Title: PSI 2018/2019 - Condensed Matter Review - Lecture 13

Date: Jan 23, 2019 11:30 AM

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Abstract:

Temporal fluctuations $A(t)$

$$\Delta \bar{A}_T^2 = \overline{(A(t) - \bar{A})^2} \leq \|A\| \, \mathrm{Tr}\, \bar{\phi}^2$$

ϕ initial state

$$\bar{\phi} = \sum_n \Pi_n \phi \Pi_n$$

$|E_n\rangle\langle E_n|$

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Noether
Klein-Gordon
Casimir
Cross section
SZ

4) Feynman prop.
5) LSZ assumptions → 2nd order
6) Kallen-Lehmann
Gauge
1) Dirac eqn
2) Spinors

7) Qtz Dirac
8) Anti particles
9) Causality
10) Wick
11) Fermionic rules
12) Yukawa diags
13) Qtz Maxwell

G5) QED lag

↳ is vacuum energy

$$E(r) = \frac{1}{2}\sum_{n\in\mathbb{N}} \omega_n - \frac{n\pi}{r}$$

Temporal Fluctuations $A(t)$

$$\Delta A_T^2 = \overline{(A(t)-\bar{A})^2} \leq \|A\| \operatorname{Tr} \tilde{\phi}^2$$

ϕ initial state

$\bar{\phi} = \sum_n \Pi_n \phi \Pi_n$

$|E_n\rangle\langle E_n|$

- $\tilde{\phi}_t := \phi_t - \bar{\phi} = \sum_{m\neq n} \phi_{mn} e^{i(E_m - E_n)t} |E_n\rangle\langle E_m|$
- $\boxed{\operatorname{tr} \tilde{\phi}_t = 0}$
- $\mathcal{H}^+ = \operatorname{supp}(\phi)$; P^+ projector onto $\mathcal{H}^+$
- δ_A norm of A on $\mathcal{H}^+$ $\quad \max_{\psi\in\mathcal{H}^+} \langle\psi|A|\psi\rangle - \min_{\psi\in\mathcal{H}^+} \langle\psi|A|\psi\rangle \equiv a_{max} - a_{min}$
- $\tilde{A} := A - a_{min} \Rightarrow 0 \leq \langle\psi|\tilde{A}|\psi\rangle \leq \delta_A \quad \psi\in\mathcal{H}^+$

$$\Delta A_T^2 = \overline{(A(t)-\bar{A})^2} = \overline{\operatorname{Tr}}$$

(3) Qtz Maxwell | | | GR | SAM

- $\tilde{\phi}_t := \phi_t - \bar{\phi} = \sum_{m \neq n} \phi_{mn} e^{i(E_m - E_n)t} |E_n\rangle\langle E_m|$

$$\boxed{\mathrm{tr}\,\tilde{\Phi}_t = 0}$$

- $\mathcal{H}^+ = \mathrm{supp}(\phi)$; P^+ projector onto $\mathcal{H}^+$
- δ_A norm of A on H^+ $\quad \max_{\psi \in \mathcal{H}^+} \langle\psi|A|\psi\rangle - \min_{\psi \in \mathcal{H}^+} \langle\psi|A|\psi\rangle \equiv a_{max} - a_{min}$
- $\tilde{A} := A - a_{min} \Rightarrow 0 \leq \langle\psi|\tilde{A}|\psi\rangle \leq \delta_A \quad \psi \in \mathcal{H}^+$

$$\Delta A_T^2 = \overline{(A(t)-\bar{A})^2} = \overline{(\mathrm{Tr}\, A(\phi_t - \bar{\phi}))^2} = \overline{(\mathrm{Tr}\,\tilde{A}\tilde{\phi}_t)^2} = \sum_{\substack{m\neq n \\ j\neq k}} \tilde{A}_{jk}\phi_{kj}\tilde{A}_{nm}\phi_{mn}\, \overline{e^{i(E_j - E_k + E_m - E_n)t}}$$

$E_j - E_k \neq E_n - E_m \Rightarrow$ $= 0$

Non deg. gaps $= 0$ iff $j = n$, $k = m$

$$= \sum_{n\neq m} \tilde{A}_{nm}\phi_{mn}\tilde{A}_{mn}\phi_{nm} \leq \sum_{nm} |\tilde{A}_{nm}|^2 |\phi_{nm}|^2$$

$$\leq \sum_{n\neq m} |\tilde{A}_{nm}|^2 \bar{\phi}_{nn}\bar{\phi}_{mm} = \sum_{n,m} \langle E_m|\tilde{A}\bar{\phi}|E_n\rangle\langle E_n|\tilde{A}\bar{\phi}|E_m\rangle$$

$$= \sum_m \langle E_m|\tilde{A}\bar{\phi}^2|E_m\rangle = \overset{11}{T_r}[\tilde{A}^2\bar{\phi}^2] \overset{\text{Exercise}}{=} \delta_A^2\, T_r[\bar{\phi}^2]$$

$$e^{i(E_j - E_k + E_m - E_n)t}$$

$$-E_m \Rightarrow \qquad = 0$$

$$f \qquad j = n, \quad k = m$$

raise

$$[\bar{\phi}^2]$$

$$\Pr\left(|x-\mu| > k\sigma\right) \leq \frac{1}{k^2} \qquad k \geq 0$$

$$\Pr\left(|x-\mu| > \sqrt{\sigma}\right) \leq \sigma \qquad k = \frac{1}{\sqrt{\sigma}}$$

$$\mathrm{Tr}\,\bar{\phi}^2 = \mathrm{Tr}\left[\phi_{nn}^2\right]$$

$$\Pr\left(|x-\mu| > k\sigma\right) \leq \frac{1}{k^2} \qquad k \geq 0$$

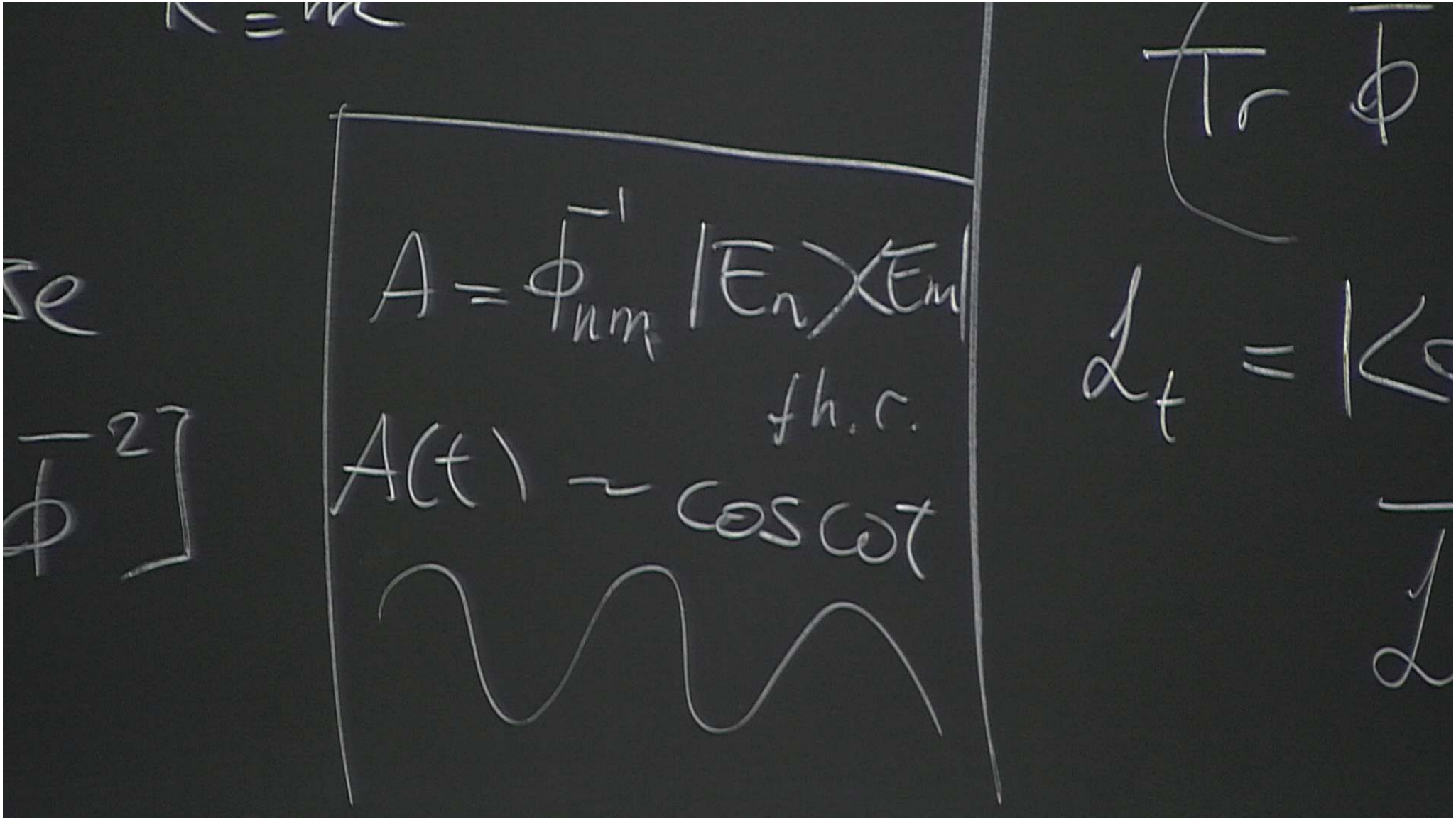
$$\Pr\left(|x-\mu| > r\sigma\right) \leq \sigma \qquad k = \frac{1}{r\sigma}$$

$$\mathrm{Tr}\,\bar{\phi}^2 = \mathrm{Tr}\left[\phi_{nn}^2\right] \sim \frac{1}{n}$$

$\dim \mathcal{H}$

$$\mathcal{L}_t = |\langle \phi | \phi_t \rangle|^2$$

$$\overline{\mathcal{L}_t} = \mathrm{tr}\,\bar{\phi}^2$$


$A = \phi^{-1}_{nm} |E_n\rangle\langle E_m|$
$A(t) \sim \cos\omega t$
h.c.

$$\underline{\Phi} = \sum_i p_i |\phi_i\rangle\langle\phi_i|$$

$$\underline{\Phi}_t = U_t \underline{\Phi} \qquad S_\alpha(\underline{\Phi}_t)$$

$\underline{\Phi}$ pure $\xRightarrow{\text{Unitary}}$ $\underline{\Phi}_t$ pure

Therm $\longrightarrow \frac{e^{-\beta H}}{Z_1}$

$$S_1 = -\sum_i p_i \log p_i \sim N$$

$$p_i = Z_1^{-1} e^{-\beta E_i}$$

$\bar{\Phi} = \bar{\Sigma}_i p_i |\phi_i\rangle\langle\phi_i|$

$\bar{\Phi}_t = U_t \bar{\Phi}$ $\qquad S_\alpha(\bar{\Phi}_t)$

$\bar{\Phi}$ pure $\xrightarrow{\text{Unitary}}$ $\bar{\Phi}_t$ pure

Therm $\longrightarrow \frac{e^{-\beta H}}{Z_1}$ $\qquad S_1 = -\bar{\Sigma}_i p_i \log p_i \sim N \sim$ Vdume law

$p_i = Z_1^{-1} e^{-\beta E_i}$

$\bar{\Phi} = \sum_i p_i |\phi_i\rangle\langle\phi_i|$

$\bar{\Phi}_t = U_t \bar{\Phi}$ $\quad S_\alpha(\bar{\Phi}_t)$

$\bar{A} = Tr[A\bar{\Phi}] = \sum_n P_n A_{nn}$

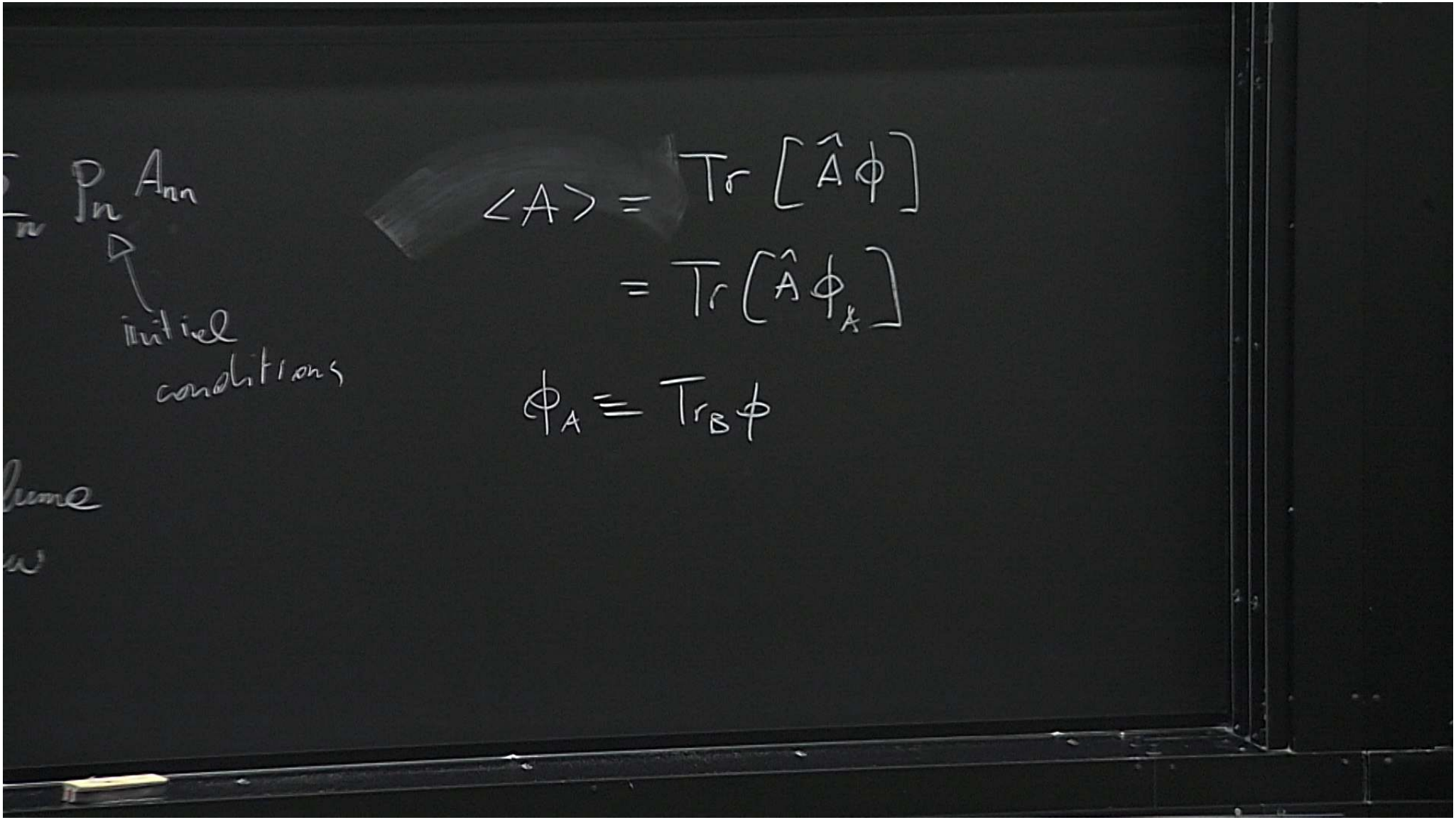
P_n: initial conditions

$\bar{\Phi}$ pure $\xrightarrow{\text{Unitary}}$ $\bar{\Phi}_t$ pure

Therm $\longrightarrow \frac{e^{-\beta H}}{Z_1}$

$S_1 = -\sum_i p_i \log p_i \sim N \sim$ Volume law

$p_i = Z_1^{-1} e^{-\beta E_i}$


P_n A_{nn}
initial conditions
$\langle A \rangle = Tr[\hat{A}\phi]$
$= Tr[\hat{A}\phi_A]$
$\phi_A \equiv Tr_B \phi$

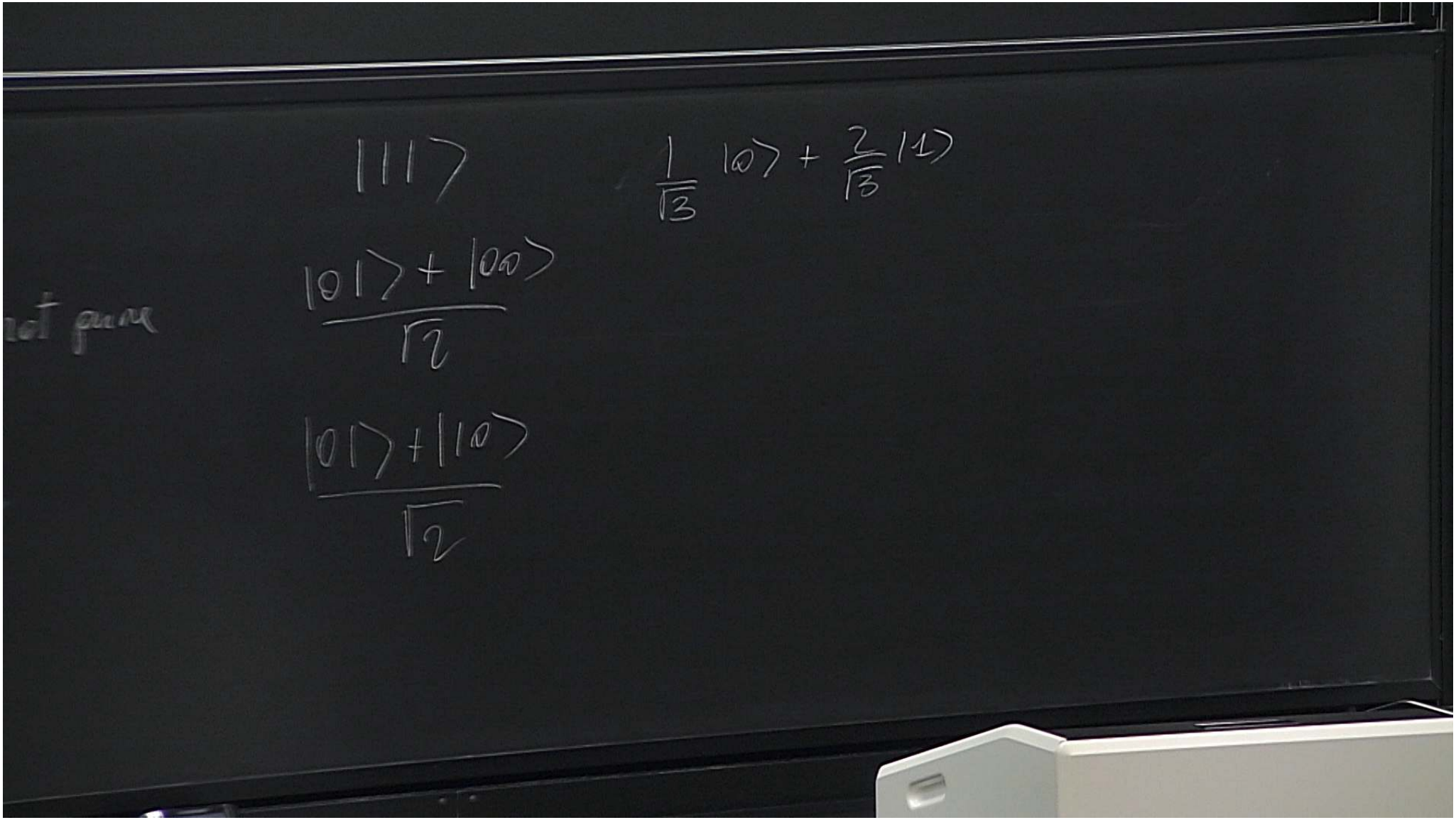
$|\Phi\rangle = |\Phi_A\rangle |\Phi_B\rangle \longrightarrow \Phi_t = U_t \Phi$ pure

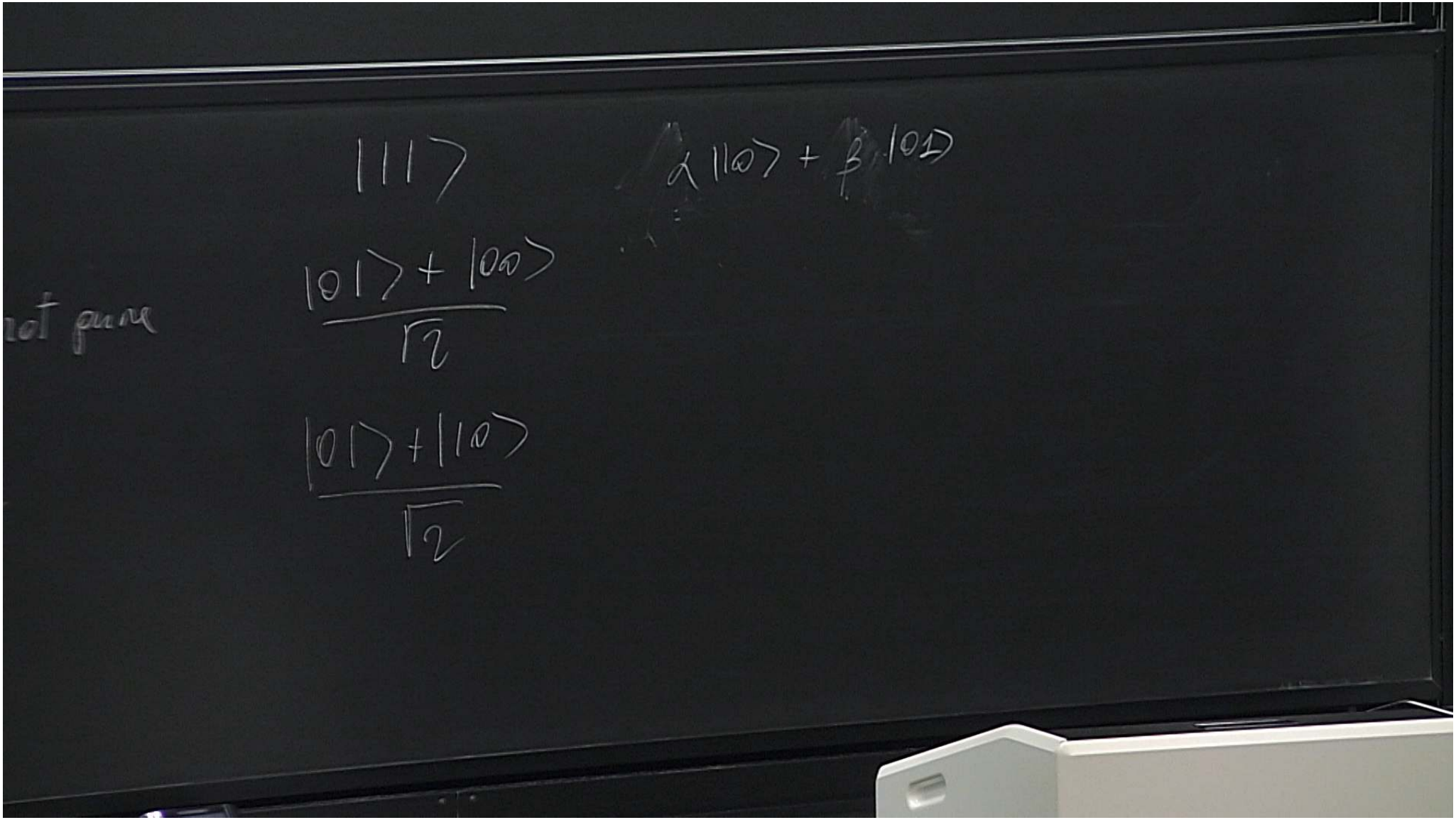
$\phi_A(t) = \boxed{Tr_B \, U_t \phi}$ not pure

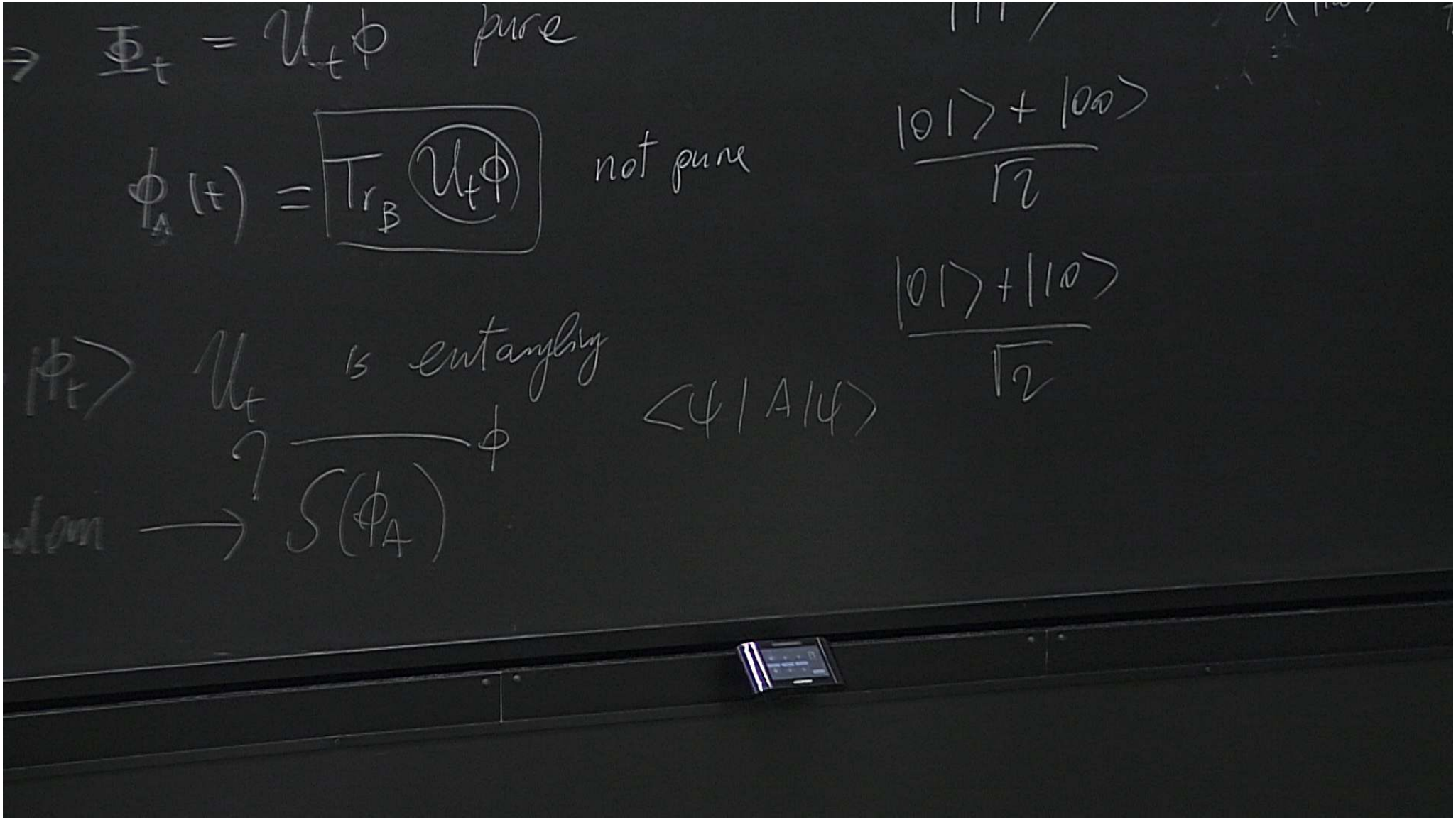
Φ_A pure

- H is ergodic $|\phi\rangle \longrightarrow |\phi_t\rangle$ U_t is entangling

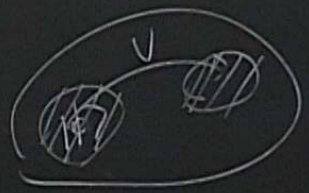
- $|\phi\rangle \in \mathcal{H}$ random $\overset{?}{\longrightarrow} S(\phi_A)$


|11⟩
$\frac{1}{\sqrt{3}}|0\rangle + \frac{2}{\sqrt{3}}|1\rangle$
$\frac{|01\rangle + |00\rangle}{\sqrt{2}}$
$\frac{|01\rangle + |10\rangle}{\sqrt{2}}$


|11⟩
α|10⟩ + β|01⟩
|01⟩ + |00⟩ / √2
|01⟩ + |10⟩ / √2


$\Phi_t = U_t \phi$ pure
$\phi_A(t) = \mathrm{Tr}_B (U_t \phi)$ not pure
$\frac{|01\rangle + |00\rangle}{\sqrt{2}}$
$\frac{|01\rangle + |10\rangle}{\sqrt{2}}$
$|\phi_t\rangle$ U_t is entangling
ϕ
$\langle \psi | A | \psi \rangle$
$S(\phi_A)$

$\Phi_t = \mathcal{U}_t \phi$ pure



$|11\rangle$

$\alpha|10\rangle + \beta$

$\phi_A(t) = \boxed{\mathrm{Tr}_B\, \mathcal{U}_t \phi}$ not pure

$\frac{|01\rangle + |00\rangle}{\sqrt{2}}$

$\frac{|01\rangle + |10\rangle}{\sqrt{2}}$

$|\phi_t\rangle$ $\mathcal{U}_t$ is entangling

$\langle\psi|A|\psi\rangle$

? ϕ

$\longrightarrow S(\phi_A)$