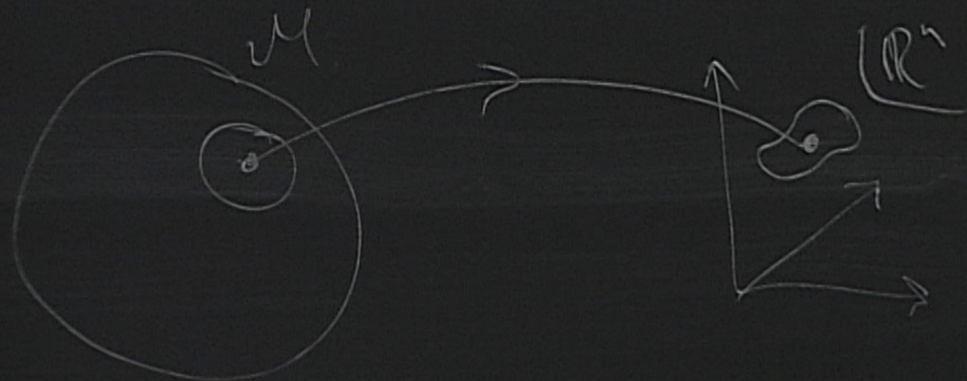
Title: PSI 2018/2019 - Gravitational Physics - Lecture 1

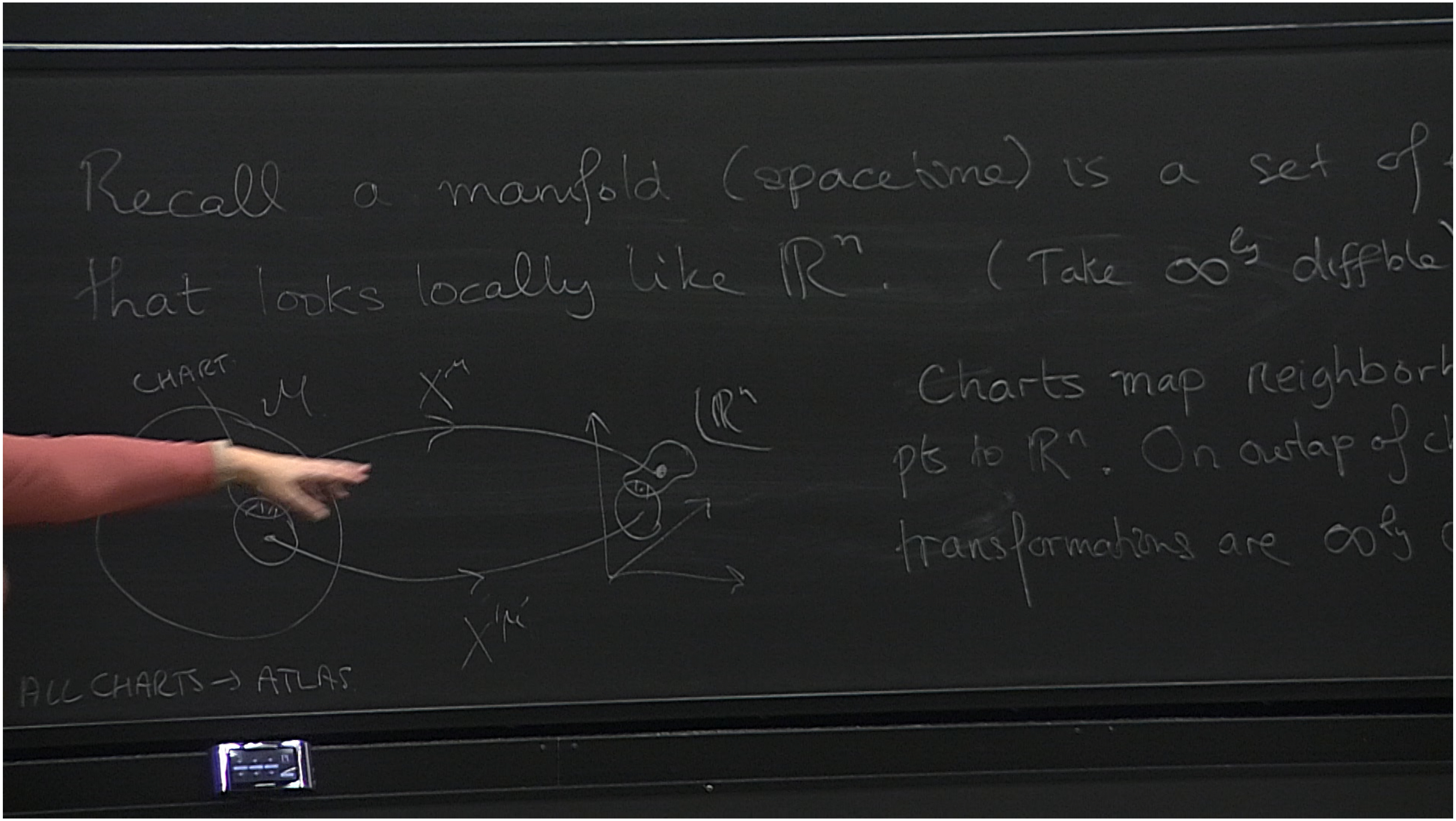
Date: Jan 28, 2019 10:15 AM

URL: http://pirsa.org/19010051

Abstract:

Recall a manifold (spacetime) is a set of e
that looks locally like $\mathbb{R}^n$.




Recall a manifold (spacetime) is a set of
that looks locally like $\mathbb{R}^n$. (Take ∞ly diff'ble
CHART
$\mathcal{M}$
X^μ
$\mathbb{R}^n$
$X'^{\mu'}$
Charts map neighbor
pts to $\mathbb{R}^n$. On overlap of c
transformations are ∞ly
ALL CHARTS → ATLAS

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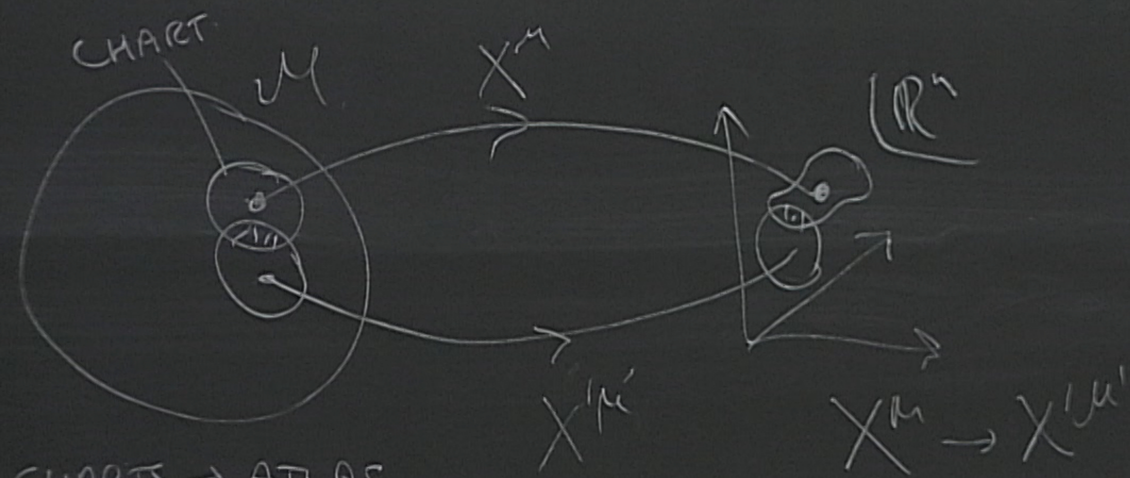

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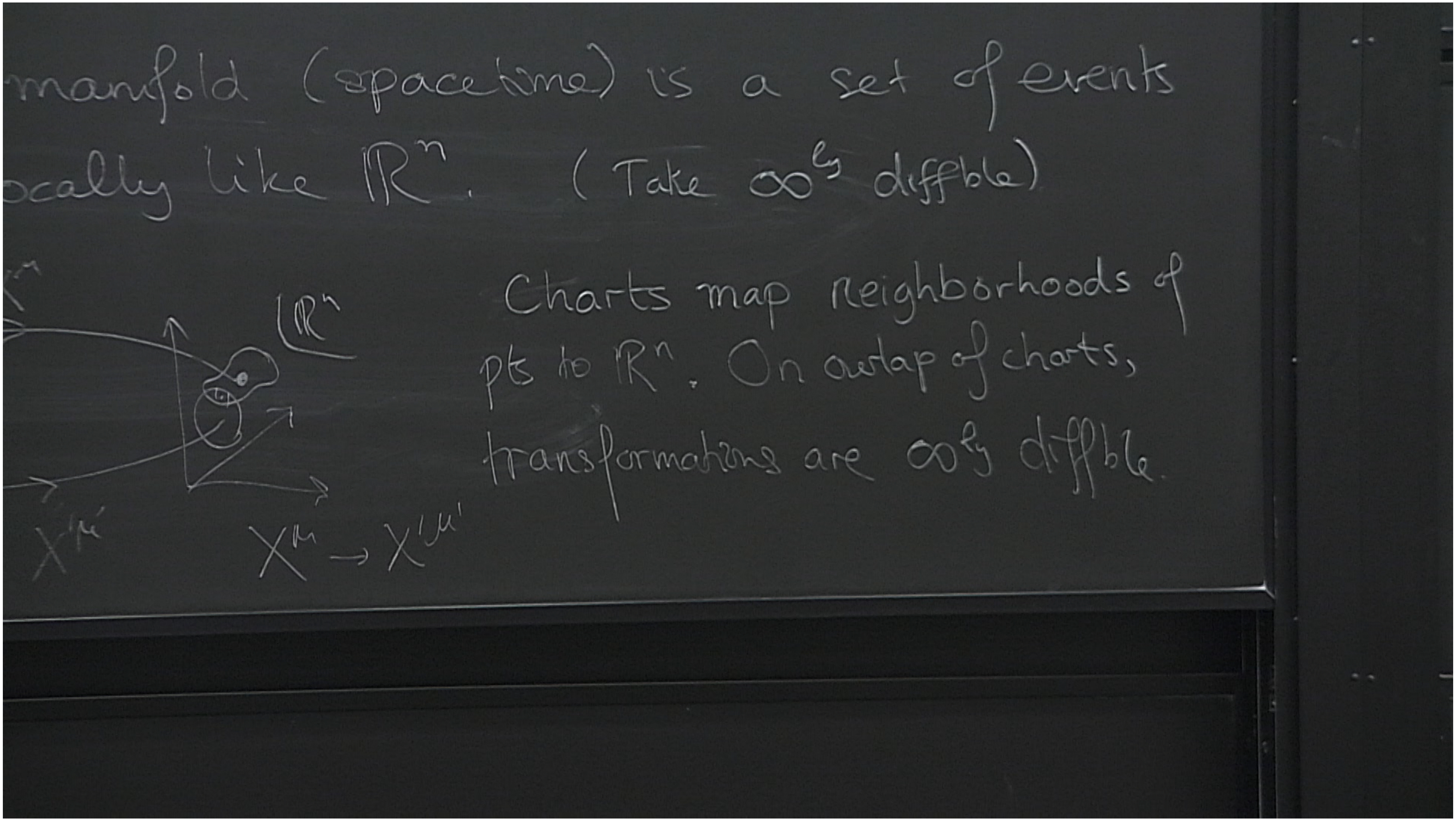
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ocally like $\mathbb{R}^n$. (Take ∞ly diff'ble)
$\mathbb{R}^n$
x'^μ
$x^\mu \to x'^\mu$
Charts map neighborhoods of
pts to $\mathbb{R}^n$. On overlap of charts,
transformations are ∞ly diff'ble.

Functions: $C^\infty(\mathcal{M})$ - collections of ∞^{ly} diffble fns on $\mathcal{M}$

$$f : \mathcal{M} \to \mathbb{R}$$

$$x^\mu \mapsto f(x^\mu)$$

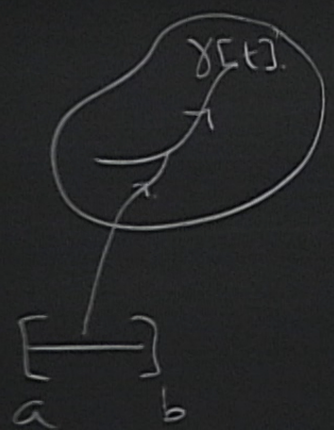
(use charts to define diff)

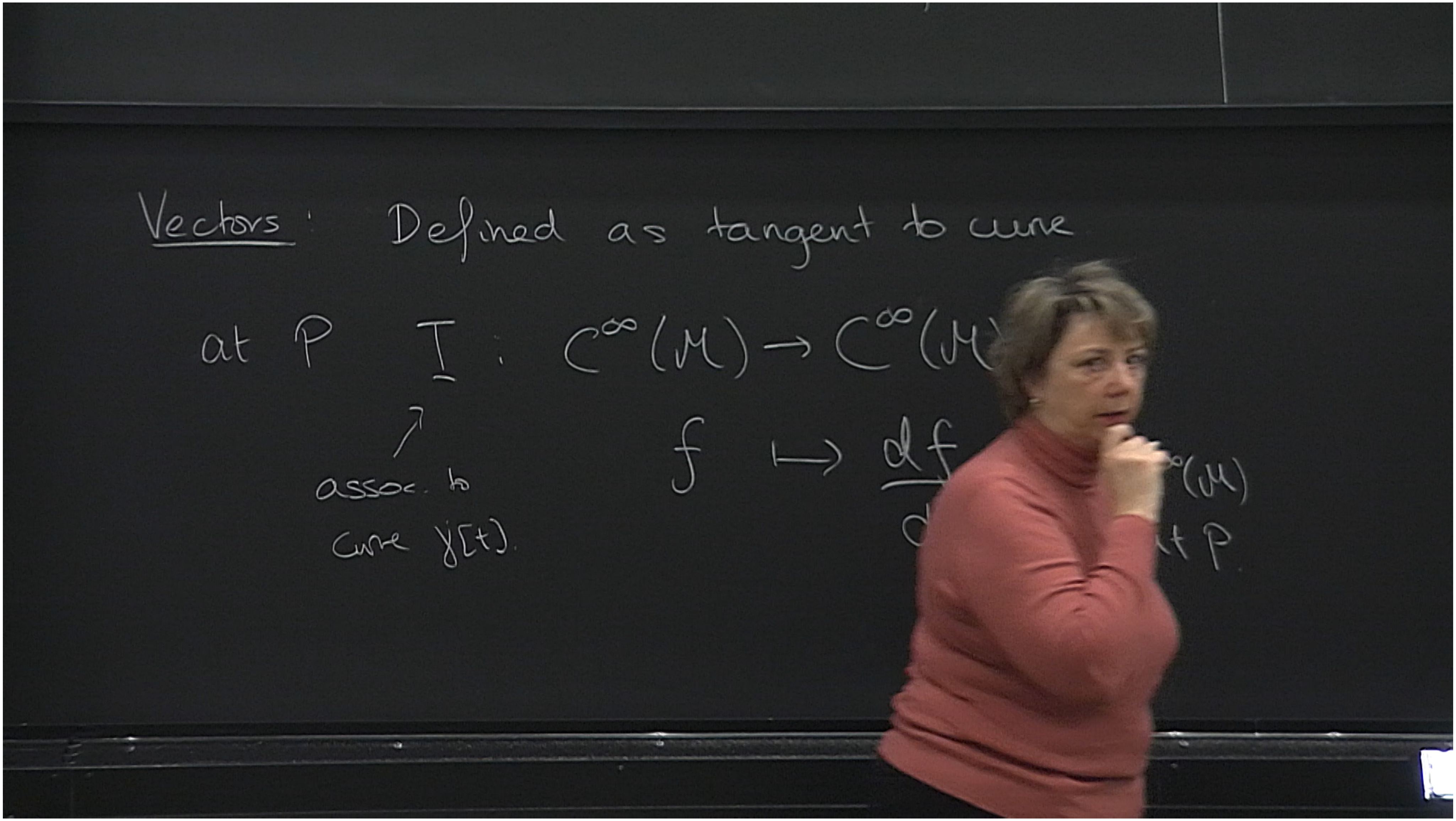
Curve

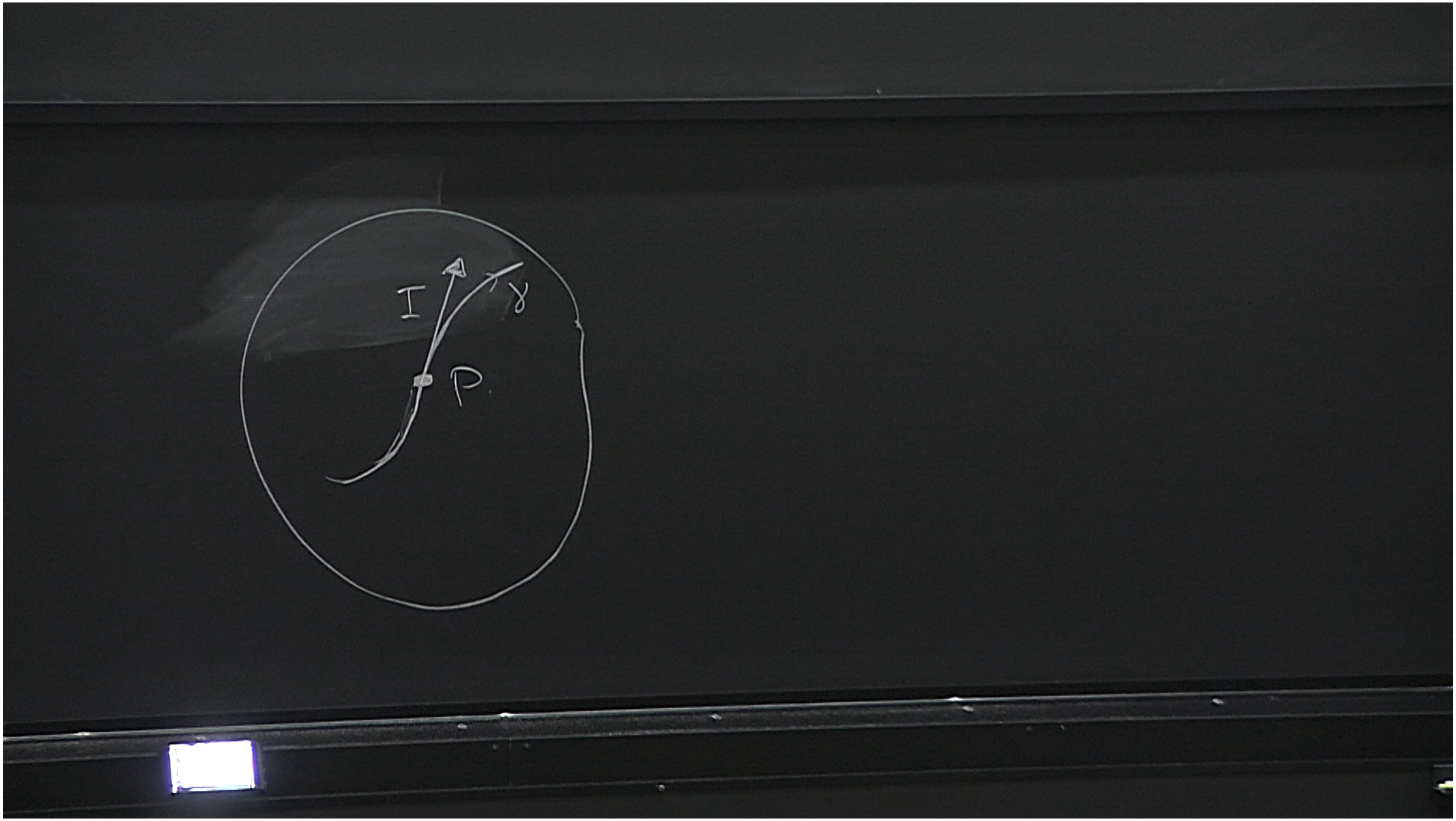
Curves: These are a map from $\mathbb{R}$ (or subset) to $\mathcal{M}$

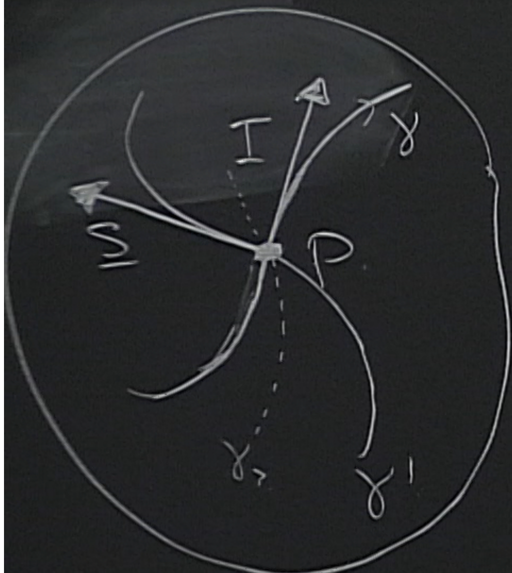
$\gamma(t)$ is the curve on $\mathcal{M}$

Can also write as $X^\mu(t)$ in local chart.




Vectors: Defined as tangent to curve
at P
T : $C^\infty(M) \to C^\infty(M)$
assoc. to
curve $\gamma[t]$
$f \mapsto$ df
(M)
at P


I
γ
P



These operators form a vector space associated to P.

Add vectors by composing curves in chart; scale by taking $\gamma[\lambda t]$ $\underline{I} \to \underline{I}/\lambda$.

TANGENT SPACE $T_p(M)$.

Collection of tgt spaces gives
tangent bundle $T(\mathcal{M})$

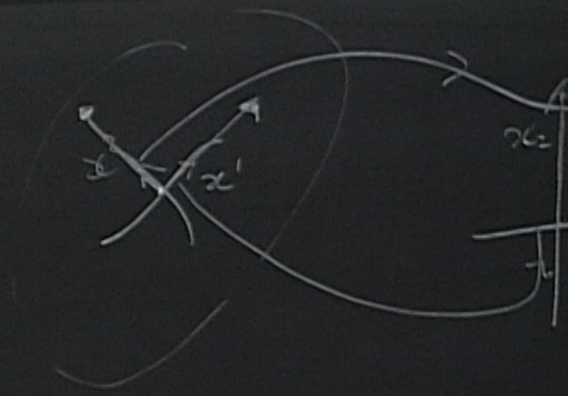
<u>Covectors</u>: Maps from tgt space to $\mathbb{R}$.

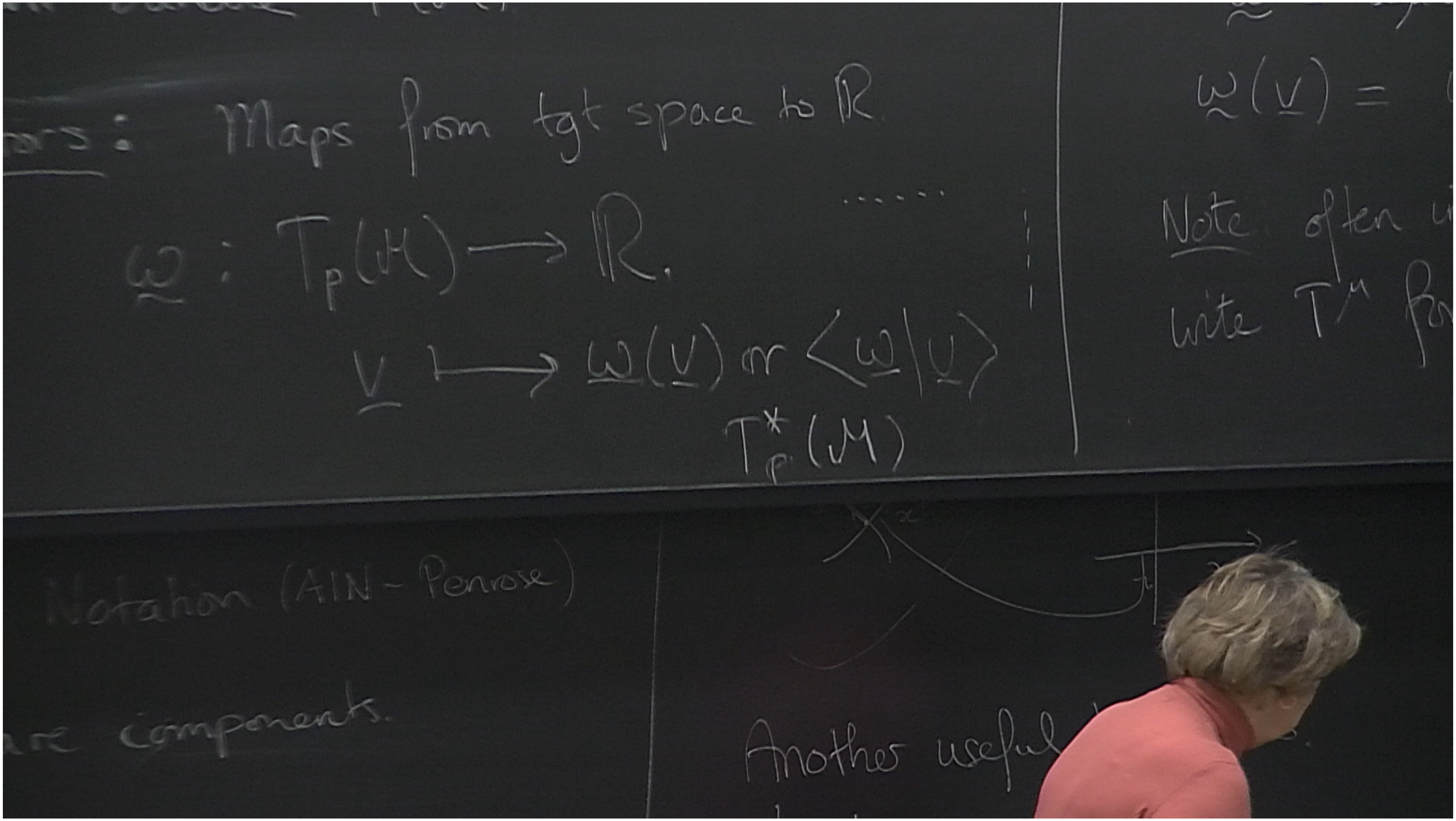
$$\underset{\sim}{\omega} : T_p(\mathcal{M}) \longrightarrow \mathbb{R}.$$

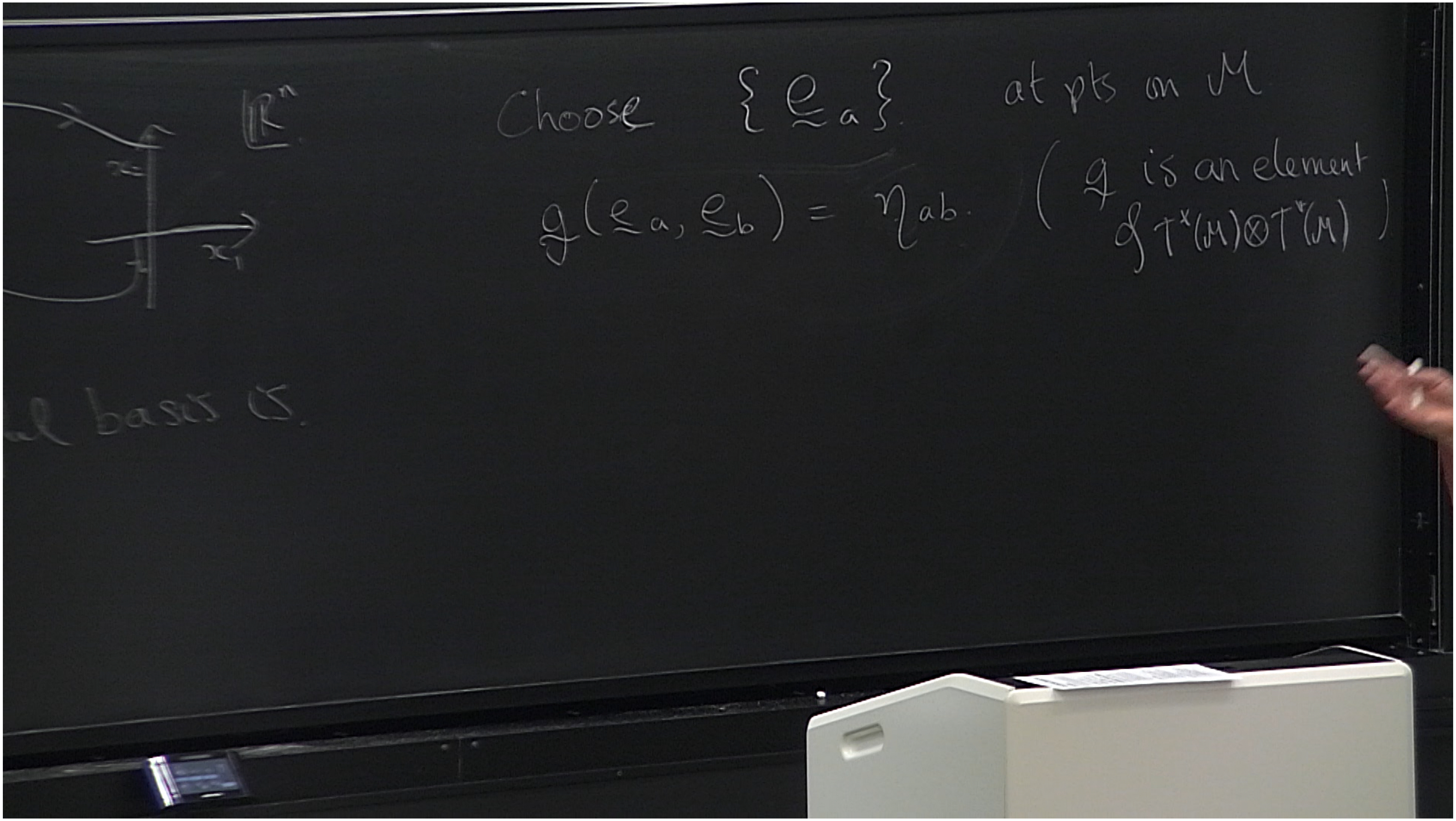
Writing T^μ is called Abstract Index Notation (AIN – Penrose)

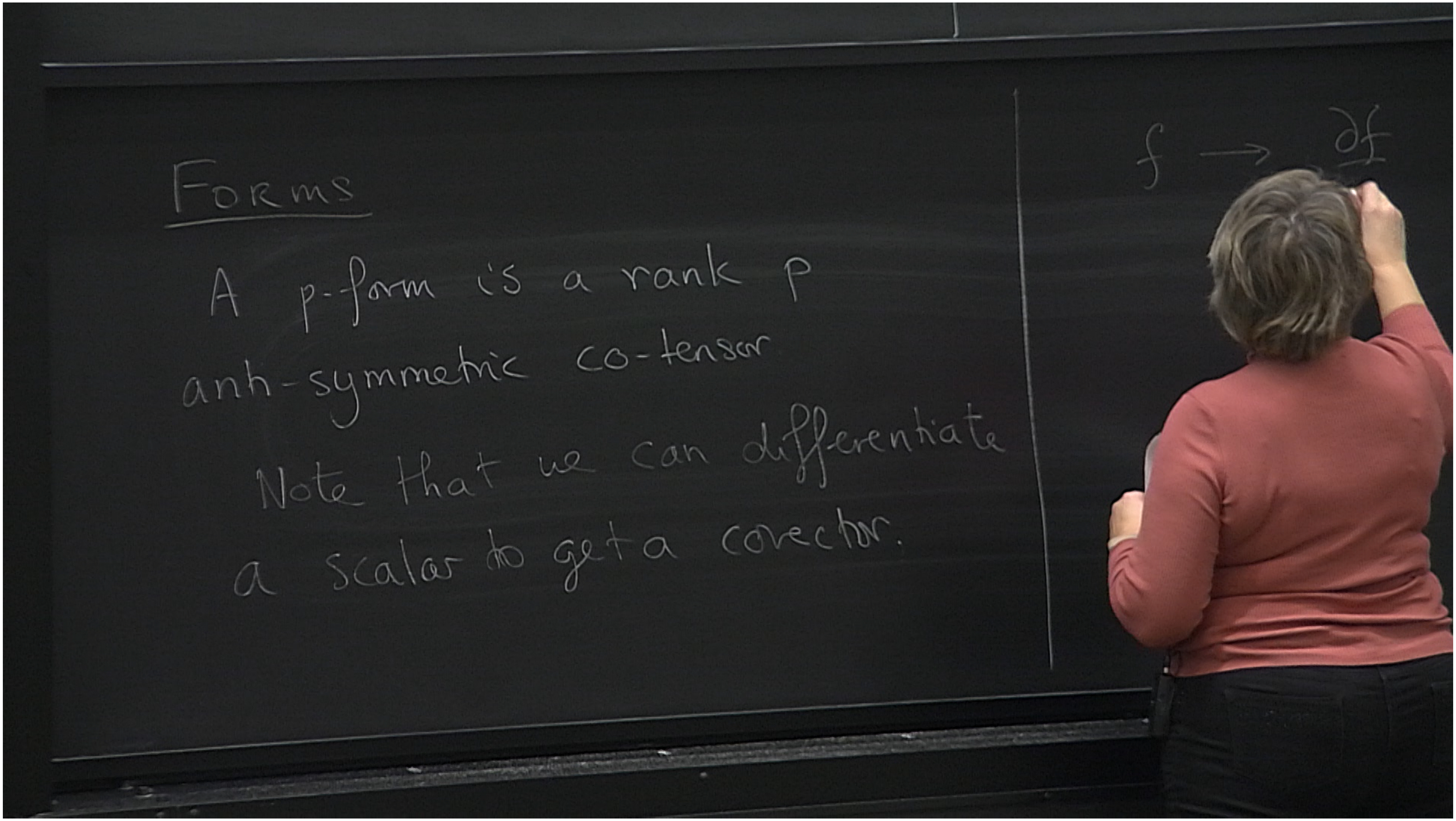
T^μ are components.

$\frac{\partial}{\partial x^\mu}$ is basis – coord basis.




Maps from tgt space to ℝ
ω : T_p(M) → ℝ.
V ↦ ω(V) or ⟨ω|V⟩
T*_p(M)
ω(V) =
Note: often
write T^μ
Notation (AIN - Penrose)
are components.
Another useful


$\mathbb{R}^n$
x_2
x_1
Choose $\{ e_a \}$ at pts on $\mathcal{M}$
$g(e_a, e_b) = \eta_{ab}$. (g is an element of $T^*(\mathcal{M}) \otimes T^*(\mathcal{M})$)
basis is.

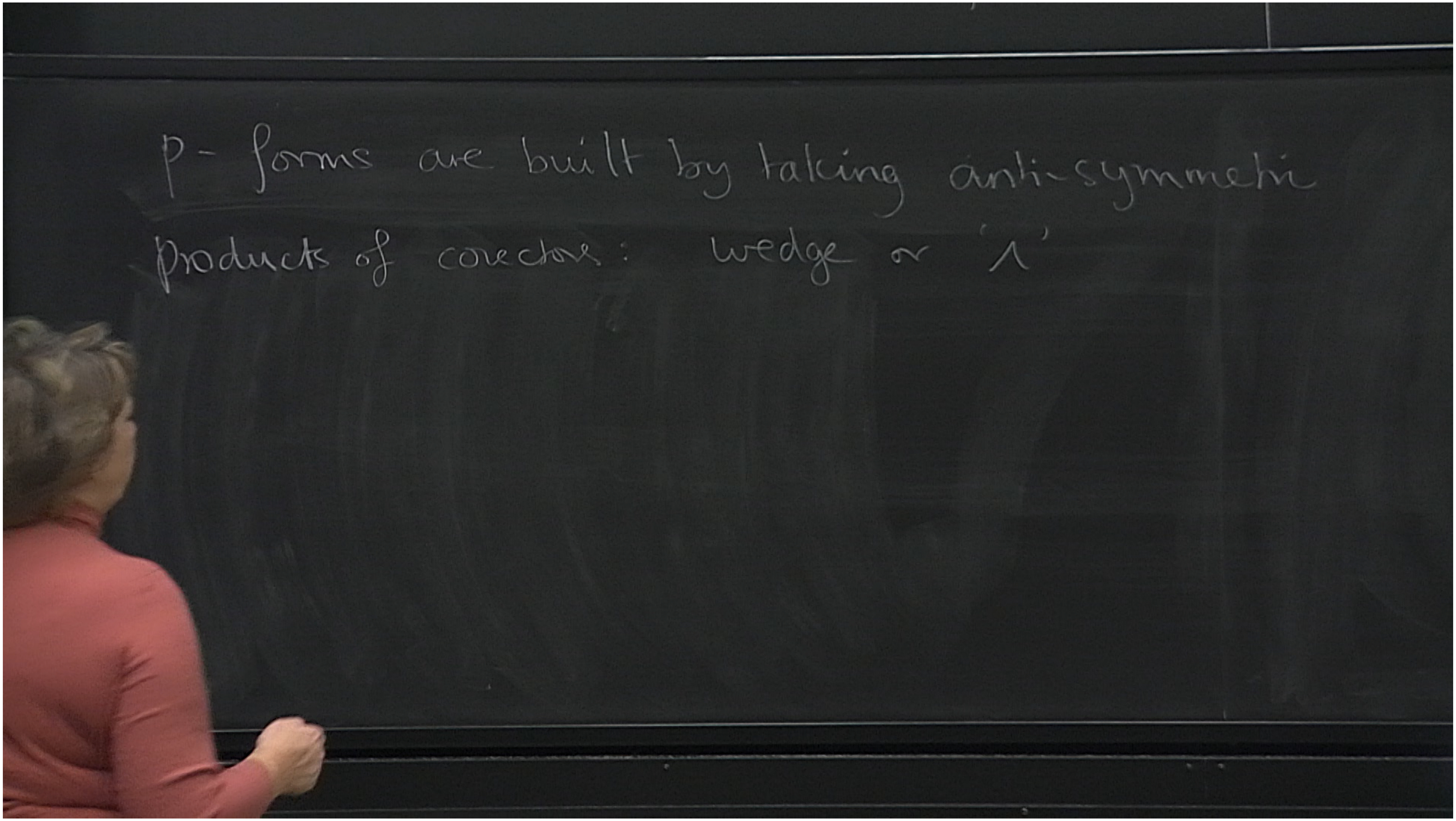

Forms
A p-form is a rank p
anti-symmetric co-tensor
Note that we can differentiate
a scalar to get a covector.
$f \longrightarrow \partial f$

$$f \longrightarrow \frac{\partial f}{\partial x^\mu}\, d x^\mu = \underline{d} f$$

$\underline{d} x^\mu$ forms a basis for $T^*(M)$

$$\left\langle \underline{d} x^\mu \middle| \frac{\partial}{\partial x^\nu} \right\rangle = \frac{\partial x^\mu}{\partial x^\nu} = \delta^\mu_\nu .$$

dual basis


p- forms are built by taking anti-symmetri
products of covectors: wedge or '$\wedge$'

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$$\underline{\omega}, \underline{\lambda} \in T_p^*(M), \quad \underline{\omega} \wedge \underline{\lambda} = \underline{\omega} \otimes \underline{\lambda} - \underline{\lambda} \otimes \underline{\omega}$$

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in cpts

$$\left[\underline{A}^{(p)} \wedge \underline{B}^{(q)}\right]_{a_1 \cdots a_{p+q}} = \frac{(p+q)!}{p!\,q!} A_{[a_1 \cdots a_p} B_{a_{p+1}}$$

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in cpts $$\left[\underset{\sim}{A}^{(p)} \wedge \underset{\sim}{B}^{(q)}\right]_{a_1 \dots a_{p+q}} = \frac{(p+q)!}{p!\,q!} A_{[a_1 \dots a_p} B_{a_{p+1}}$$

$$\underset{\sim}{A}^{(p)} \wedge \underset{\sim}{B}^{(q)} = (-)^{pq}\, \underset{\sim}{B} \wedge \underset{\sim}{A}$$

Note: cannot have a form of rank greater than dimension of manifold.

$\ldots a_{p+q}]$.

Co-ord transfms

$$X^\mu \to X'^{\mu'}$$

on overlapping chart

$\underset{\sim}{T}$ is geometric, so indep of chart, but components of $\underset{\sim}{T}$ change.

$$\underset{\sim}{T} = T^\mu \frac{\partial}{\partial X^\mu} = T'$$

$$= T^\mu \frac{\partial X^{\mu'}}{\partial X^\mu} \frac{\partial}{\partial X'}$$

ms

art

so indep

mponents of I change.

$$I = T^{\mu}\frac{\partial}{\partial x^{\mu}} = T'^{\mu'}\frac{\partial}{\partial x'^{\mu'}}$$

$$= T^{\mu}\frac{\partial x^{\mu'}}{\partial x^{\mu}}\frac{\partial}{\partial x'^{\mu'}}$$

$$T'^{\mu'} = \frac{\partial x'^{\mu'}}{\partial x^{\mu}} T^{\mu}$$

contra-
variant

simly:

$\frac{\partial}{\partial X'^{\mu'}}$

T^{μ} contra-variant

simly: $\omega'_{\mu'} = \frac{\partial X^{\mu}}{\partial X'^{\mu'}} \omega_{\mu}$ COVARIANT.

? Construct n-form ??

$$\frac{1}{n!} \epsilon_{\mu\nu\lambda\rho}\, dx^{\mu} \wedge dx^{\nu} \wedge dx^{\lambda} \wedge dx^{\rho}$$

$n = 4$

$$= \frac{1}{n!} \det\left(\frac{\partial X}{\partial X'}\right) \epsilon_{\mu'\nu'\lambda'\rho'}\, dx'^{\mu'} \wedge dx'^{\nu'} \wedge dx'^{\lambda'} \wedge dx'^{\rho'}$$

DENSITY.

I change

With a metric can define a 4 (or n) - form tensor:

$$\det(g_{\mu\nu}) \xrightarrow[x \to x']{} \det(g'_{\mu'\nu'}) = \det\left[\frac{\partial x^\mu}{\partial x^{\mu'}} \frac{\partial x^\nu}{\partial x^{\nu'}} g_{\mu\nu} \right.$$

$$= \det\left(\frac{\partial x}{\partial x'}\right)^2 \det g$$

DENSITY

Define $\mathcal{E}_{\mu\nu\lambda\rho} = \sqrt{\det g}\; \epsilon_{\mu\nu\lambda\rho} = \sqrt{\det g'}\; \epsilon_{\mu'\nu'\lambda'\rho'}$

$$\times \frac{\partial x'^{\mu'}}{\partial x^{\mu}} \text{ etc}$$

With $\mathcal{E}$ define Hodge dual $*$

$$* : \Lambda^{(p)} \longrightarrow \Lambda^{(n-p)} \qquad (*A)_{a_1 \cdots a_{n-p}}$$

$$\underline{A} \longmapsto * \underline{A} \qquad = \frac{1}{p!} \mathcal{E}_{a_1 \cdots a_{n-p}}{}^{a_{n-p+1} \cdots a_n} A_{a_{n-p+1} \cdots a_n}$$