Title: Gauge theory and boundaries: a complicated relationship

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Abstract: <p>I argue that we do not understand gauge theory as well as we think when boundaries are present. I will briefly explain the conceptual and technical issues that arise at the boundary. I will then propose a tentative resolution, which requires us to think of theories not in spacetime, but in field-space. </p>

Gauge and boundary: a complicated relationship[1]

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January 17, 2019

[1] Based on joint work with A. Riello and F. Hopfmüller

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The big picture

Local gauge theories underlie the standard model. How well do we understand them?

I want to claim in this talk that, in the presence of spatial boundaries, not very well.

Boundaries are very important! All we have access to are bounded regions.

For local gauge theories, we have technical difficulties when using boundaries: in the calculation of entanglement entropy (e.g. 'replica trick' [Holzhey, Larsen, Wilczek'94] doesn't always work) and in the computation of Noether charges (relies on arbitrary choices).

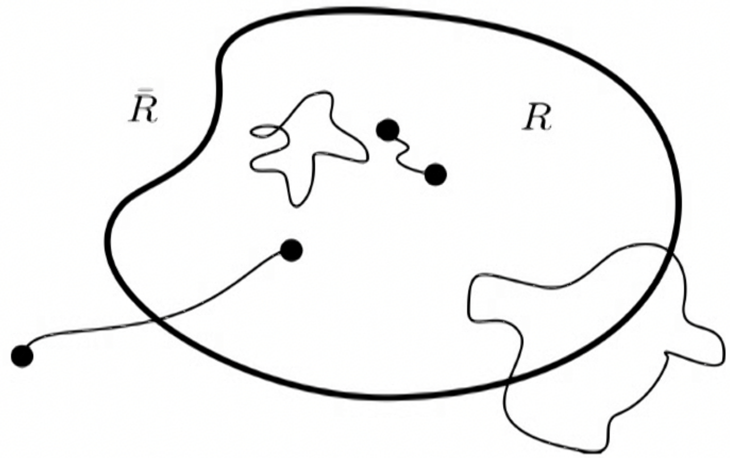
One attempted solution: 'edge-modes'—new physical degrees of freedom that appear at any boundary [Donnelly&Freidel '16].

I will present an alternative resolution: it requires us to think of theories not in spacetime, but in field-space. [DeWitt, Vilkowisky, etc.]

(everything here will be classical, but there are quantum implications: e.g. entanglement entropy)

Summary: the source of the problems

The physical degrees of freedom of local gauge theories can be non-local (think of Wilson loops). [Buividovich&Polikarpov,'08]



Gauge-invariant configuration space doesn't decompose:

$$(\text{Gauge inv})_{R\cup\overline{R}} \neq (\text{Gauge inv})_R \times (\text{Gauge inv})_{\overline{R}}$$

How do we glue regions in a gauge theory?

Summary: solutions

Challenge: have some regional notion of 'physical' (i.e. gauge-invariant) which behaves well under composition of regions.

New independent physical degrees of freedom have been proposed to live at boundaries (even abstract ones) to solve some of these problems.
(See, e.g. 'edge-modes' [Donnelly&Freidel '16, Balachandran '94, Regge&Teitelboim '74])

I will show they are not necessary: we can use the fields at our disposal to solve these problems.

Essentially, we can repeat the original motivation for a gauge-connection, but now in field-space.

When we do this, a mathematical object with many of the same properties as the 'edge-modes' emerges:

The field-space connection-form, ϖ [VAR-PIE]

Why gauge?

Why not gauge-fix? Doesn't a gauge-fixed description contain all the physics?

Rovelli: Because we need to couple regions. [Rovelli '13]

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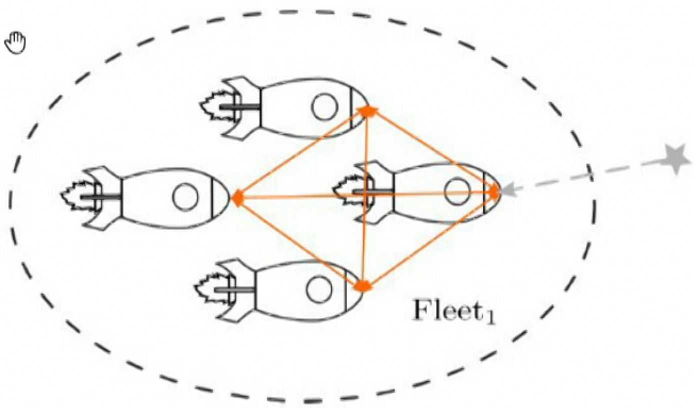

$L_1 = \sum_{i=1}^{N}(\dot{q}^1_{i+1} - \dot{q}^1_i)^2$,

symmetry: $\delta_1 q^1_i = q^1_i + f_1(t)$.

Gauge-inv. variables: $\overline{q}^1_i := q^1_{i+1} - q^1_i$

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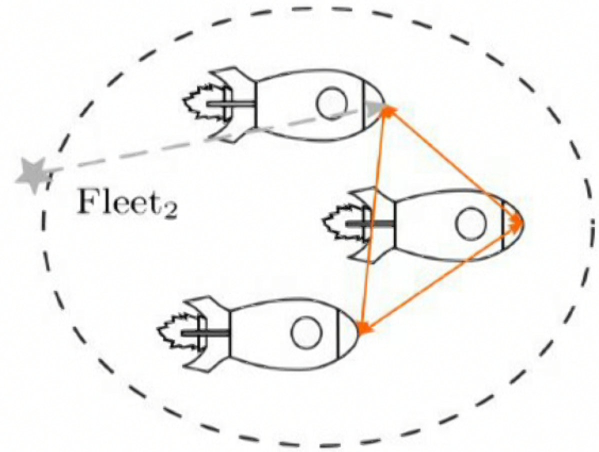

$L_2 = \sum_{j=1}^{M} (\dot{q}_j^2 - \dot{q}_{j-1}^2)^2,$

symmetry: $\delta_2 q_j^2 = q_j^2 + f_2(t)$.

Gauge-inv. variables: $\overline{q}_j^2 := q_{j+1}^2 - q_j^2$

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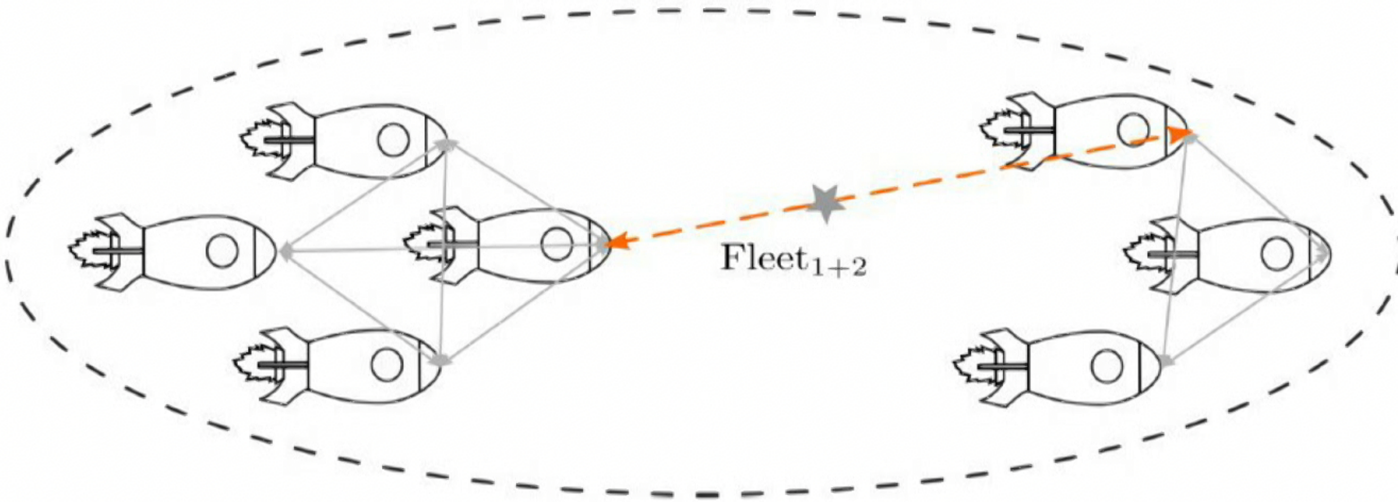


$L = L_1 + L_2 + L_{\text{int}}$ where $L_{\text{int}} := (\dot{q}^1_N - \dot{q}^2_1)$. Cannot express L_{int} in terms of $\overline{q}^1, \overline{q}^2$.

Coupling between regions/squadrons requires retention of gauge-variant variables.

Rovelli: gauge is a 'handle' for coupling.

An analogy: the dictionary extinction event.

Two regions in the world. In region 1, an edict to only speak Brazilian Portuguese. In region 2, an edict to only speak German. Each can perfectly communicate internally.

They throw away foreign lang. dictionaries (everything described in same language). But 1 and 2 can't communicate if they meet.

Here: edicts = gauge-fixing, and dictionaries = gauge-transformations. For single region, gauge-fixed description loses nothing.

But some possibility of gauge transformation must be kept if we know ourselves to be provincial.

Gauge-fixing in field theories

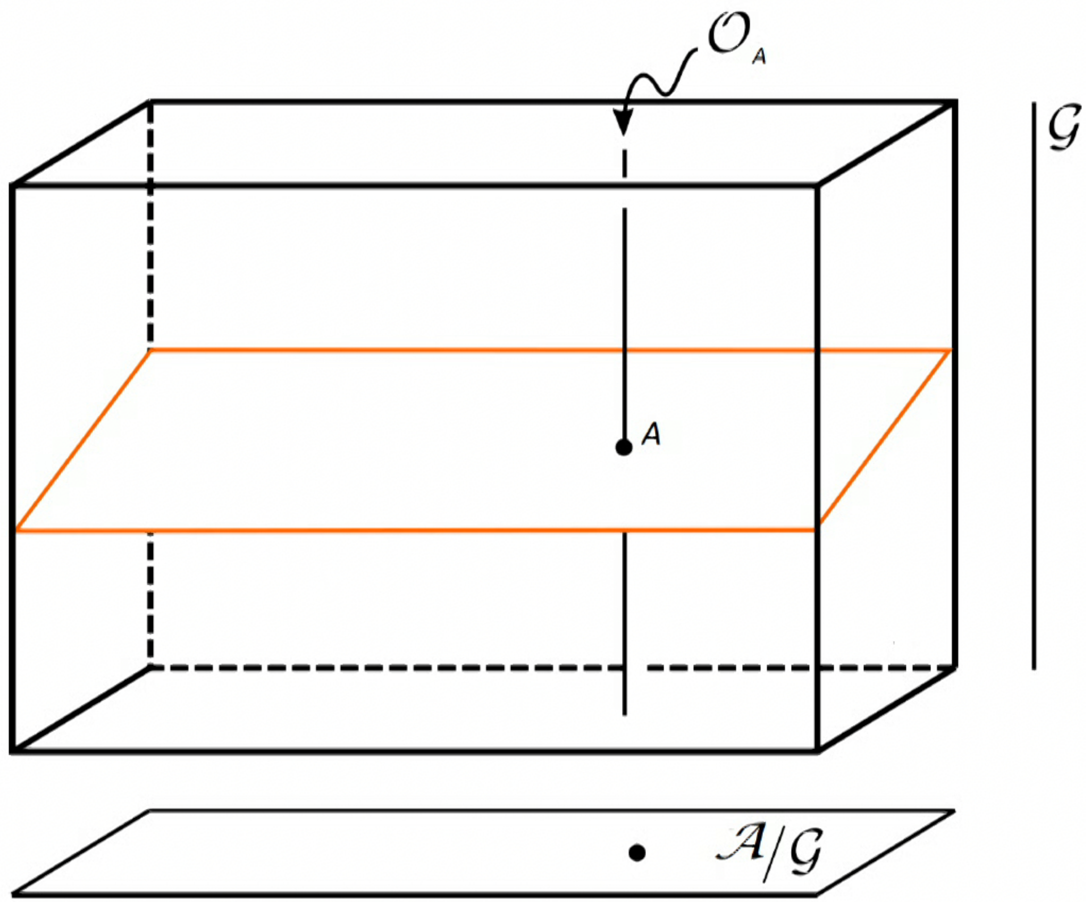
Let $A_\mu \to\ A^\lambda_\mu := A_\mu + \partial_\mu\lambda$ for $A_\mu \in \Lambda^1(R)$ (EM potential) and $\lambda \in C^\infty(R, \mathbb{R})$.

A gauge-fixing condition (Landau): $\quad \mathsf{f}(A^\lambda) := \partial^\mu(A^\lambda_\mu) = 0 \Rightarrow \nabla^2\lambda = -\partial^\mu A_\mu$

solve for λ, obtain field-*dependent* $\lambda^{\mathsf{f}}(A)$, and gauge-fixed $A^{\mathsf{f}}_\mu = A_\mu + \partial_\mu\lambda^{\mathsf{f}}(A)$.

Gauge-fixing requires existence and uniqueness of solution $\lambda^{\mathsf{f}}(A)$.

Gauge-fixing in field theories



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Gauge-fixing requires existence and uniqueness of solution $\lambda^{\mathsf{f}}(A)$.

- For closed region (R compact and $\partial R = \emptyset$, or $R = \mathring{R}$, e.g. $R = S^3$.)

Only solution for $\mathsf{f}((A^{\mathsf{f}})^{\lambda'}) = \partial^\mu (A_\mu^{\mathsf{f}} + \partial_\mu \lambda') = \nabla^2 \lambda' = 0$ is $\lambda^{\mathsf{f}'} \equiv$ const.

Gauge-related configurations are mapped to the same gauge-fixed configuration.

For closed regions, gauge-fixed description is completely gauge-invariant. ✓

Gauge-fixing and boundaries

- What about for bounded regions? Start with $\partial^\mu A_\mu = 0$

Again, obtain: $f(A^\lambda) \Rightarrow \nabla^2\lambda = -\partial^\mu A_\mu$. But now, requires boundary conditions for λ.

For two (Dirichlet) boundary conditions such that $\lambda_{|\partial R} - \lambda'_{|\partial R} \neq$ const., i.e. for $\delta\lambda_{|\partial R} \neq$ const., obtain solutions: $\lambda^{\mathrm{f}}(x) - \lambda^{\mathrm{f}'}(x) \neq$ const.

This means that if we change the gauge at the boundary,

$$A^{\mathrm{f}'}(x) = A^{\mathrm{f}}(x) + \partial_\mu(\lambda^{\mathrm{f}} - \lambda^{\mathrm{f}'})(x) \neq A^{\mathrm{f}}(x) \qquad \text{for some} \quad x \in \mathring{R}$$

Gauge related configurations are **not** mapped to the same gauge-fixed configuration.

(P.S. Bdary conditions are imposed independently of A, not as function $f_{\partial R}(A_{|\partial R}) = 0$).

The source of 'edge-modes'?

If each gauge-fixed configuration corresponds to a unique physical state, and

if $\mathcal{G}_{\mathring{R}} \subset \mathcal{G}$ is the subgroup of gauge transformations such that $\lambda_{|\partial R} = \mathrm{Id}$, then

it would seem we are left with $\mathcal{G}/\mathcal{G}_{\mathring{R}}$ gauge dofs that "become physical".

Could these be Rovelli's 'handles'? [Donnelly&Freidel '16]

But isn't gauge just redundant "fluff"? Why would it be special at the boundary?

Outline

Where we are and where we want to go.

What do we have so far?

- There seems to be a relationship between: inter-regional coupling, non-trivial change of boundary gauge conditions ($\delta\lambda_{|\partial R} \neq$ const.), and boundary gauge degrees of freedom that "turn physical"

What do we want?

- To talk about regional physical processes but still be able to compose them. Cannot couple regional gauge-invariant functionals, only gauge-covariant ones. [Rovelli '13]

Gauge-fixing lets us talk about regional physical processes but loses regional covariance properties (loses Rovelli's 'handle').

So, we want a decomposition of a process into a "physical description" part, and a "covariant 'handle' for coupling" part.

The gist of it: an analogy

Standard story for motivating the gauge-connection:

For ψ a (complex)-scalar field,

Terms like $\partial_\mu \psi$ not covariant under $\psi \to e^{i\lambda(x)}\psi$.

(that is, $\partial_\mu \psi \to e^{i\lambda(x)}(\partial_\mu \psi + i(\partial_\mu \lambda)\psi)$)

Solution: $\partial_\mu \to D_\mu = \partial_\mu + A_\mu$, where A_μ is a connection-form.

Transformation properties, $A_\mu \to A_\mu - \partial_\mu \lambda$, s.t. $D_\mu \psi \to e^{i\lambda(x)} D_\mu \psi$.

The gist of it: an analogy

Heuristically, we have configuration spaces associated to regions:

$\Phi := (\text{Fields})_{R\cup\overline{R}}\,;\ \Phi_R := (\text{Fields})_R\,;\ \Phi_{\overline{R}} := (\text{Fields})_{\overline{R}}$, where

$$(\text{Gauge inv})_{R\cup\overline{R}} \neq (\text{Gauge inv})_R \times (\text{Gauge inv})_{\overline{R}}$$

According to our gauge-fixed analysis, how do we allow $\delta\lambda_{|\partial R} \neq \text{const.}$?

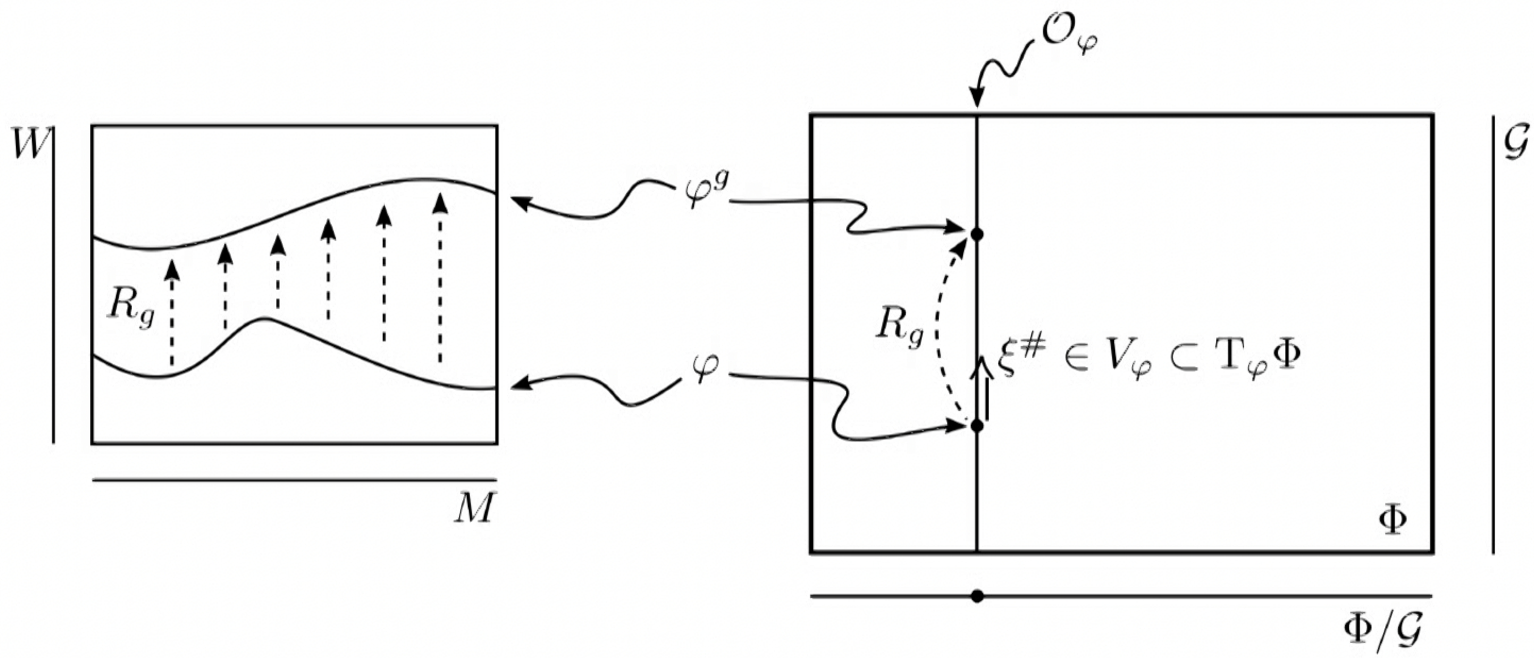
Replace point-dependent gauge transformations $\lambda(x)$, by transformations depending on the configuration, $\lambda[A;x)$ (DeWitt mixed-dependence notation)

Allows bdary conditions imposed as functions of A: $f_{\partial R}(A_{|\partial R}) = 0$ (unified view)

Gauge-covariance for $\lambda[A;x)$ (i.e. s.t $\delta\lambda \neq 0$). In analogy to extending covariance under spacetime derivatives ($\partial_\mu \to D_\mu$), under field-space variations:

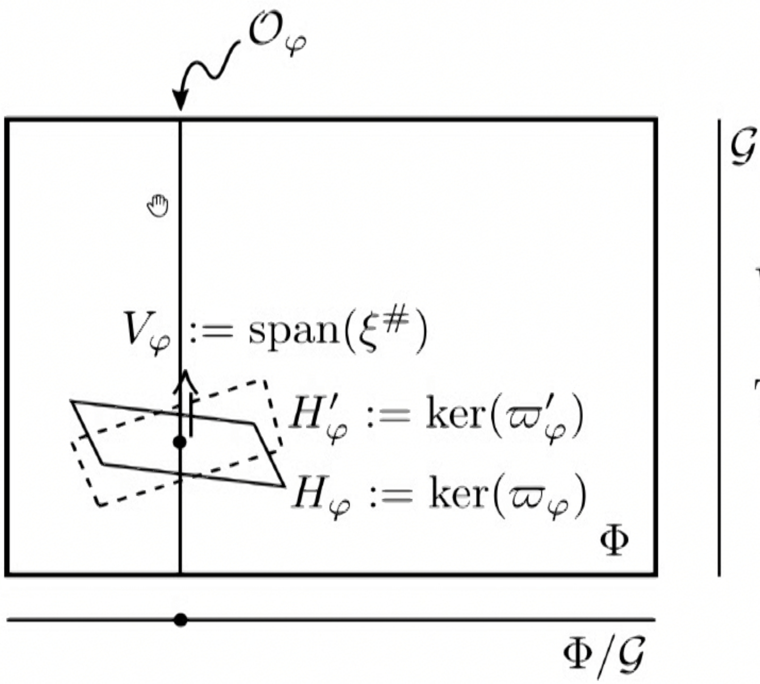
$$\delta(e^{i\theta[\varphi;x)}\psi) = e^{i\theta[\varphi;x)}(\delta\psi + i(\delta\theta)\psi) \text{ and we require } \delta_\varpi(e^{i\theta}\psi) = e^{i\theta}\delta_\varpi\psi$$

Field-space as a principal fiber bundle



$\varphi \in C^\infty(M, W) = \Phi$	field value (e.g. section of vector bundle over M)
$g \in C^\infty(M, G) = \mathcal{G}$	gauge parameter, will be generalized to depend on field.
$\xi \in \mathrm{Lie}(\mathcal{G})$	"infinitesimal" gauge parameter
$\xi^\#$	infinitesimal gauge transformation (vector field in field-space, Φ).

Gauge vs physical: a relative notion



$$V_\varphi = \mathrm{T}_\varphi \mathcal{O}_\varphi$$

$$\begin{aligned} \mathrm{T}_\varphi \Phi &= H_\varphi \oplus V_\varphi \\ &= H'_\varphi \oplus V_\varphi \end{aligned}$$

The gauge structure identifies the **vertical** subspaces, canonically

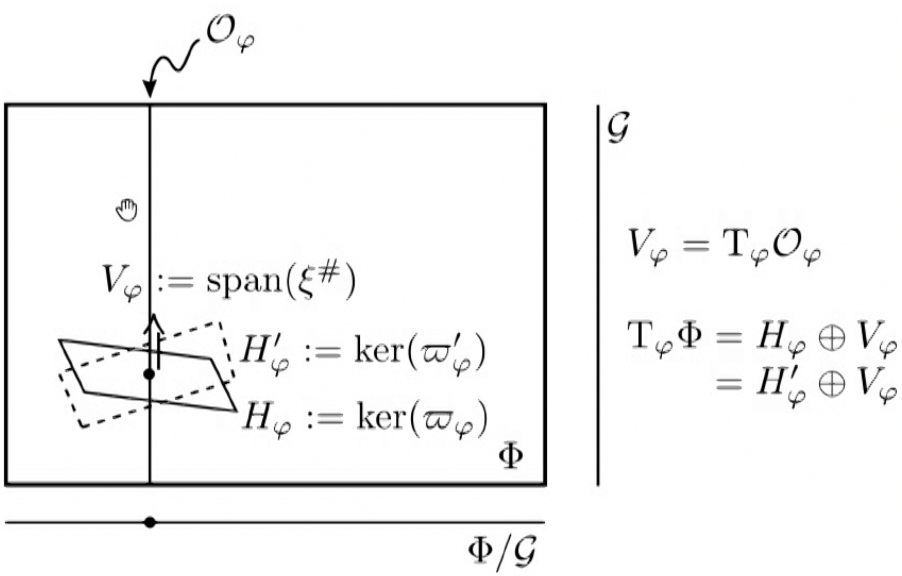
(i.e. pure-gauge transformations are well-defined)

Their **horizontal** complements identify physical variations but are noncanonical

Thus, a general field variation $\mathbb{X} \in T_\varphi \Phi$ has no canonical decomposition into pure gauge and physical parts.

(if it's $\mathbb{double-struck}$, i.e. $\mathbb{X}, \mathbb{L}_{\mathbb{X}}, \mathbb{d}$, etc, it is a field-space object)

A covariant gauge frame: the connection ϖ [Var-Pie]



Covariant choice of horizontal (physical) subspaces encoded in a a field-space connection-form,

$$\varpi \in \Lambda^1(\Phi, \mathrm{Lie}(\mathcal{G}))$$

This is a field-space 1-form, transforming covariantly under gauge transformations:

$$\varpi(\xi^\#) = \xi \qquad \text{(vertical projector)}$$
$$\mathbb{L}_{\xi^\#}\varpi = [\varpi, \xi] + \mathbb{d}\xi \quad \text{(covariance)}$$

$[\cdot,\cdot]$ is commutator for $\mathfrak{g}$ (and $\mathbb{d}\xi \neq 0$ only if gauge transformation is field-dependent)

What we wanted

Given a "process", $\mathbb{X} \in T_\varphi \Phi$ and a ϖ, we have a physical component, $\widehat{H}(\mathbb{X})$, and the gauge-covariant handle, to be used for gluing, $\widehat{V}(\mathbb{X})$. What we wanted.

The covariance condition does not require $\xi_{|\partial R} = 0$, nor $\varphi_{|\partial R}$ to be fixed.

If the condition can be satisfied, then

$$(R_g)_*(\widehat{H}_\varphi(\mathbb{X}_\varphi)) = \widehat{H}_{R_g \varphi}((R_g)_* \mathbb{X}_\varphi)$$

1-1 map between horizontal fields along full gauge-orbit. They all project to the same base-space vector.

In other words, $\widehat{H}$ will mend a gap left by standard gauge-fixings.

(Recall, for the Abelian case, $\partial^\mu(A_\mu) = 0$ for $\delta\lambda_{|\partial R} \neq$const.: $\widehat{\mathsf{f}}(A^\lambda) \neq \widehat{\mathsf{f}}(A^{\lambda'})$)

How does $\widehat{V}$ work for coupling?

A covariant coupling of physical processes

Suppose we have two regions: $M = R \cup \overline{R}$, sharing the boundary ∂R.

Each has a field-space, e.g.: Φ_R, and let $\chi_R : \Phi \to \Phi_R$ be the restriction.

Defining $\mathbb{X}_R := \chi_R(\mathbb{X})$, the following results hold: [HG,Riello&Hopfmuller'18]

- The difference, at the boundary, between two descriptions of phys. processes (the "translation"):

$$(\widehat{H}_R(\mathbb{X}_R) - \widehat{H}_{\overline{R}}(\mathbb{X}_{\overline{R}}))_{|\partial R} = \overbrace{(\widehat{V}_{\overline{R}}(\mathbb{X}_{\overline{R}}) - \widehat{V}_R(\mathbb{X}_R))_{|\partial R}}^{\text{"translation" at the boundary}} =: \xi^{\#}_{\partial R}$$

  is not only vertical, but also intrinsic to the boundary (i.e.not only in $V \subset T\Phi$ but also in $V_{\partial R} \subset T\Phi_{\partial R}$), where $\xi_{\partial R} \in \mathrm{Lie}(\mathcal{G})_{\partial R}$.

  I.e. we can prove difference is due to a mode gauge-valued and parallel to the boundary (like an edge-mode).

- The global "purely physical" component of a process matches the sum of the local "purely physical" components of the projections if and only if $\xi^{\#}_{\partial R} = 0$. I.e.

$$\widehat{H}_R(\mathbb{X}_R) + \widehat{H}_{\overline{R}}(\mathbb{X}_{\overline{R}}) = \widehat{H}(\mathbb{X}) \qquad \text{iff} \qquad \xi^{\#}_{\partial R}(\mathbb{X}_{\overline{R}}, \mathbb{X}_R) = 0$$

In other words, there exists an injective map:

$$\text{“global physical modes”} / \text{“local physical modes”} \to \text{“edge-modes”}$$

but the edge-modes are not independent of the regional field configurations. Therefore they are not new, independent degrees of freedom.

The connection-form, ϖ, itself does not need new dofs to be introduced.

For certain types of ϖ (constructed from local matter fields), $\xi^{\#}_{\partial R} = 0$ (for $\mathbb{X}$, $\mathbb{X}_R = \xi_R(\mathbb{X})$, etc).

Let's see an example of a ϖ constructed from a gauge-field. (here in general $\xi^{\#}_{\partial R} \neq 0$)

An example: ϖ from a supermetric

Recall $\varpi_\varphi \Leftrightarrow H_\varphi \subset T_\varphi\Phi$. Is there a natural notion of "horizontality"?

PFB structure not enough, but in presence of supermetric $\mathbb{G}$ on Φ: $H_\varphi := (V_\varphi)^\perp$.

Is there a natural supermetric?

Pure gauge-theory: kinetic term provides unique gauge-compatible ultralocal supermetric:

$$K = \mathbb{G}_R(\dot{A}, \dot{A}) = \int_R g^{ij}\mathrm{Tr}(\dot{A}_i\dot{A}_j), \Rightarrow \ \mathbb{G}_R(\mathbb{X}, \mathbb{Y}) = \int_R g^{ij}\mathrm{Tr}(\mathbb{X}_i\mathbb{Y}_j)$$

Singer-DeWitt connection (SdW)

$$\mathrm{D}^2_A\varpi = D^i_A \mathbb{d} A_i \qquad \text{where} \qquad \mathrm{D}^i_A := \partial^i + [A^i, \cdot]$$ [Singer'78,'81, DeWitt'03]

$$n_i \mathrm{D}^i_A \varpi_{|\partial R} = n_i \mathbb{d} A^i_{|\partial R}$$ field-dependent, covariant bdary conditions (**new**)

No limitation on $A^a_{i\,|\partial R}$ nor on $g(x)_{|\partial R}$: gauge-related horizontal vectors project identically to $\Phi/\mathcal{G}$.

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General properties, gluing and example

A technical problem: Noether charges

Relation to BRST, field-space curvature, etc.

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Symplectic charges

$L = \mathcal{L}(\varphi)\mathrm{d}^d x$ is a Lagrangian (spacetime-) *density*. Define the presymplectic potential $\theta \in \Lambda^1(\Phi) \otimes \Lambda^{d-1}(M)$:

$$\mathbb{d}L = \mathrm{EL}_I(\varphi)\mathbb{d}\varphi^I + \mathrm{d}\theta(\varphi)$$

where d is the spacetime exterior derivative, and $\mathrm{EL}_I(\varphi)$ are the (densitized) Euler-Lagrange equations for φ^I.

Noether: for $\xi \in \mathrm{Lie}(\mathcal{G})$ s.t. $\mathbb{L}_{\xi^\#} L = 0$, Noether gauge charges:

$$Q[\xi] := \int_R \theta(\mathrm{D}_A \xi) \approx \int \mathrm{d}j_\xi = \oint_{\partial R} \mathrm{Tr}(E(x)\xi(x))$$

For Yang-Mills, where the electric-field E is the pull-back of $*F$ to ∂R, $\approx$ means "up to terms proportional to $\mathrm{EL}_I(\varphi)$" and the current density is given by $j_\xi = \mathrm{Tr}(E(x)\xi(x))$.

Two problems

- Ambiguity:

$$\theta \mapsto \theta + d\alpha \Rightarrow Q[\xi] + \oint_{\partial R} \alpha(\mathrm{D}_A \xi)$$

Which charges are physical? How to tell pure gauge transformations ($Q = 0$) from physical symmetries ($Q \neq 0$)?

- Field-independent gauge invariance (under ξ s.t. $\mathbb{d}\xi = 0$) $\not\Rightarrow$ Field-dependent gauge invariance (under ξ s.t. $\mathbb{d}\xi \neq 0$ at boundary):

$$\mathbb{L}_{\xi^\#}\theta \approx \mathrm{d}\,\mathrm{Tr}(E\mathbb{d}\xi) \neq 0$$

gauge-variant charges if we allow changing boundary conditions.

Both problems related to the physical meaning of gauge-transformations at boundary.

Horizontal symplectic charges

It turns out that using ϖ, replace $\mathbb{d} \to \mathbb{d}_H$ and find unique definition

$$\theta_H := \mathrm{Tr}\,(E\mathbb{d}_H A) \approx \theta - \mathrm{d}\,\mathrm{Tr}\,(E\varpi)$$

With this definition, $\mathbb{L}_{\xi^\#}\theta_H = 0$, and

$$\Omega_H := \mathbb{d}_H\theta_H = \mathbb{d}\theta_H \approx \Omega - \mathbb{d}\mathrm{d}\,\mathrm{Tr}\,(E\varpi)$$

(Ω_H is still $\mathbb{d}$-exact—giving a good symplectic form— and modification from Ω is also d-exact—i.e. exists at boundary)

It turns out:

$$Q_H[\xi] = \int_R \theta_H(\mathrm{D}_A\xi) \approx j_\xi - j_{\varpi(\xi^\#)} = 0$$

(because of property $\varpi(\xi^\#) = \xi$)

- So where did the electric (or color) charge go?

Reducible configurations: towards global charges

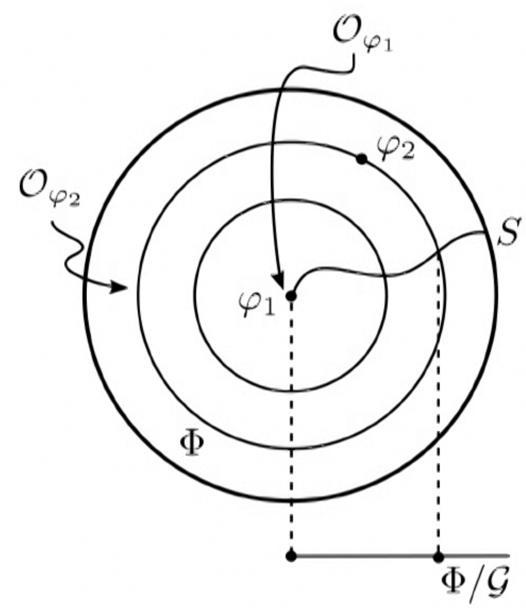
Field-space Φ is not a bona-fide principal fiber bundle.

Quotient space $\Phi/\mathcal{G}$ is a stratified manifold.

That is because there exist reducible configurations:

$$\exists\ \varphi^*, \xi^* \neq 0 \mid (\xi^*)^\#_{|\varphi^*} = 0$$

e.g. for YM: $\exists\ A^*, \xi^* \neq 0 \mid D_{A^*}\xi^* = 0$



Certain gauge-directions are in the "blind-spot" of certain field-configurations.

Global charges from the SdW connection

For Yang-Mills, charges appear only when we add matter, ψ, to the field-space.

Say ψ in fundamental representation $\mathrm{SU}(N)$:

$$\xi^{\#} = \int (\mathrm{D}_A \xi) \frac{\mathbb{d}}{\mathbb{d}A} - (\xi\psi) \frac{\mathbb{d}}{\mathbb{d}\psi}$$

Then (given the corresponding Lagrangian)

$$\theta_H = \mathrm{Tr}\,(E \mathbb{d}_H A) + \overline{\psi} \mathbb{d}_H \psi$$

Then $Q^H_{\mathrm{SdW}}[\xi] = 0$, except at $A^*, \xi^* \neq 0$ s.t. $\mathrm{D}_{A^*}\xi^* = 0$ when:

$$Q^H_{\mathrm{SdW}}[\xi^*] = \int_R \mathrm{Tr}\,(J\xi^*) \approx \oint \mathrm{Tr}\,(E\xi^*)$$

where J is the matter current density (and $j_\xi = J_a \xi^a$). In the Abelian case, recovers electric flux.

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The connection-form

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A technical problem: Noether charges

Relation to BRST, field-space curvature, etc.

Conclusions

Section, foliation, and distribution (for closed R).

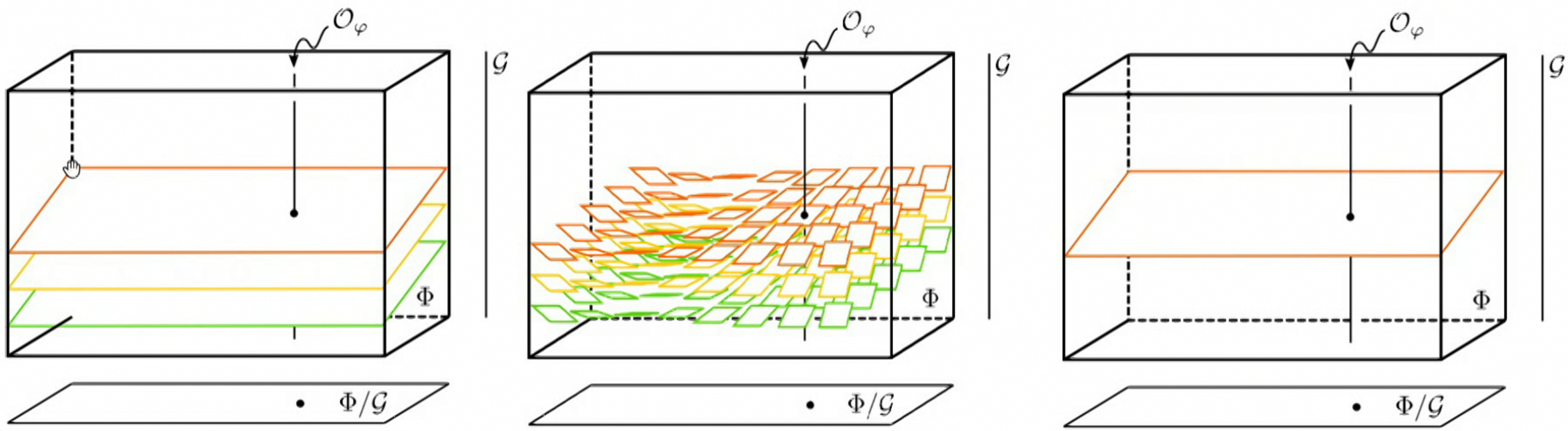

Figure: A visual illustration of the different structures provided respectively by: the rigid vertical transformations compatible with BRST symmetry, the horizontal distribution given by the connection-form, and gauge-fixing.

Relation to BRST (for closed R)

ϖ is graded (1-form on field-space).

since $\varpi(\xi^{\#}) = \xi$, ϖ is a place-holder for gauge-transformations.

ϖ avoids counting as physical boundary dofs which are fundamentally gauge.

For vertical derivative, $\mathbb{d}_V$:

$$\mathbb{d}_V \varpi = -\frac{1}{2}[\varpi, \varpi] \qquad \text{2-form}$$

$$\mathbb{d}_V A = \mathrm{D}_\varpi A \qquad \text{1-form}$$

BRST ghost, η is graded

BRST-transf. replaces generators of gauge-transf. by ghosts, e.g. $\xi^a \to \eta^a$

BRST ghosts avoid counting as physical dofs which are fundamentally gauge.

For BRST transformation s:

$$s\eta = -\frac{1}{2}[\eta, \eta] \qquad \text{gh}\# = 2$$

$$sA = \mathrm{D}_\eta A \qquad \text{gh}\# = 1$$

EXTRA: $\quad \mathbb{d}_H = \mathbb{d}\varpi + \frac{1}{2}[\varpi, \varpi] =: \mathbb{F} \quad$ curvature!

(If horizontal distribution is non-integrable (not a section))

relates [Thierry-Mieg'80, Bonora&Cotta-Ramusino'83] to [Singer'78, Vilkowiski'86, DeWitt'03] and generalizes to bdary.

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When introducing boundaries in gauge theories a number of issues arise.

On the conceptual side, we need to find a way to couple the physical information residing in adjacent regions.

(e.g. regional gauge-fixed (or gauge-invariant) dofs don't couple well)

On the technical side, we don't want gauge degrees of freedom at boundaries to become physical.

(e.g. by carrying physical charges).

Introducing a covariant structure in each regional field-space, we are able to resolve these questions simultaneously, without the need for new degrees of freedom playing the role of handles at the boundaries (as in [Donnelly&Freidel '16]).

But there are more things to be done.

Open problems

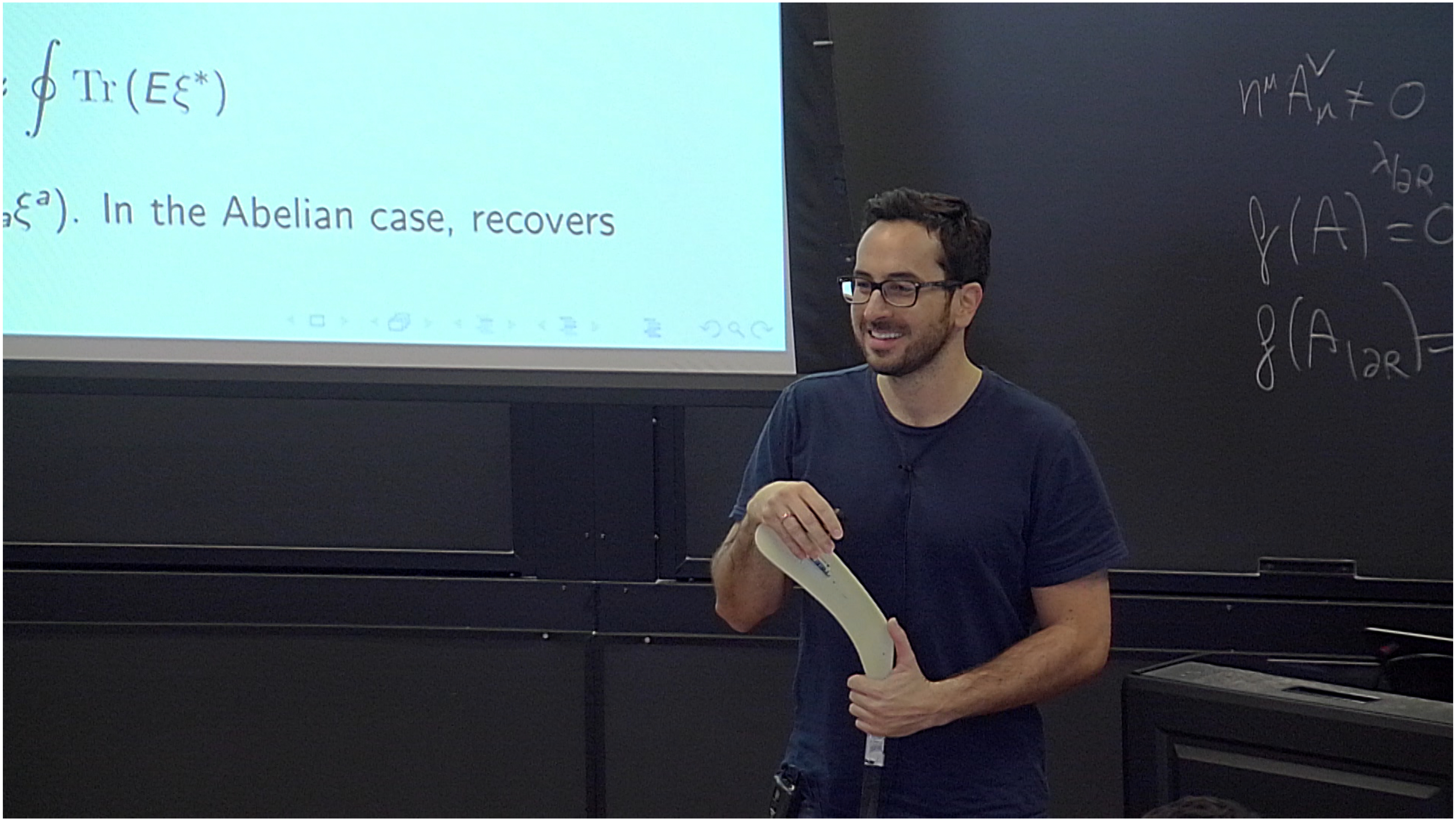
- Calculation of entanglement entropy.
- Extension to the diffeomorphisms (for GR).
- Relation to BV-BFV framework?
- Adapt formalism to restrictions on field-space, e.g. when stringent boundary conditions are imposed on field-content, to describe particular subsystems.

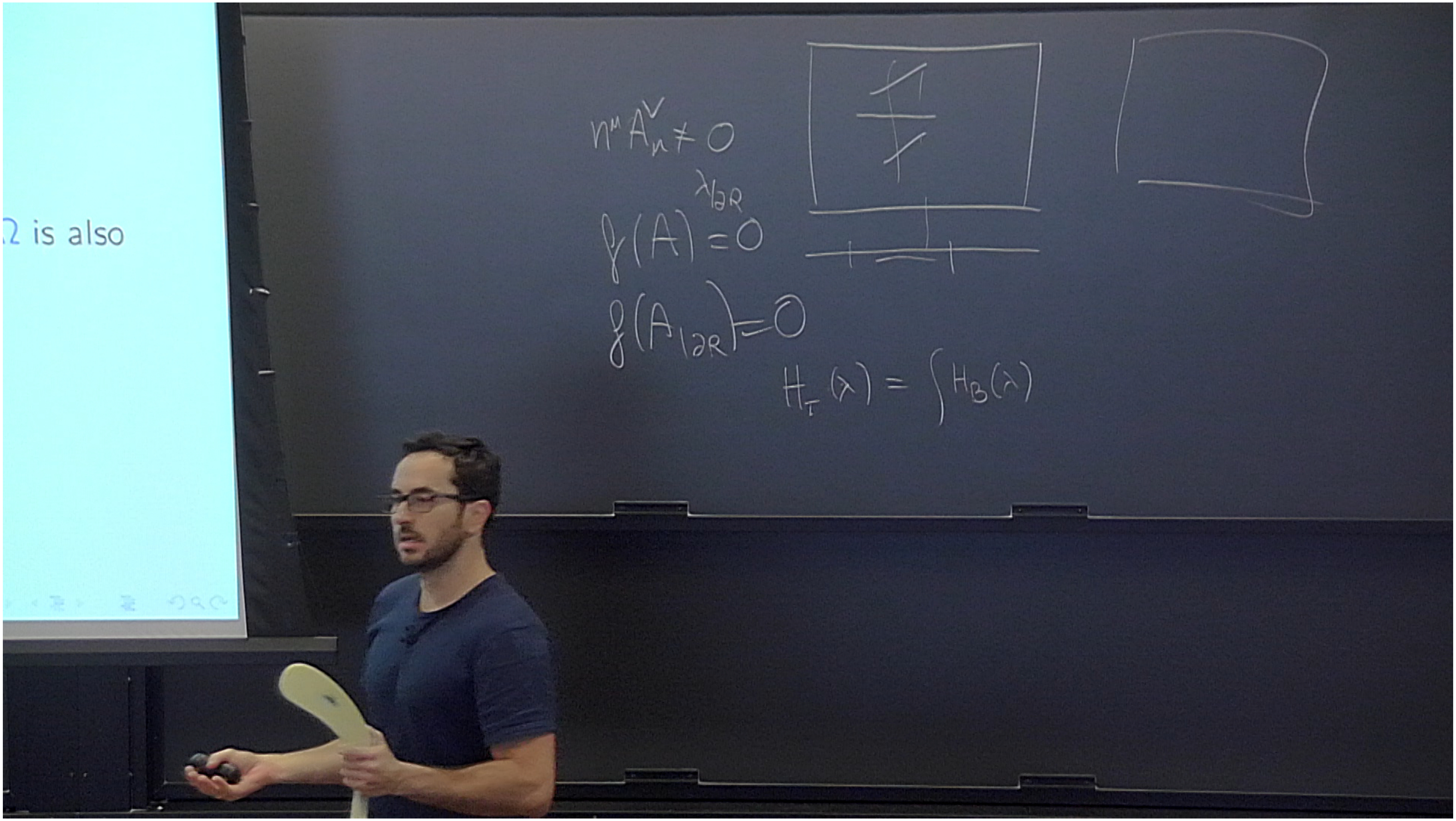
  (changes derivation of ϖ and structure of $\Phi/\mathcal{G}$)

  - Have something to say about e.g. supertranslation charges? [Strominger et al '13-'18]

(Haven't talked about matter-based ϖ: The Higgs connection. Other interesting open opportunities.)

THANK YOU


$\oint \mathrm{Tr}(E\xi^*)$
ξ^a). In the Abelian case, recovers
$n^\mu A^\nu_\mu \neq 0$
$\lambda_{|\partial R}$
$g(A) = 0$
$g(A_{|\partial R})$


Ω is also
$n^\mu A_\mu^\nu \neq 0$
$\lambda_{|\partial R}$
$f(A) = 0$
$f(A_{|\partial R}) = 0$
$H_T(\lambda) = \int H_B(\lambda)$