

Title: PSI 2018/2019 - Condensed Matter Review - Lecture 13

Date: Jan 23, 2019 11:30 AM

URL: <http://pirsa.org/19010044>

Abstract:

Temporal fluctuations $A(t)$

$$\Delta A_T^2 = \overline{(A(t) - \bar{A})^2} \leq \|A\| T_c \bar{\phi}^2$$

ϕ initial state

$$\bar{\phi} = \sum_n \pi_n \phi \pi_n$$

$$\uparrow$$
$$|E_n \rangle \langle E_n|$$

Temporal fluctuations $A(t)$

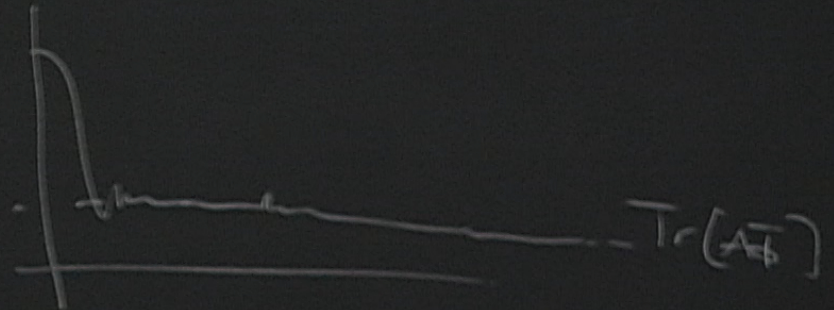
$$\Delta A_T^2 = \overline{(A(t) - \bar{A})^2} \leq \|A\| \text{Tr} \bar{\phi}^2$$

ϕ initial state

$$\bar{\phi} = \sum_n \pi_n \phi \pi_n$$

$$\uparrow$$

$$|E_n\rangle \langle E_n|$$



Noether Klein-Gordon Casimir Cross section SZ	1) Feynman prop. 2) LSZ assumptions → 2nd order 3) Kallen-Lehmann Gung 1) Dirac eqn 2) Spinors	7) Qtz Dirac 8) Antiparticles 9) Causality 10) Wick 11) Fermionic rules 12) Yukawa dogs 13) Qtz Maxwell	65) QED log	11 → Is vacuum energy $E(r) = \sum_{k \in N} \frac{1}{2} \omega_k \frac{n \pi}{r}$
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Temporal fluctuations $A(t)$

$$\Delta A_T^2 = \overline{(A(t) - \bar{A})^2} \leq \|A\| \text{Tr } \bar{\Phi}^2$$

ϕ initial state

$$\bar{\Phi} = \sum_n \Pi_n \phi \Pi_n$$

↑
 $|E_n\rangle \langle E_n|$

- $\tilde{\Phi}_t := \phi_t - \bar{\phi} = \sum_{m \neq n} \phi_{mn} e^{i(E_m - E_n)t} |E_n\rangle \langle E_m|$
- $\text{tr } \tilde{\Phi}_t = 0$

- $\mathcal{H}^+ = \text{supp}(\phi)$; P^+ projector onto $\mathcal{H}^+$

- δ_A norm of A on $\mathcal{H}^+$

$$\max_{\psi \in \mathcal{H}^+} \langle \psi | A | \psi \rangle - \min_{\psi \in \mathcal{H}^+} \langle \psi | A | \psi \rangle = a_{\max} - a_{\min}$$

- $\tilde{A} := A - a_{\min} \Rightarrow 0 \leq \langle \psi | \tilde{A} | \psi \rangle \leq \delta_A \quad \psi \in \mathcal{H}^+$

$$\Delta A_T^2 = \overline{(A(t) - \bar{A})^2} = \text{Tr}$$

• $\tilde{\phi}_t := \phi_t - \bar{\phi} = \sum_{m \neq n} \phi_{mn} e^{i(E_m - E_n)t} |E_n\rangle \langle E_m|$

$\boxed{\text{tr } \tilde{\phi}_t = 0}$

• $\mathcal{H}^+ = \text{supp}(\phi)$; P^+ projector onto $\mathcal{H}^+$

• S_A norm of A on H^+ $\max_{\psi \in \mathcal{H}^+} \langle \psi | A | \psi \rangle - \min_{\psi \in \mathcal{H}^+} \langle \psi | A | \psi \rangle = a_{\max} - a_{\min}$

• $\tilde{A} := A - a_{\min} \Rightarrow 0 \leq \langle \psi | \tilde{A} | \psi \rangle \leq S_A \quad \psi \in \mathcal{H}^+$

$$\begin{aligned}
\Delta A_T^2 &= \overline{(A(t) - \bar{A})^2} = \overline{(\text{Tr } A(\phi_t - \bar{\phi}))^2} = \overline{(\text{Tr } \tilde{A} \tilde{\Phi}_t)^2} = \sum_{\substack{m \neq n \\ j \neq k}} \tilde{A}_{jk} \phi_{kj} \tilde{A}_{nm} \phi_{mn} e^{i(\underbrace{E_j - E_k}_{\neq 0} + \underbrace{E_n - E_m}_{\neq 0})t} \\
&= \sum_{n \neq m} \tilde{A}_{nm} \phi_{mn} \tilde{A}_{mn} \phi_{nm} \leq \sum_{nm} |\tilde{A}_{nm}|^2 |\phi_{nm}|^2 \quad \text{Non deg gaps} = 0 \text{ iff } \begin{matrix} j=n \\ k=m \end{matrix} \\
&\leq \sum_{nm} |\tilde{A}_{nm}|^2 \bar{\phi}_{nn} \bar{\phi}_{mm} = \sum_{nm} \underbrace{\langle E_n | \tilde{A} \bar{\Phi} | E_n \rangle}_{\text{Exercise}} \langle E_m | \tilde{A} \bar{\Phi} | E_m \rangle \\
&= \sum_m \langle E_m | \tilde{A} \bar{\Phi}^2 | E_m \rangle = \text{Tr} [\tilde{A}^2 \bar{\Phi}^2] = S_A^2 \text{Tr} [\bar{\Phi}^2]
\end{aligned}$$

$$e^{i(\underline{E}_i - \underline{E}_k + \underline{E}_m - \underline{E}_n)t}$$

$$-E_m \Rightarrow \text{---} = 0$$

$$f \quad \begin{matrix} j=n \\ k=m \end{matrix}$$

raise

$$[\bar{\phi}^2]$$

$$Pr(|x - \mu| > k\sigma) \leq \frac{1}{k^2} \quad k \geq 0$$

$$Pr(|x - \mu| > \sqrt{\sigma}) \leq \sigma \quad k = \frac{1}{\sqrt{\sigma}}$$

$$Tr \bar{\phi}^2 = Tr \left[\phi_{nn}^2 \right]$$

$\overline{(E-t)}$

$= 0$

$$P_r(|x - \mu| > k\sigma) \leq \frac{1}{k^2} \quad k \geq 0$$

$$P_r(|x - \mu| > \sqrt{\sigma}) \leq \sigma \quad k = \frac{1}{\sqrt{\sigma}}$$

$$\overline{\text{Tr } \Phi^2} = \text{Tr} \left[\Phi_{nn}^2 \right] \sim \frac{1}{n} \int \text{dim } \mathcal{H}$$

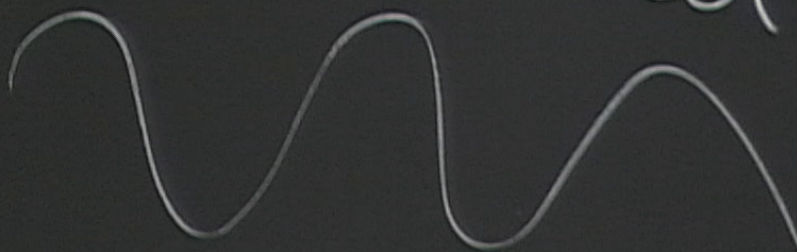
$$\mathcal{L}_t = |\langle \Phi | \Phi_t \rangle|^2$$
$$\overline{\mathcal{L}_t} = \text{Tr } \Phi^2$$

$$K = \hbar \omega$$

$$A = \Phi_{nm}^{-1} |E_n\rangle \langle E_m|$$

f.h.c.

$$A(t) \sim \cos \omega t$$



$$\text{Tr } \phi$$

$$\mathcal{L}_t = \langle \mathcal{L} \rangle$$

$$\underline{\Phi} = \sum_i \textcircled{p_i} |\phi_i\rangle\langle\phi_i|$$

$$\underline{\Phi}_t = \mathcal{U}_t \underline{\Phi} \quad S_\alpha(\underline{\Phi}_t)$$

$$\underline{\Phi} \text{ pure} \xRightarrow{\text{Unitary}} \underline{\Phi}_t \text{ pure}$$

$$\xrightarrow{\text{Therm}} \frac{e^{-\beta H}}{\mathcal{Z}_1}$$

$$S_1 = -\sum_i p_i \log p_i \sim N$$

$$p_i = \mathcal{Z}_1^{-1} e^{-\beta E_i}$$

$$\bar{\Phi} = \sum_i (p_i) |\phi_i\rangle\langle\phi_i|$$

$$\bar{\Phi}_t = \mathcal{U}_t \bar{\Phi} \quad S_\alpha(\bar{\Phi}_t)$$

$$\bar{\Phi} \text{ pure} \xRightarrow{\text{Unitary}} \bar{\Phi}_t \text{ pure}$$

$$\xrightarrow{\text{Therm}} \frac{e^{-\beta H}}{\mathcal{Z}_1}$$

$$S_1 = -\sum_i p_i \log p_i \sim N \sim \text{Volume law}$$

$$p_i = \frac{1}{\mathcal{Z}_1} e^{-\beta E_i}$$

$$\bar{\Phi} = \sum_i (p_i) |\phi_i\rangle\langle\phi_i|$$

$$\bar{\Phi}_t = \mathcal{U}_t \bar{\Phi} \quad S_\alpha(\bar{\Phi}_t)$$

$$\begin{array}{ccc} \bar{\Phi} \text{ pure} & \xRightarrow{\text{Unitary}} & \bar{\Phi}_t \text{ pure} \\ & \text{Then} & \checkmark \\ & \longrightarrow & \frac{e^{-\beta H}}{Z_1} \end{array}$$

$$\bar{A} = \text{Tr}[A \bar{\Phi}] = \sum_n p_n A_{nn}$$

initial conditions

$$S_1 = -\sum_i p_i \log p_i \sim N \sim \text{Volume law}$$

$$p_i = \frac{1}{Z_1} e^{-\beta E_i}$$

P_{In} A_{nn}
 $\uparrow$
 initial conditions

$$\begin{aligned}
 \langle A \rangle &= \text{Tr} [\hat{A} \phi] \\
 &= \text{Tr} [\hat{A} \phi_A]
 \end{aligned}$$

$$\phi_A \equiv \text{Tr}_B \phi$$

$$|\Phi\rangle = |\Phi_A\rangle |\Phi_B\rangle \longrightarrow \Phi_t = U_t \phi \quad \text{pure}$$

$$\downarrow$$

$$\Phi_A \quad \text{pure}$$

$$\phi_A(t) = \boxed{\text{Tr}_B (U_t \phi)} \quad \text{not pure}$$

- H is ergodic $|\phi\rangle \longrightarrow |\phi_t\rangle$ U_t is entangling
- $|\phi\rangle \in \mathcal{H}$ random $\longrightarrow ? S(\phi_A)$

$$|111\rangle$$

$$\frac{1}{\sqrt{3}}|10\rangle + \frac{2}{\sqrt{3}}|11\rangle$$

$$\frac{|01\rangle + |00\rangle}{\sqrt{2}}$$

$$\frac{|01\rangle + |10\rangle}{\sqrt{2}}$$

not pure

$$|11\rangle$$

$$\alpha |110\rangle + \beta |101\rangle$$

$$\frac{|01\rangle + |00\rangle}{\sqrt{2}}$$

$$\frac{|01\rangle + |10\rangle}{\sqrt{2}}$$

$$\Rightarrow \Phi_t = U_t \phi \quad \text{pure}$$

$$\phi_A(t) = \boxed{\text{Tr}_B (U_t \phi)} \quad \text{not pure}$$

$|\phi_t\rangle$ U_t is entangling

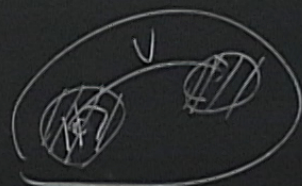
dom $\rightarrow$ $S(\phi_A)$

$$\langle \psi | A | \psi \rangle$$

$$\frac{|01\rangle + |00\rangle}{\sqrt{2}}$$

$$\frac{|01\rangle + |10\rangle}{\sqrt{2}}$$

$$\Phi_t = U_t \phi \quad \text{pure}$$



$$\phi_A(t) = \text{Tr}_B (U_t \phi)$$

not pure

ϕ_t U_t is entangling
 ϕ
 $\rightarrow S(\phi_A)$

$$\langle \psi | A | \psi \rangle$$

$$|11\rangle$$

$$\frac{|01\rangle + |00\rangle}{\sqrt{2}}$$

$$\frac{|01\rangle + |10\rangle}{\sqrt{2}}$$

$$\alpha |10\rangle + \beta$$