Title: PSI 2018/2019 - Standard Model Review - Lecture 4

Date: Jan 31, 2019 09:00 AM

URL: http://pirsa.org/19010050

Abstract:

2) Spinors
3) Spin 1
(3) Qtz Maxwell
GR SM

Nonabelian Higgs Model

$$\mathcal{L} = (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(\Phi) \quad , \qquad \Phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = \text{complex doublet}$$

$$- \frac{1}{2} \mathrm{Tr}\left[F_{\mu\nu} F^{\mu\nu} \right]$$

$$V(\Phi) = -\mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$$

$$D_\mu \Phi = (\partial_\mu + i g T^a A^a_\mu) \Phi$$

$$V(\Phi) = -\mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$$

$$D_\mu \Phi = (\partial_\mu + i g T^a A^a_\mu) \Phi$$

Minimize potential $\quad \Phi^\dagger \Phi = \frac{v^2}{2} \qquad v = \sqrt{\frac{\mu^2}{\lambda}}$

Write $\quad \Phi(x) = \exp\left(i T^a \xi^a(x)/v \right) \begin{pmatrix} 0 \\ \frac{v + h(x)}{\sqrt{2}} \end{pmatrix}$ $\quad$ 4 real fields $h(x)$, $\xi^{a=1,2,3}(x)$

Gauge transformation $\quad \Phi \to e^{-i \xi^a T^a / v} \Phi = \begin{pmatrix} 0 \\ \frac{v + h(x)}{\sqrt{2}} \end{pmatrix}$

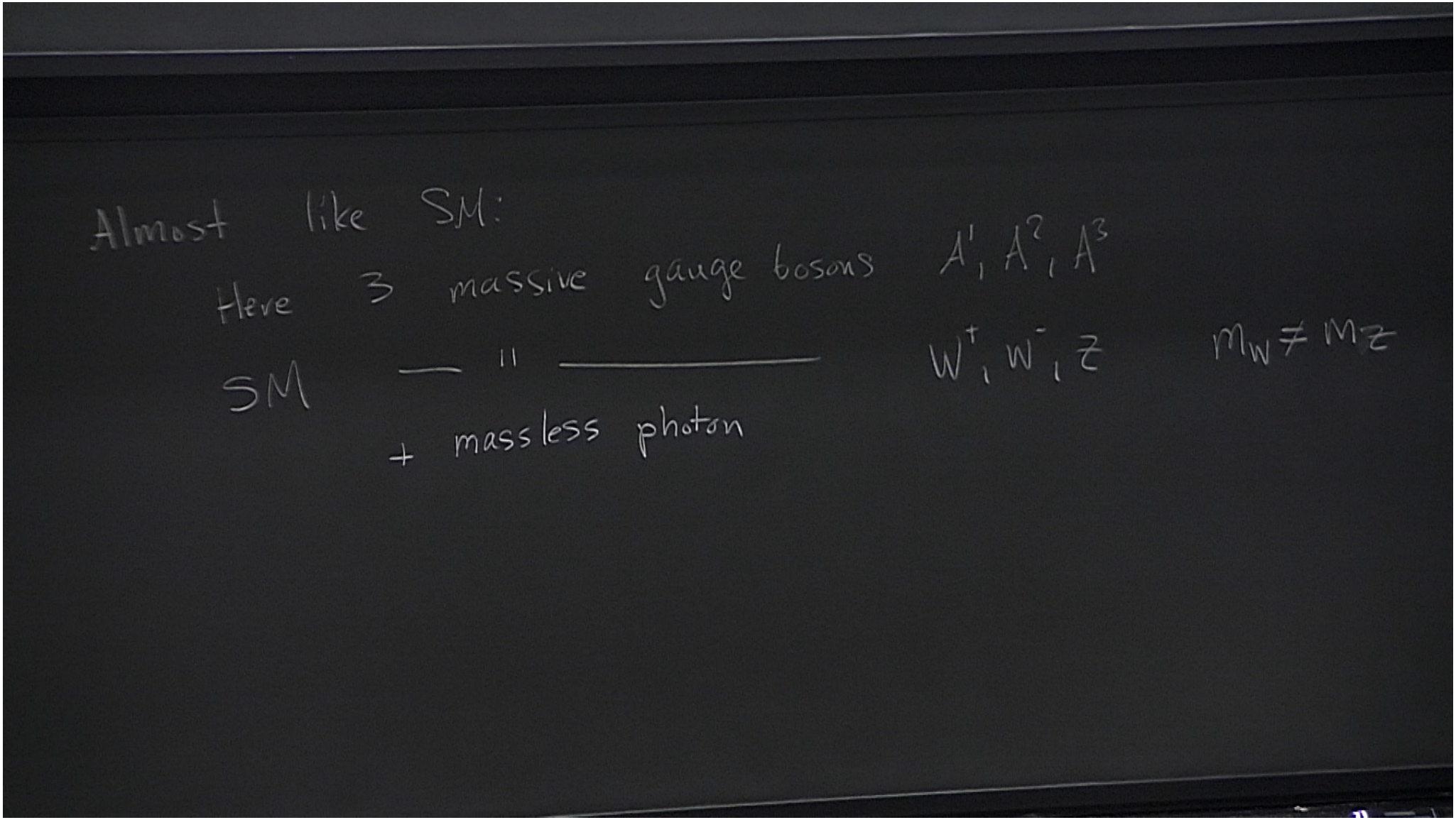
$$= \frac{1}{2}(\partial_\mu h)^2 + \frac{m_A^2}{2}\left((A_\mu^1)^2 + (A_\mu^2)^2 + (A_\mu^3)^2\right)\left(1+\frac{h}{v}\right)^2 \qquad m_A = \frac{gv}{2}$$

Unbroken theory:

2x3 gauge d.o.f. + 2x2 scalar (real) d.o.f. = 10

Broken theory

3x3 gauge d.o.f. + 1 scalar d.o.f. = 10


Almost like SM:
Here 3 massive gauge bosons A^1, A^2, A^3
SM — || — W^+, W^-, Z $m_W \neq m_Z$
+ massless photon

Standard Model Lagrangian

Gauge group (W, Z, γ + gluons)

D.o.f. (fermions)

What are quantum numbers for fermions/scalar.

Gauge group: $G = \underbrace{SU(3)_C}_{QCD} \times$

C = color

Gauge group: $G = \underbrace{SU(3)_C}_{\text{QCD}} \times \underbrace{SU(2)_L \times U(1)_Y}_{\text{electroweak}}$

C = color

L = left, Y = hypercharge

Fields	Quantum numbers $(SU(3)_C, SU(2)_L, U(1)_Y)$
$Q_L^i = \begin{pmatrix} u_L^i \\ d_L^i \end{pmatrix}$	$(3, 2, \frac{1}{6})$
u_R^i	$(3, 1, \frac{2}{3})$
d_R^i	$(3, 1, -\frac{1}{3})$

$i = 1, 2, 3$ labels generation

Gauge bosons:

			Couplings
$SU(3)_C$:	gluon fields g_μ^A	$(A = 1, \ldots, 8)$	g_s
$SU(2)_L$:	W_μ^a	$(a = 1, 2, 3)$	g
$U(1)_Y$:	B_μ		g'

$$\mathcal{L}_{SM} = \mathcal{L}_{gauge} + \mathcal{L}_{fermion} + \mathcal{L}_{scalar} + \mathcal{L}_{Yukawa}$$

$$\mathcal{L}_{gauge} = -\frac{1}{2}\mathrm{Tr}\left(g_{\mu\nu}\, g^{\mu\nu}\right) - \frac{1}{2}\mathrm{Tr}\left(W_{\mu\nu} W^{\mu\nu}\right) - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

$$\mathcal{L}_{fermion} = \sum_{\text{fermions } \psi} \bar{\psi}\, i \not{D}\, \psi \qquad \text{(mass terms forbidden due to chiral gauge interaction)}$$

$$\mathcal{L}_{scalar} = (D_\mu H)^\dagger (D^\mu H) - V(H) \qquad V(H) = -\mu^2 H^\dagger H + \lambda (H^\dagger H)^2$$

Scalar Lagrangian:

Work in unitary gauge $H = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix}$, $v = \sqrt{\frac{\mu^2}{\lambda}}$

$$D_\mu H = \left(\partial_\mu + \frac{ig}{2}\sigma^a W_\mu^a + i\frac{g'}{2} B_\mu \right) \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix}$$

$$= \left(\partial_\mu + \frac{i}{2}\begin{pmatrix} gW_\mu^3 + g'B_\mu & g(W_\mu^1 - iW_\mu^2) \\ g(W_\mu^1 + iW_\mu^2) & -gW_\mu^3 + gB_\mu \end{pmatrix} \right)$$

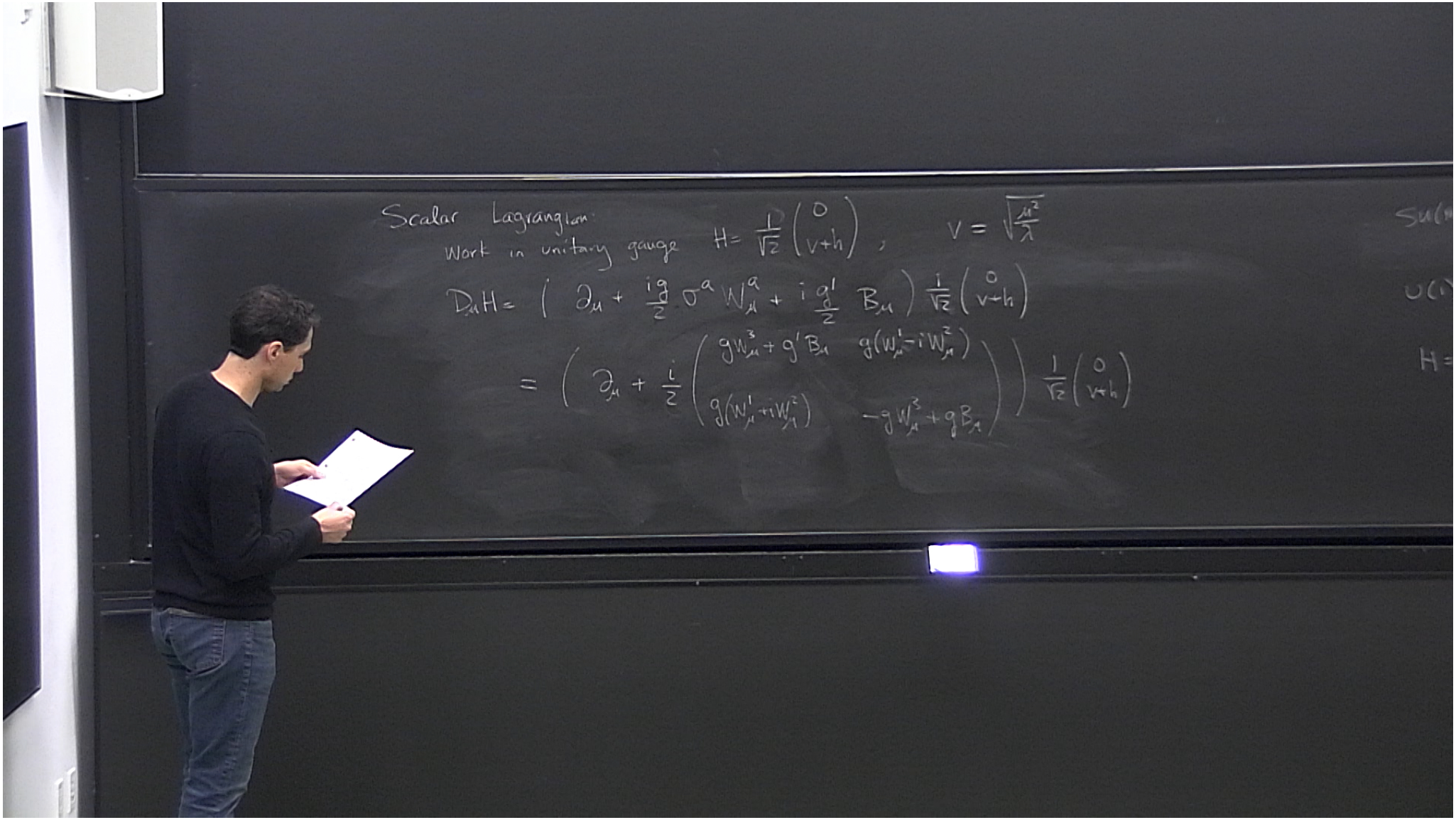
$SU(N)$: $\underline{1}$ = trivial rep

N = fundamental

$U(1)$: 0 = trivial rep

Q = charge Q

$H = (1, 2, \frac{1}{2})$


Scalar Lagrangian:
Work in unitary gauge $H = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix}$, $v = \sqrt{\frac{\mu^2}{\lambda}}$
$D_\mu H = \left(\partial_\mu + \frac{ig}{2}\sigma^a W_\mu^a + i\frac{g'}{2} B_\mu \right) \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix}$
$= \left(\partial_\mu + \frac{1}{2}\begin{pmatrix} gW_\mu^3 + g'B_\mu & g(W_\mu^1 - iW_\mu^2) \\ g(W_\mu^1 + iW_\mu^2) & -gW_\mu^3 + gB_\mu \end{pmatrix} \right) \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix}$

2) Spinors
3) Spin $\frac{1}{2}$

(3) a tz Maxwell

GR SM

Scalar Lagrangian:

Work in unitary gauge $H = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix}$, $v = \sqrt{\frac{\mu^2}{\lambda}}$

$$D_\mu H = \left(\partial_\mu + \frac{ig}{2}\sigma^a W_\mu^a + i\frac{g'}{2} B_\mu \right) \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix}$$

$$= \left(\partial_\mu + \frac{i}{2}\begin{pmatrix} gW_\mu^3 + g'B_\mu & g(W_\mu^1 - iW_\mu^2) \\ g(W_\mu^1 + iW_\mu^2) & -gW_\mu^3 + gB_\mu \end{pmatrix} \right) \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix}$$

$$(D_\mu H)^\dagger (D^\mu H) = \frac{1}{2}(\partial_\mu h)^2 + \frac{g^2 v^2}{8}\left((W_\mu^1)^2 + (W_\mu^2)^2 \right)\left(1+\frac{h}{v}\right)^2$$

$$+ \frac{1}{8} v^2 \left(g W_\mu^3 - g' B_\mu \right)^2 \left(1+\frac{h}{v}\right)^2$$

Define:

$$Z_\mu = \cos\theta_W \, W_\mu^3 - \sin\theta_W \, B_\mu$$

$$A_\mu = \sin\theta_W \, W_\mu^3 + \cos\theta_W \, B_\mu$$

$$W_\mu^\pm = \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp i W_\mu^2 \right)$$

θ_W = weak mixing angle

$c_W = \cos\theta_W$

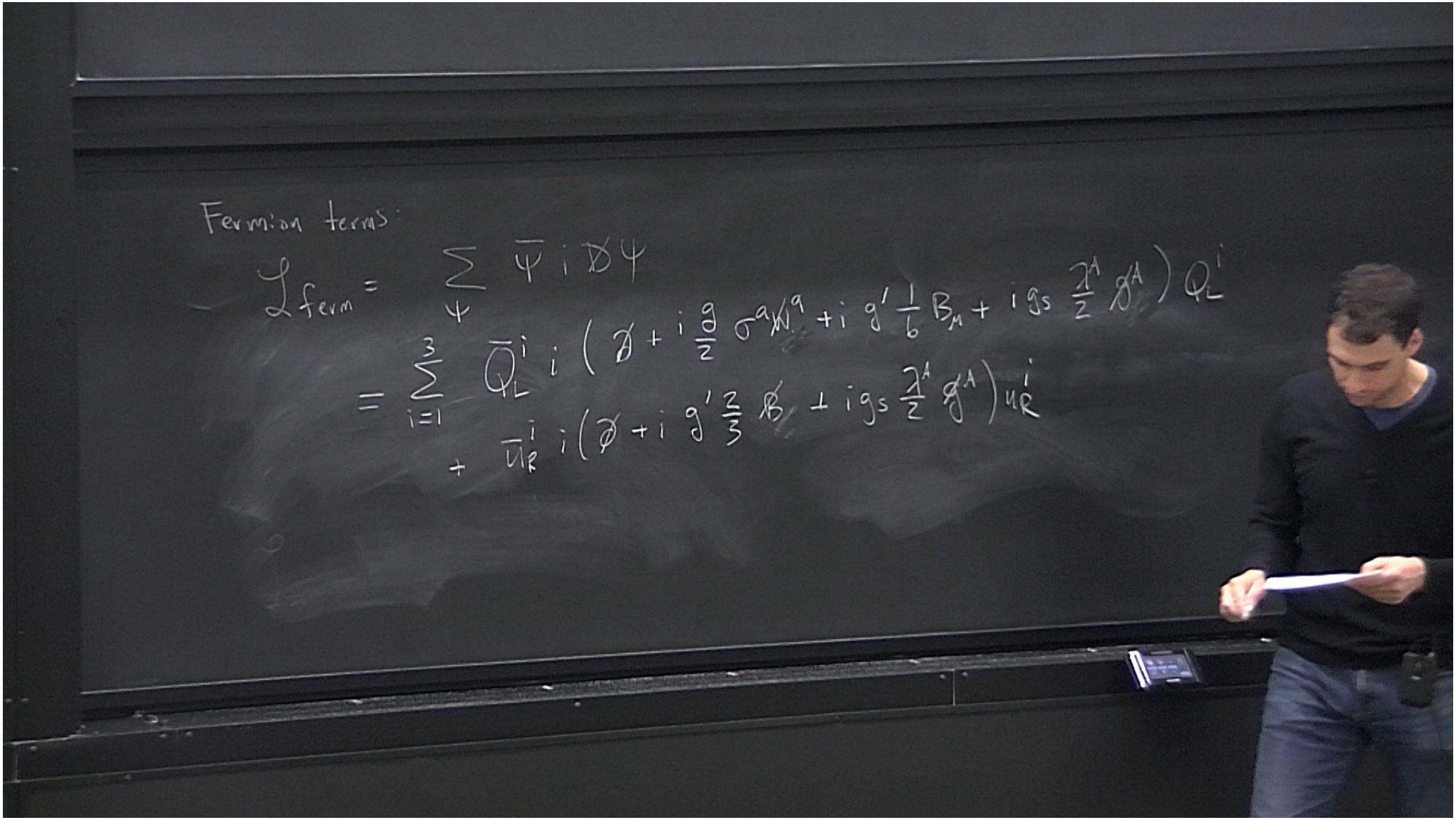
$s_W = \sin\theta_W$

$$(D_\mu H^\dagger)(D^\mu H) = \frac{1}{2}(\partial_\mu h)^2 + \frac{g^2 v^2}{4} W_\mu^+ W^{-\mu}\left(1+\frac{h}{v}\right)^2 + \frac{1}{8}\frac{g^2 v^2}{c_W^2} Z_\mu Z^\mu \left(1+\frac{h}{v}\right)^2$$

$$m_W = \frac{gv}{2}$$

$$m_Z = \frac{gv}{2c_W}$$

Three parameters m_W, m_Z, $\cos\theta_W$ $\rightarrow$ $\rho = \dfrac{m_W^2}{m_Z^2 c_W^2} = 1$


Fermion terms:
$\mathcal{L}_{ferm} = \sum_{\psi} \bar{\psi}\, i \not{D} \psi$
$= \sum_{i=1}^{3} \bar{Q}_L^i\, i \left(\not{\partial} + i \frac{g}{2} \sigma^a \not{W}^a + i g' \frac{1}{6} \not{B} + i g_s \frac{\lambda^A}{2} \not{g}^A \right) Q_L^i$
$+ \bar{u}_R^i\, i \left(\not{\partial} + i g' \frac{2}{3} \not{B} + i g_s \frac{\lambda^A}{2} \not{g}^A \right) u_R^i$

Fermion terms:

$$\mathcal{L}_{ferm} = \sum_{\psi} \bar{\psi}\, i \not{D} \psi$$

$$= \sum_{i=1}^{3} \bar{Q}_L^i\, i \left(\not{\partial} + i \frac{g}{2} \sigma^a \not{W}^a + i g' \frac{1}{6} \not{B} + i g_s \frac{\lambda^A}{2} \not{g}^A \right) Q_L^i$$

$$+ \bar{u}_R^i\, i \left(\not{\partial} + i g' \frac{2}{3} \not{B} + i g_s \frac{\lambda^A}{2} \not{g}^A \right) u_R^i$$

$$+ \bar{d}_R^i\, i \left(\not{\partial} + i g' \left(-\frac{1}{3} \right) \not{B} + i g_s \frac{\lambda^A}{2} \not{g}^A \right) d_R^i$$

$$+ \bar{L}_L^i \, i\left(\not{\partial} + i g'\left(-\tfrac{1}{2}\right)\not{B} + i\frac{g}{2}\sigma^a \not{W}^a\right) L_L^i$$

$$+ \bar{e}_R^i \, i\left(\not{\partial} + i g'(-1)\not{B}\right) e_R^i$$

$\not{\partial}$ terms. $\quad \bar{Q}_L^i \not{\partial} Q_L^i + \bar{u}_R^i \not{\partial} u_R^i + \bar{d}_R^i \not{\partial} d_R^i$

$$= \bar{u}_L^i \not{\partial} u_L^i + \bar{d}_L^i \not{\partial} d_L^i + \bar{u}_R^i \not{\partial} u_R^i + \bar{d}_R^i \not{\partial} d_R^i = \bar{u}^i \not{\partial} u^i + \bar{d}^i \not{\partial} d^i$$

gluon terms: $\quad -g_s g_\mu^A \left(\bar{u}^i \gamma^\mu \frac{\lambda^A}{2} u^i + \bar{d}^i \gamma^\mu \frac{\lambda^A}{2} d^i\right)$

