Title: PSI 2018/2019 - Standard Model Review - Lecture 2

Date: Jan 29, 2019 09:00 AM

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Abstract:

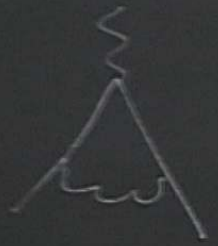
$$D_{\mu\nu}(k) = \frac{-i}{k^2 - m_A^2 + i\varepsilon}\left(\eta_{\mu\nu} - \frac{k_\mu k_\nu}{m_A^2}\right) \longrightarrow \text{large } k \qquad i\,\frac{k_\mu k_\nu}{k^2}$$

Divergences in QED: , are d

Renormalize wavefns., electric charge, electron mass

a $L-a$

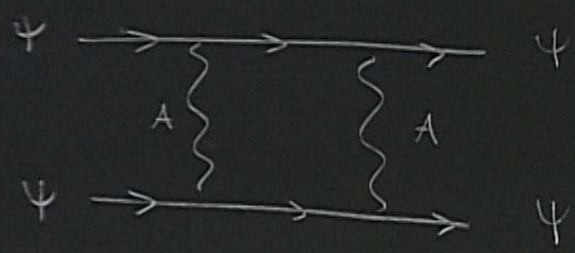
$i \quad \frac{k_\mu k_\nu}{k^2}$



are divergent → absorbed into parameters of theory.

charge. electron mass

Consider 4-point function in massive QED at 1-loop:



$$\sim g^4 \int_{\Lambda} \frac{d^4k}{(2\pi)^4} \left(\frac{1}{k}\right)^2 \left(\frac{k_\mu k_\nu}{k^2}\right)^2$$

keep divergent term

Λ = momentum cut-off

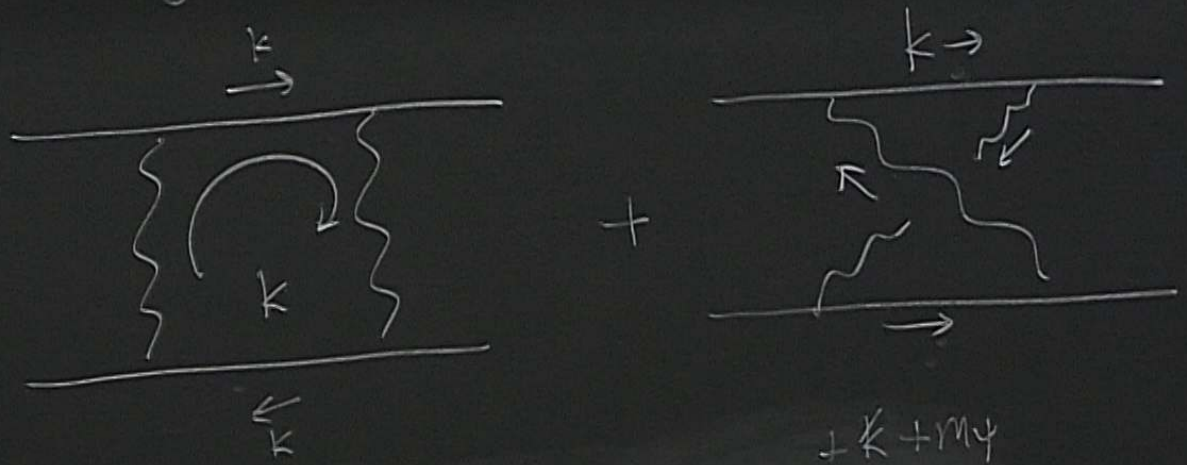
massive QED at 1-loop: $\psi\psi \to \psi\psi$

$$\int_{\Lambda} \frac{d^4k}{(2\pi)^4} \left(\frac{1}{k}\right)^2 \left(\frac{k_\mu k_\nu}{k^2}\right)^2 \sim g^4 \Lambda^2$$

keep divergent term

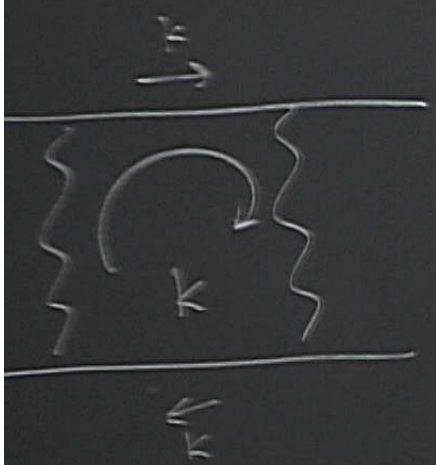
Λ = momentum cut-off

Other diagrams cancel quadratic divergence.



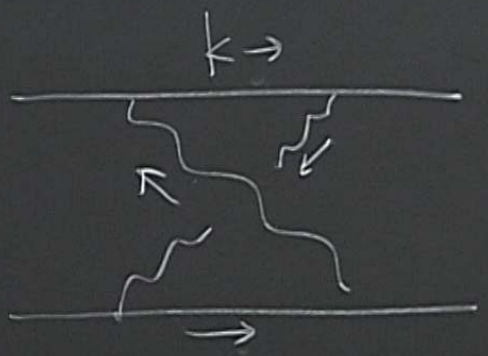
$$-\frac{\not{k}+m_\psi}{k^2-m_\psi^2} \qquad \frac{+\not{k}+m_\psi}{k^2-m_\psi^2}$$

iagrams cancel quadratic divergence. Still log divergent:



$$-\frac{\not{k}+m_\psi}{k^2-m_\psi^2}$$

$+$



$$\frac{+\not{k}+m_\psi}{k^2-m_\psi^2}$$

$$\propto \quad g^4 m_\psi^2 \log \Lambda^2$$

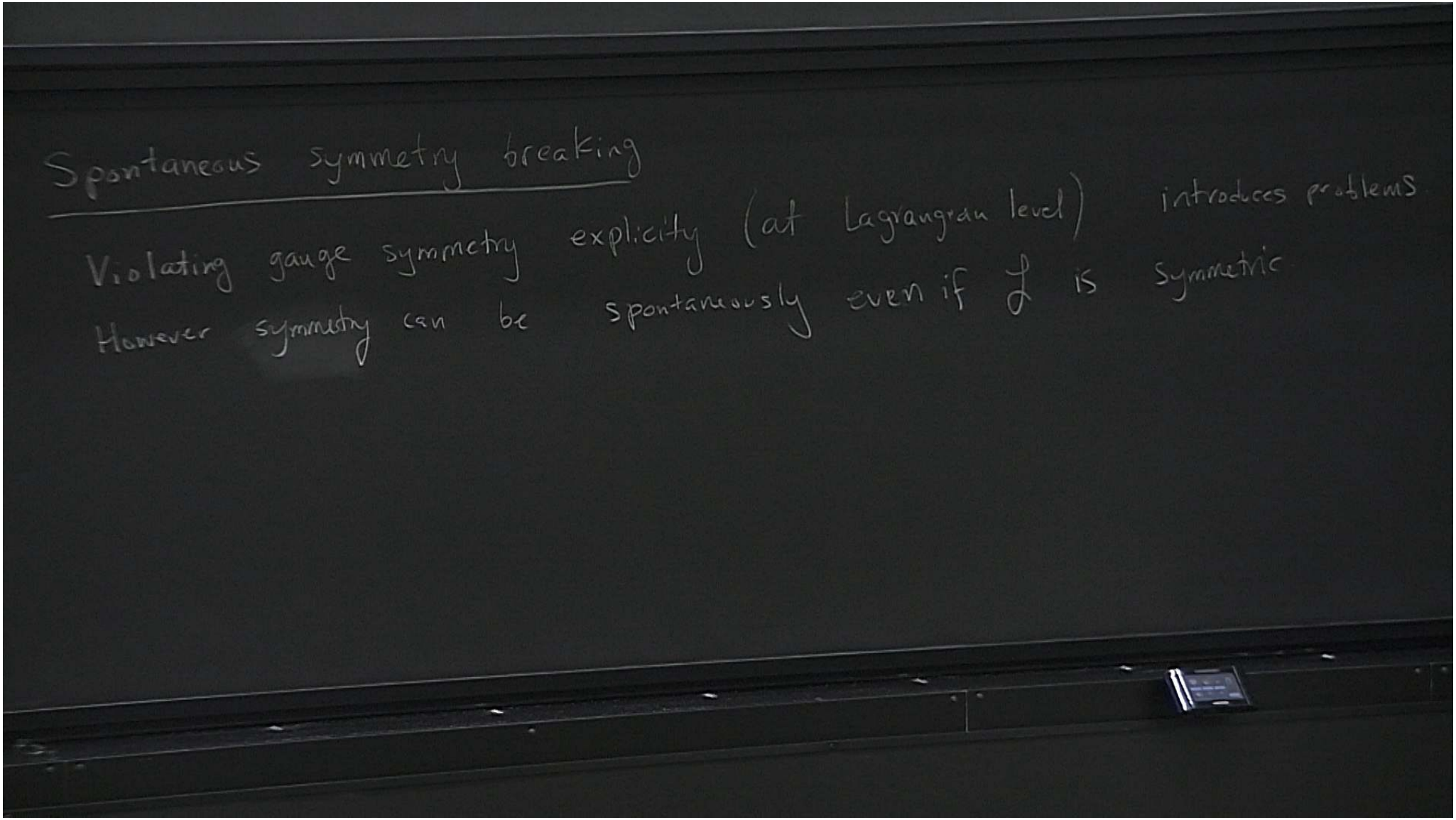
Divergence absorbed by counter term:

Need dimension-6 term in $\mathcal{L}$:

$$\mathcal{L} = c\,\bar{\psi}\psi\,\bar{\psi}\psi$$

c has mass dimension -2 $\Rightarrow$ nonrenormalizable theory.

$i\mathcal{M}(\psi\psi \to \psi\psi) =$ + ...


Spontaneous symmetry breaking
Violating gauge symmetry explicity (at Lagrangian level) introduces problems
However symmetry can be spontaneously even if $\mathcal{L}$ is symmetric

Start with a theory that is invariant under a symmetry.
Ground state where symmetry is broken may have lower e
compared to state where symmetry is unbroken.

Let U be a symmetry transformation that leaves

Suppose we have two states:

$$|A\rangle = a_A^\dagger |0\rangle$$

$$|B\rangle = a_B^\dagger |0\rangle$$

er a symmetry.

may have lower energy

is unbroken.

ation that leaves Hamiltonian H invariant: $H \to UHU^\dagger = H$

$a_\uparrow^\dagger |0\rangle$

$a_B^\dagger |0\rangle$

Want $|A\rangle, |B\rangle$ related by U: $U|A\rangle = |B\rangle$

First suppose: $U a_A^\dagger U^\dagger = a_B^\dagger$

Then $U|A\rangle = U a_A^\dagger \underbrace{U^\dagger U}_{\mathbb{1}} |0\rangle = a_B^\dagger \underbrace{U|0\rangle}_{} = a_B^\dagger |0\rangle = |B\rangle$

Assume vacuum invariant under U.

$U|0\rangle = |0\rangle$

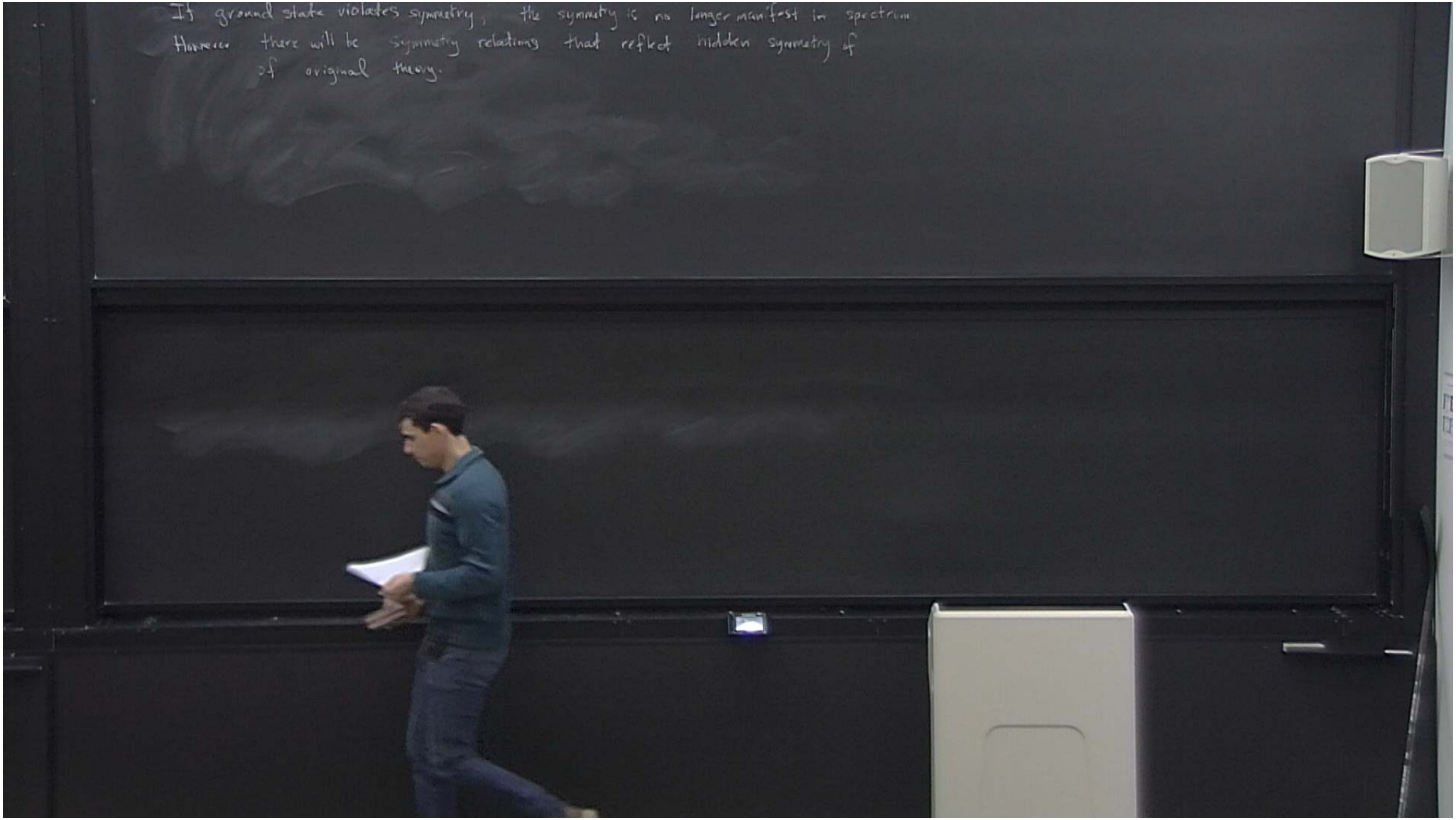
$U|A\rangle = |B\rangle$ only if vacuum is invariant under U.

If true, then $E_A = E_B$:

$$E_A = \langle A|H|A\rangle = \langle B|\underbrace{U H U^\dagger}_{H}|B\rangle = \langle B|H|B\rangle = E_B$$

If ground state preserves symmetry, then the symmetry will be manifest in the spectrum of states.

If ground state violates symmetry, the symmetry is no longer manif
However there will be symmetry relations that reflect hidden s
of original theory.


If ground state violates symmetry, the symmetry is no longer manifest in spectrum
However there will be symmetry relations that reflect hidden symmetry of
of original theory.

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: symmetry relations that reflect hidden symmetry of
l theory.

Discrete symmetry

Theory with real scalar field ϕ and $\mathbb{Z}_2$ symmetry ($\phi \to -\phi$)

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}M^2\phi^2 - \frac{\lambda}{4}\phi^4$$

No linear/cubic terms.

Discrete symmetry

Theory with real scalar field ϕ and $\mathbb{Z}_2$ symmetry ($\phi \to -\phi$)

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}\mu^2 \phi^2 - \frac{\lambda}{4}\phi^4$$

No linear/cubic terms.

$$H = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}|\vec{\nabla}\phi|^2 + V(\phi), \qquad V(\phi) = \frac{1}{2}\mu^2 + \frac{\lambda}{4}\phi^4$$

try

ed scalar field ϕ and $\mathbb{Z}_2$ symmetry ($\phi \to -\phi$)

$$-(\partial_\mu \phi)^2 - \frac{1}{2}\mu^2 \phi^2 - \frac{\lambda}{4}\phi^4$$

near/cubic terms.

$$\frac{1}{2}|\vec{\nabla}\phi|^2 + V(\phi), \qquad V(\phi) = \frac{1}{2}\mu^2 + \frac{\lambda}{4}\phi^4$$

Hamiltonian density.

Discrete symmetry

Theory with real scalar field ϕ and Z_2 symmetry ($\phi \to -$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}\mu^2 \phi^2 - \frac{\lambda}{4}\phi^4$$

No linear/cubic terms.

$$H = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}|\vec{\nabla}\phi|^2 + V(\phi), \qquad V(\phi) = \frac{1}{2}\mu^2\phi^2 + \frac{\lambda}{4}\phi^4$$

2) Spinors
3) spin $\frac{1}{2}$

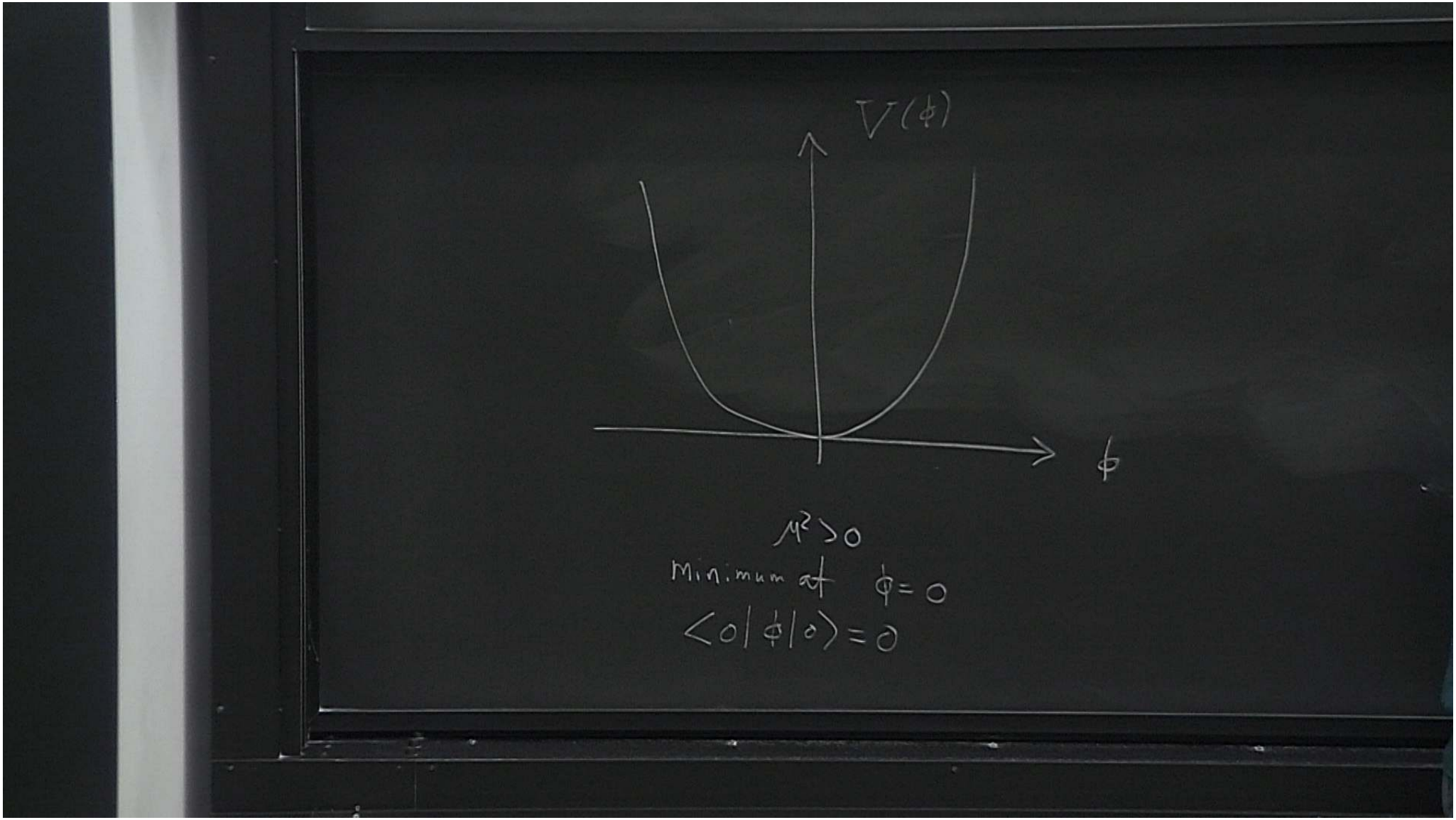
Yukawa diags
(3) Qtz Maxwell
4) $A^\mu = 4 \times 4$

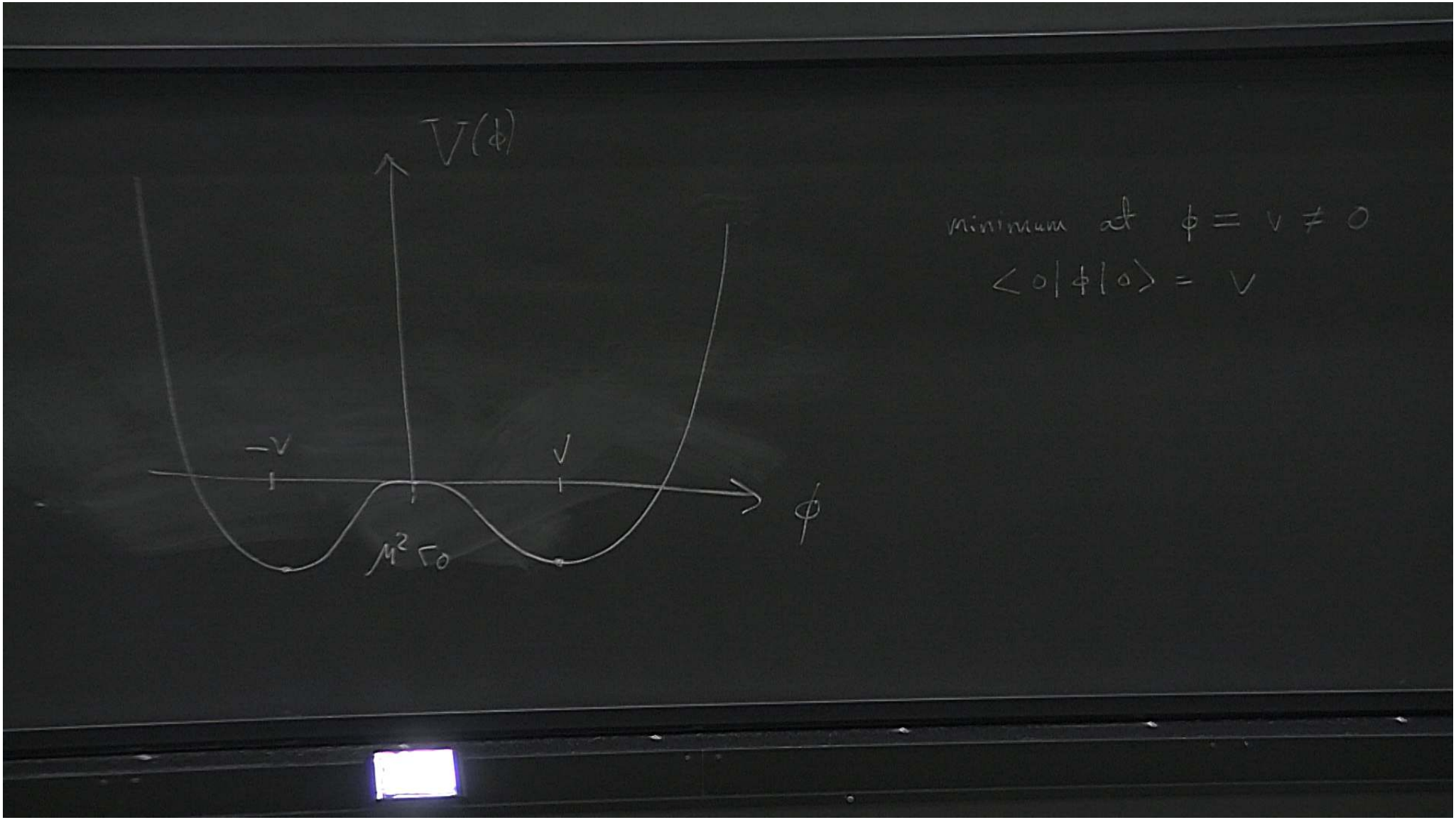
Energy minimized for ϕ = const $(\dot{\phi}, \vec{\nabla}\phi = 0)$

Minimize $V(\phi)$

$\lambda > 0$ (H unbounded from below if $\lambda < 0$)

μ^2 can have either sign

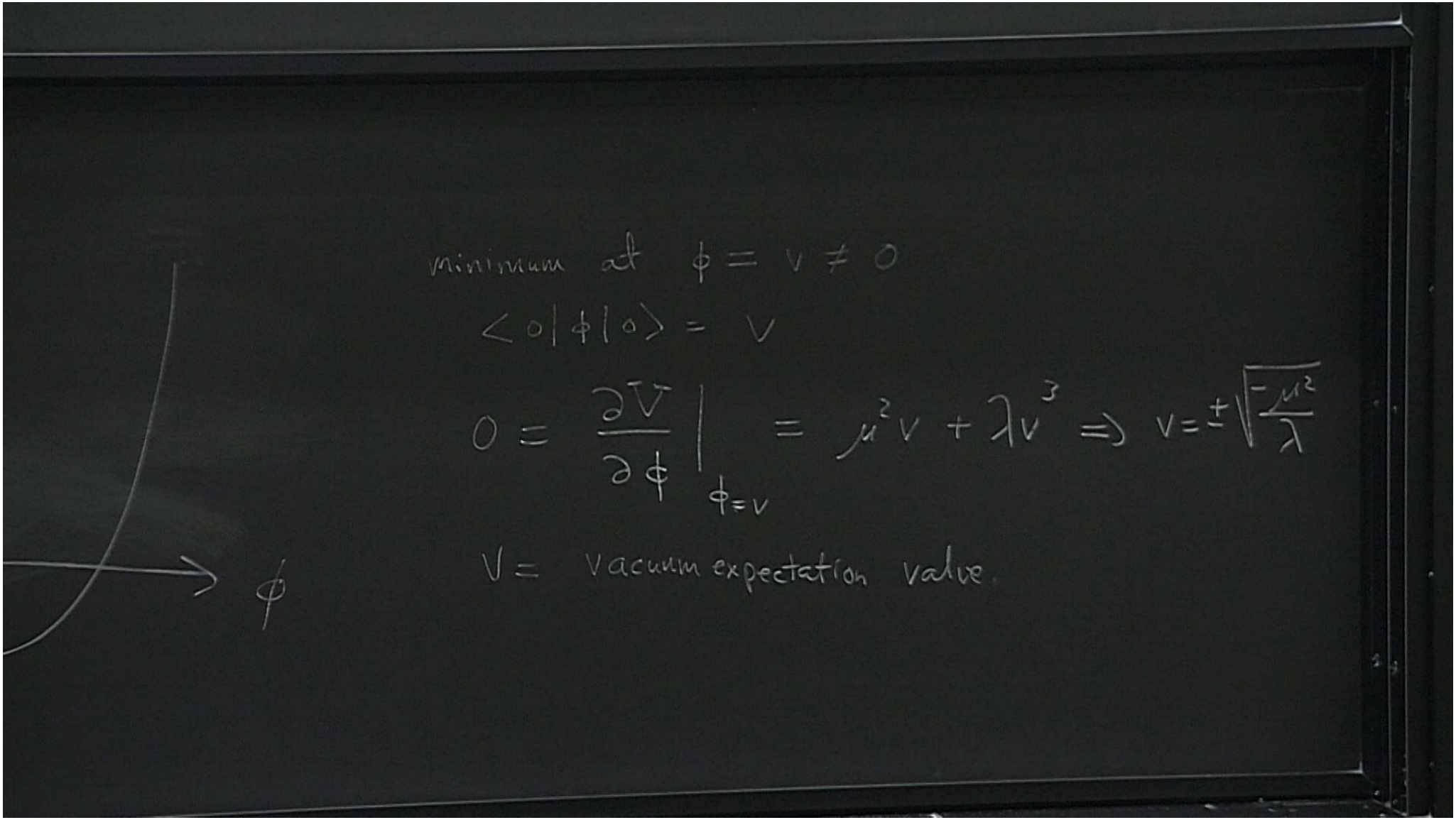

V(φ)
φ
μ² > 0
Minimum at φ = 0
⟨0|φ|0⟩ = 0

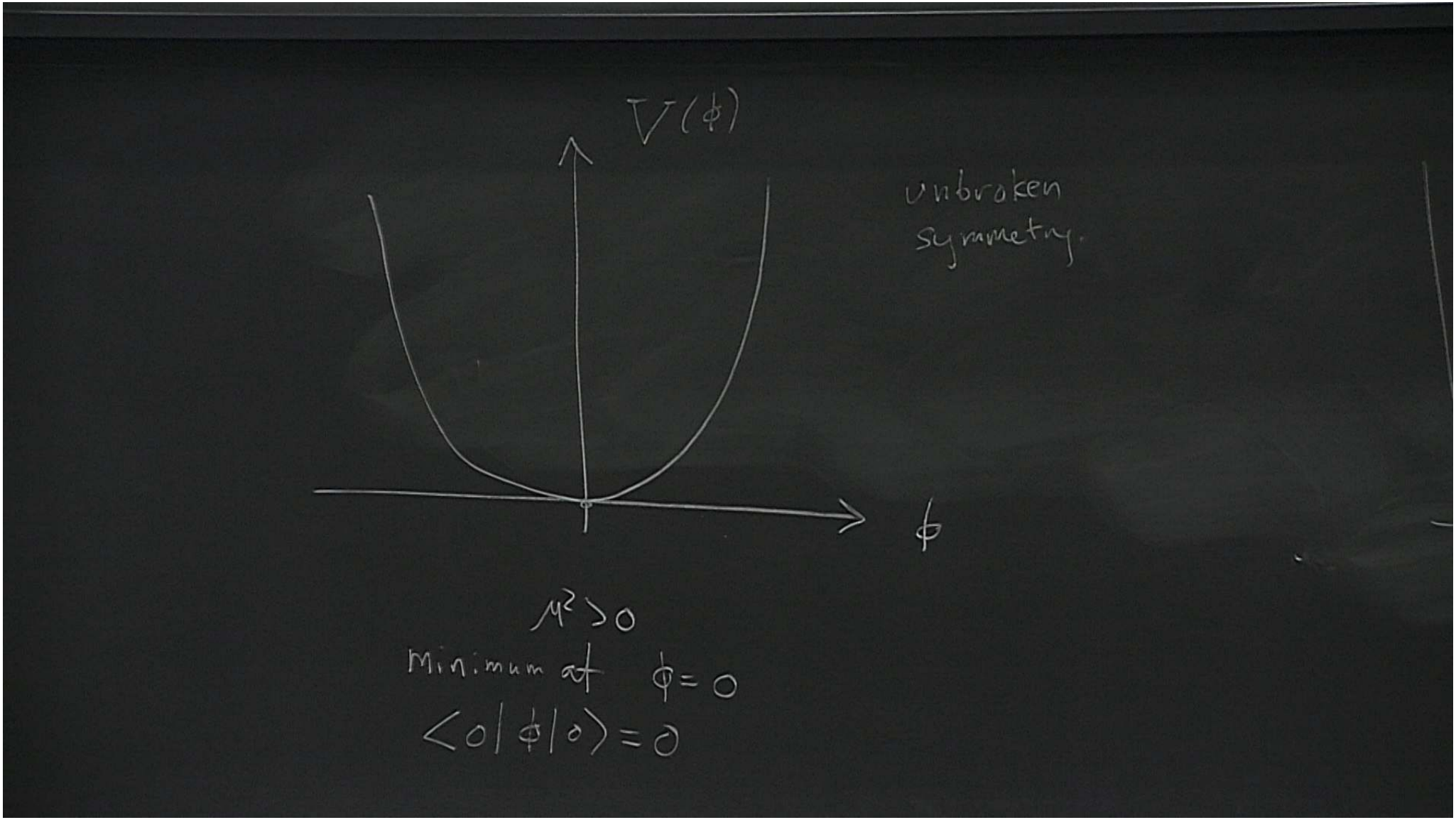

V(φ)
-v
v
φ
μ² < 0
minimum at φ = v ≠ 0
<0|φ|0> = v

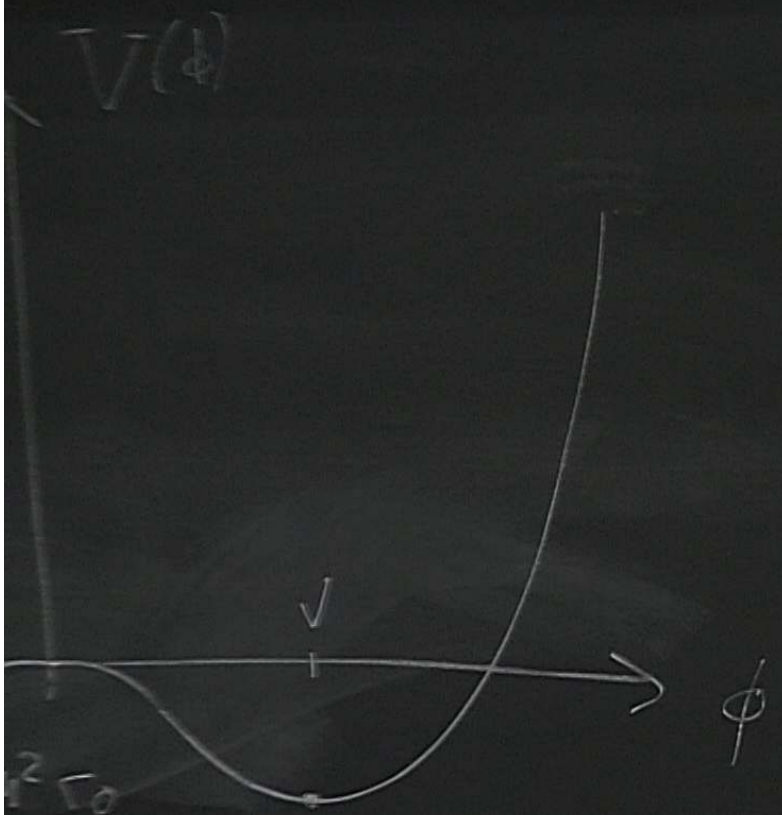
minimum at $\phi = v \neq 0$

$\langle 0|\phi|0\rangle = v$

$$0 = \left.\frac{\partial V}{\partial \phi}\right|_{\phi=v} = \mu^2 v + \lambda v^3 \Rightarrow v = \pm\sqrt{\frac{-\mu^2}{\lambda}}$$


minimum at $\phi = v \neq 0$
$\langle 0|\phi|0\rangle = v$
$0 = \left.\frac{\partial V}{\partial \phi}\right|_{\phi=v} = \mu^2 v + \lambda v^3 \Rightarrow v = \pm\sqrt{\frac{-\mu^2}{\lambda}}$
ϕ
$V =$ vacuum expectation value.


V(φ)
φ
unbroken
symmetry.
μ² > 0
Minimum at φ = 0
⟨0|φ|0⟩ = 0



broken symmetry.

minimum at $\phi = v \neq 0$

$\langle 0|\phi|0\rangle = v$

$$0 = \frac{\partial V}{\partial \phi}\bigg|_{\phi=v} = \mu^2 v + \lambda v^3 \Rightarrow$$

V = vacuum expectation value.

Physical states are small oscillations about the ground state.

Expand around $\phi = v$:

Define shifted field: $\phi = v + \phi'$ (broken case):

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi')^2 - V(v+\phi') = \frac{1}{2}(\partial_\mu \phi')^2 - \frac{1}{2}\mu^2(v+\phi')^2 - \frac{1}{4}\lambda(v+\phi')^4$$

$$= \frac{1}{2}(\partial_\mu \phi')^2 - (-\mu^2)\phi'^2 - \lambda v \phi'^3 - \frac{\lambda}{4}\phi'^4$$

oscillations about the ground state.

$= v + \phi'$ (broken case)

$= \frac{1}{2}(\partial_\mu \phi') - \frac{1}{2}\mu^2 (v+\phi')^2 - \frac{1}{4}\lambda (v+\phi')^4$

$- \lambda v \phi'^3 - \frac{\lambda}{4}\phi'^4$

(1) No Z_2 symmetry is broken for ϕ'

$= v + \phi'$ (broken case):

$= \frac{1}{2}(\partial_\mu \phi') - \frac{1}{2}\mu^2 (v+\phi')^2 - \frac{1}{4}\lambda (v+\phi')^4$

$- \lambda v \phi'^3 - \frac{\lambda}{4}\phi'^4$

(1) No Z_2 symmetry is broken for ϕ'

(2) Relations between parameters are remnant of original symmetry.

metry; the symmetry is no longer manifest in spectrum.

ory relations that reflect hidden symmetry of

Mass of ϕ': $m_{\phi'}^2 = -2\mu^2 = 2\lambda v^2$

Still only two parameters: λ, μ^2 or $m_{\phi'}^2, v$

$\Rightarrow$ 3 interactions in $\mathcal{L}$ defined in terms of 2 parameters.

Abelian global symmetry

Complex scalar ϕ

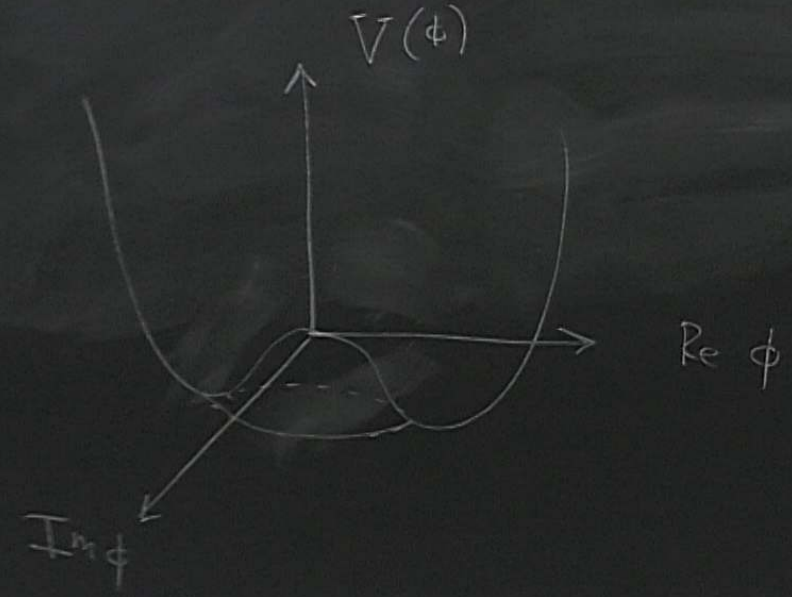
$$\mathcal{L} = (\partial_\mu \phi^\dagger)(\partial^\mu \phi) + \mu^2 \phi^\dagger \phi - \lambda(\phi^\dagger \phi)^2$$

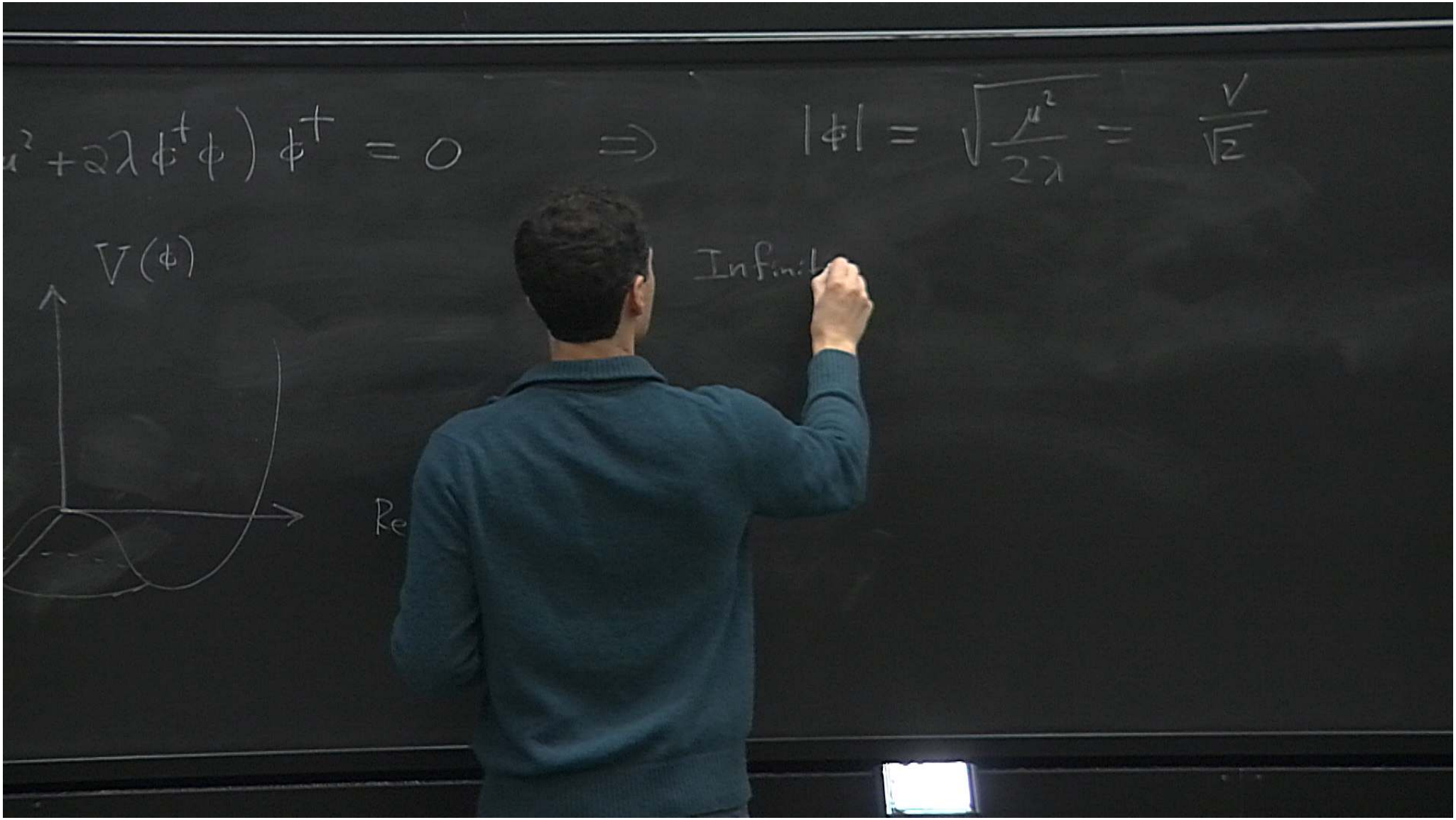
$\mathcal{L}$ is invariant under $U(1)$ symmetry: $\phi \to e^{i\alpha}\phi$

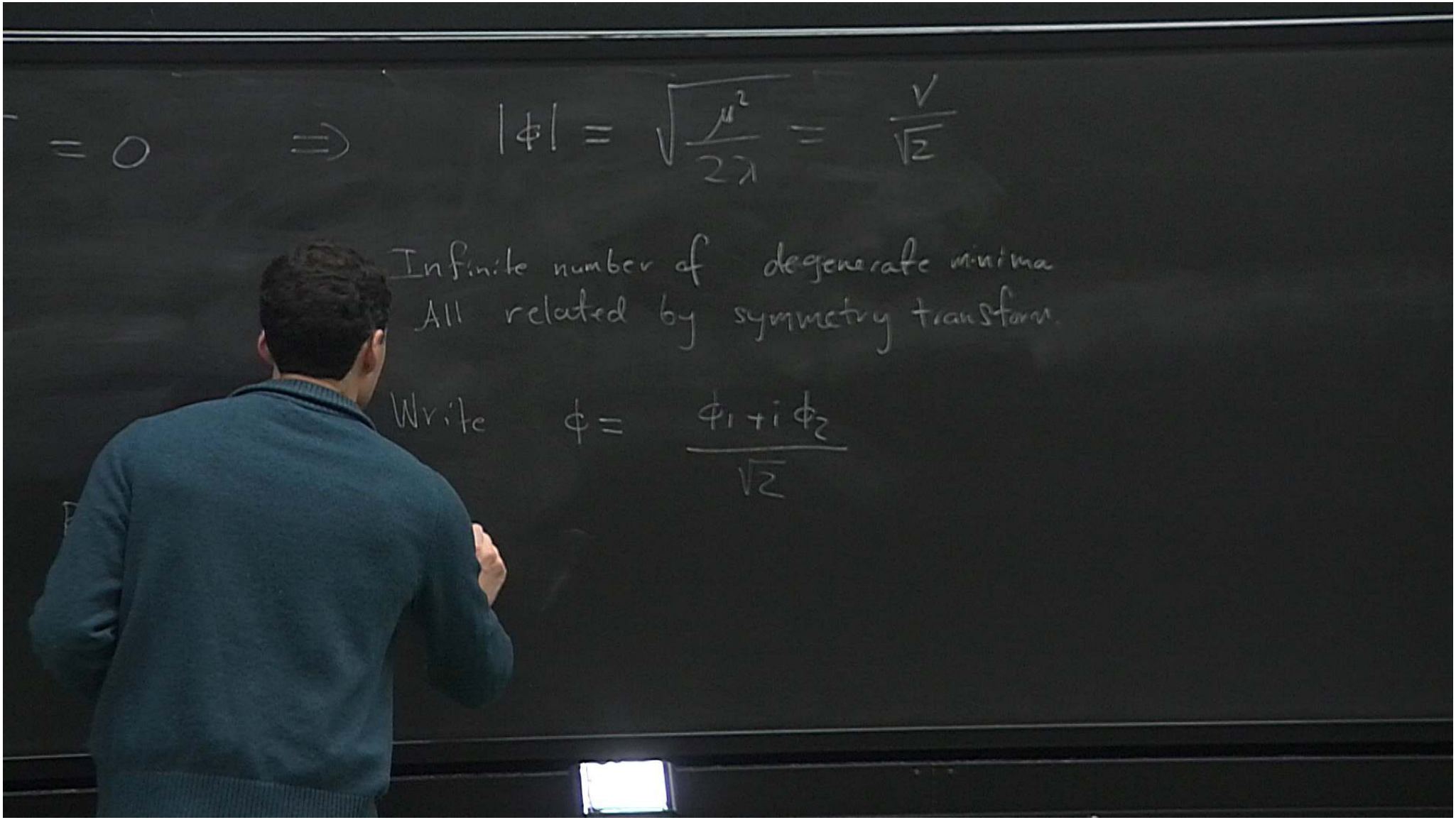
Assume $\mu^2 > 0$ → spontaneous symmetry breaking

$$V(\phi) = -\mu^2 \phi^\dagger \phi + \lambda(\phi^\dagger \phi)^2$$

$$\frac{\partial V}{\partial \phi} = \left(-\mu^2 + 2\lambda \phi^\dagger \phi\right) \phi^\dagger = 0 \quad \Rightarrow \quad |\phi| = \sqrt{\frac{\mu^2}{2\lambda}} = \frac{v}{\sqrt{2}}$$




$(\mu^2 + 2\lambda\phi^\dagger\phi)\,\phi^\dagger = 0 \quad \Rightarrow \quad |\phi| = \sqrt{\frac{\mu^2}{2\lambda}} = \frac{v}{\sqrt{2}}$
$V(\phi)$
Infinit
Re

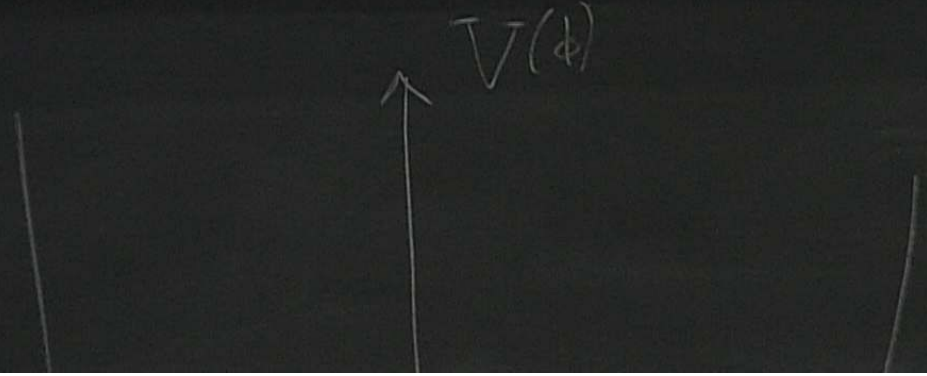

$= 0 \quad \Rightarrow \quad |\phi| = \sqrt{\frac{\mu^2}{2\lambda}} = \frac{v}{\sqrt{2}}$
Infinite number of degenerate minima
All related by symmetry transform.
Write $\phi = \frac{\phi_1 + i\phi_2}{\sqrt{2}}$

Infinite number of degenerate minima
All related by symmetry transform.

Write $\phi = \frac{\phi_1 + i\phi_2}{\sqrt{2}}$

Free to assume only ϕ_1 gets vacuum expectation value (vev)

$\langle 0|\phi_1|0\rangle = \frac{v}{\sqrt{2}} = \sqrt{\frac{\mu^2}{\lambda}}$, $\langle 0|\phi_2|0\rangle = 0$



broken symmetry.

Minimum at $\phi = v \neq 0$

$\langle 0|\phi|0\rangle = v$

Expand $\mathcal{L}$ in terms of shifted fields:

$$\phi_1 = v + \phi_1' \quad , \qquad \phi_2 = \phi_2'$$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi_1')^2 + \frac{1}{2}(\partial_\mu \phi_2')^2 - \mu^2 \phi_1'^2 - \lambda v \phi_1' (\phi_1'^2 + \phi_2'^2) -$$

... of shifted fields:

$$v + \phi_1' \,, \qquad \phi_2 = \phi_2'$$

$$+ \frac{1}{2}(\partial_\mu \phi_2')^2 - \mu^2 \phi_1'^2 - \lambda v \phi_1' (\phi_1'^2 + \phi_2'^2) - \frac{\lambda}{4}(\phi_1'^2 + \phi_2'^2)^2$$

shifted fields:

$$\phi_2 = \phi_2'$$

$$(\partial_\mu \phi_2')^2 - \mu^2 \phi_1'^2 - \lambda v \phi_1' \left(\phi_1'^2 + \phi_2'^2\right) - \frac{\lambda}{4}\left(\phi_1'^2 + \phi_2'^2\right)^2$$

$2\mu^2$

0 $\quad \phi_2'$ is massless. No energy cost to travel along minimum.

Goldstone's theorem: For every symmetry that is broken spontaneously, there is one massless degree of freedom (Goldstone boson) for for every generator that is no longer a symmetry.

U(1) had one generator → one Goldstone boson.