Title: Application of the black hole-ï¬,uid analogy: identiï¬•cation of a vortex ï¬,ow through its characteristic waves

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Abstract: <p>Black holes are like bells; once perturbed they will relax through the emission of characteristic waves. The frequency spectrum of these waves is independent of the initial perturbation and, hence, can be thought of as a `fingerprint' of the black hole. Since the 1970s scientists have considered the possibility of using these characteristic modes of oscillation to identify astrophysical black holes. With the recent detection of gravitational waves, this idea has started to turn into reality.
Inspired by the black hole-fluid analogy, we demonstrate the universality of the black-hole relaxation process through the observation of characteristic modes emitted by a hydrodynamical vortex flow. The characteristic frequency spectrum is measured and agrees with theoretical predictions obtained using techniques developed for astrophysical black holes. Our findings allow for the first identification of a hydrodynamical vortex flow through its characteristic waves.
The flow velocities inferred from the observed spectrum agree with a direct flow measurement. Our approach establishes a non-invasive method, applicable to vortex flows in fluids and superfluids alike, to identify the wave-current interactions and hence the effective field theories describing such systems.</p>

Silke Weinfurtner
The University of Nottingham

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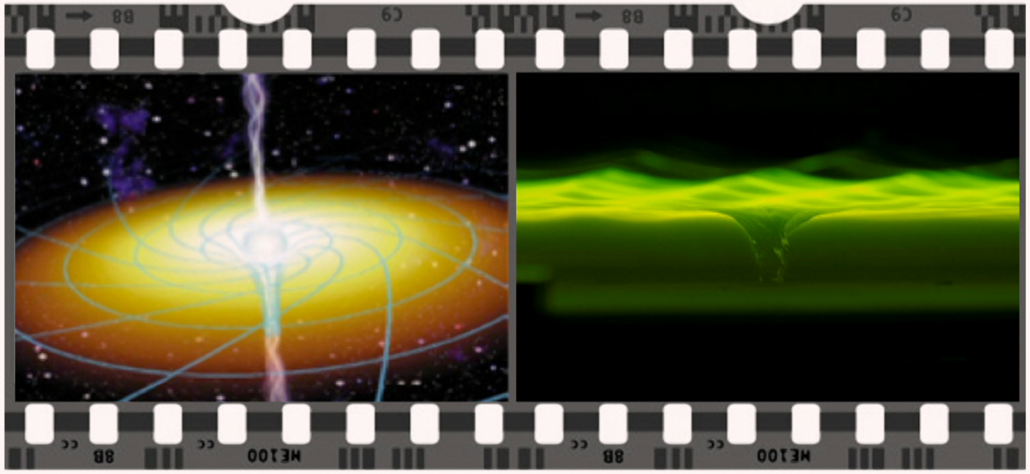
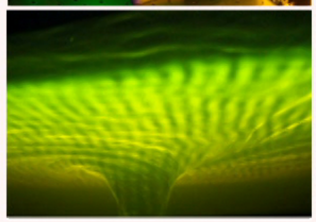
Application of the black hole-fluid analogy:
The black hole relaxation process in a bathtub



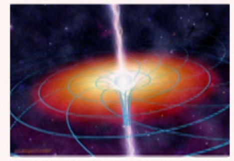
Analogue Simulators

There exists a **broad class of systems**



Fluctuations described by an **effective Relativistic Classical and Quantum Field Theory** in **flat** or **curved** spacetimes.

Astrophysical systems



Rotating Black Holes

Black Holes

Cosmological spacetimes

Quantum Vacuum

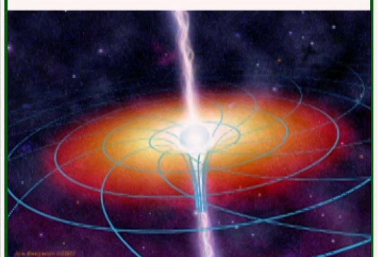
Simulators for classical and quantum field theory in CS

Possibility for experimental verification of some of the exotic effects predicted to occur in our universe…

Black Holes

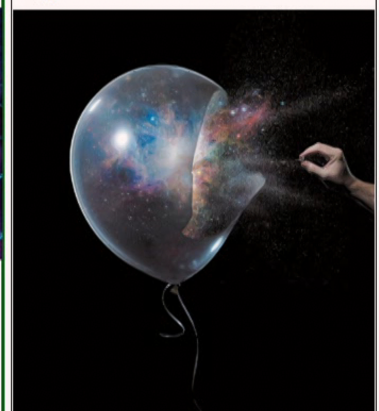
e.g. Hawking radiation

Rotating Black Holes



e.g. Superradiance, Light bending Ringdown

Cosmological spacetimes



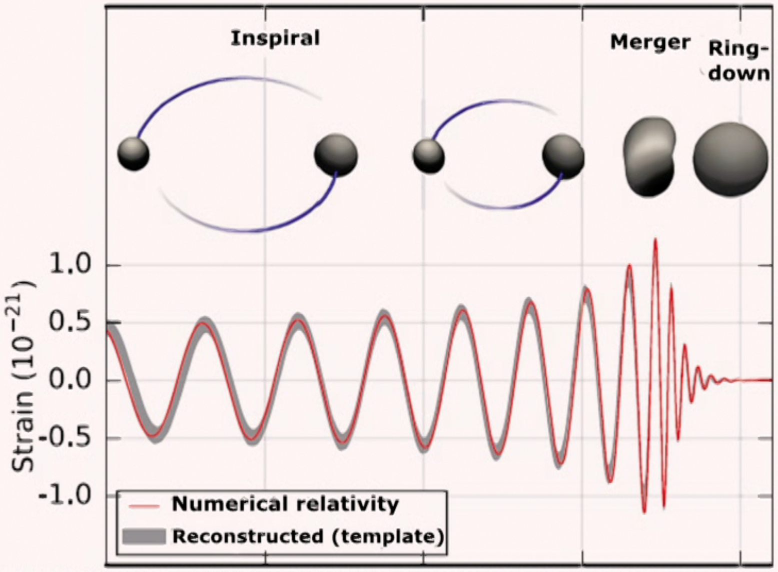
e.g. Early Universe Processes

Quantum Vacuum

e.g. The False Vaccuum Decay

Fluid Dynamics

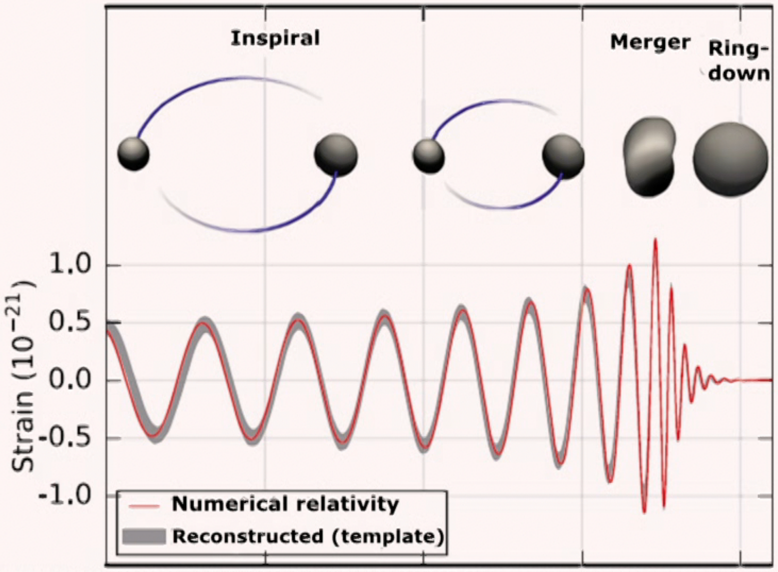
Gravitational Physics

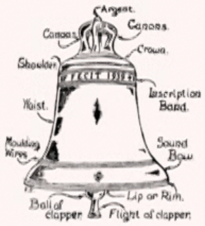


Why is this interesting?

Fluid Dynamics

Gravitational Physics





Do vortex flows ring down when out of equilibrium?

If yes, can use the ringdown modes to probe vortex flows?

Light-cones/Horizons in fluid flows?

The beginnings of analogue gravity

1981: W.G. Unruh: Experimental Black Hole Evaporation?

Analogue gravity systems:

The <u>equations of motion for linear perturbations</u> in an analogue/effective/emergent gravity system can be simplified to

$$\frac{1}{\sqrt{-g}}\partial_a\left(\sqrt{-g}g^{ab}\partial_b\psi\right)=0$$

defining an effective/acoustic/emergent metric tensor:

$$g_{ab}\propto\begin{bmatrix}-\left(c^2(\mathbf{x},t)-v^2(\mathbf{x},t)\right) & -\vec{v}^T(\mathbf{x},t)\\ -\vec{v}(\mathbf{x},t) & \mathbf{I}_{d\times d}\end{bmatrix}$$

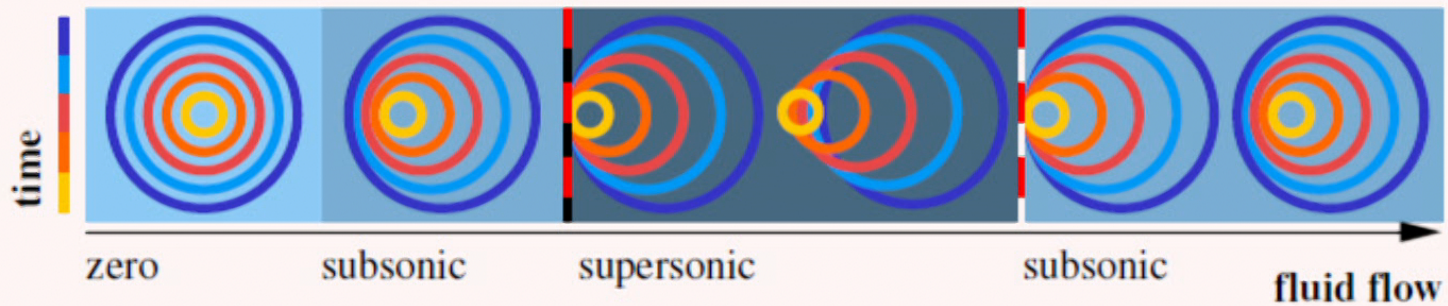
<u>Where do we expect such a behavior?</u>

Broad class of systems with various dynamical equations, e.g. electromagnetic waveguide, fluids, ulatra-cold gas of Bosons and Fermions.

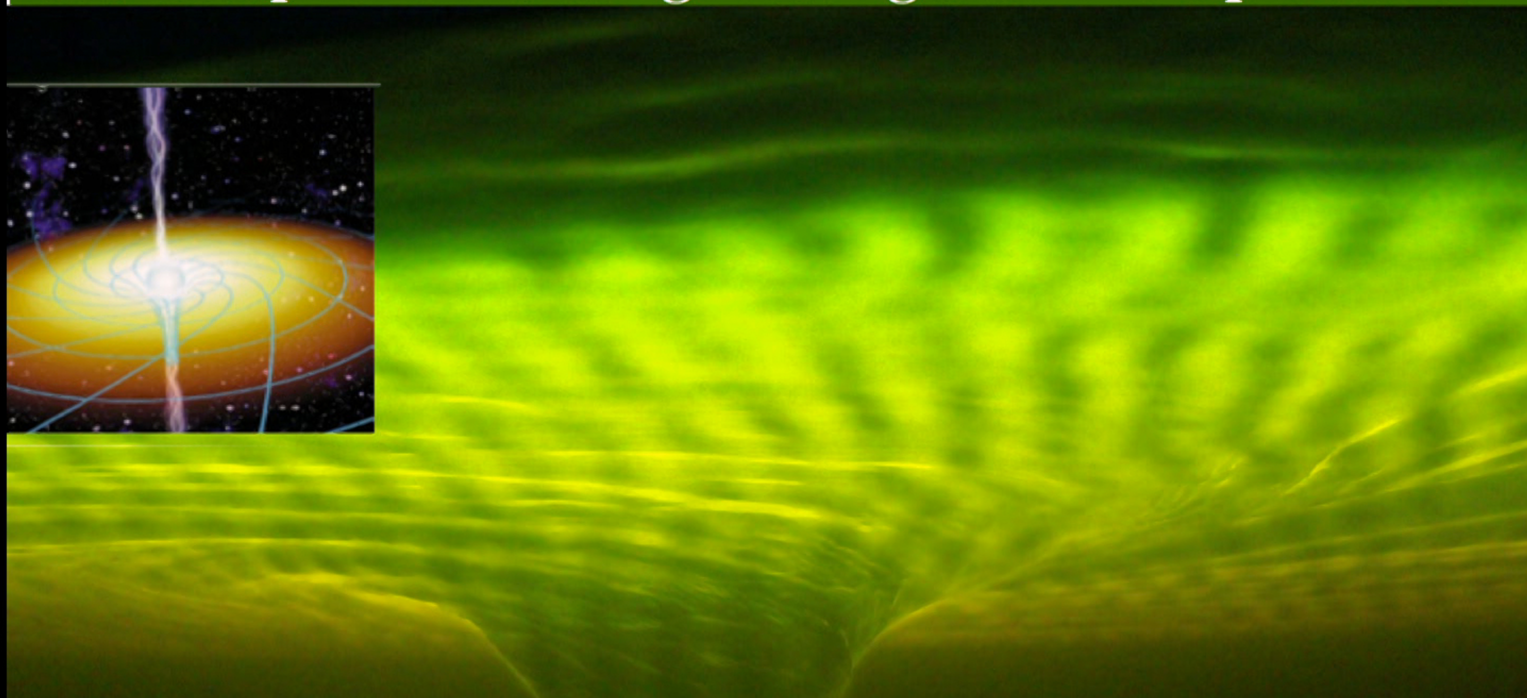
In example below: Fluid dynamics derived from conservation laws:

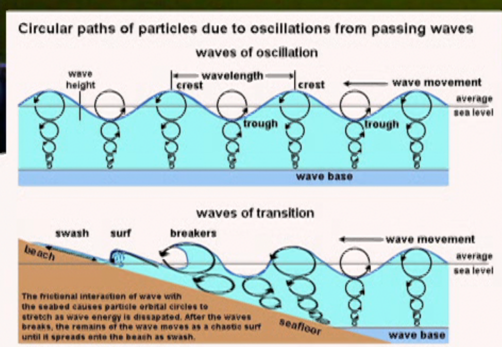
$$\frac{\partial\rho}{\partial t}+\nabla\cdot(\rho\,\mathbf{v})=0 \quad \textit{Continuity equation}$$

$$\rho\frac{D\mathbf{v}}{Dt}=-\nabla p \quad \textit{Euler equation}$$



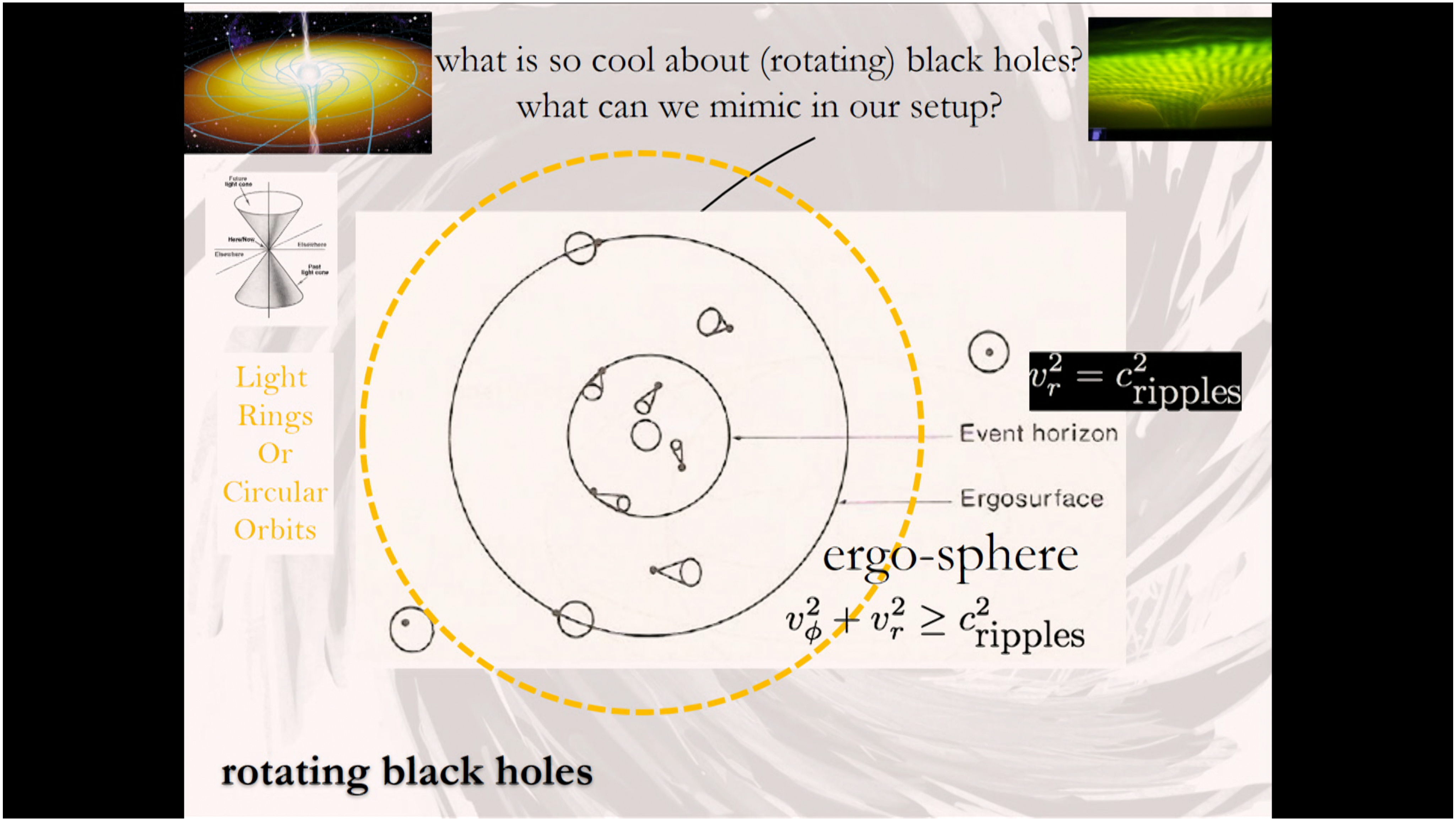
Ideal setup for simulating rotating black hole processes





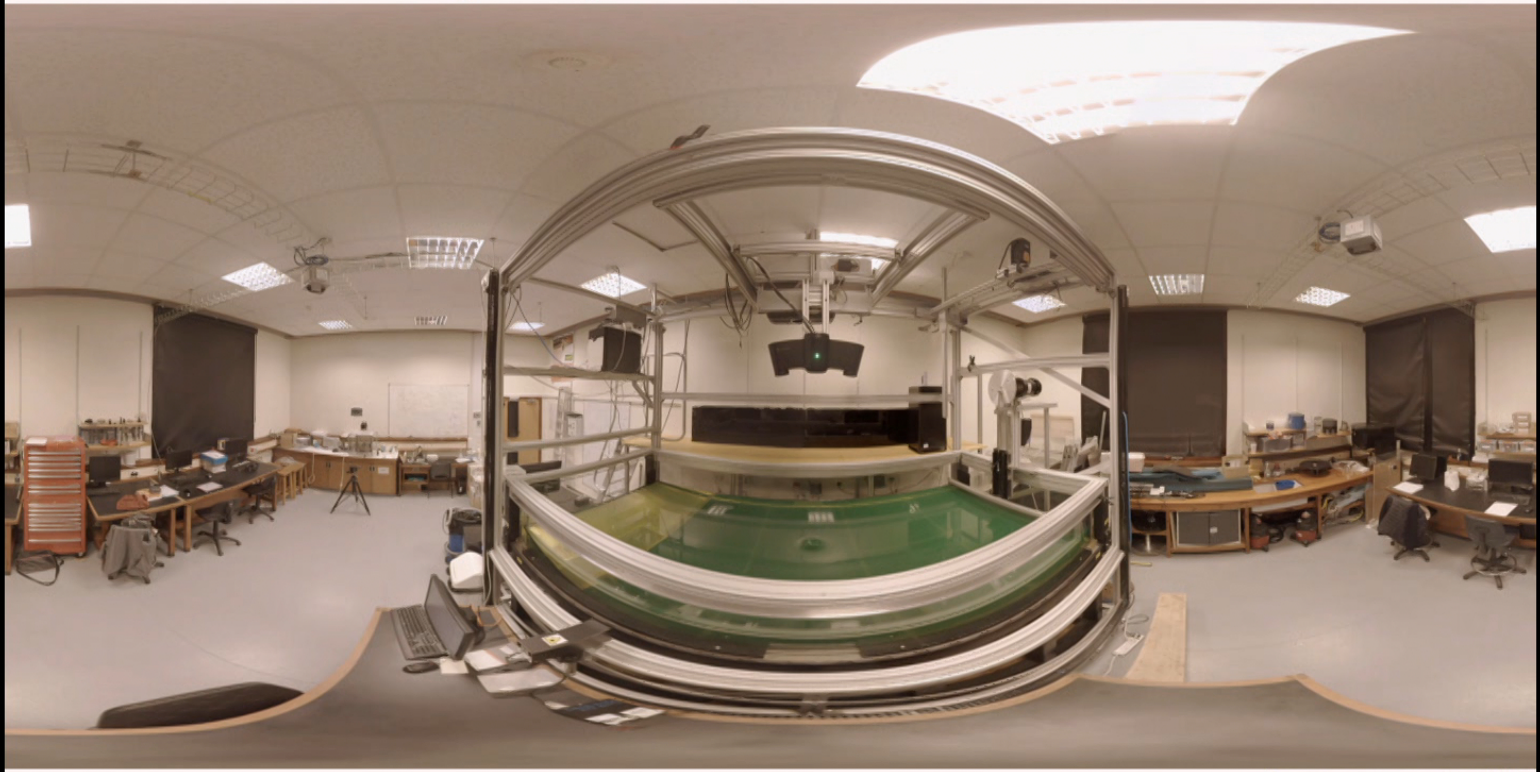
Gravity wave analogs of black holes;
Schützhold, Ralf; Unruh, William G;
2002 Phys Rev D

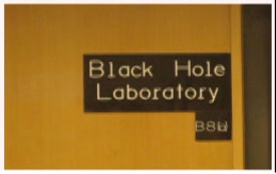
$$g_{\mu\nu} = \left(\frac{h}{\tilde{g}}\right) \begin{pmatrix} -\tilde{g}h + v_{\parallel}^2\Big|_{z=h} & -\mathbf{v}_{\parallel}\Big|_{z=h} \\ -\mathbf{v}_{\parallel}\Big|_{z=h} & \mathbf{I}_{2\times 2} \end{pmatrix}$$



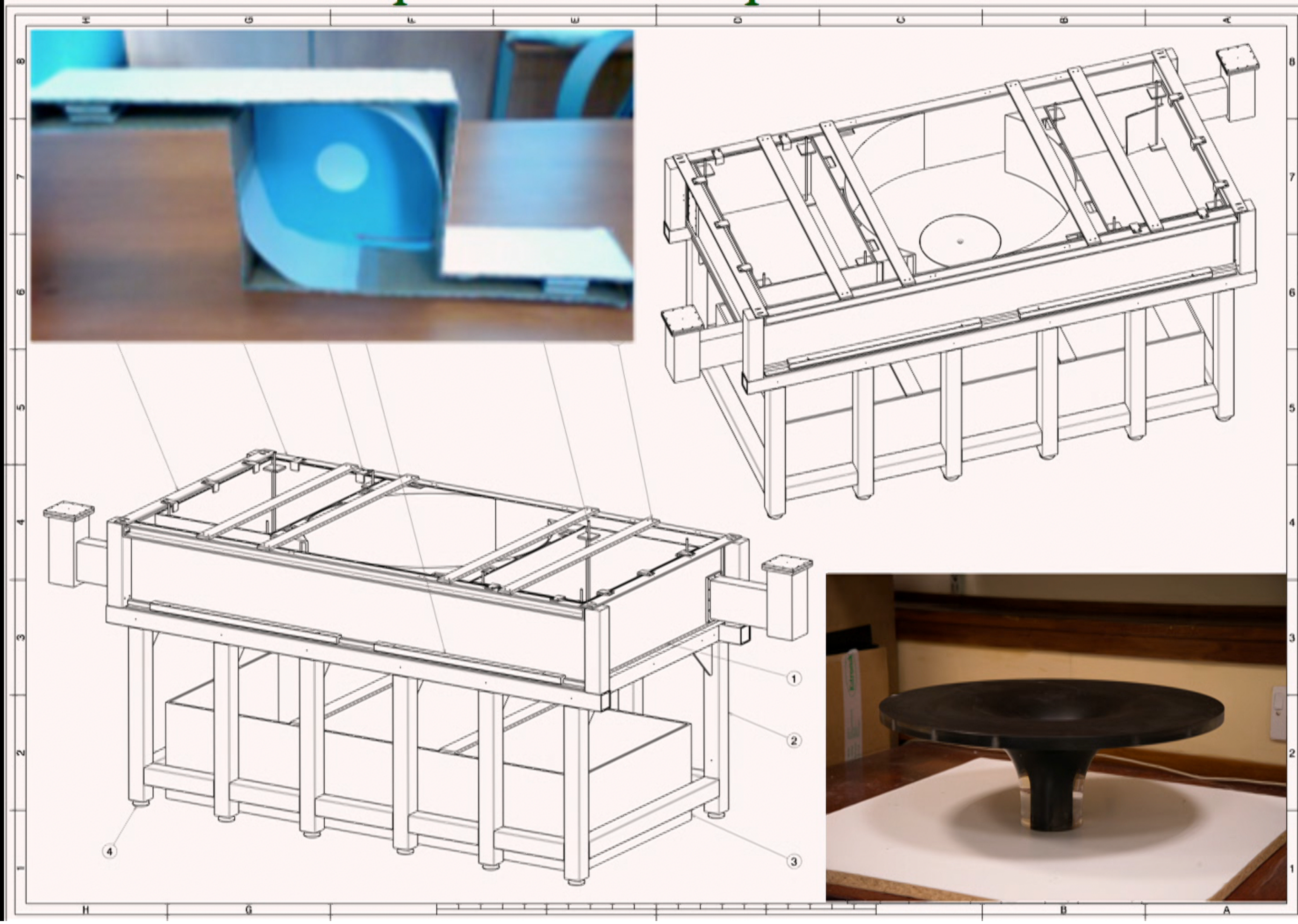
what is so cool about (rotating) black holes?
what can we mimic in our setup?
Future light cone
Here/Now
Elsewhere
Elsewhere
Past light cone
Light
Rings
Or
Circular
Orbits
$v_r^2 = c^2_{\mathrm{ripples}}$
Event horizon
Ergosurface
ergo-sphere
$v_\phi^2 + v_r^2 \geq c^2_{\mathrm{ripples}}$
rotating black holes

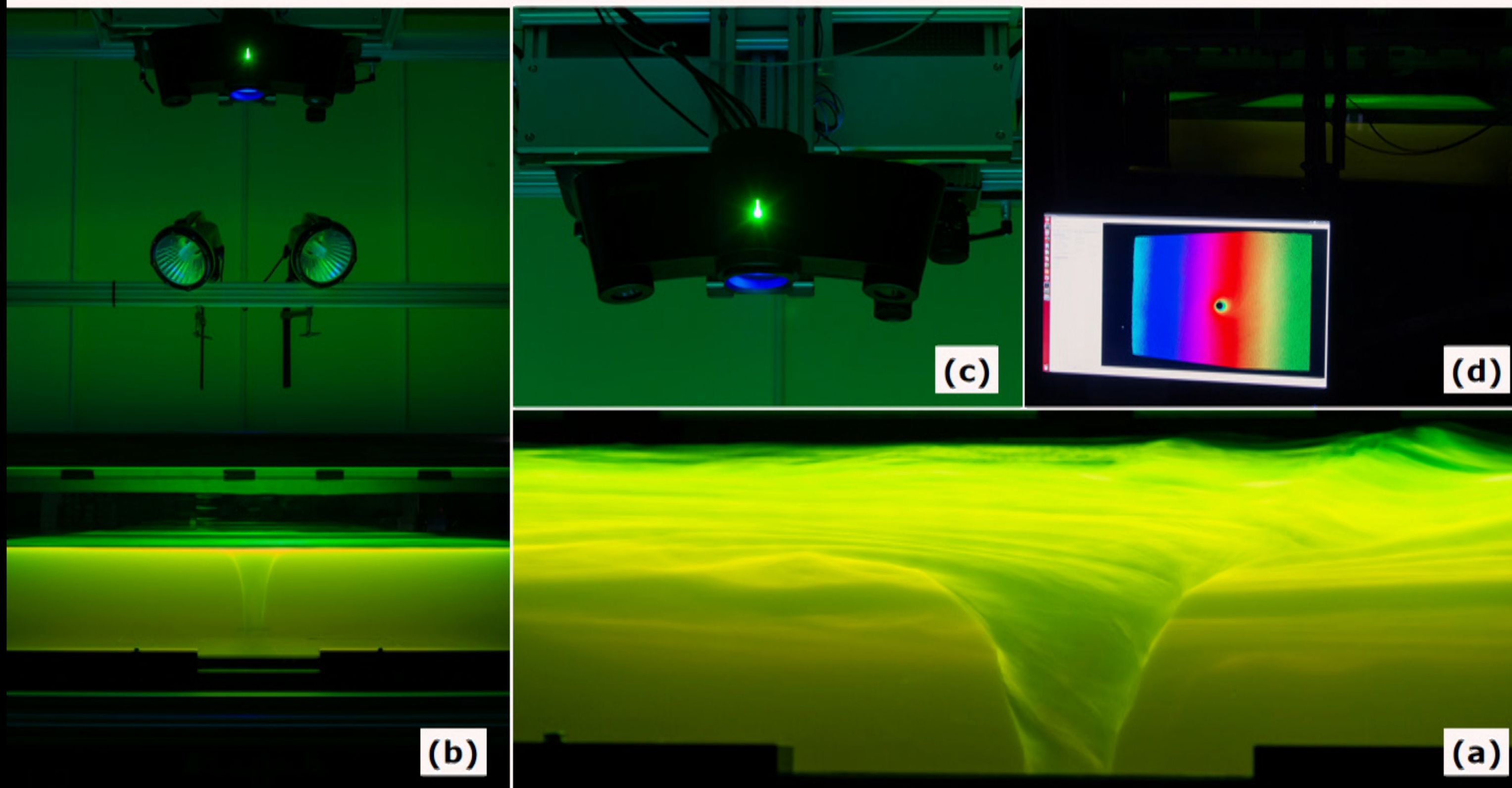
How to set up a hydrodynamic rotating black hole?





Schematics of experimental setup



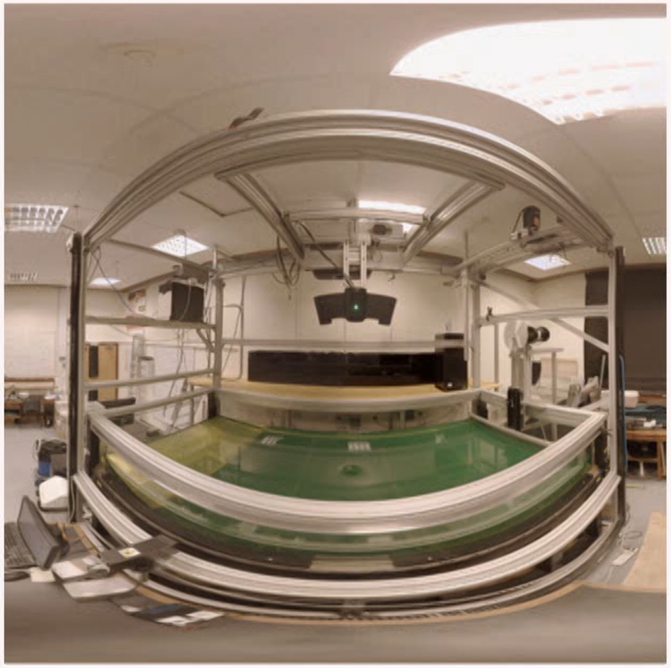


EnShape

New technologies:
In collaboration with EnShape (Jena, Germany) we have developed a new (fast and high accuracy) surface wave detection mechanism. Pending patent application.

Unruh vortex

The goal was to set up an Unruh vortex flow:
To investigate the black hole relaxation process we set up a **vortex flow out of equilibrium**. We call such a restless vortex flow an Unruh vortex: the German word **'Unruhe'** means **restless** and it was chosen in acknowledgment of W.G. Unruh, the founder of analogue gravity studies.

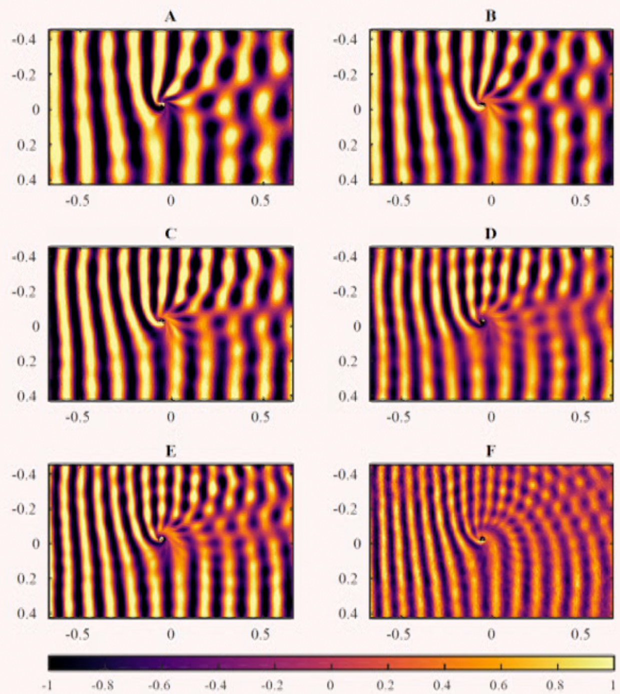


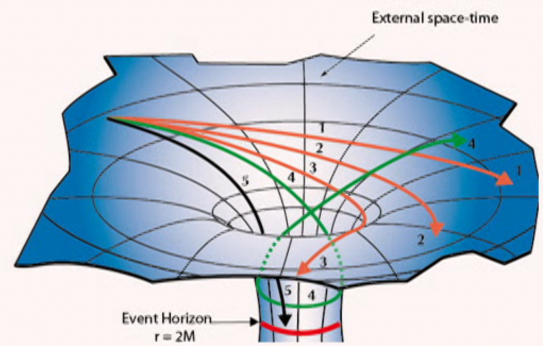
Implementation straightforward:
Water is pumped continuously from one corner at a flow rate of 15±1 l/min. The sink hole is covered until water raises to a height of 10.00±0.05cm. Water is then allowed to drain again, leading to the formation of an Unruh vortex. We recorded the perturbations of the free surface when the flow was in a quasi-stationary state at a water depth of 5.55±1cm. The water elevation was recorded using a Fast-Checkerboard Demodulation method. The entire procedure was repeated 25 times.

What are the theoretically expected ringdown frequencies?

Interlude: a path on an analogue curved spacetime

pattern ↔ geodesics (generalisation of a straight line in curved spacetimes)





Waves on a vortex: rays, rings and resonances

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NATURE PHYSICS | LETTER

Rotational superradiant scattering in a vortex flow

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Affiliations | Contributions | Corresponding author

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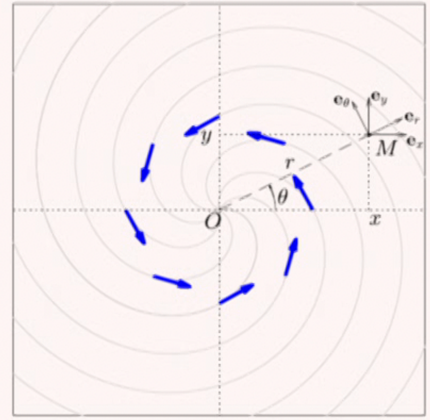
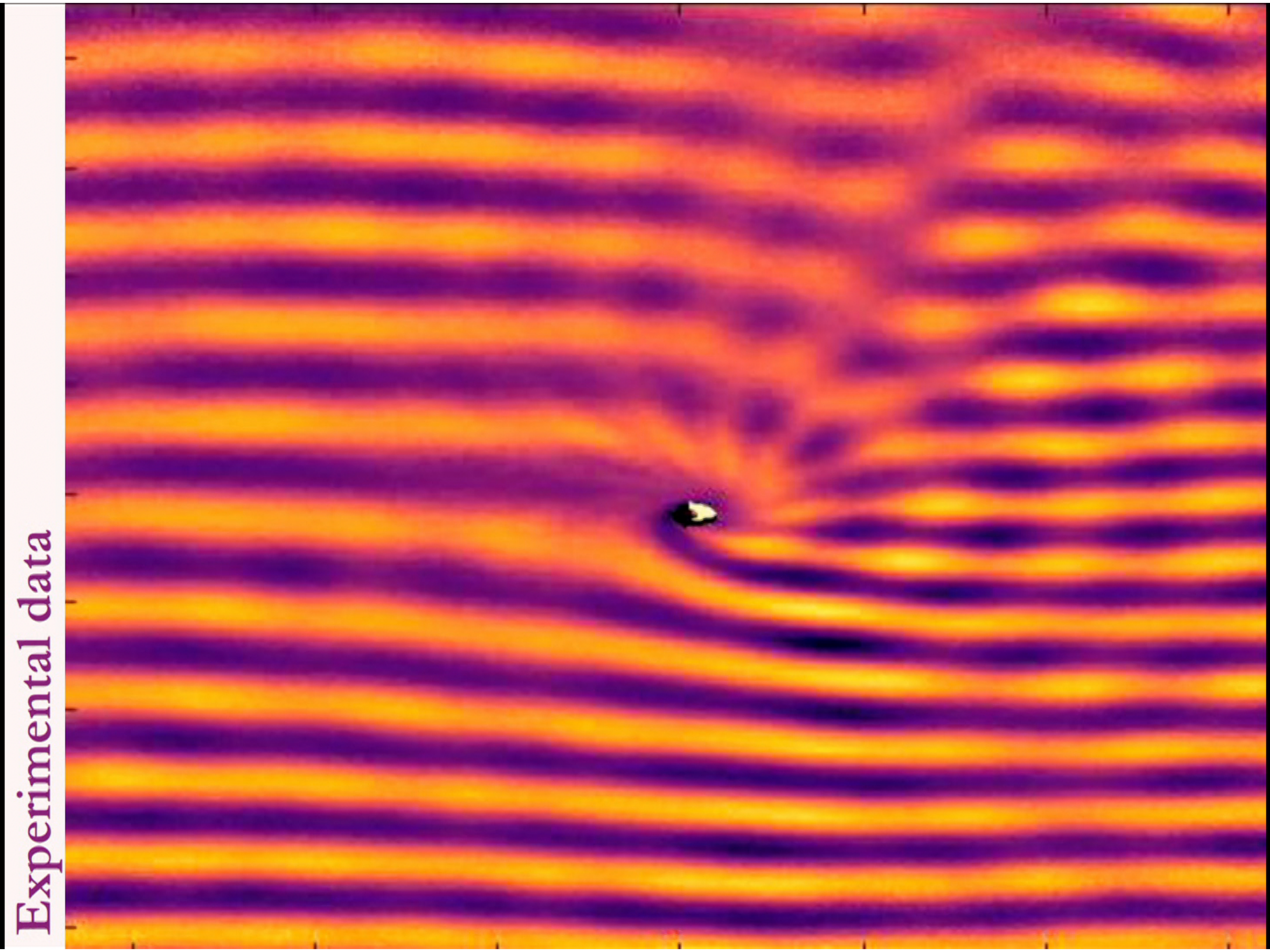
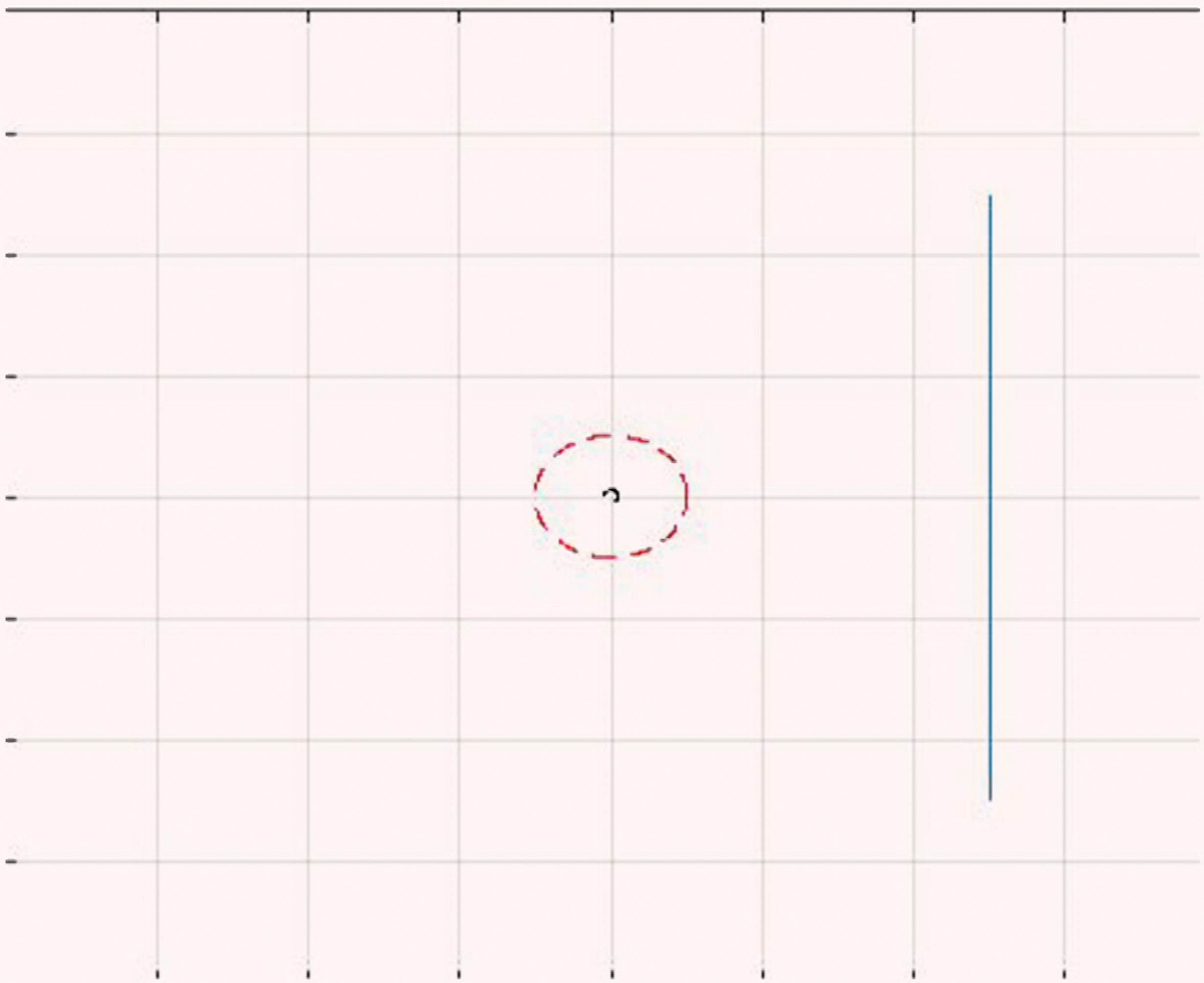



Experimental data

Wave fronts interacting with analogue rotating bh



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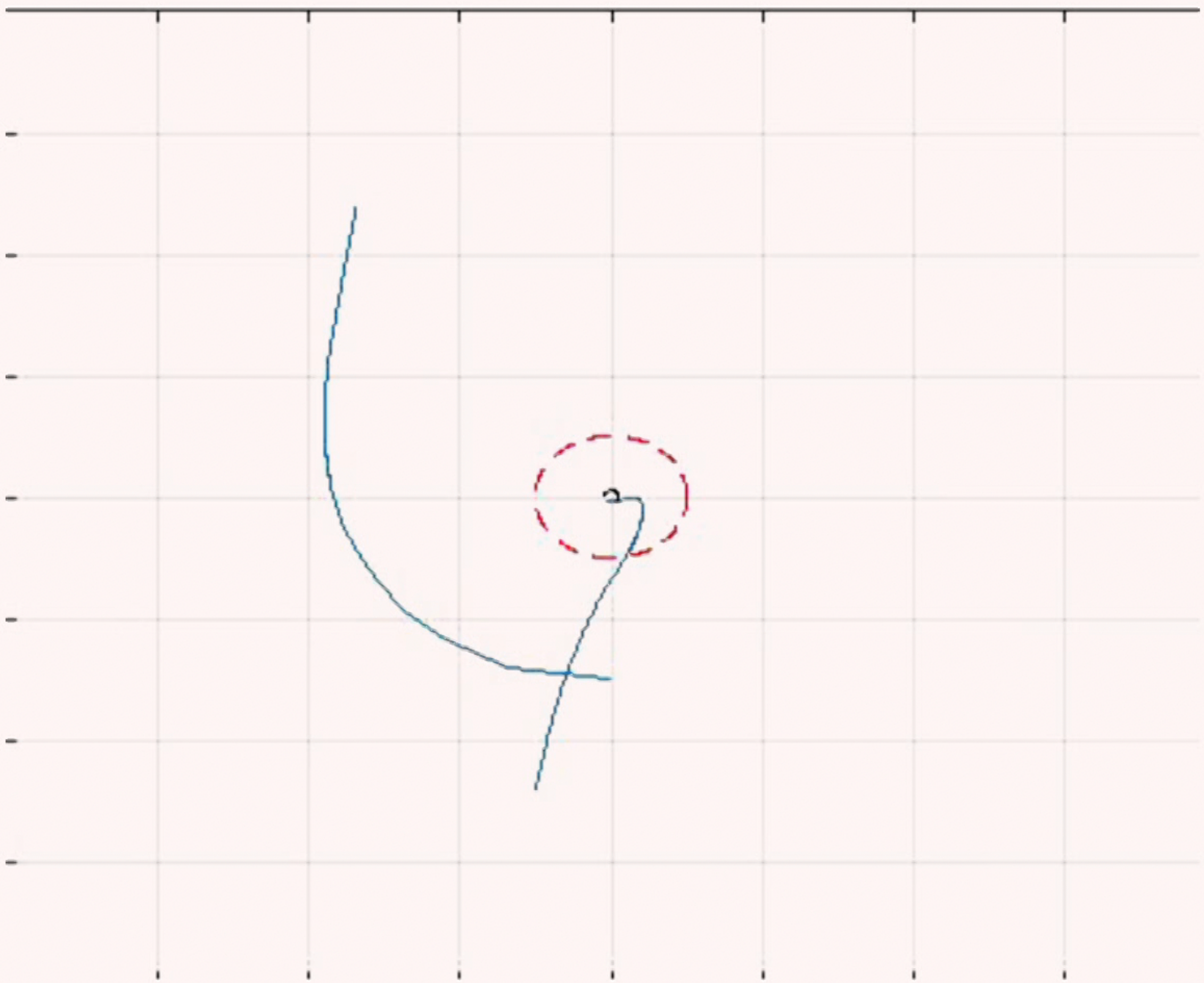

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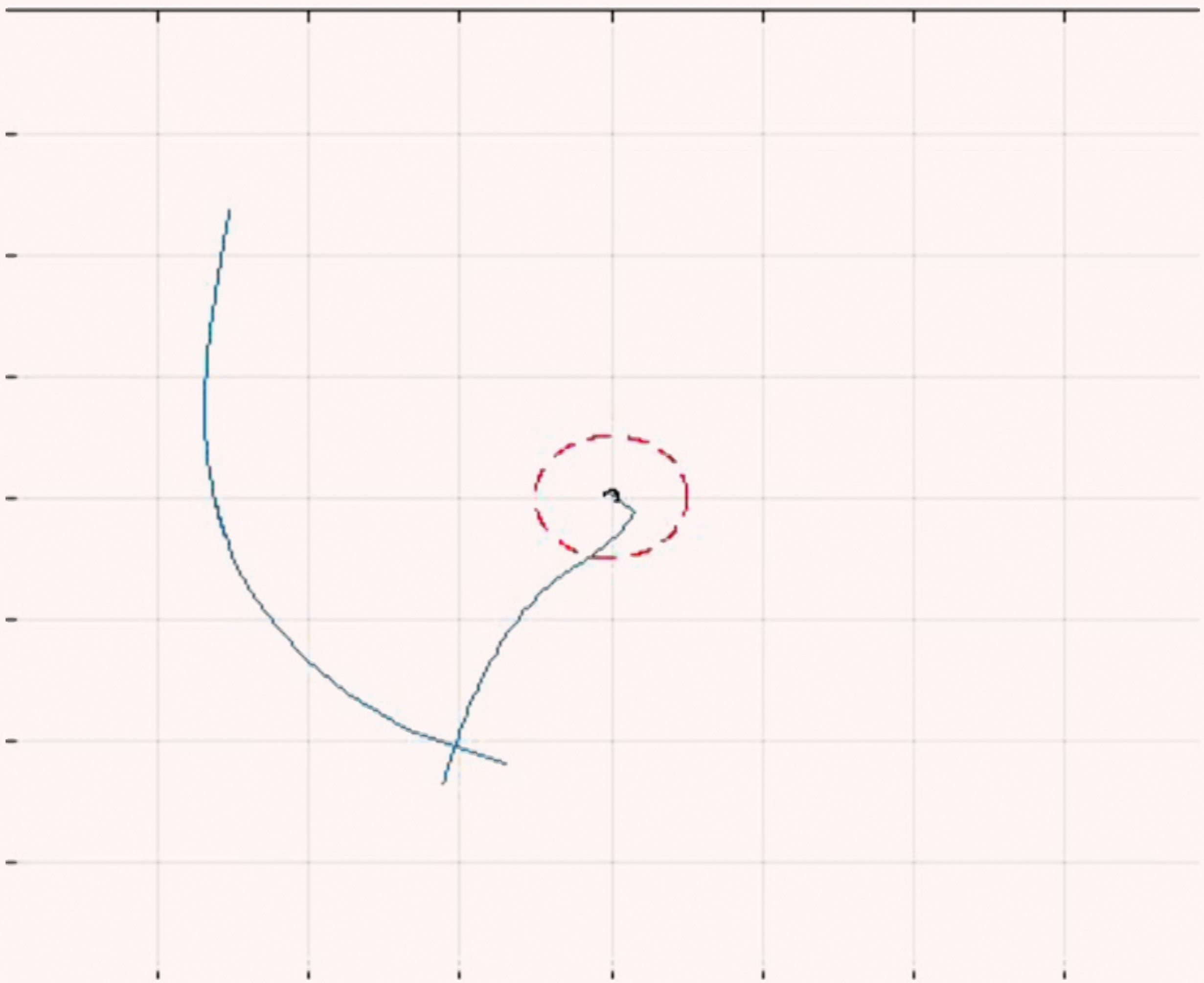

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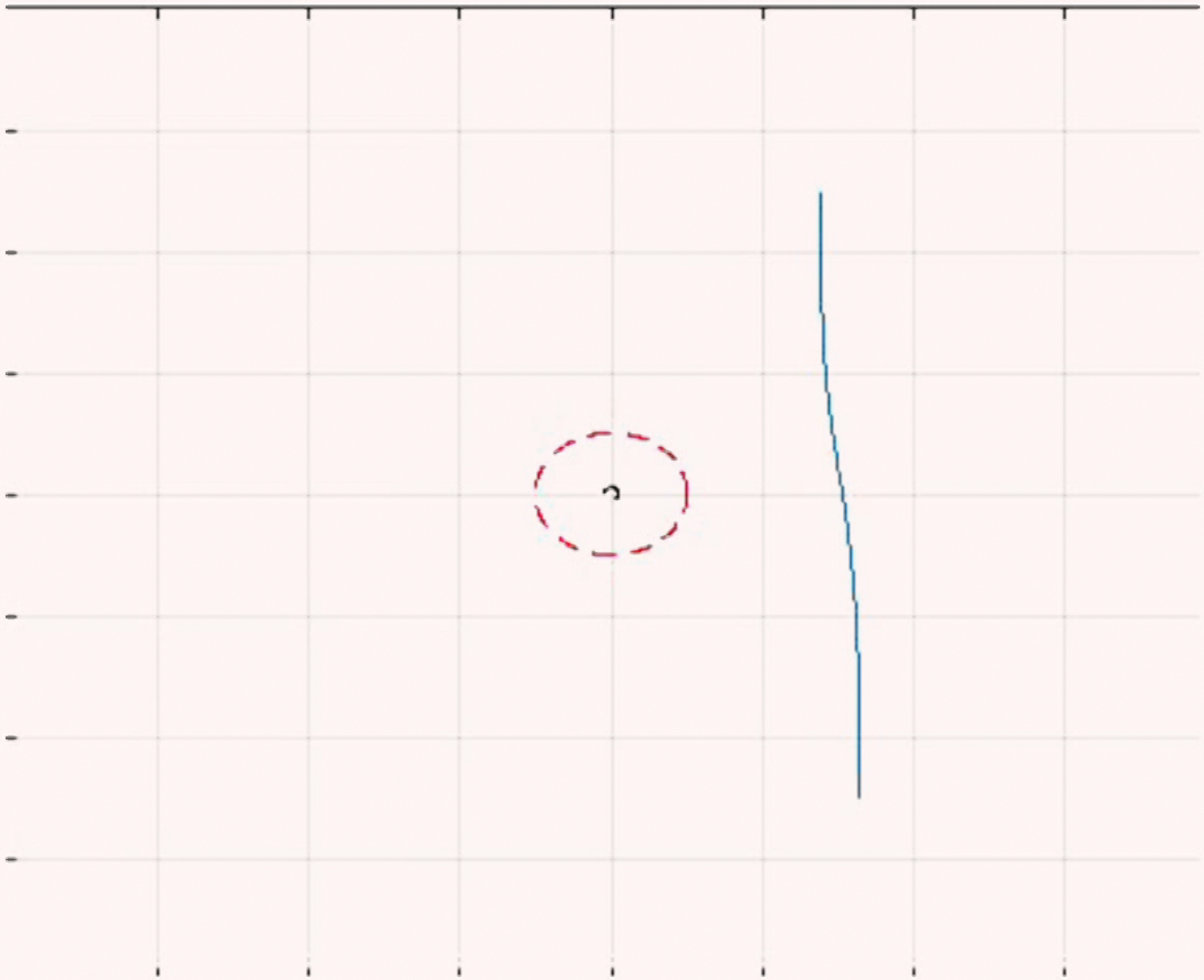
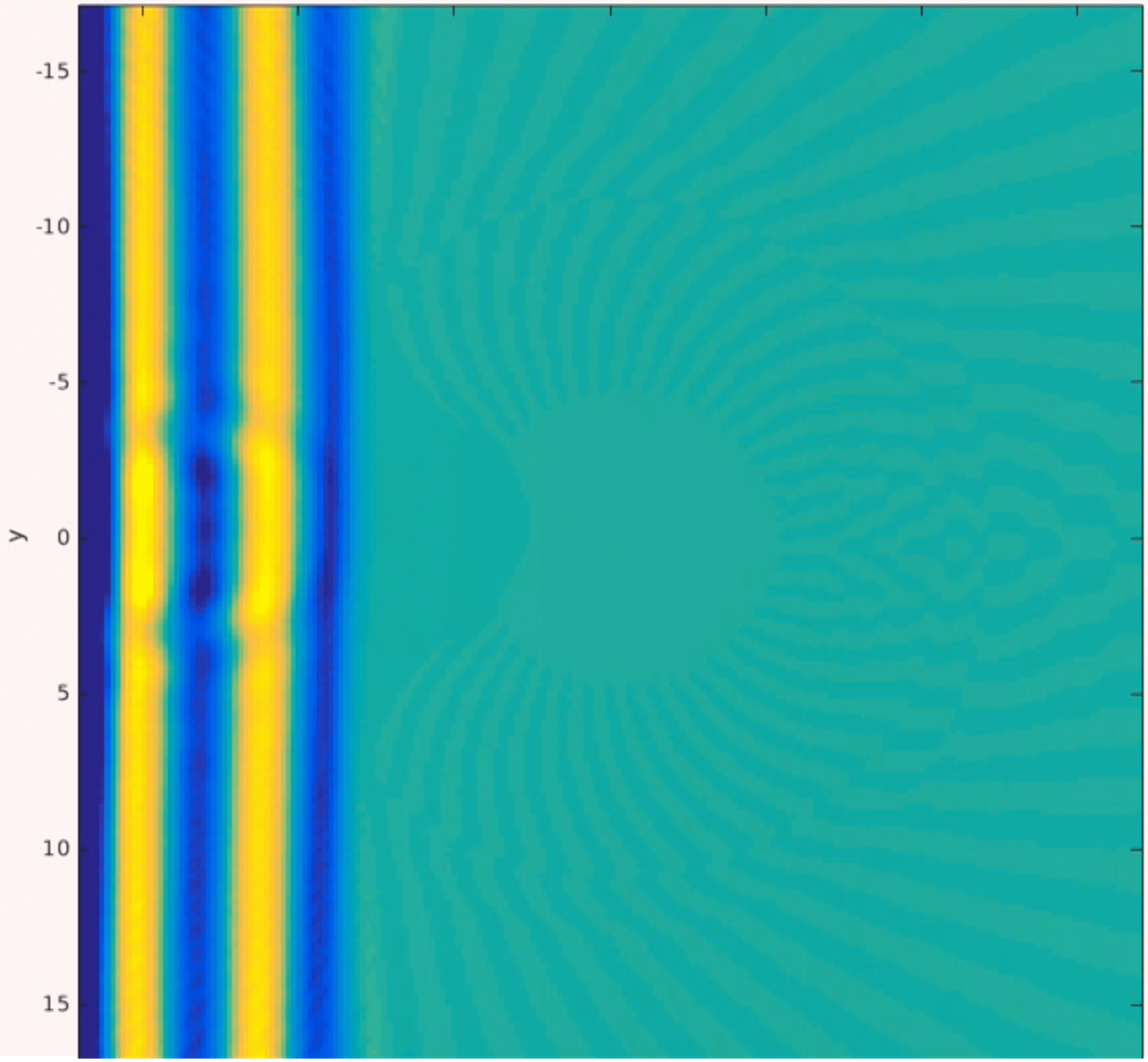


numerical simulations



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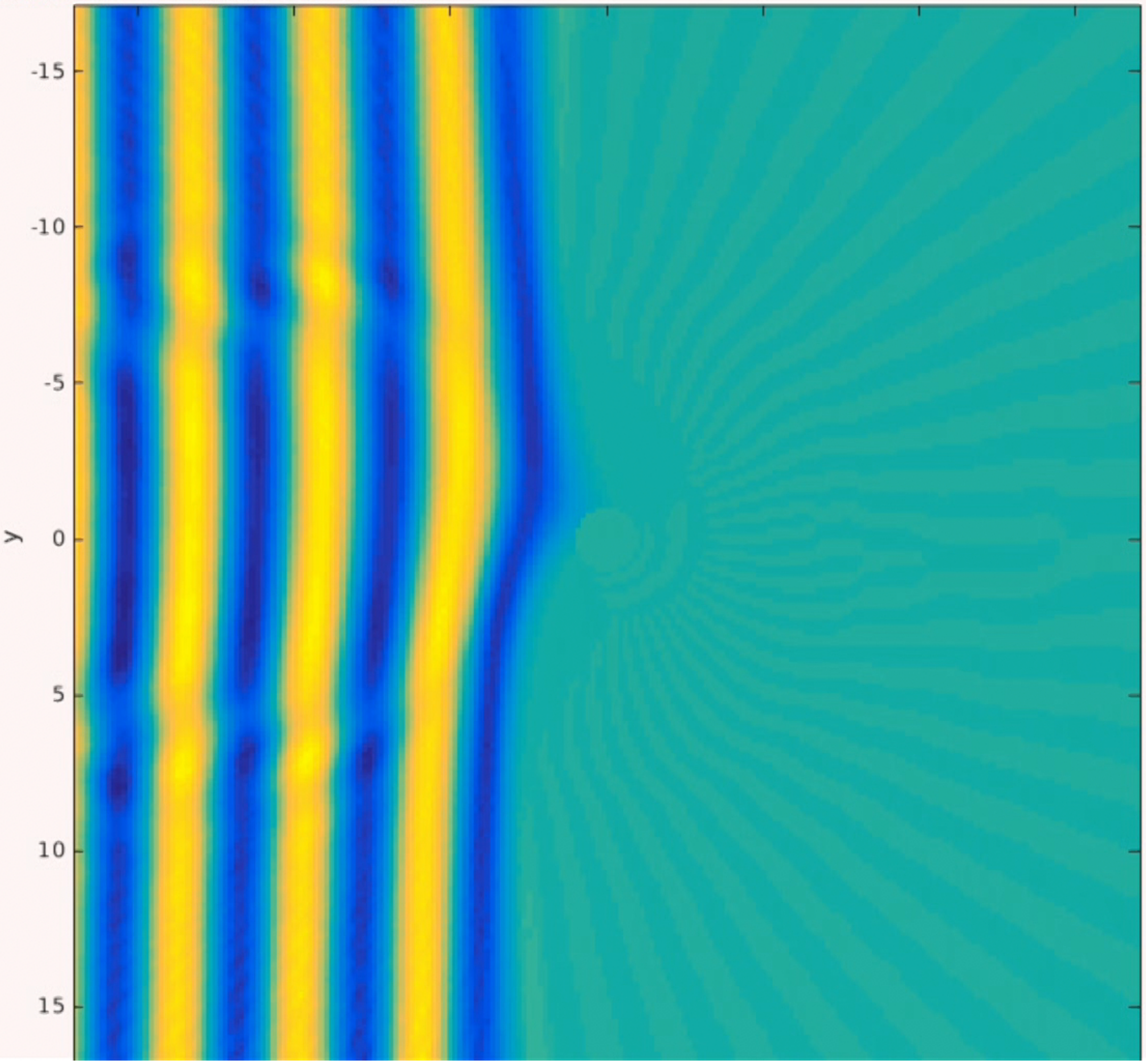

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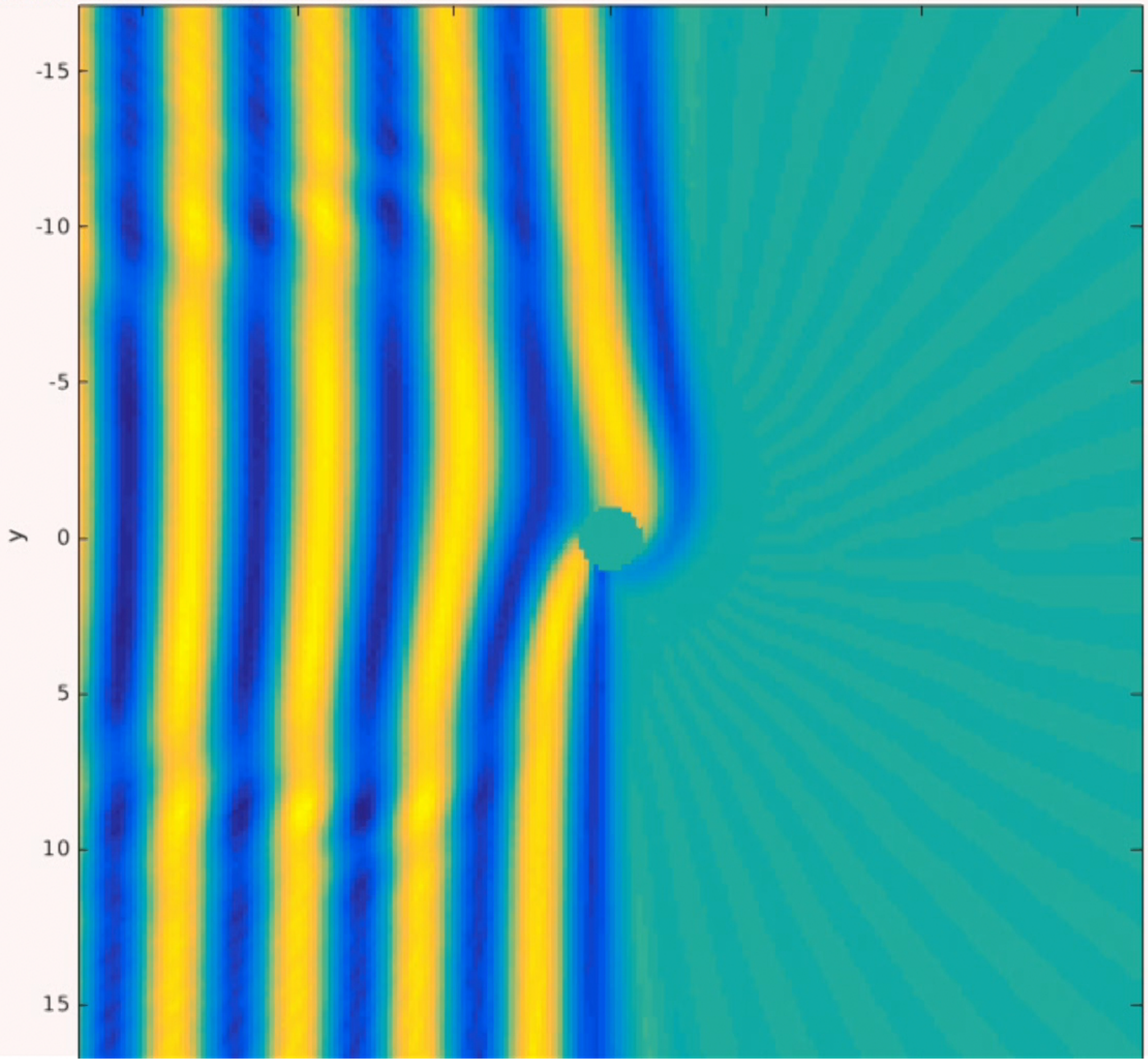

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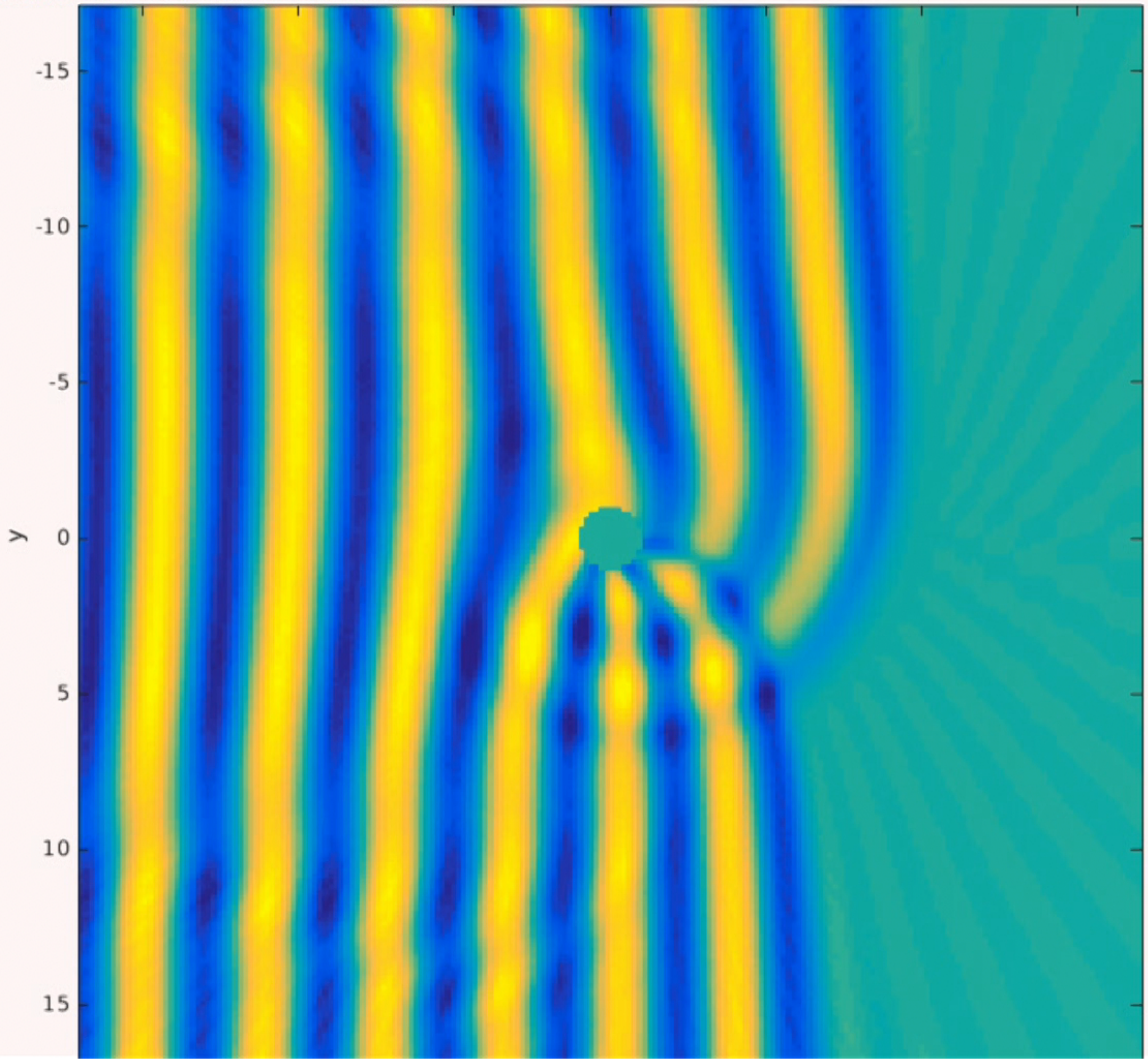

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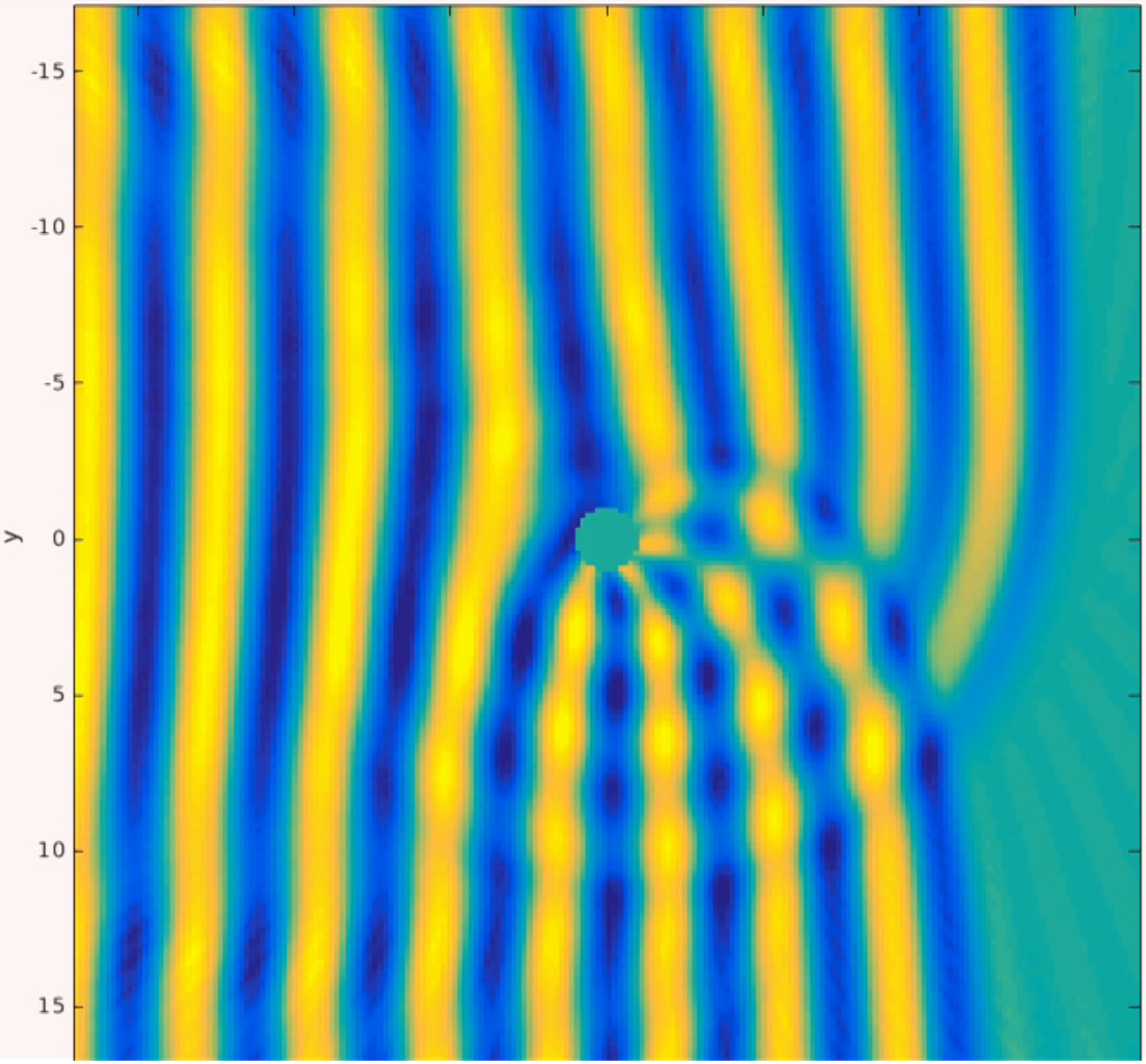

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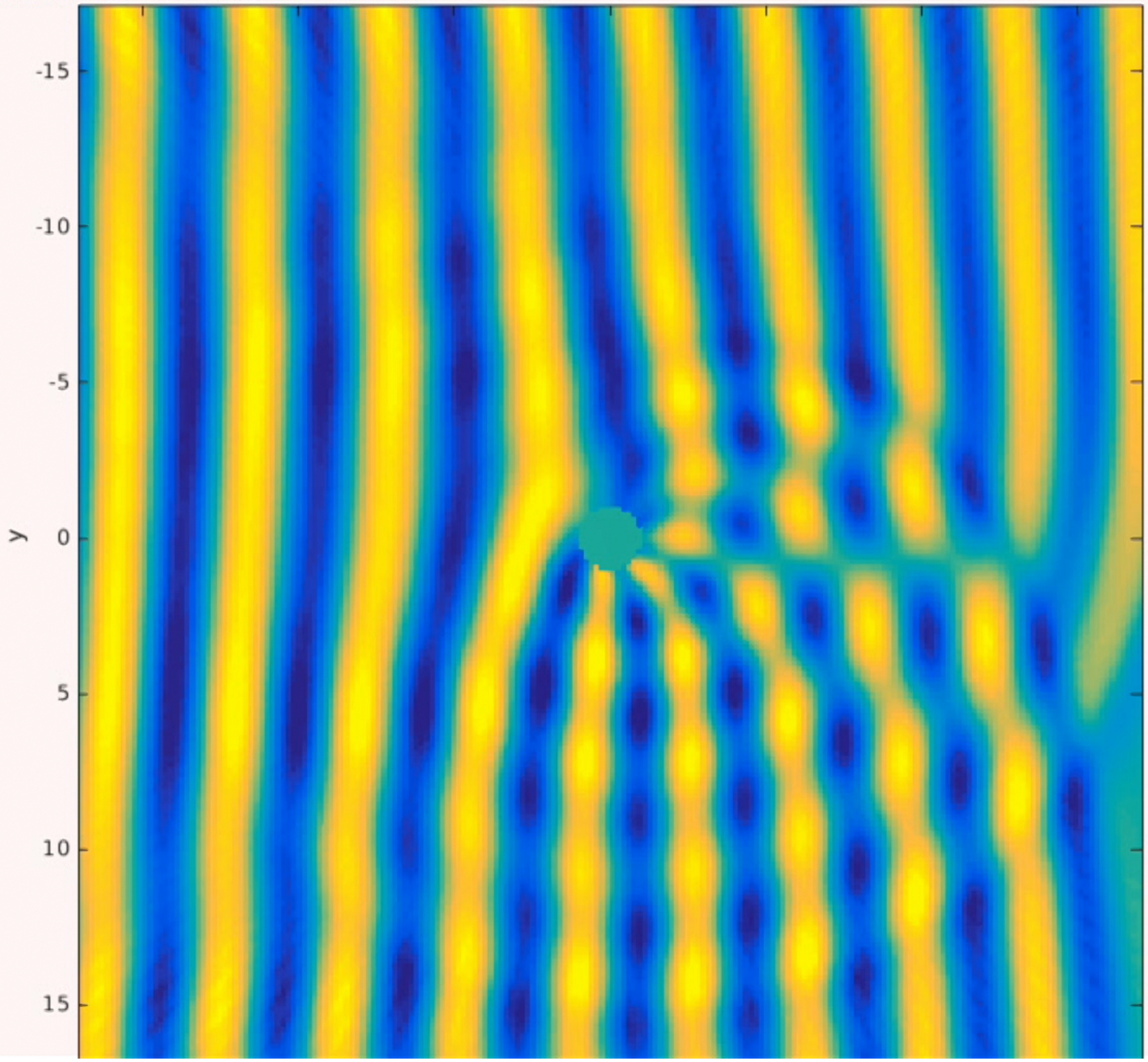


Emergent effective field theory in the system

Effective field theory / dynamical equations for small perturbations /surface waves $\phi = \Phi - \Phi_0$ on a background flow $\mathbf{v_0} = \nabla\Phi_0$

$$\mathcal{D}_t^2\phi - i(g\boldsymbol{\nabla} - \gamma\boldsymbol{\nabla}^3)\tanh(-ih_0\boldsymbol{\nabla})\phi + 2\nu\nabla^2\mathcal{D}_t\phi = 0$$

γ is the ratio of surface tension and the fluid density

ν the viscosity of the fluid

$$\mathcal{D}_t^2\phi + F(-i\boldsymbol{\nabla})\phi - 2\nu\Gamma(-i\boldsymbol{\nabla})\mathcal{D}_t\phi = 0$$

In the absence of a background flow, the solutions are plane waves

$$\omega^2 = F(\mathbf{k}) - 2i\nu\omega\Gamma(\mathbf{k})$$

$F(\mathbf{k}) = (gk + \gamma k^3)\tanh(h_0 k)$

$\Gamma(k) = k^2$

Geodesic motion in our system

Effective field theory / dynamical equations for small perturbations /surface waves are given:

$$\mathcal{D}_t^2\phi + F(-i\boldsymbol{\nabla})\phi - 2\nu\Gamma(-i\boldsymbol{\nabla})\mathcal{D}_t\phi = 0$$

Ray tracing methods: gradient expansion valid if the background flow Changes over a scale significantly larger than the wavelength.

$$\phi = A(\mathbf{x})\exp\left(i\frac{S(\mathbf{x})}{\epsilon}\right) \qquad \partial \to \epsilon\partial$$

At leading order in epsilon we get the Hamilton-Jakobi equation:

$$(\partial_t S_0 + \mathbf{v_0}\cdot\boldsymbol{\nabla}S_0)^2 - F(\boldsymbol{\nabla}S_0) = 0$$

Substituting $\boldsymbol{\nabla}S_0$ by the wave vector $\mathbf{k}$ and $\partial_t S$ by $-\omega$

$$\mathcal{H} = -\frac{1}{2}(\omega - \mathbf{v}\cdot\mathbf{k})^2 + \frac{1}{2}F(\mathbf{k}) \qquad \dot{t} = -\frac{\partial\mathcal{H}}{\partial\omega}, \quad \dot{\omega} = \frac{\partial\mathcal{H}}{\partial t}, \quad \dot{x}_j = \frac{\partial\mathcal{H}}{\partial k_j} \quad \text{and} \quad \dot{k}_j = -\frac{\partial\mathcal{H}}{\partial x_j} \qquad \mathcal{H} = 0$$

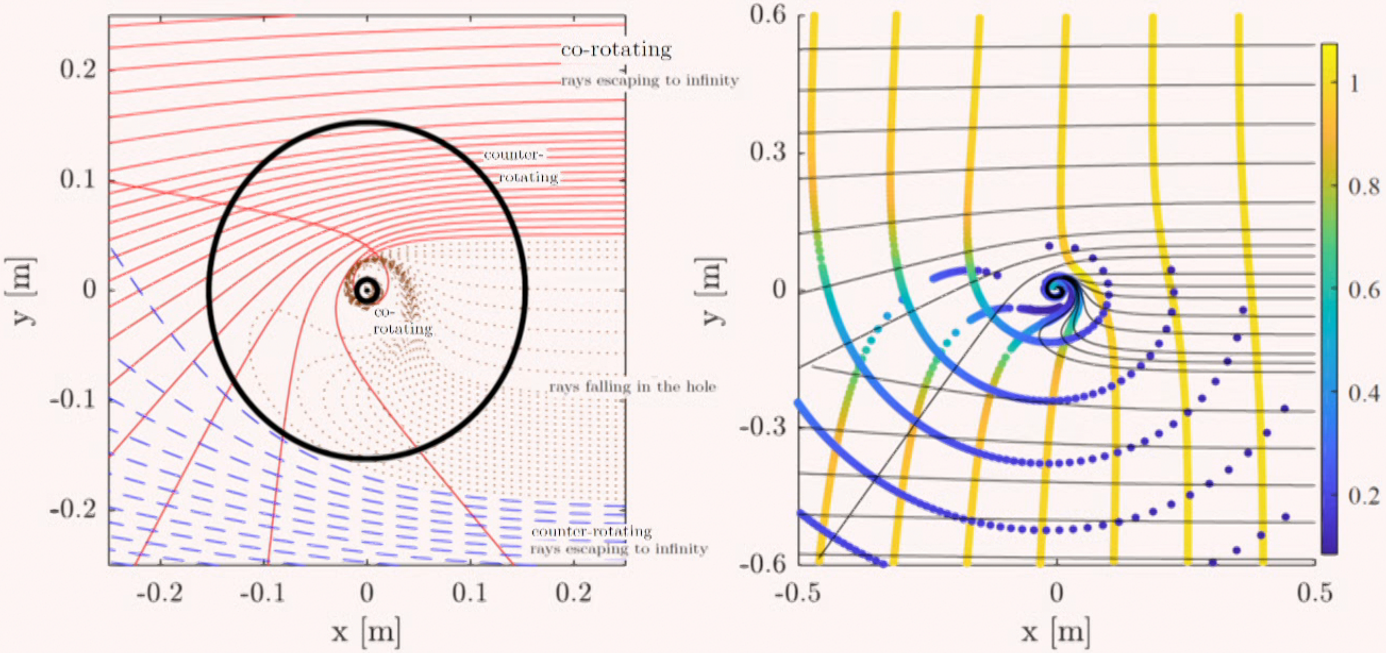
Rays around vortex flows

Assume an irrotational vortex flow: $\mathbf{v}_0 = -\frac{D}{r}\mathbf{e}_r + \frac{C}{r}\mathbf{e}_\theta$

$C = 1.6 \times 10^{-2}\mathrm{m}^2\mathrm{s}^{-1}$, $D = 1 \times 10^{-3}\mathrm{m}^2\mathrm{s}^{-1}$ and $h = 0.06\mathrm{m}$

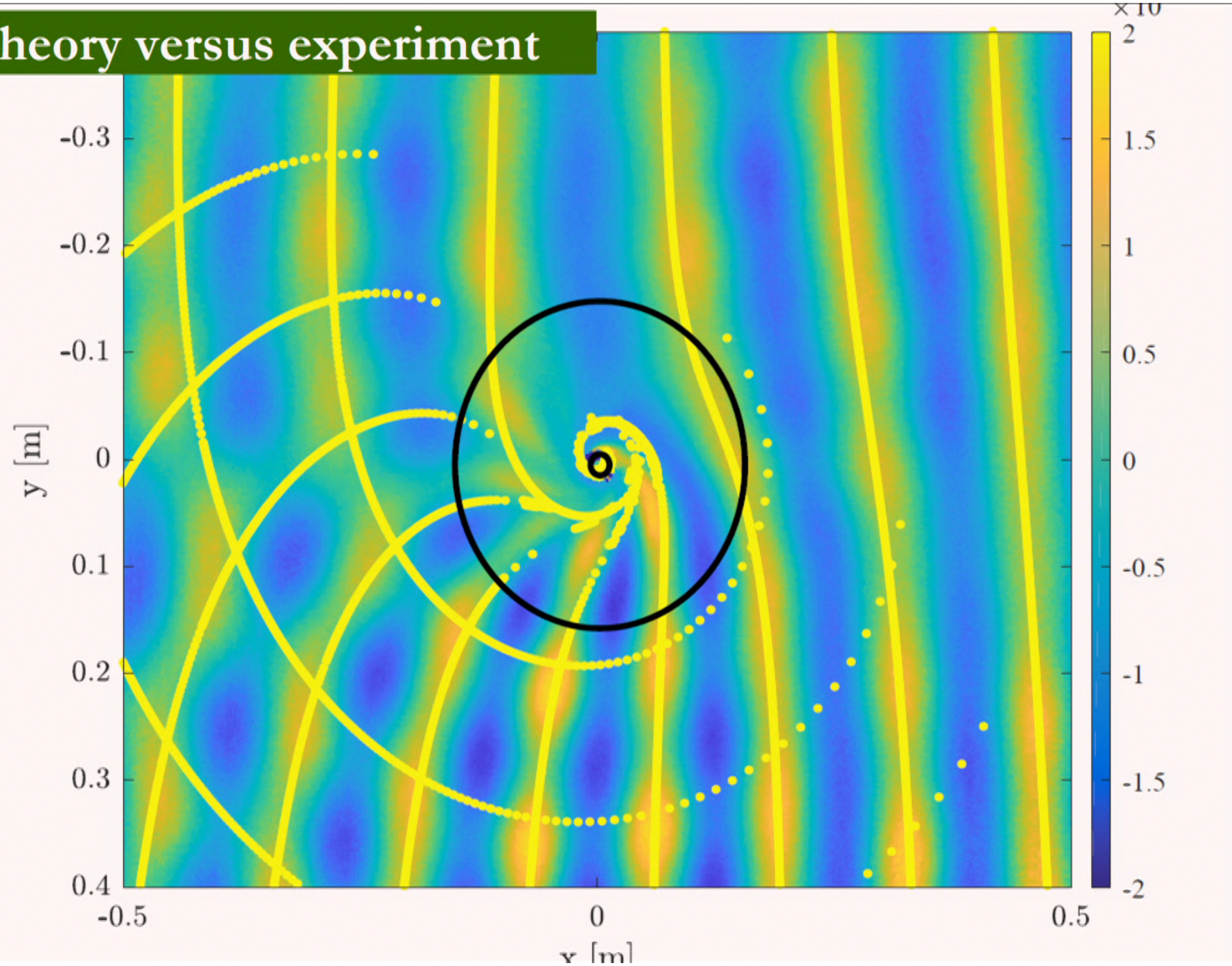
Surface wave in the deep water regime: $\omega = 19.8$ rad/s $\quad hk_{\mathrm{in}} \simeq 2.4$

Circular orbits for co- and counter-rotating wave: $r = 1$ cm and $r = 15.3$ cm

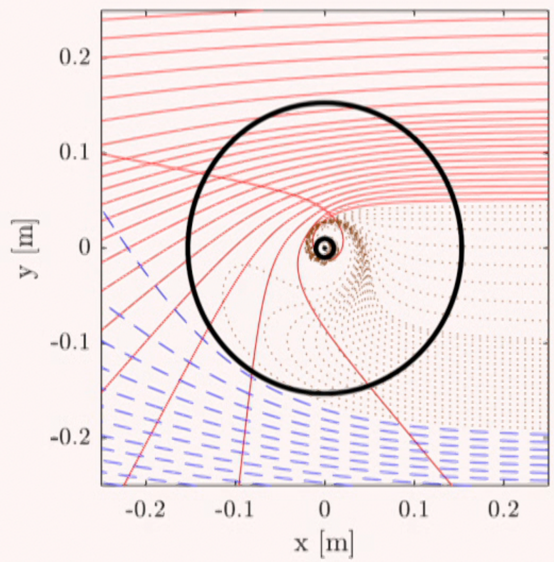


(a) Congruence of rays **(b)** Reconstructed eikonal wavefront

Theory versus experiment



Can we estimate the real part of the ringdwon using the concept of light-rings?



Quasinormal modes in the analogue gravity system

Dispersive media -> dynamical equations second order in time, but higher order in space (as already discussed previously, for the characteristics)

A circular orbit is an equilibrium point in the radial direction. This means that it is a critical point of the Hamiltonian for (r, k_r)

$$\mathcal{H} = 0, \quad \frac{\partial \mathcal{H}}{\partial r} = 0 \quad \text{and} \quad \frac{\partial \mathcal{H}}{\partial k_r} = 0.$$

Hamiltonian constraint $\mathcal{H} = 0$ gives a relation between ω and m on the circular orbit

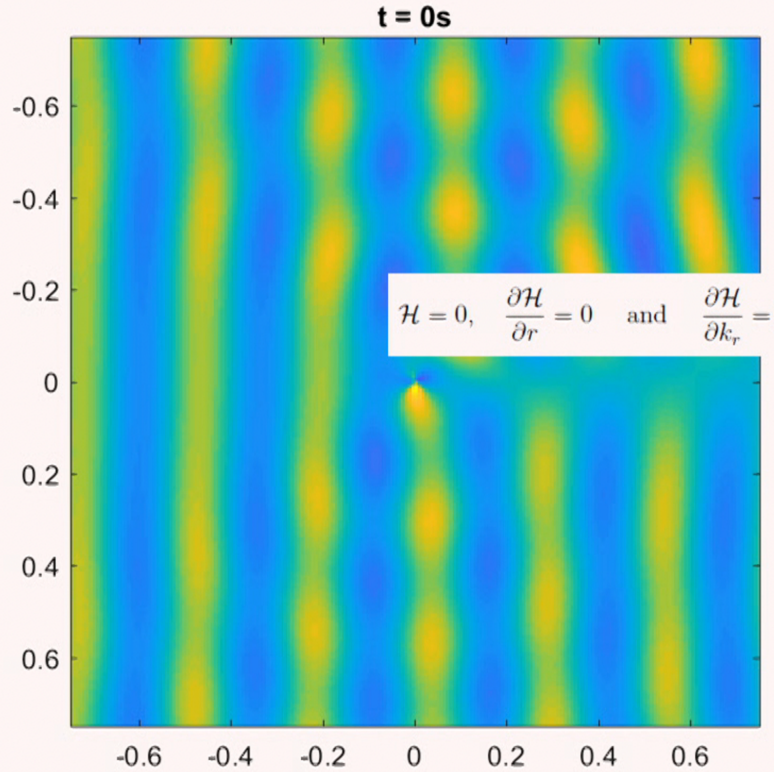
$$\omega = \omega_\star(m)$$

Quasi-normal or ringdown modes: $\boxed{\omega_{\mathrm{QNM}}(m) = \omega_\star(m) - i\Lambda(m)\left(n + \frac{1}{2}\right)}$

Quasinormal modes in the analogue gravity system

Dispersive media -> dynamical equations second order in time, but higher order in space (as

A circular ... it is a critical poi...



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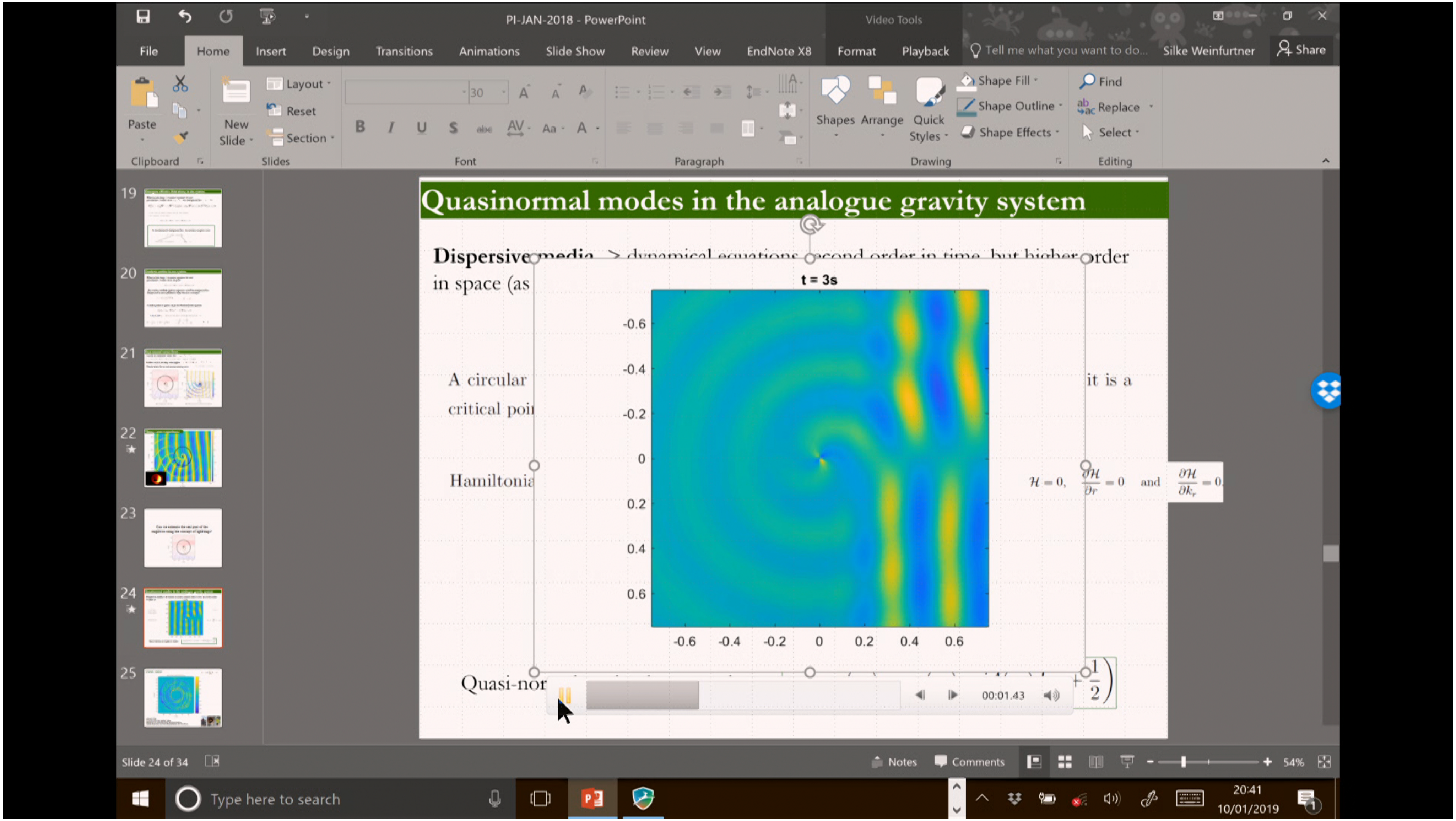
Hamiltonia... bit

Quasi-normal or ringdown modes: $$\omega_{\mathrm{QNM}}(m) = \omega_\star(m) - i\Lambda(m)\left(n + \frac{1}{2}\right)$$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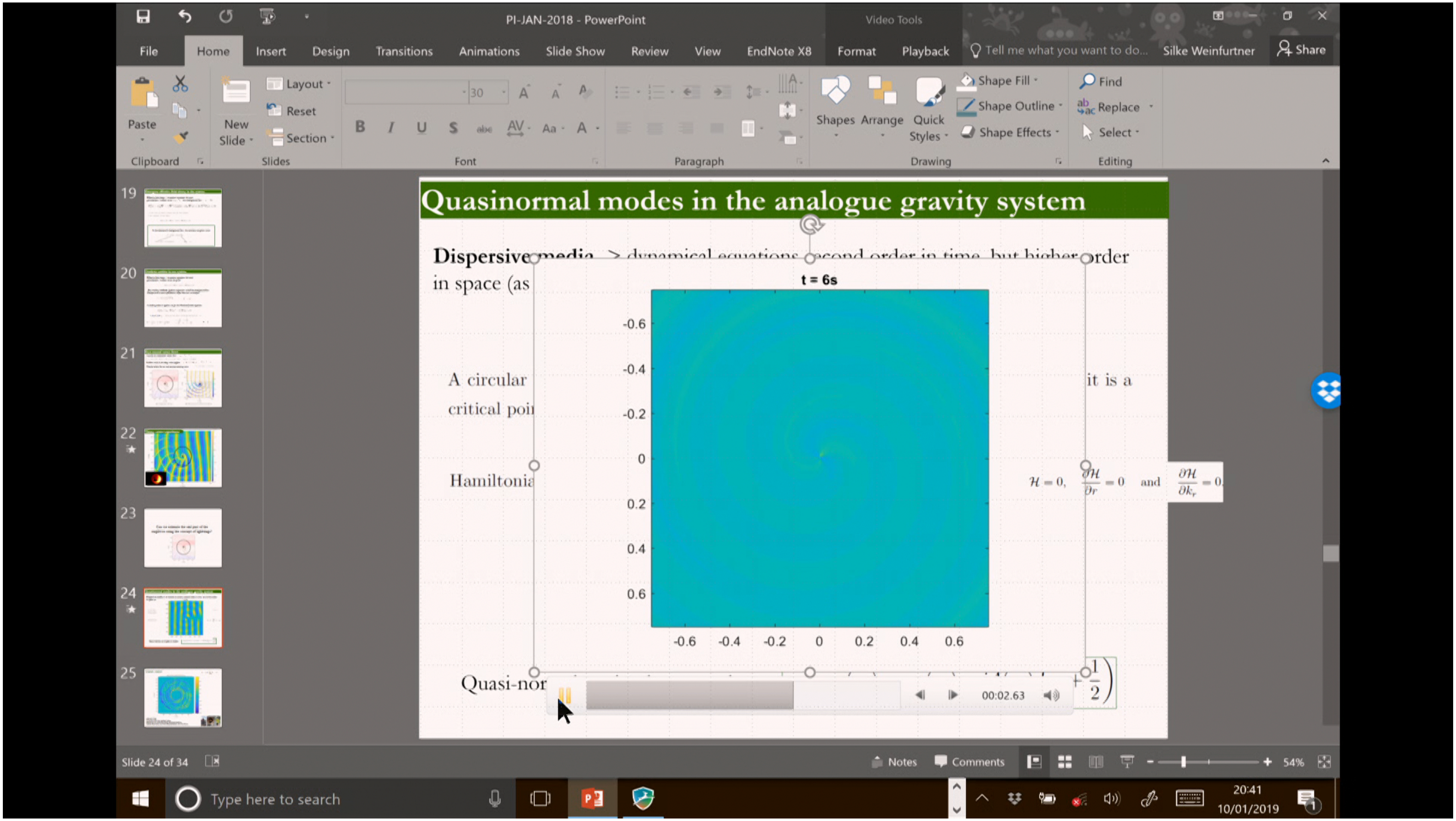

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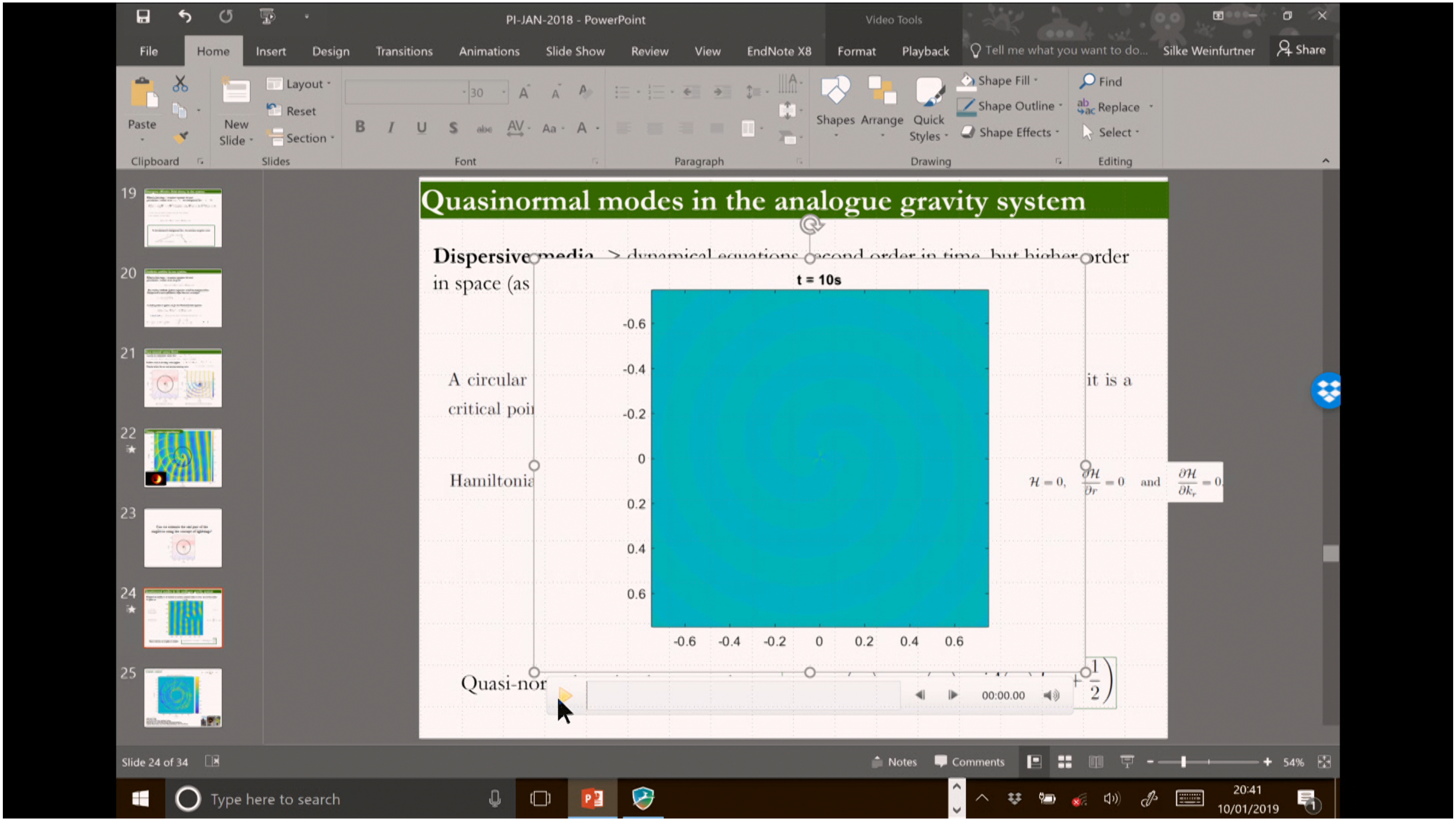

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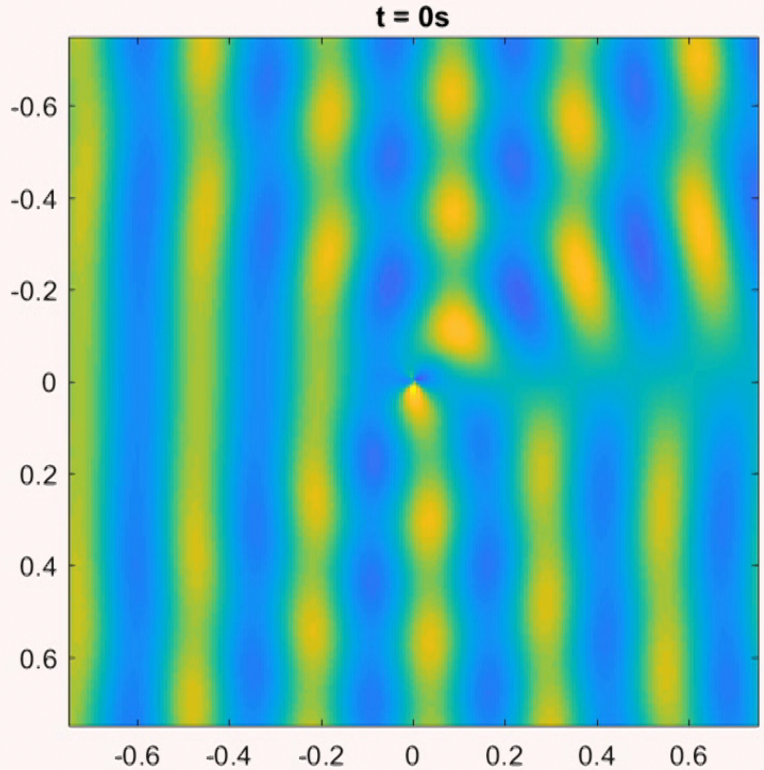
Quasinormal modes in the analogue gravity system

Dispersive media -> dynamical equations second order in time, but higher order in space (as



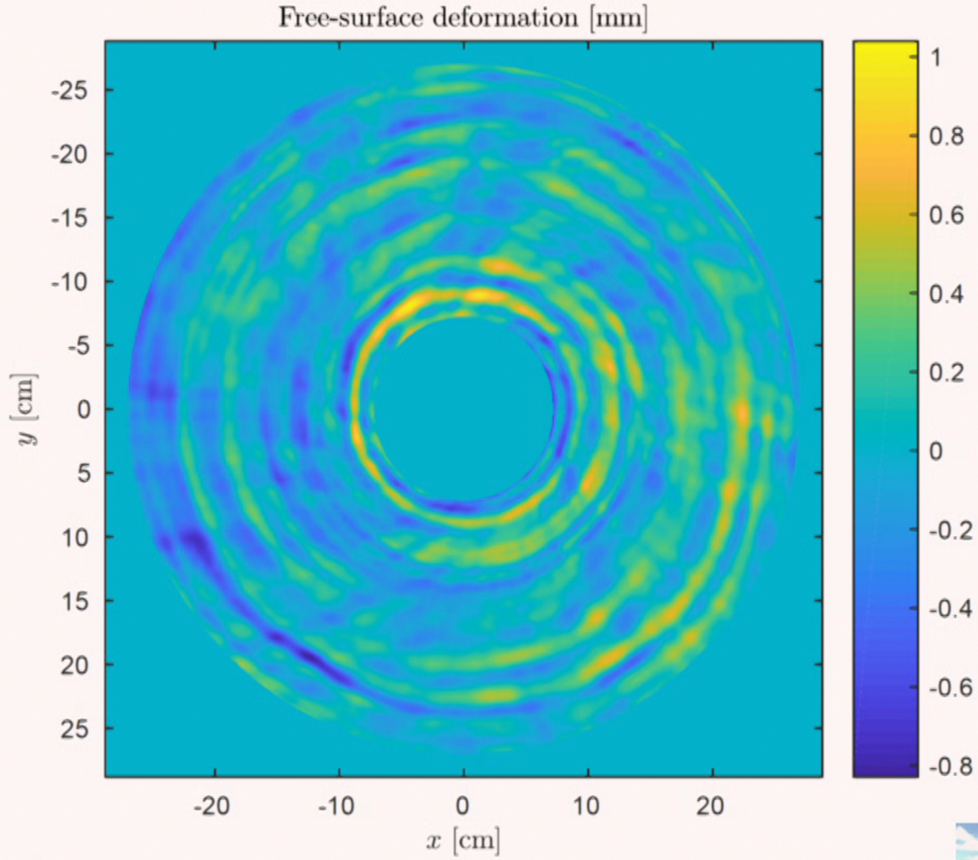
A circular … it is a critical poi…

Hamiltonia… $\mathcal{H} = 0, \quad \frac{\partial \mathcal{H}}{\partial r} = 0$ and

Quasi-normal or ringdown modes: $\boxed{\omega_{\mathrm{QNM}}(m) = \omega_\star(m) - i\Lambda(m)\left(n + \frac{1}{2}\right)}$

Unruh vortex

$$\delta h(t,r,\theta) = \mathrm{Re}\left[\sum_{m\in\mathbb{Z}} \delta h_m(t,r) e^{im\theta}\right]$$



arXiv:1811.07858

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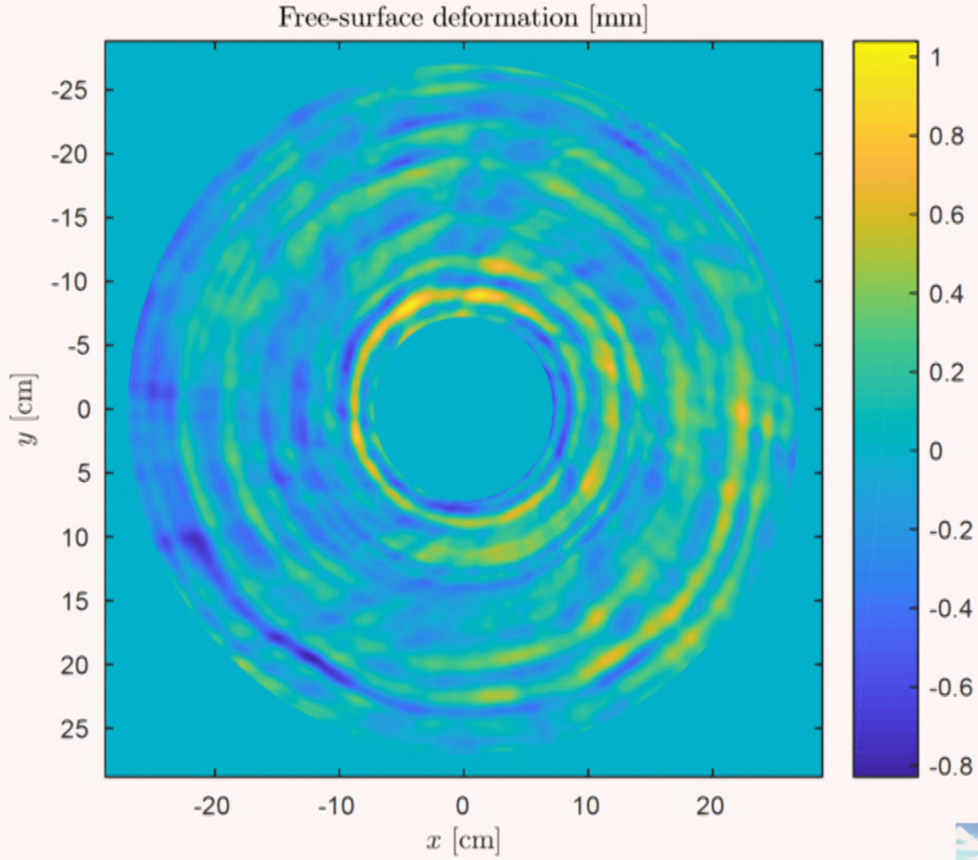

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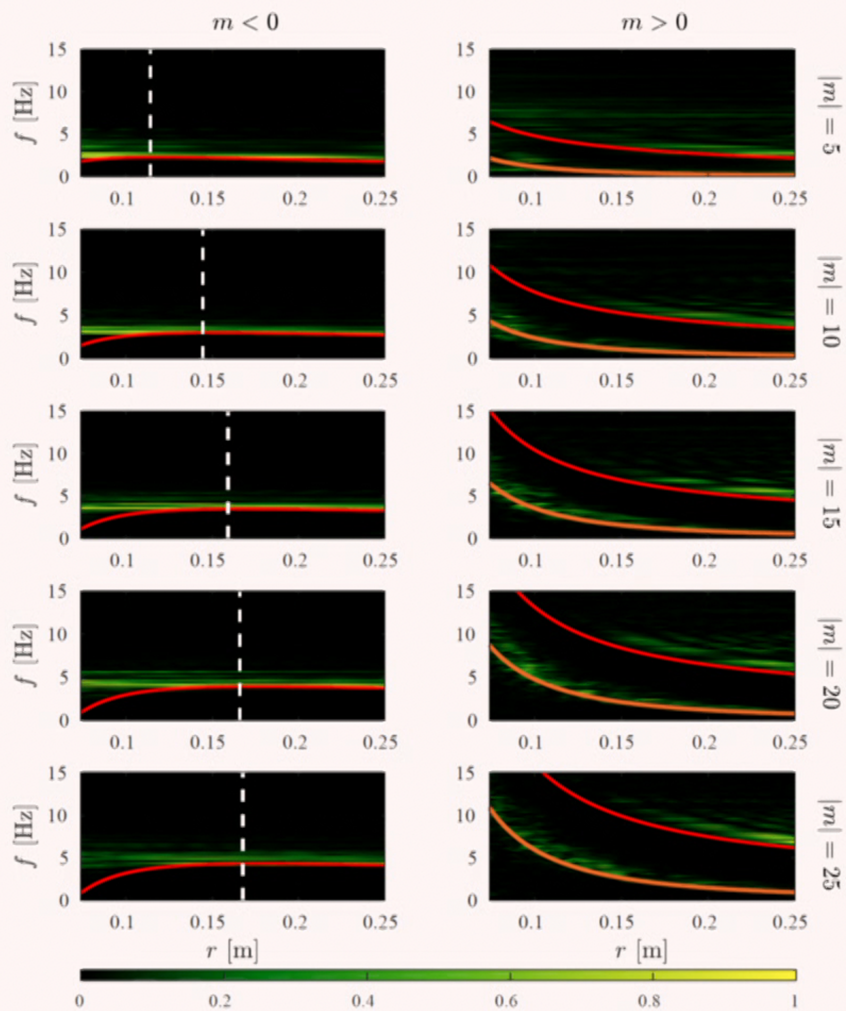
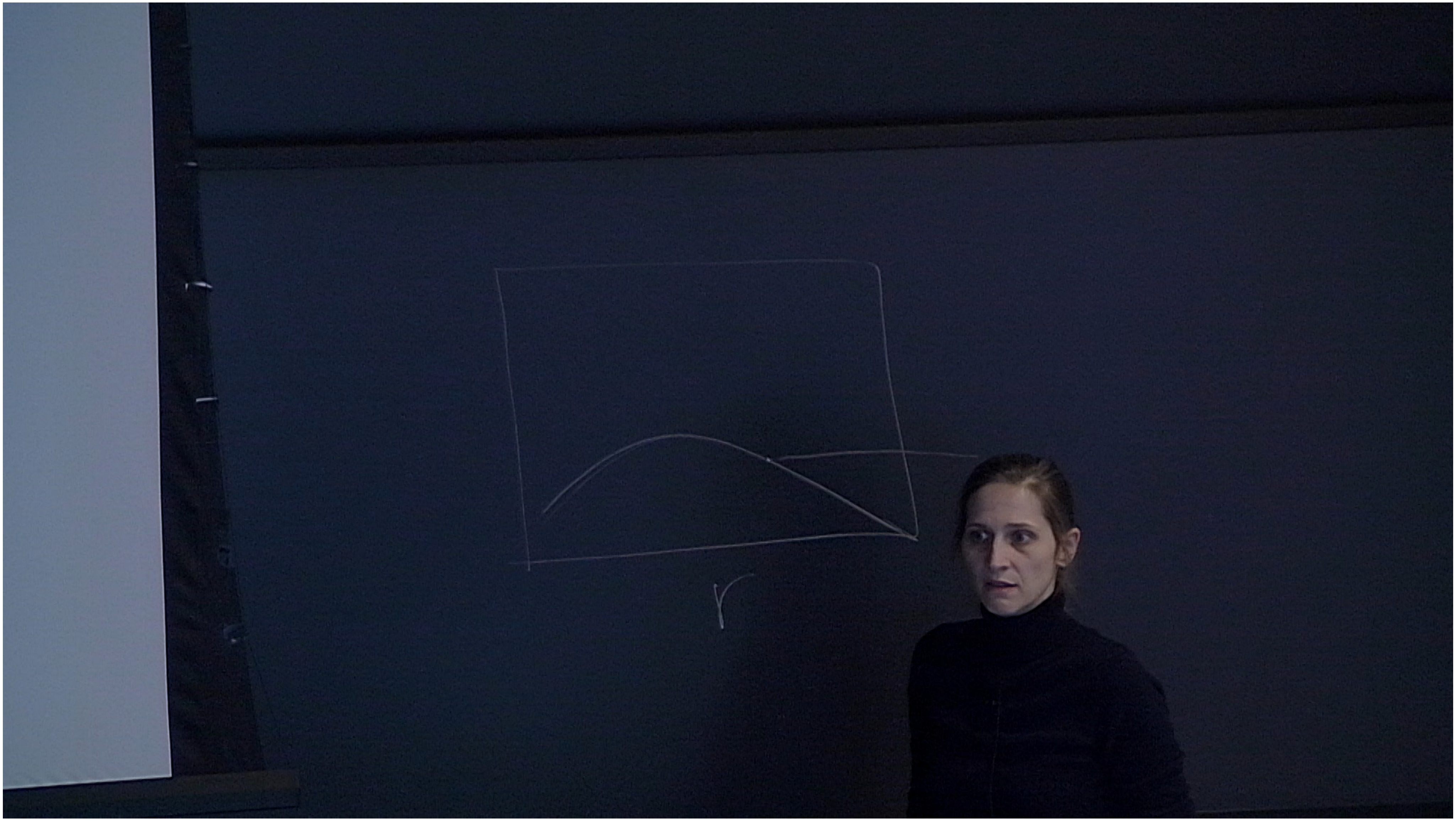
Normalized Power Spectral Density

$$PSD(f, r_i, m) \propto \left|\tilde{h}(f, r_i, m)\right|^2$$

$$\omega = \omega_\star(m)$$

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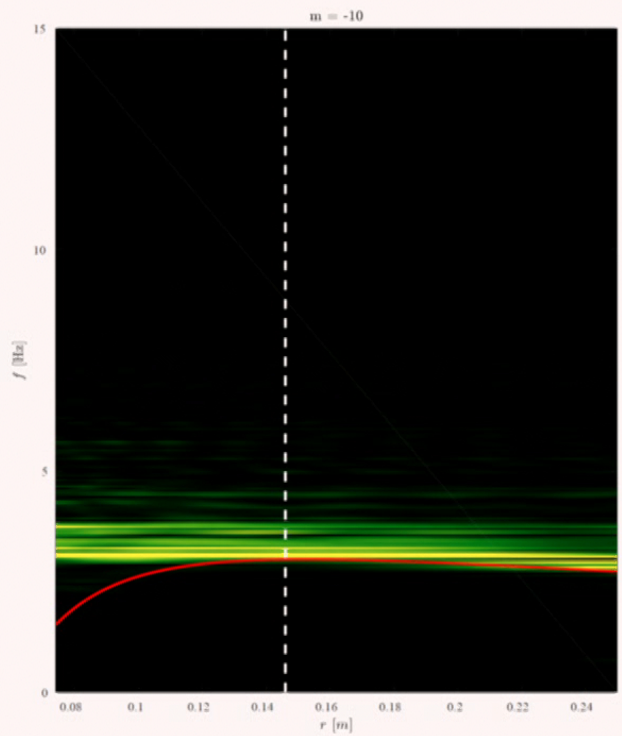
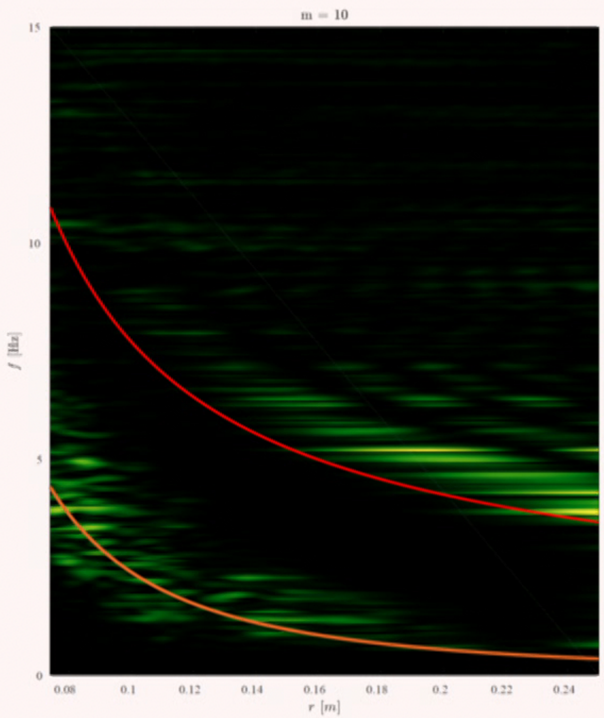
Normalized Power Spectral Density

$$PSD(f, r_i, m) \propto \left|\tilde{h}(f, r_i, m)\right|^2$$

$\omega = \omega_\star(m)$ $\quad m > 0$ $\qquad\qquad m < 0$

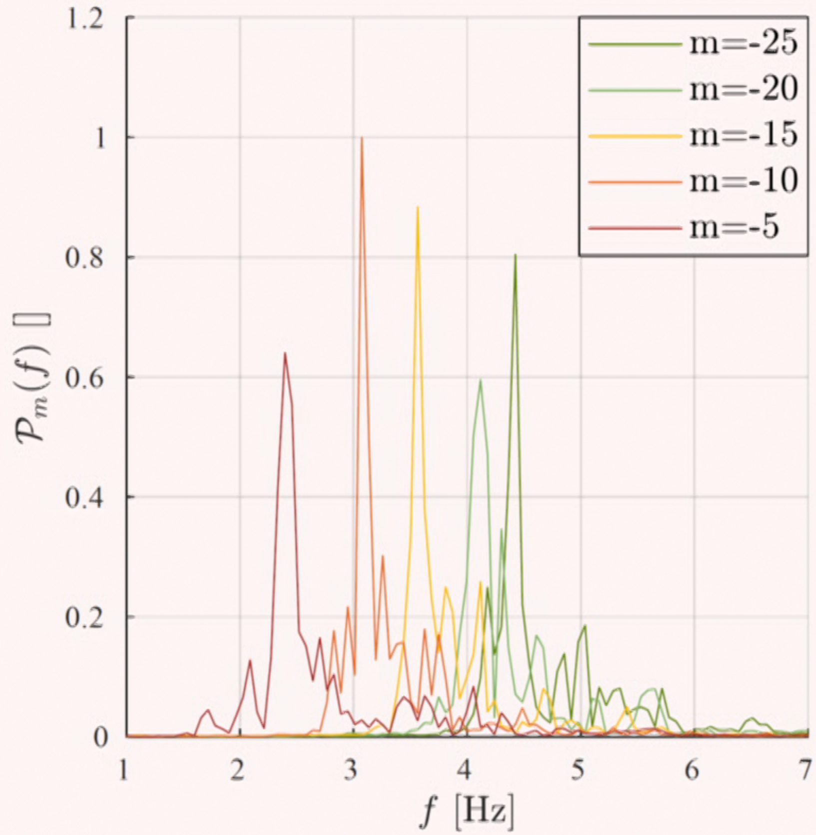




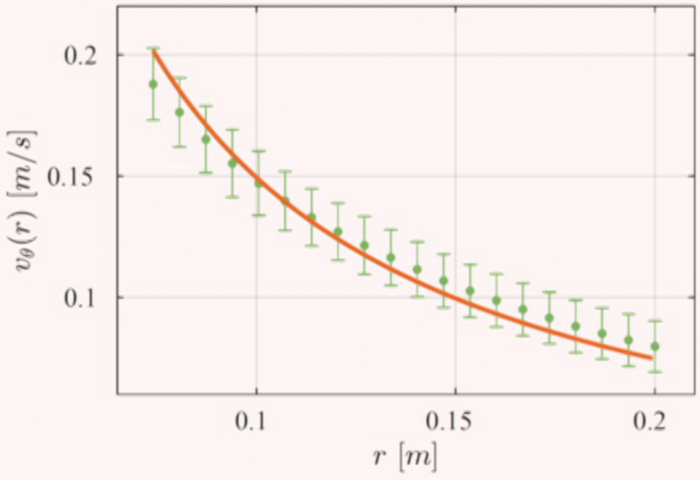
This gives a new interpretation of the LR condition:
critical points of the Hamiltonian are also critical points of the dispersion relation.

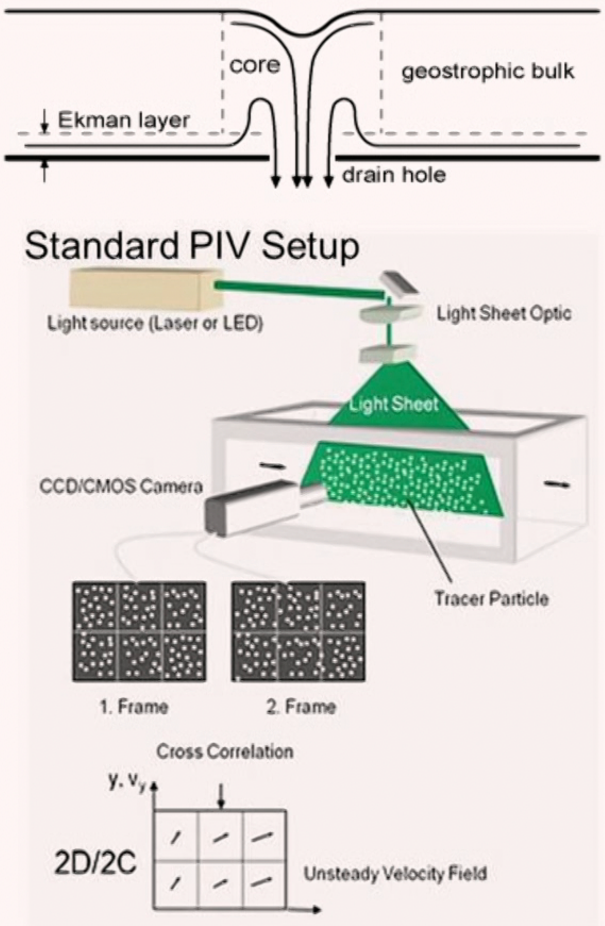
$$\mathcal{H} = 0, \quad \frac{\partial \mathcal{H}}{\partial r} = 0 \quad \text{and} \quad \frac{\partial \mathcal{H}}{\partial k_r} = 0. \qquad \rightarrow \qquad \frac{\partial \mathcal{H}}{\partial k_r} = \pm F(k)\frac{\partial \omega_d}{\partial k_r} \quad \text{and} \quad \frac{\partial \mathcal{H}}{\partial r} = \pm F(k)\frac{\partial \omega_d}{\partial r}$$

Typical radius-averaged Power Spectral Density



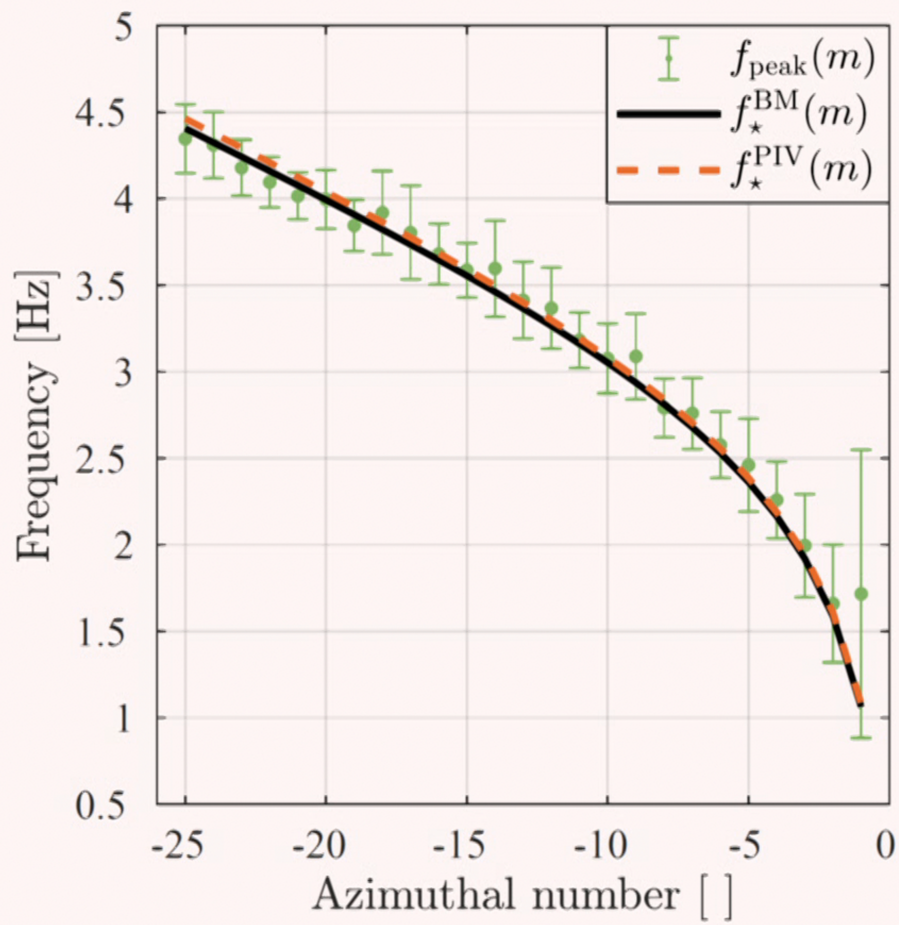
Independent fluid flow measurments: PIV



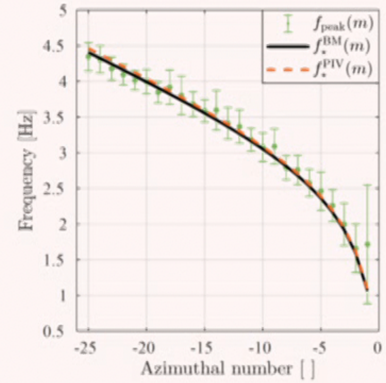


Characteristic spectrum of the Unruh vortex

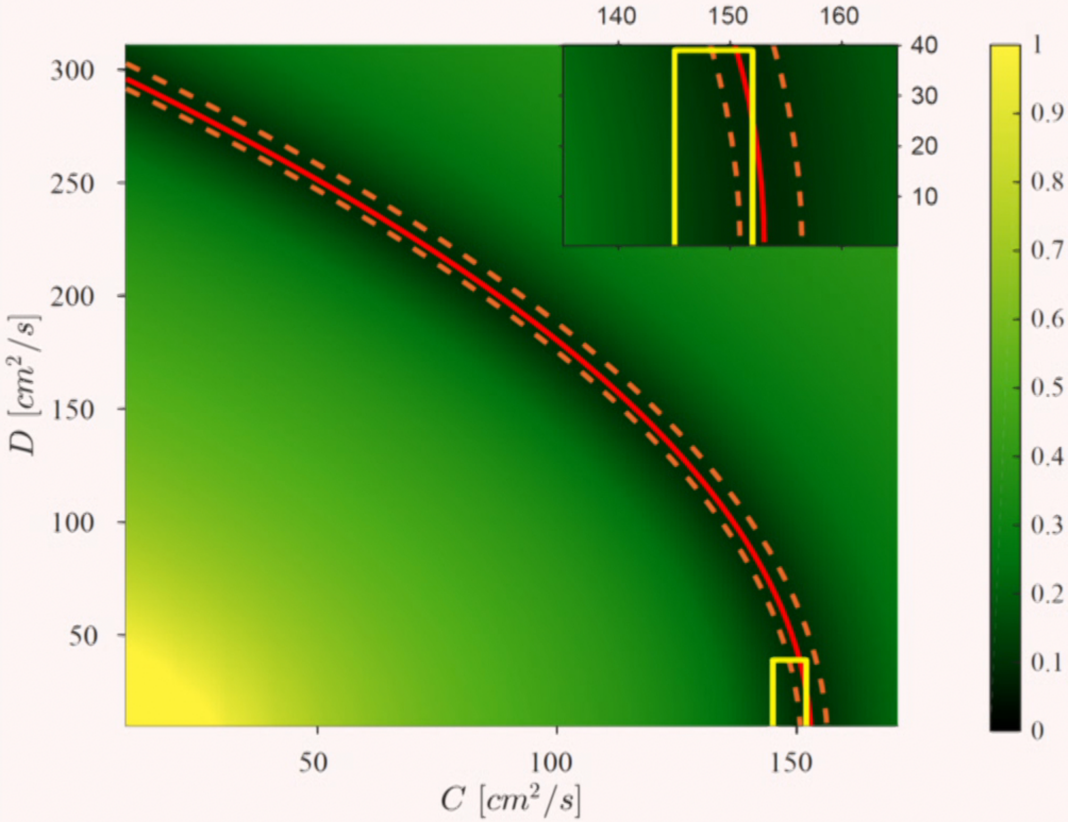
Real part of the lightring modes

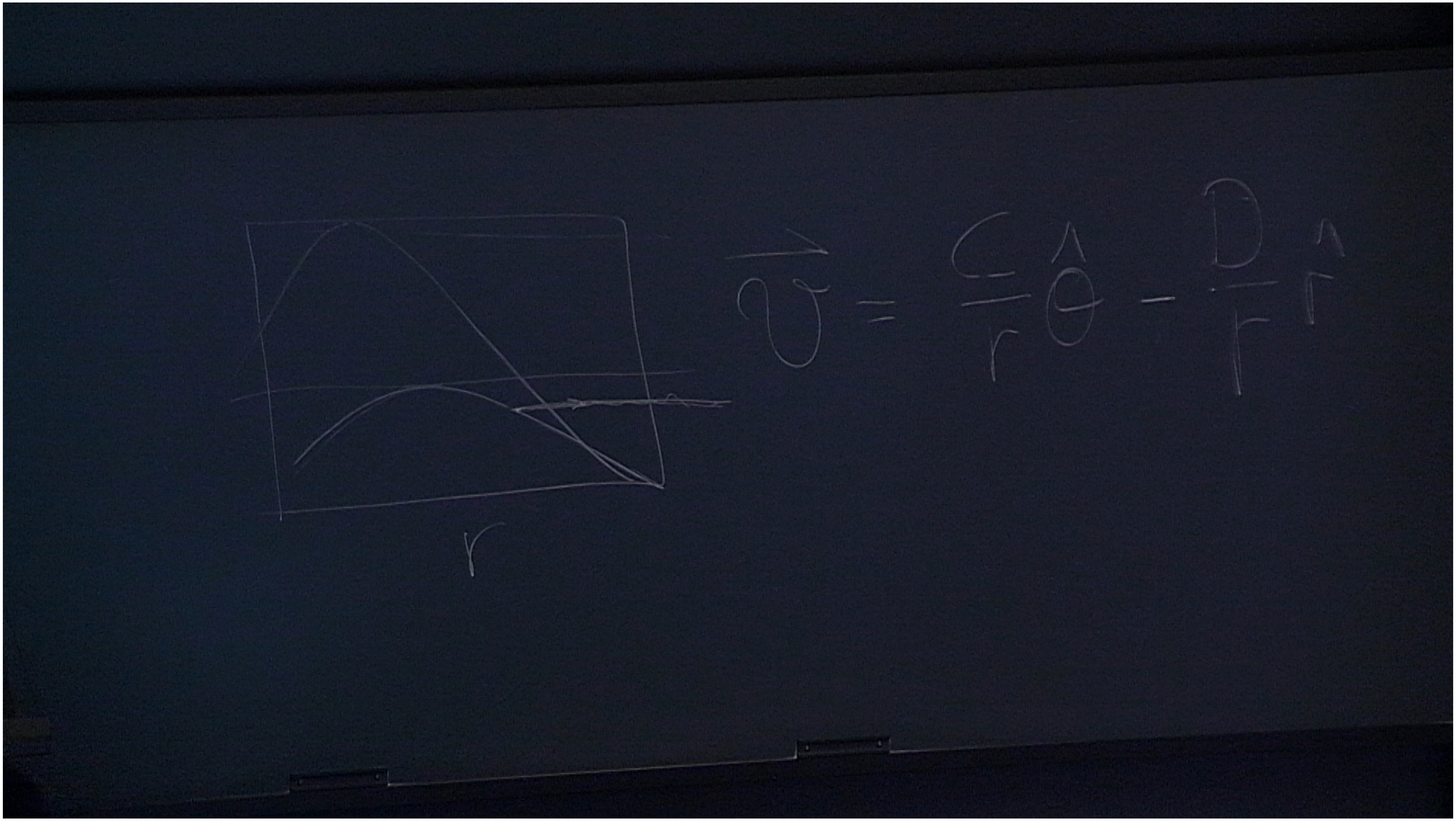


Flow characterisation



95% bootstrap confidence interval




$\vec{v} = \frac{C}{r}\hat{\theta} - \frac{D}{r}\hat{r}$
r

What about Quasi-bound states?

The effective field theory is much more difficult than a single propagating degree of freedom and/or effective Lorentz symmetry breaking:

a) Rotational vortex core -> extra degrees of freedom /different potential

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Editors' Suggestion

Black Hole Quasibound States from a Draining Bathtub Vortex Flow

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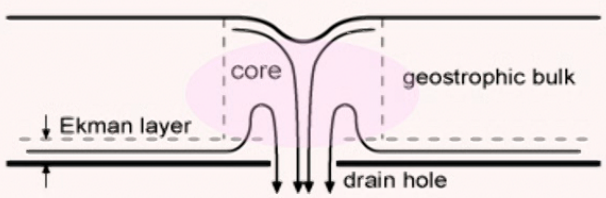
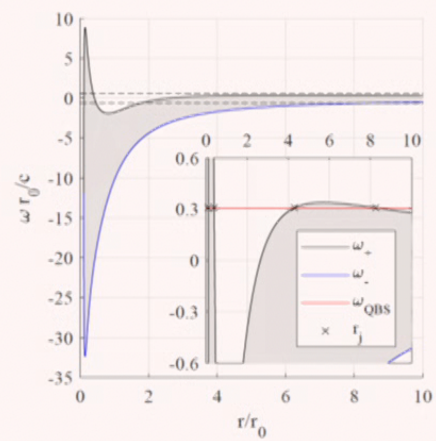
$$(\partial_t + \mathbf{v}\cdot\nabla)^2\phi + \Omega_v^2\phi - c^2\nabla^2\phi = 0$$





Summary

- **Superradiance is not only universal, it seems incredibly robust to me, and transferring some of the insight gained from working on superradiance for the last 6 years, and given that** our understanding of the exterior of rotating black holes seems decent, a black hole undergoes superradience if:
  - the black holes rotates: $\omega - m\Omega < 0$

  - the black holes is a **partially** absorbent

- **Application of black hole-fluid analogy: identification of a vortex through its characteristics waves (arXiv:1811.07858)**
  This tool enables us to explore fluid and superfluid vortex flows through its characteristic waves, similar to the idea of probing GR (possible deviations from GR) through QN/LRMs, we test wave-current (hence the background flow) through it's QN/LRM spectrum.

Can we apply our findings to superfluid flows?