

Title: PSI 2018/2019 - Condensed Matter Review - Lecture 14

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Abstract:

Averages in $\mathcal{H}$

$$|\psi\rangle \rightarrow \langle\psi|\hat{A}|\psi\rangle \equiv A_\psi$$

$$\rho_\psi \rightarrow \text{Tr}_\psi A_\psi$$

ψ random in $\mathcal{H}$

$$\langle \psi | A | \psi \rangle = \int d\psi \langle \psi | A | \psi \rangle = \int dU U | \psi_0 \rangle \langle \psi_0 | U^\dagger$$

$$| \psi \rangle = U | \psi_0 \rangle$$

ψ random in $\mathcal{H}$

$$\mathbb{E}(|\psi\rangle\langle\psi|) = \int |\psi\rangle\langle\psi| d\psi = \int dU U |\psi_0\rangle\langle\psi_0| U^\dagger$$

$$|\psi\rangle = U |\psi_0\rangle$$

$$\mathcal{H} \simeq \mathbb{C}^n$$

$\mathcal{U}(n)$ unitary ops on $\mathbb{C}^n$

$\mathcal{H}$ ψ random in $\mathcal{H}$

$$\langle \psi | A | \psi \rangle = A_\psi \quad \mathbb{E}(\psi) = \int |\psi\rangle\langle\psi| d\psi = \int dU \, U |\psi_0\rangle\langle\psi_0| U^\dagger$$

A_ψ

$$|\psi\rangle = U |\psi_0\rangle$$

$$\mathbb{E}(X) = \int dU \, \Gamma_U X \Gamma_U^\dagger$$

$$\mathcal{H} \simeq \mathbb{C}^n$$

$U \in \mathcal{U}(n)$ unitary

Γ_U operator on $\mathbb{C}^n$

Averages in $\mathcal{H}$

$$|\psi\rangle \rightarrow \langle\psi|\hat{A}|\psi\rangle = A_\psi$$

$$\rho_\psi \rightarrow \mathbb{E}_\psi A_\psi$$

$$\begin{aligned} dU &= d(V^\dagger W) \\ &= d(WV) \\ &\text{for } V \in \mathcal{U} \end{aligned}$$

ψ random

$$\mathbb{E}(\psi) =$$

$$|\psi\rangle =$$

$$\mathcal{H} = \mathbb{C}^n$$

$U \in \mathcal{U}(n)$ unitary

$\mathcal{U}$ operator

$$\langle \psi_0 | U^\dagger$$

$$T_U \times T_U^\dagger$$

Schur's Lemma

T_U be a unitary rep. of $\mathcal{U}$

A Hermitian operator

that is in the commutant of T_U

for all U , $[A, T_U] = 0$

$$\langle \psi_0 | U^\dagger$$

$$T_U \times T_U^\dagger$$

Schur's Lemma

T_U be a unitary rep. of $\mathcal{U}$

A Hermitian operator

that is in the commutant of T_U

for all U , $[A, T_U] = 0$

$$A = \sum_i \lambda_i \Pi_i$$

$$\mathcal{H} = \bigoplus_i V_i \quad \text{inv. subspaces of } T_U$$

Schur's Lemma

Γ_U be a unitary rep. of $\mathcal{U}$

A Hermitian operator

that is in the commutant of Γ_U

for all U , $[A, \Gamma_U] = 0$

$$\left\{ \begin{array}{l} A = \sum_i \lambda_i \Pi_i \end{array} \right. \xrightarrow{\text{irr. rep}} A = \lambda \mathbb{1}$$

$$\left\{ \begin{array}{l} \mathcal{H} = \bigoplus_i V_i \end{array} \right. \text{ inv. subspaces of } \Gamma_U$$

for $V \in \mathcal{U}$

Γ_U operator on $\mathbb{C}^n$

$$VE(x) = V \int dU \Gamma_U \times \Gamma_U^\dagger = \int dU \underbrace{VU \times U^\dagger V^\dagger}_{VU=W} = \int d(V^\dagger W) W \times W^\dagger V$$

$$V \in \Gamma_U$$

$$\boxed{dU = d(v^+ w) = d(wv) \text{ for } v \in \mathcal{U}}$$

$$\mathcal{H} = \mathbb{C}^n$$

$$U \in \mathcal{U}(n) \text{ unitary}$$

$$\Gamma_U \text{ operator on } \mathbb{C}^n$$

$$\left. \begin{array}{l} A = \sum_i \lambda_i T_i \\ \mathcal{H} = \bigoplus_i V_i \end{array} \right\}$$

$$V \Gamma_U(x) = V \int dU \Gamma_U \times \Gamma_U^+ = \int dU \underbrace{V U \times U^+ V^+}_V V = \int dU \underbrace{V U \times U^+}_W V = \int dW W \times W^+ V = \Gamma(x) V$$

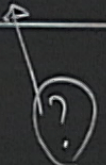
$$V \in \Gamma_U$$

$$\boxed{\Gamma(x) = \sum_i \lambda_i T_i}$$

$$V \mathbb{E}(X) = V \int dU \quad \Gamma_U \times \Gamma_U^+ = \int dU \underbrace{V U \times U^+ V^+}_W = \int d(V^+ W) W \times W^+$$

$$V \subset \Gamma_U$$

$$\boxed{\mathbb{E}(X) = \sum_i \lambda_i \pi_i}$$



$$V U X U^\dagger V^\dagger V = \int d(v^\dagger w) w x w^\dagger V = \int dw w x w^\dagger V = E(X) V$$

$$\underbrace{V U}_{V U = W}$$

$$X = |\psi_0\rangle\langle\psi_0|$$

$$\Pi_U = U \quad \text{trivial rep.}$$

$$V U X U^\dagger V^\dagger V = \int d(v^\dagger w) w x w^\dagger V = \int dw w x w^\dagger V = E(X) V$$

$$\underbrace{V U}_{V U = W}$$

$$X = |4_0\rangle\langle 4_0|$$

$$\Pi_U = U \quad \text{trivial rep.} \rightarrow \text{irreducible}$$

$$E(4) = \int dU U |4_0\rangle\langle 4_0| U^\dagger = \lambda \mathbb{1}$$

$$\int dU \underbrace{VU}_{VU=W} X U^\dagger V^\dagger V = \int d(V^\dagger W) W X W^\dagger V = \int dW W X W^\dagger V = E(X) V$$

$$X = |\psi_0\rangle\langle\psi_0|$$

$$\Pi_U = U \quad \text{trivial rep.} \rightarrow \text{irreducible}$$

$$\text{Tr } E(\psi) = \int dU U |\psi_0\rangle\langle\psi_0| U^\dagger = \lambda \mathbb{1}$$

$$V E(x) = V \int dU \quad \Gamma_U \times \Gamma_U^+ = \int dU \quad \underbrace{V U \times U^+ V^+}_W V = \int dU \dots$$

$$V < \Gamma_U$$

$$\int dU = 1$$

$$\boxed{E(x) = \sum_i \lambda_i \pi_i}$$

↑
(?)

$$X = 10$$

$$\Gamma_U =$$

$$E(4)$$

$$V E(X) = V \int dU \quad \Pi_U X \Pi_U^\dagger = \int dU \underbrace{V U X U^\dagger V^\dagger}_V V = \int dU V X V$$

$$V \in \Pi_U$$

$$\int dU = 1$$

$$E(X) = \sum_i \lambda_i \Pi_i$$

$$1 = \text{Tr} \Pi_U = \text{Tr} E(X)$$

$$X = 1$$

$$\Pi_U =$$

$$E(X)$$

$$\rho_X \rho_0^\dagger = \int dU \underbrace{V U X U^\dagger V^\dagger}_V V = \int dW W X W^\dagger V = E(X) V$$

$VU = W$

$$\left| \sum_i \lambda_i \pi_i \right|$$

① $\text{Tr} \rho = \text{Tr} E(\rho) = \lambda$

$$X = |\psi_0\rangle\langle\psi_0|$$

$$\rho_0 = U \text{ trivial rep.} \rightarrow \text{irreducible}$$

$$E(\rho) = \int dU U |\psi_0\rangle\langle\psi_0| U^\dagger = \lambda \mathbb{1}$$

$$\rho_X \rho_0^\dagger = \int dU \underbrace{VU}_{VU=W} X U^\dagger V^\dagger V = \int dW W X W^\dagger V = E(X) V$$

$$\left| \sum_i \lambda_i \pi_i \right|$$

(?)

$$1 = \text{Tr} \rho = \text{Tr} E(\rho) = \lambda n$$

$$X = |\psi_0\rangle\langle\psi_0|$$

$$\rho_0 = U \quad \text{trivial rep.} \rightarrow \text{irreducible}$$

$$E(\rho) = \int dU U |\psi_0\rangle\langle\psi_0| U^\dagger = \lambda \mathbb{1}$$

$$V^\dagger V = \int d(v^\dagger w) w x w^\dagger V = \int dw w x w^\dagger V = E(x) V$$

$$X = |4_0\rangle\langle 4_0|$$

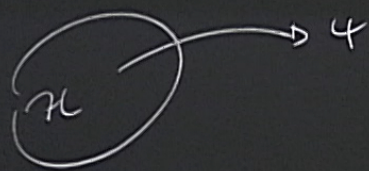
$\Pi_U = U$ trivial rep. $\rightarrow$ irreducible

$$\lambda \text{ in } \overline{E(4)} = \int dv U |4_0\rangle\langle 4_0| U^\dagger = \lambda \mathbb{1} = \frac{11}{n}$$

Completely mixed state!

$$\overline{\text{Tr}(A\psi)}^\psi = \text{Tr}[A\overline{\psi}] = \text{Tr}\left[A\frac{11}{n}\right] = \frac{1}{n} \sum_n A_{nn}$$

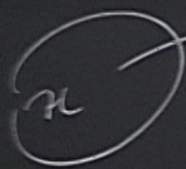
$$1 = \text{Tr} \rho = \text{Tr} E(\psi) = \text{Tr} \rho$$



How likely is it

ψ is VERY entangled?

$$1 = \ln 2 = -\text{Tr} \mathbb{E}(\psi) = \ln 2 \quad (\mathbb{E}(\psi) = \int \psi \psi^\dagger)$$



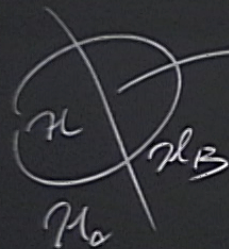
→ ψ

How lively is it

ψ is VERY entangled?

Every state in $\mathcal{M}$
is max entangled

$$1 = \text{Tr} \rho = \text{Tr} E(\rho) = \ln (E(\rho)) = \int dU \dots$$



How likely is it
 ψ is VERY entangled?

Every state in $\mathcal{H}$
 is max entangled

$$\psi \xrightarrow{\text{pure Haar}} \int_{\mathcal{H}_A} d\psi_A (\psi_A) = \int_{\mathcal{H}_A} d\psi_A \left(\text{Tr}_B (U |\psi_0\rangle \langle \psi_0| U^\dagger) \right) dU$$

$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$$

$$1 = \text{Tr} \rho = \text{Tr} E(\psi) = \lambda_n \quad (E(\psi) = \int dU \psi \psi^\dagger U)$$

→ ψ How likely is it
 ψ is VERY entangled?

Every state in $\mathcal{M}$
 is max entangled

$$\psi \xrightarrow{\text{pure Haar}} \frac{\int d\psi (\psi_A^\dagger \psi_A)}{\text{Tr} \psi_A^2} =$$

$$\int dU \left(\text{Tr}_B (U |\psi_0\rangle \langle \psi_0| U^\dagger) \right) dU =$$

$$\text{Tr } \sigma^2 = \text{Tr} [S \sigma^{\otimes 2}]$$

$\mathcal{H}$
 nyled

=

$$\boxed{\text{Tr } \sigma^2 = \text{Tr} [\mathcal{S} \sigma^{\otimes 2}]}$$

$$\sigma \in \mathcal{H} \quad \{ |i\rangle \}$$

$$\sigma^{\otimes 2} \in \mathcal{H}^{\otimes 2}$$

$$\{ |ij\rangle \}$$

$$\mathcal{S} |ij\rangle = |ji\rangle$$

$$\boxed{\text{Tr } \sigma^2 = \text{Tr} [\mathbb{S} \sigma^{\otimes 2}]}$$

$$\begin{aligned} \sigma &\in \mathcal{H} \quad \{|i\rangle\} \\ \sigma^{\otimes 2} &\in \mathcal{H}^{\otimes 2} \\ &\quad \{|ij\rangle\} \end{aligned}$$

state in $\mathcal{H}$
entangled

$$\boxed{\text{Tr } \sigma^2 = \text{Tr} [S \sigma^{\otimes 2}]}$$

$$\begin{aligned} \sigma &\in \mathcal{H} \quad \{|i\rangle\} \\ \sigma^{\otimes 2} &\in \mathcal{H}^{\otimes 2} \\ &\quad \{|ij\rangle\} \end{aligned}$$

$$dU = \text{Tr} \int dU S_A^{\otimes 1/2} U |i\rangle \langle i| U^\dagger$$

state in $\mathcal{H}$
entangled

$$\boxed{\text{Tr } \sigma^2 = \text{Tr} [S \sigma^{\otimes 2}]}$$

$$\sigma \in \mathcal{H} \quad \{ |i\rangle \}$$

$$\sigma^{\otimes 2} \in \mathcal{H}^{\otimes 2}$$

$$dU = \text{Tr} \int dU \underbrace{S_A^{\otimes 1/2}}_{\tilde{S}_A} U | \psi \rangle \langle \psi | U^\dagger = \text{Tr} \tilde{S}_A \bar{\psi} \quad \{ |ij\rangle \}$$

state in $\mathcal{H}$
entangled

$$\boxed{\text{Tr } \sigma^2 = \text{Tr} [S \sigma^{\otimes 2}]}$$

$$\sigma \in \mathcal{H} \quad \{|i\rangle\}$$

$$\sigma^{\otimes 2} \in \mathcal{H}^{\otimes 2}$$

$$\text{Tr} \int dU \underbrace{S_A^{\otimes 2}}_{\sum_A} \underbrace{U |i\rangle \langle i| U^\dagger}_{\psi^{\otimes 2}} = \text{Tr} \tilde{S}_A (\overline{\psi^{\otimes 2}}) \quad \{|i\rangle\}$$

state in $\mathcal{H}$
entangled

$$\boxed{\text{Tr } \sigma^2 = \text{Tr} [S \sigma^{\otimes 2}]}$$

$$\sigma \in \mathcal{H} \quad \{|i\rangle\}$$

$$\sigma^{\otimes 2} \in \mathcal{H}^{\otimes 2}$$

$$\text{Tr} \int dU \underbrace{S_A^{\otimes 2}}_{\tilde{S}_A} \underbrace{U | \psi \rangle \langle \psi | U^\dagger}_{\psi^{\otimes 2}} = \text{Tr} \tilde{S}_A \overline{\psi^{\otimes 2}} \quad \{|i,j\rangle\}$$

$$\psi^{\otimes 2} = \text{Tr} [\psi^{\otimes 2} \tilde{S}_A]$$

any state in $\mathcal{H}$
max entangled

$$\overline{\text{Tr} \psi_A^2}$$

$$= \text{Tr} \int dU$$

$$\underbrace{S_A^{\otimes 1/2}}_{\sum_A}$$

$$\underbrace{U \psi_A^{\otimes 2} U^\dagger}_{\psi^{\otimes 2}}$$

$$= \text{Tr} \tilde{S}_A \overline{\psi^{\otimes 2}} = \text{Tr} [\psi^{\otimes 2} \tilde{S}_A]$$

$$\boxed{\text{Tr} \sigma^2 = \text{Tr} [S \sigma^{\otimes 2}]}$$

$\sigma \in \mathcal{H}$ $\{ |i\rangle \}$
 $\sigma^{\otimes 2} \in \mathcal{H}^{\otimes 2}$

$\{ |ij\rangle \}$

$$\overline{\text{Tr}(A\psi)}^\psi = \text{Tr}[A\overline{\psi}] = \text{Tr}\left[A\left(\frac{11}{n}\right)\right] = \frac{1}{n} \sum_n A_{nn}$$

$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$$

$$\mathcal{H}^{\otimes 2} = (\mathcal{H}_A \otimes \mathcal{H}_B) \otimes (\mathcal{H}_A \otimes \mathcal{H}_B)$$

$$\overline{\psi\phi^2}$$

$$\psi$$

$$\psi\phi\psi$$

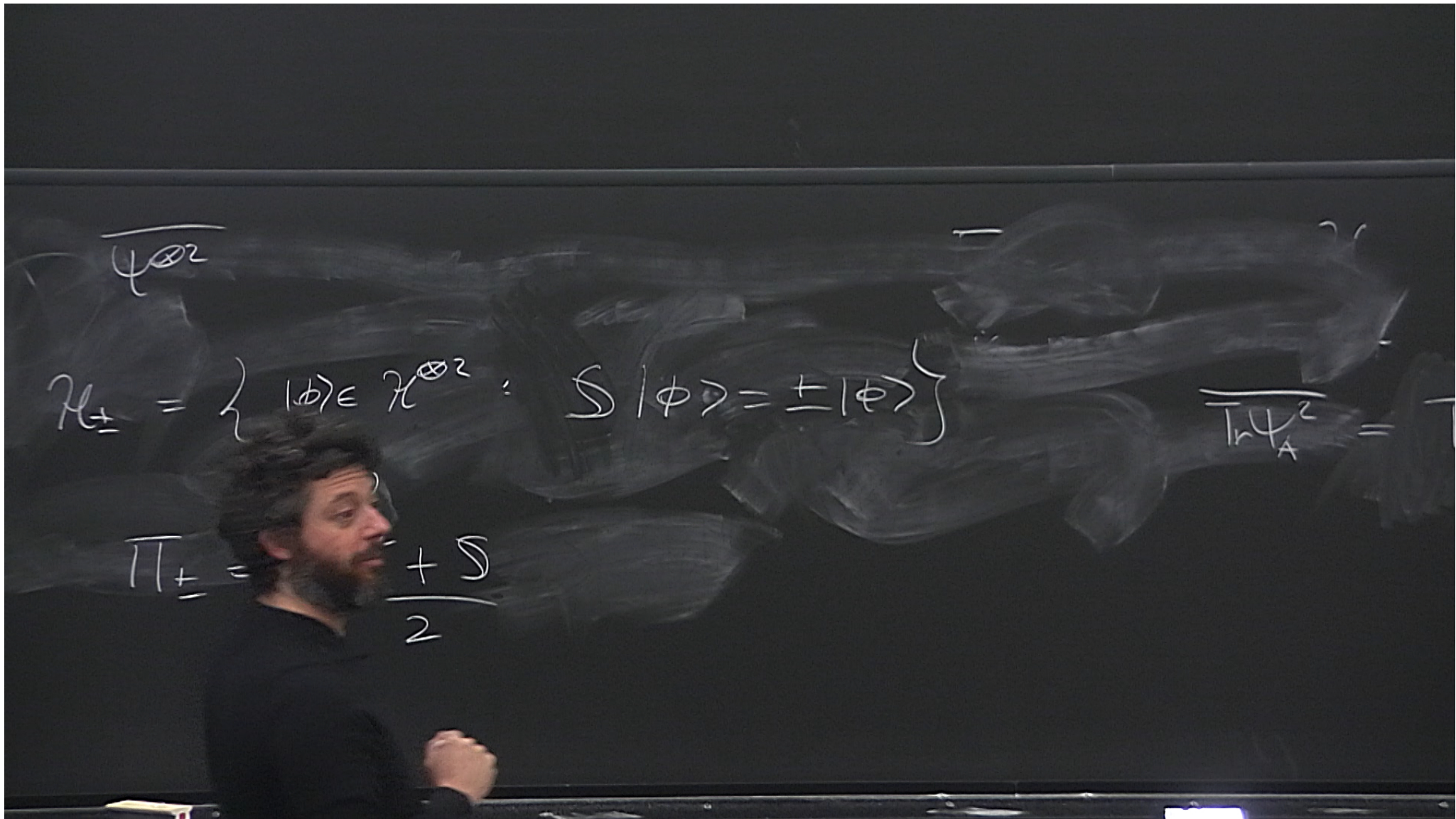
$$\overline{\psi\psi_A^2}$$

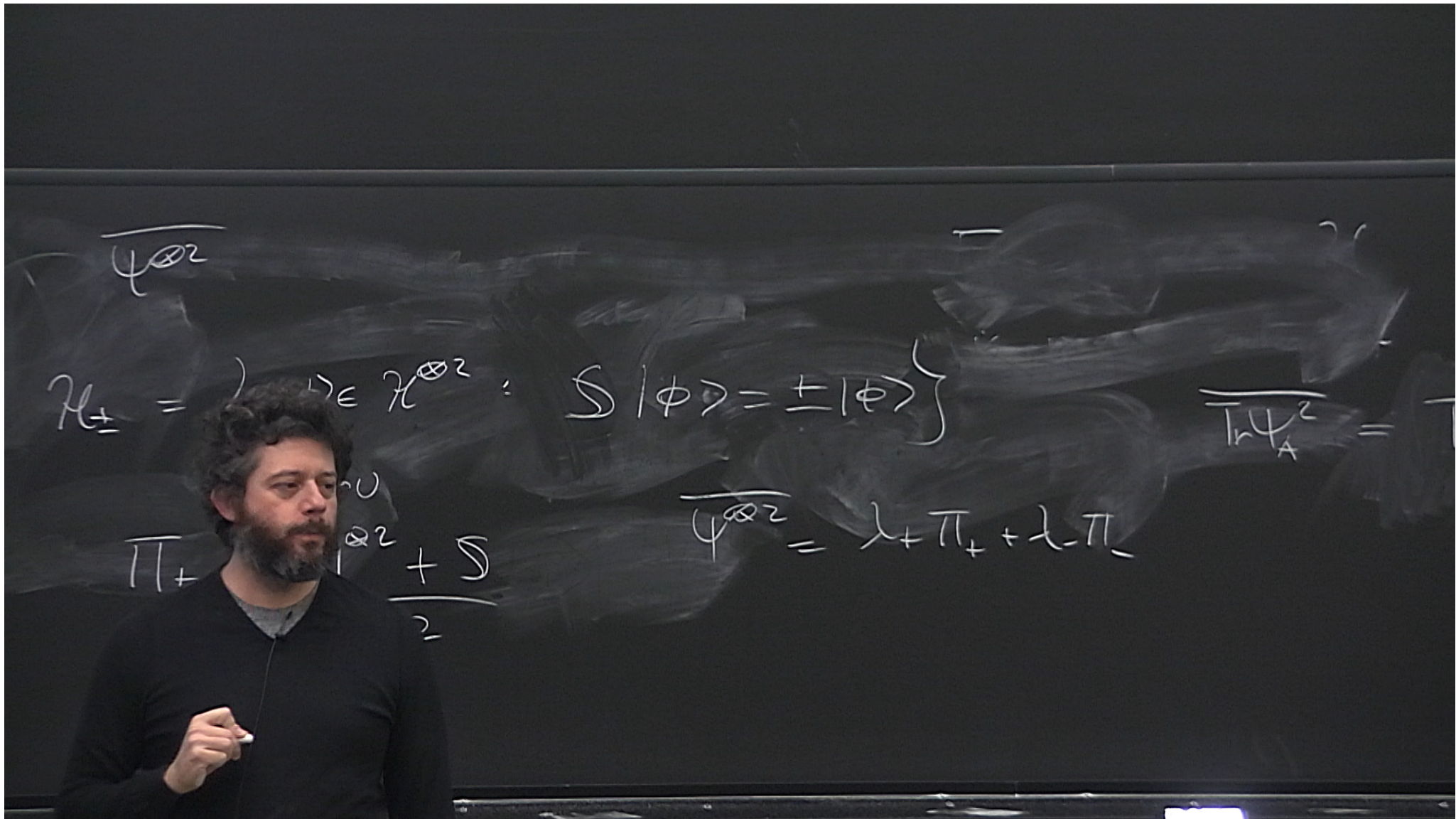
$$\overline{\psi \otimes \psi}$$

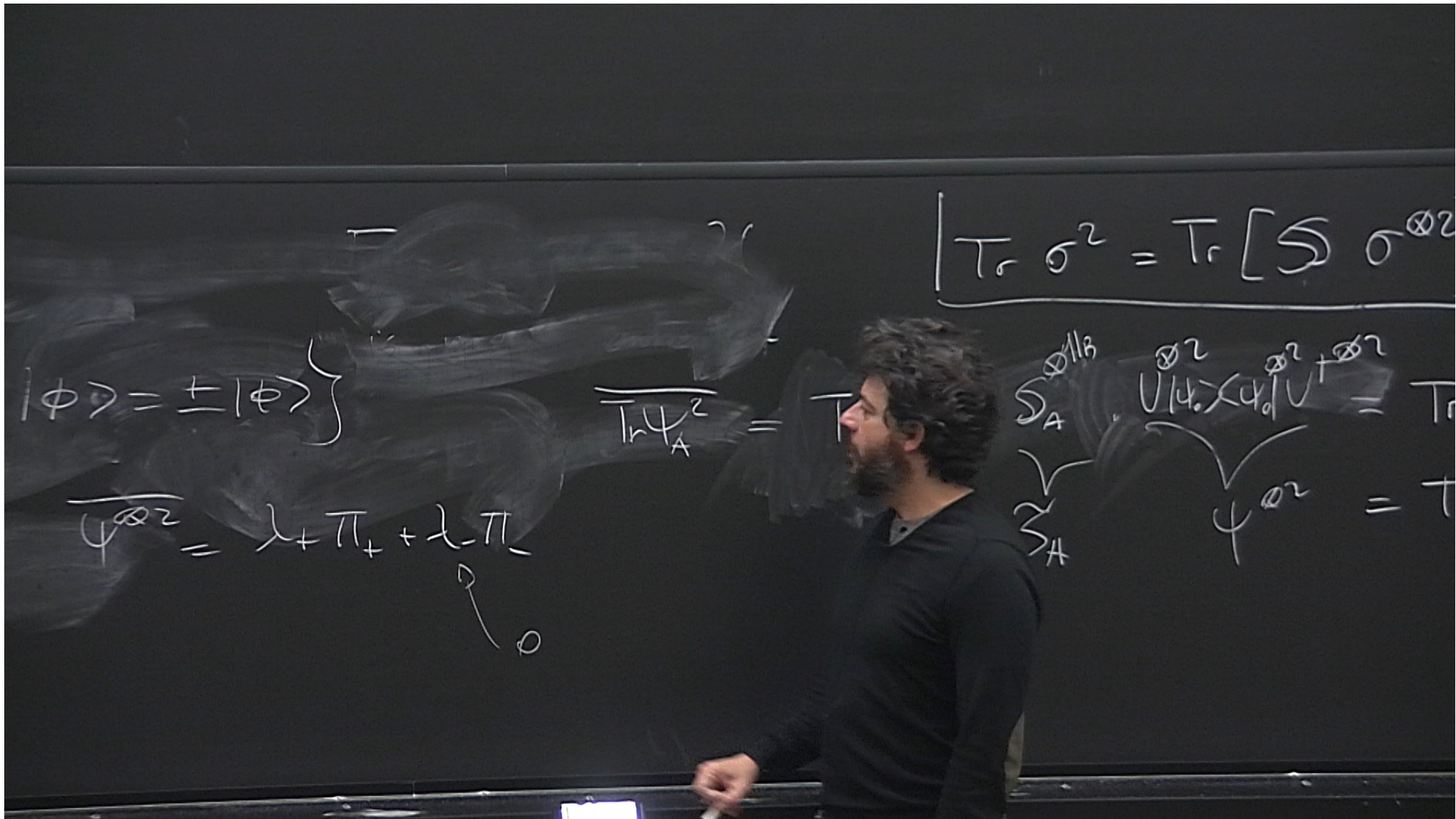
$$\mathcal{H}_{\pm} = \left\{ |\phi\rangle \in \mathcal{H}^{\otimes 2} : S|\phi\rangle = \pm |\phi\rangle \right\}$$

$U \otimes U$

$$\overline{\text{Tr} \psi_A^2} =$$







$$|\phi\rangle = \pm |\phi\rangle$$

$$\overline{\psi^{\otimes 2}} = \lambda_+ \pi_+ + \lambda_- \pi_-$$

$$\overline{\text{Tr} \psi_A^2} = \text{Tr}$$

$$\text{Tr} \sigma^2 = \text{Tr} [S \sigma^{\otimes 2}]$$

$$S_A^{\otimes 1/3} \cdot U^{\otimes 2} \cdot X \cdot U^{\otimes 2} = \text{Tr}$$

$$\begin{aligned}
 & \text{Tr } \sigma^2 = \text{Tr} \\
 & \text{Tr } \psi_A^2 = \text{Tr} \int dV \sum_A S_A \\
 & \psi^2 = \lambda + \pi_+ + \lambda - \pi_- \\
 & \frac{2}{n(n+1)}
 \end{aligned}$$

$$\boxed{\text{Tr } \sigma^2 = \text{Tr} [S \sigma^{\otimes 2}]}$$

$$\sigma \in \mathcal{H} \quad \{i|j\rangle\}$$

$$\sigma^{\otimes 2} \in \mathcal{H}^{\otimes 2}$$

$$\overline{\text{Tr } \psi_A^2} = \text{Tr} \int dU \underbrace{S_A^{\otimes 1/2}}_{\tilde{S}_A} \underbrace{U | \psi_A \rangle \langle \psi_A | U^\dagger}_{\psi^{\otimes 2}} = \text{Tr} \tilde{S}_A \overline{\psi^{\otimes 2}} \quad \{i|j\rangle\}$$

$$\psi^{\otimes 2} = \text{Tr} [\psi^{\otimes 2} \tilde{S}_A]$$

$$\frac{2}{N(N+1)} \text{Tr} S_A \otimes \mathbb{1}_B \left(\frac{\mathbb{1}^{\otimes 2} + S_{AB}}{2} \right)$$

$$\boxed{\text{Tr } \sigma^2 = \text{Tr} [S \sigma^{\otimes 2}]} \quad \begin{array}{l} \sigma \in \mathcal{H} \text{ s.t.} \\ \sigma^{\otimes 2} \in \mathcal{H}^{\otimes 2} \end{array}$$

$$\overline{\text{Tr } \psi_A^2} = \text{Tr} \int dU \underbrace{S_A^{\otimes 1B}}_{\sum_A} \underbrace{U | \psi_A \rangle \langle \psi_A | U^\dagger}_{\psi^{\otimes 2}} = \text{Tr} \tilde{S}_A (\overline{\psi^{\otimes 2}}) \{ |ij\rangle \}$$

$$\frac{1}{n(n+1)} \text{Tr} [S_A \otimes \mathbb{1}_B + \mathbb{1}_A \otimes S_B] = \frac{2}{n(n+1)} \text{Tr} S_A \otimes \mathbb{1}_B \left(\frac{\mathbb{1}^{\otimes 2} + S_{AB}}{2} \right)$$

$$\psi^{\otimes 2} = \text{Tr} [\psi^{\otimes 2} \tilde{S}_A]$$

$$S = \sum_{i,j} |i\rangle\langle j| \quad \left| \text{Tr } \sigma^2 = \text{Tr} [S \sigma^{\otimes 2}] \right| \quad \begin{array}{l} \sigma \in \mathcal{H} \text{ s.t. } \\ \sigma^{\otimes 2} \in \mathcal{H}^{\otimes 2} \end{array}$$

$$\overline{\text{Tr } \psi_A^2} = \text{Tr} \int dU \underbrace{S_A^{\otimes 2}}_{\tilde{S}_A} \underbrace{U | \psi_A \rangle \langle \psi_A | U^\dagger}_{\psi^{\otimes 2}} = \text{Tr} \tilde{S}_A \overline{\psi^{\otimes 2}} \quad \{|i,j\rangle\}$$

$$\frac{1}{n(n+1)} \text{Tr} \left[S_A^{\otimes 2} \otimes \mathbb{I}_B^{\otimes 2} + \mathbb{I}_A^{\otimes 2} \otimes S_B^{\otimes 2} \right] = \frac{2}{n(n+1)} \text{Tr} S_A^{\otimes 2} \otimes \mathbb{I}_B^{\otimes 2} \left(\frac{\mathbb{I}^{\otimes 2} + S_{AB}}{2} \right)$$

$$S = \sum_{i,j} |i\rangle\langle j| \quad \left| \text{Tr } \sigma^2 = \text{Tr} [S \sigma^{\otimes 2}] \right| \quad \begin{array}{l} \sigma \in \mathcal{H} \\ \sigma^{\otimes 2} \in \mathcal{H}^{\otimes 2} \end{array}$$

$$\overline{\text{Tr } \psi_A^2} = \text{Tr} \int dU \underbrace{S_A^{\otimes 2}}_{\tilde{S}_A} \underbrace{U | \psi_A \rangle \langle \psi_A | U^\dagger}_{\psi^{\otimes 2}} = \text{Tr} \tilde{S}_A \overline{\psi^{\otimes 2}} \quad \{|i,j\rangle\}$$

$$\frac{1}{n(n+1)} \text{Tr} [S_A^{\otimes 2} \mathbb{1}_B^{\otimes 2} + \mathbb{1}_A^{\otimes 2} S_B^{\otimes 2}]$$

$$n_A \quad n_B^2 \quad n_A^2 \quad n_B =$$

$$n_A n_B (n_A + n_B)$$

$$\frac{2}{n(n+1)} \text{Tr} S_A^{\otimes 2} \mathbb{1}_B^{\otimes 2} \left(\frac{\mathbb{1}^{\otimes 2} + S_{AB}}{2} \right)$$

$$S = \sum_{i,j} |i\rangle\langle j| \quad \left| \text{Tr } \sigma^2 = \text{Tr} [S \sigma^{\otimes 2}] \right| \quad \begin{array}{l} \sigma \in \mathcal{H} \text{ s.t.} \\ \sigma^{\otimes 2} \in \mathcal{H}^{\otimes 2} \end{array}$$

$$\overline{\text{Tr } \psi_A^2} = \text{Tr} \int dU \underbrace{S_A^{\otimes 2}}_{\tilde{S}_A} \underbrace{U | \psi_A \rangle \langle \psi_A | U^\dagger}_{\psi^{\otimes 2}} = \text{Tr} \tilde{S}_A \overline{\psi^{\otimes 2}} \quad \{|i,j\rangle\}$$

$$\frac{1}{n(n+1)} \text{Tr} [S_A^{\otimes 2} \mathbb{1}_B^{\otimes 2} + \mathbb{1}_A^{\otimes 2} S_B^{\otimes 2}]$$

$$\begin{array}{cccc} n_A & n_B^2 & n_A^2 & n_B \\ \hline n_A n_B & (n_A + n_B) & & \end{array}$$

$$\frac{2}{n(n+1)} \text{Tr} S_A^{\otimes 2} \mathbb{1}_B^{\otimes 2} \frac{(\mathbb{1}^{\otimes 2} + S_{AB})}{2}$$

$$P_{ur} = \frac{n_A + n_B}{n_A n_B + 1}$$

$$S = \sum 100$$

$$\Pi_{\pm} = \frac{11^{22} + 5}{2}$$

$$\overline{\psi^2} = \frac{1}{2} \frac{\lambda + \pi_+ + \lambda - \pi_-}{n(n+1)}$$

$$\overline{P_{ur}}^4 = \frac{n_A + n_B}{n_A n_B + 1}$$

$$S = \sum (1)$$

$$\overline{\ln \psi_A^2}$$

$$U \otimes U = \frac{1}{2} \otimes 2 + 5$$

$$\overline{\psi^{\otimes 2}} = \frac{1}{2} \frac{\lambda + \pi_+ + \lambda - \pi_-}{n(n+1)}$$

$$\frac{1}{n(n+1)}$$

$$\overline{P_{ur}}^4 = \frac{n_A + n_B}{n_A n_B + 1} \approx \frac{1}{n_A}$$

$$1 \ll n_A \ll n_B$$

$U \otimes U$

$$\Pi_{\pm} = \frac{11^{22} + 5}{2}$$

$$\overline{\psi^{\otimes 2}} = \frac{1}{2} \frac{1}{n(n+1)} (\lambda_+ \pi_+ + \lambda_- \pi_-)$$

$$\overline{\ln \psi_A^2}$$

$$\frac{1}{n(n+1)}$$

$$\overline{P_{ur}}^4 = \frac{n_A + n_B}{n_A n_B + 1} \approx \frac{1}{n_A} \sim \frac{1}{2^{100}}$$

$$1 \ll n_A \ll n_B$$

$$S = \sum |u\rangle \langle u| \quad |T$$

$$\overline{\text{Tr} \Psi_A^2} = \text{Tr} \int dU$$

$$\overline{\Pi_{\pm}} = \frac{11^{22} + 5}{2}$$

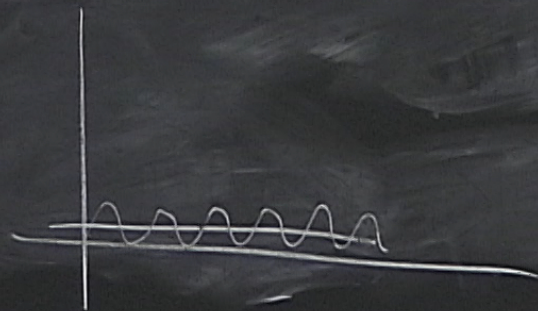
$$\overline{\Psi^{22}} = \frac{1}{2} \frac{1}{n(n+1)} (\lambda + \pi_+ + \lambda - \pi_-)$$

$$\frac{1}{n(n+1)} \text{Tr} \left[S_A \otimes 1_B + 1_A \otimes S_B \right]$$

$n_A \quad n_B^2 \quad n_A^2 \quad n_B$
 $n_A n_B (n_A + n_B)$

$$\overline{P_{ur}}^4 = \frac{n_A + n_B}{n_A n_B + 1} \approx \frac{1}{n_A} \sim \frac{1}{2^{100}}$$

$$1 \ll n_A \ll n_B$$



$$\overline{\psi^{\otimes 2}} = \frac{1}{2} \frac{\lambda + \pi_+ + \lambda - \pi_-}{n(n+1)}$$

$$S = \sum |i\rangle \langle j|$$

$$\overline{\text{Tr} \psi_A^2} = \text{Tr}$$

$$\frac{1}{n(n+1)} \text{Tr} \left(S_A^{\otimes 2} \right)$$