in the early universe by density fluctuations are candidates for microcanonical equilibrium

Begin with a matter distribution that is homogeneous over cosmological scales and in thermal equilibrium;



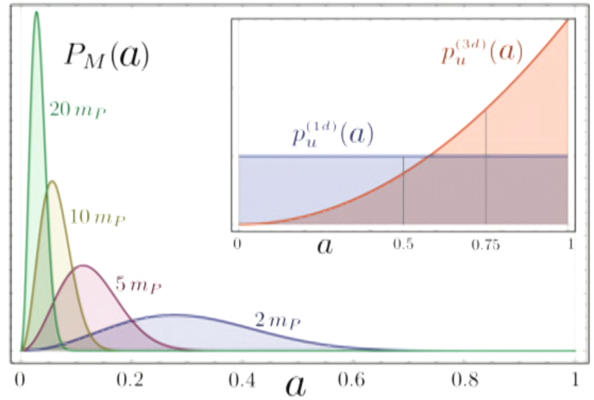
for a density perturbation near the critical collapse threshold, matter within its own Hubble radius is trapped, and a BH forms.

The BH remains effectively isolated due to the cosmic expansion of the surrounding matter, and accretion does not spin up the BH.

Bekenstein-Hawking entropy and the spin of microcanonical BHs

Spin ensemble assumptions:
- BH entropy, microstates
- microcanonical population

$$P_M(a) = \frac{e^{A(M,a)/4\ell_P^2} a^2}{\int_0^1 e^{A(M,a')/4\ell_P^2} a'^2 da'}$$



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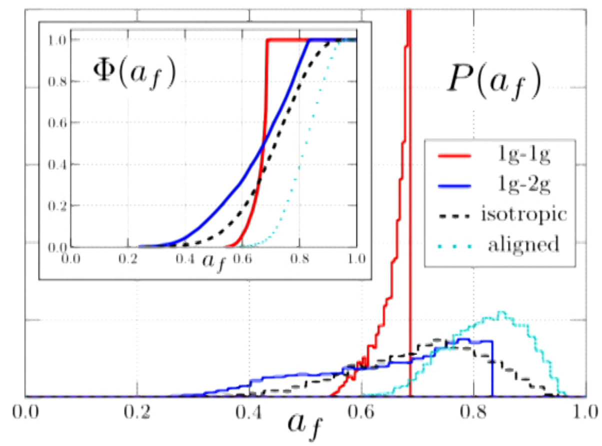

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