Title: PSI 2018/2019 - Condensed Matter Review - Lecture 15

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Abstract:

Quantum Complexity

and the emergence of irreversibility

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Work based on

Emergent irreversibility and entanglement spectrum statistics
Claudio Chamon, A.H., Eduardo R. Mucciolo
Phys. Rev. Lett. 112, 240501 (2014)

Irreversibility and Entanglement Spectrum Statistics in Quantum Circuits
Daniel Shaffer, Claudio Chamon, A.H. Eduardo R. Mucciolo
J. Stat. Mech. (2014) P12007

Two-component Structure in the Entanglement Spectrum of Highly Excited States
Zhi-Cheng Yang, Claudio Chamon, A.H. , Eduardo R. Mucciolo
Phys. Rev. Lett. 115, 267206 (2015)

Entanglement Complexity, Thermalization, and Many-Body Localization
Zhi-Cheng Yang, A. H., Salvatore M. Giampaolo, Eduardo R. Mucciolo, Claudio Chamon
Phys. Rev. B 96, 020408 (2017)

- Let us start with a video shown in reverse…

- Unitarity = Time Reversal
- Entropy is constant with unitary evolution
- Entanglement Entropy is not!
- Does increase in entropy mean irreversibility?
- Where does irreversibility come from?

IRREVERSIBILITY IN QUANTUM PHYSICS

- Irreversibility comes from entanglement complexity
- This notion tells us something about how hard it is to disentangle by local changes once one has fixed what local means = TPS
- Quantum complexity is a notion relative to a TPS
- What are the laws of quantum complexity?

QUANTUM COMPLEXITY

- FIRST LAW OF QUANTUM COMPLEXITY: The average entanglement complexity across all the TPS is constant.
- SECOND LAW OF QUANTUM COMPLEXITY: In every TPS, universal quantum evolution always entangles the state in the most possible complex way.

THE LAWS OF QUANTUM COMPLEXITY

RANDOM QUANTUM CIRCUIT (RQC)

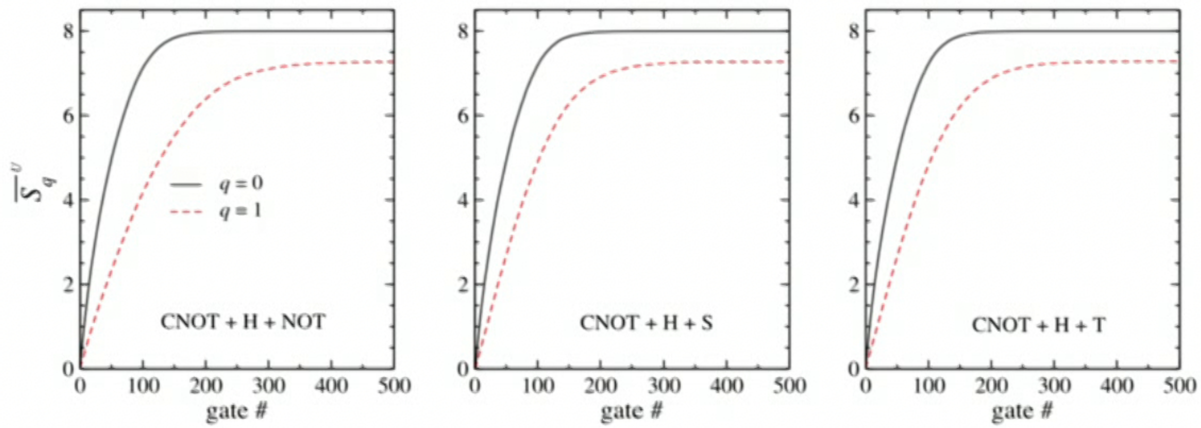
- CONSIDER A SYSTEM OF N SPINS 1/2 INITIALIZED IN A PRODUCT STATE
- WE DRAW SOME RANDOM GATES BETWEEN RANDOM QUBITS AND MONITOR THE ENTANGLEMENT (HEATING)
- AT SOME POINT THE ENTANGLEMENT BECOMES MAXIMAL AND EQUILIBRATES
- THEN WE TRY TO REVERSE THE EVOLUTION AND DISENTANGLE THE STATE (COOLING)

Quantum Gates

- **NOT** gate: flips the state of the qubit
$|0\rangle \to |1\rangle$ and $|1\rangle \to |0\rangle$
- **H** (Hadamard) gate:
$|0\rangle \to \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ and $|1\rangle \to \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$
- Phase gates $\mathbf{P}_\delta$: gives a state dependent phase
$|0\rangle \to |0\rangle$ and $|1\rangle \to e^{i\delta}|1\rangle$
The phase gate with $\delta = \pi/4$ is called **T**
The phase gate with $\delta = \pi/2$ is called **S**.
- **CNOT** gate or controlled-NOT gate:
$|00\rangle \to |00\rangle$, $|01\rangle \to |01\rangle$
$|10\rangle \to |11\rangle$, $|11\rangle \to |10\rangle$

UNIVERSAL AND NOT SETS OF GATES

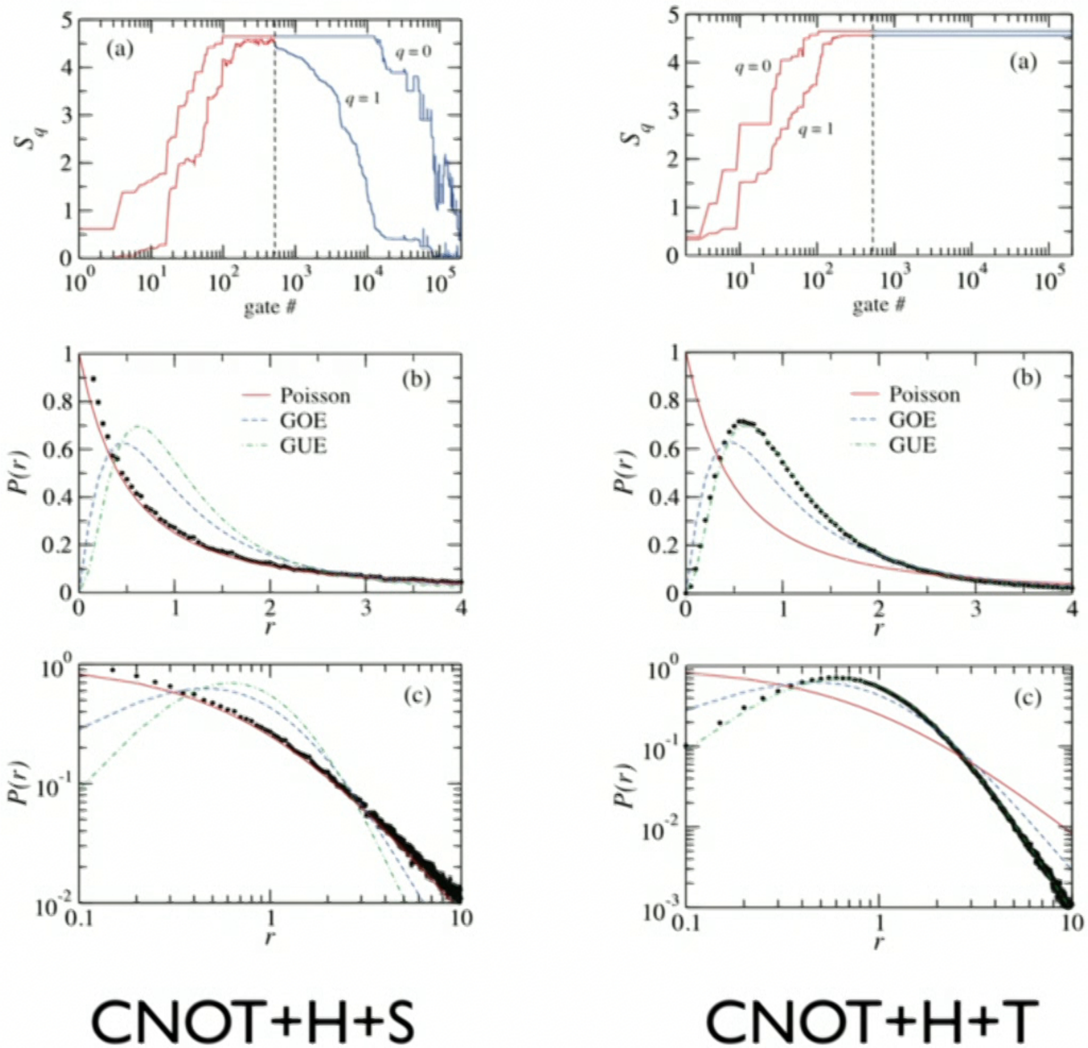
- (H,CNOT,S) dense in Clifford Group, **NOT UNIVERSAL**
- (H,CNOT,NOT) = **NOT UNIVERSAL**
- (H,CNOT,T) = **UNIVERSAL**
- Entanglement Heating by RQC



Reversing the evolution: *Disentangling the state*

Metropolis algorithm:

- (1) Compute the entanglement entropies (all bipartitions)
- (2) Choose a gate randomly
- (3) Apply the gate
- (4) Recompute the entanglement entropies
  - If entropies are reduced, add gate to reverse list
  - If entropies increase, keep gate with Boltzmann probability or discard it and go back to (2)
- (5) Stop when entanglement entropies are zero


(a)
q = 0
q = 1
S_q
gate #
(b)
Poisson
GOE
GUE
$P(r)$
r
(c)
CNOT+H+S
CNOT+H+T

STATISTICAL COMPLEXITY

Be $\{p_i\}$ the populations of the reduced density matrix

$$r_i = \frac{p_{i-1} - p_i}{p_i - p_{I+1}} \qquad P(r) = R^{-1} \sum_{I=1}^{R} \langle \delta(r - r_i) \rangle$$

- Complexity as adherence of the statistics of the gaps to random matrix theory

$$P_{WD} = \frac{1}{Z} \frac{(r + r^2)^{\beta}}{(1 + r + r^2)^{1+3\beta/2}} \qquad P_{\text{Poisson}}(r) = \frac{1}{(1+r)^2}$$

Operator Set	Level Spacing Statistics	Reversible?
$\{CNOT, H, NOT\}$	Poisson	Yes
$\{CNOT, H, S\}$	Poisson	Yes
$\{CNOT, H, T\}$	GUE	No

Irreversibility and Complexity

- Irreversibility arises not just because there are many ways of entangling vs few ways of disentangling.
- Even with active disentangling (Metropolis) it may be difficult
- Complex entanglement means that searching for a disentangler requires exhaustive search
- Complex entanglement is a property of the statistics of the entanglement gaps.
- Simple entanglement means that a disentangler can be found by some principle (annealing)
- This also characterizes universal computation as ergodic and non-universal as non complex
- A classification of EC for quantum circuits is to be done

Heisenberg Spin chain

- What happens for a spin system described by a Hamiltonian?
- Time evolution is given by exp(-iHt)
- We can also study the entanglement complexity of the eigenstates

$$H = J\sum_{i=1}^{L-1}(\sigma_i^x\sigma_{i+1}^x + \sigma_i^y\sigma_{i+1}^y + \Delta\sigma_i^z\sigma_{i+1}^z + z_i\sigma_i^z + x_i\sigma_i^x)$$

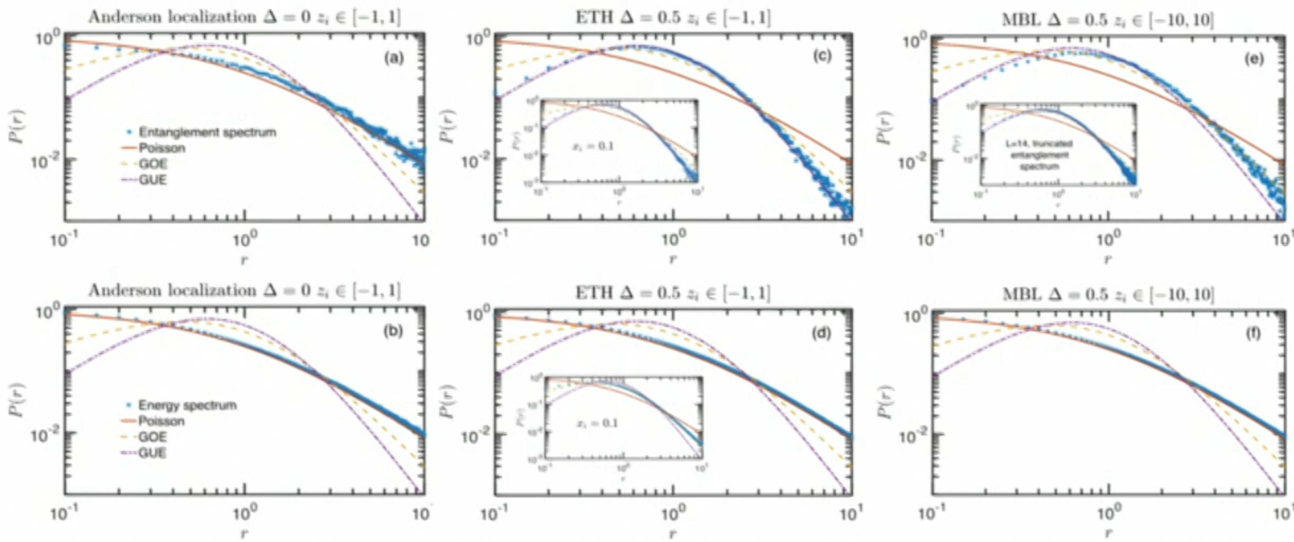
$\Delta = x_i = 0, z_i \in [-1,1]$ XX model (AA localization)

$\Delta = 0.5, z_i \in [-1,1]$ ETH

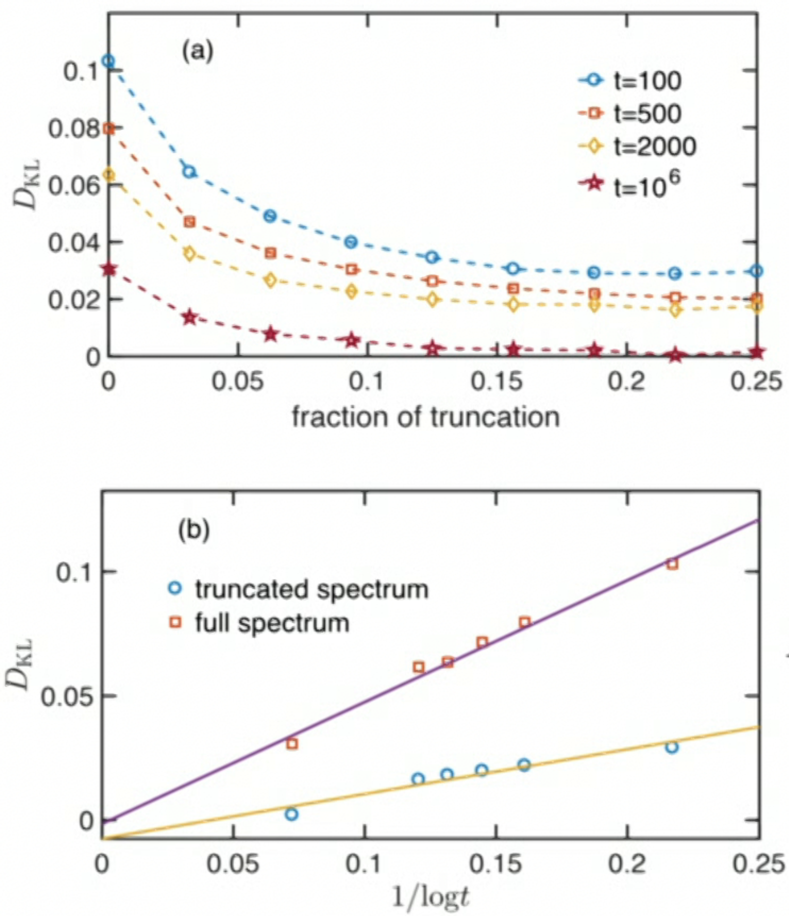
$\Delta = 0.5, z_i \in [-10,10]$ MBL

- XX is integrable and localized
- ETH thermalizes in subsystems
- MBL has quasi-local integrals of the motion and slow entanglement production

ENTANGLEMENT COMPLEXITY AND DYNAMICAL PHASES



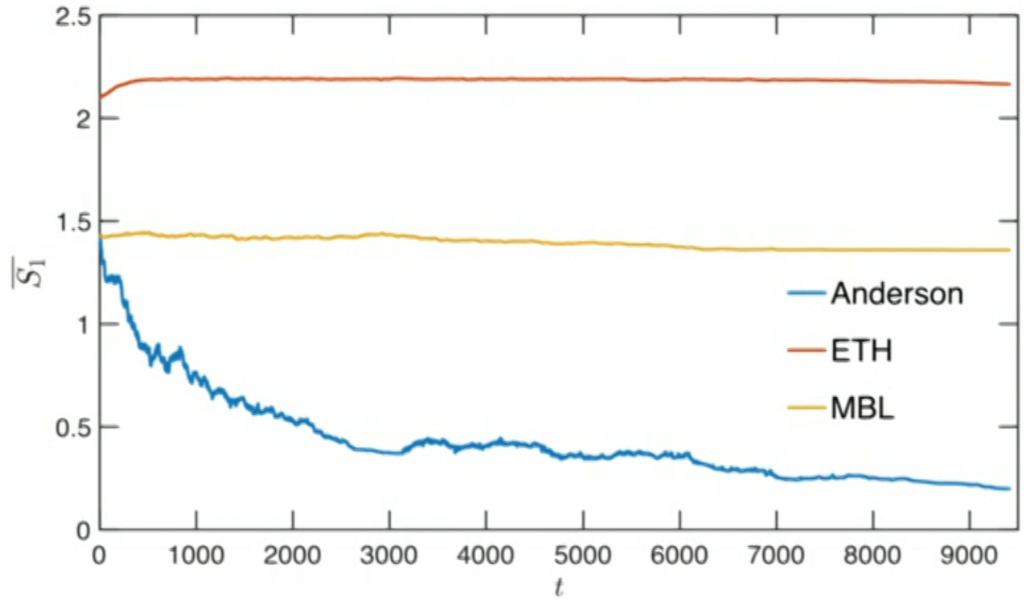
- XX model: free fermions/ integrable, Anderson localized
- ETH: thermalizes
- MBL: Hybrid behavior



- MBL spectrum approaches WD if we truncate heavy weights
- At long times, it approaches WD with vanishing truncation
- Entanglement in MBL is universal at large times

$$D_{\mathrm{KL}}(p\|q) = \sum_i p_i \log(p_i/q_i)$$

Disentangling and Complexity



- XX can be disentangled
- ETH is complex and cannot be disentangled
- MBL is hybrid in its disentangling behavior. In spite of low entanglement its pattern is complex

- How do we quantify complexity? Can we use the KL distance?
- What type of equilibration does MBL reach at exponential times?
- How robust are these results with respect to the TPS? Is there a phase transition driven by the TPS?
- What is the role of time fluctuations of the entropy in the effectiveness of cooling?
- What is the relationship between conserved quantities and entanglement complexity?
- Can we understand eigenselection (TPS selection) from the point of view of Entanglement Complexity?

Open Problems