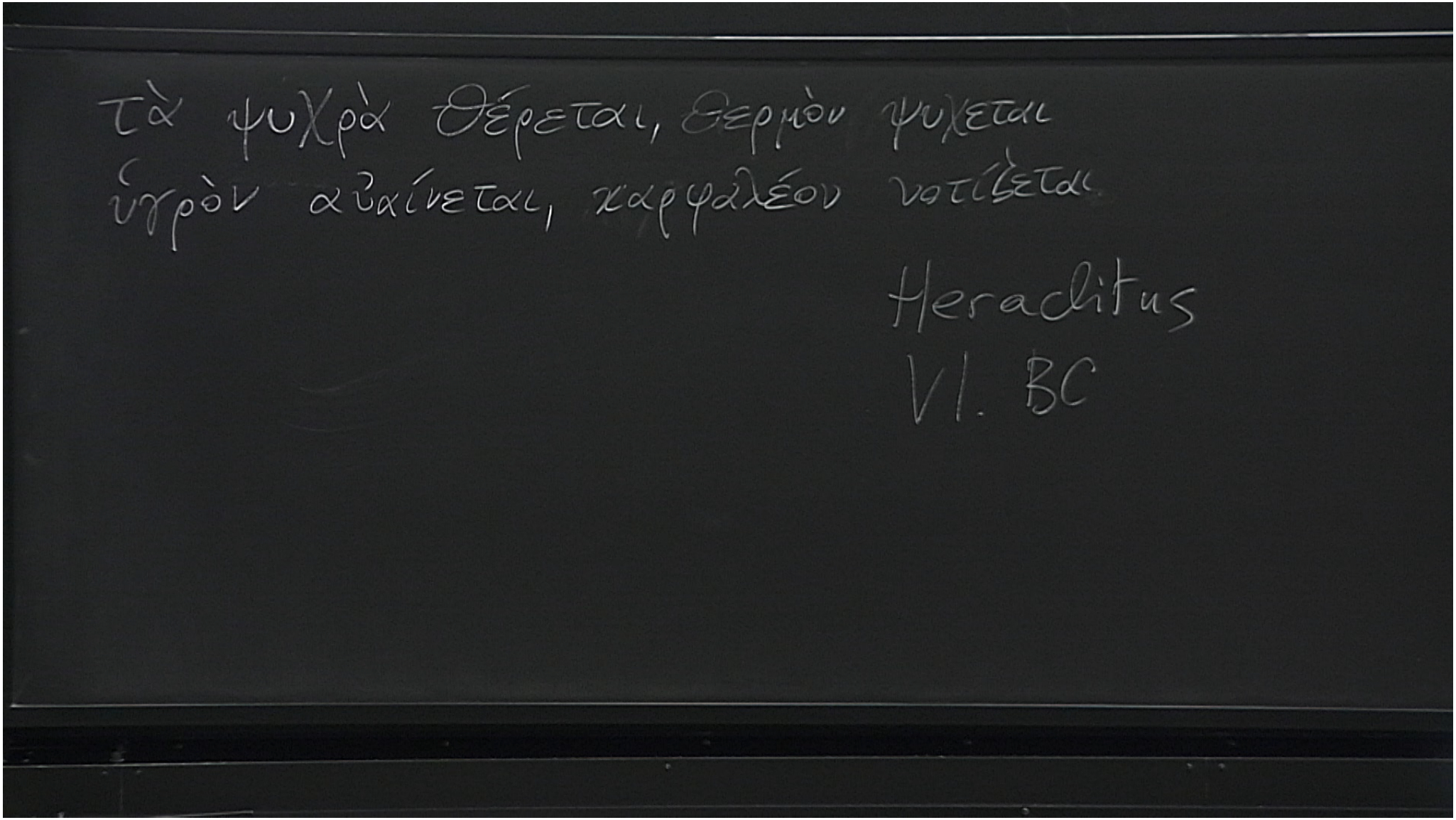
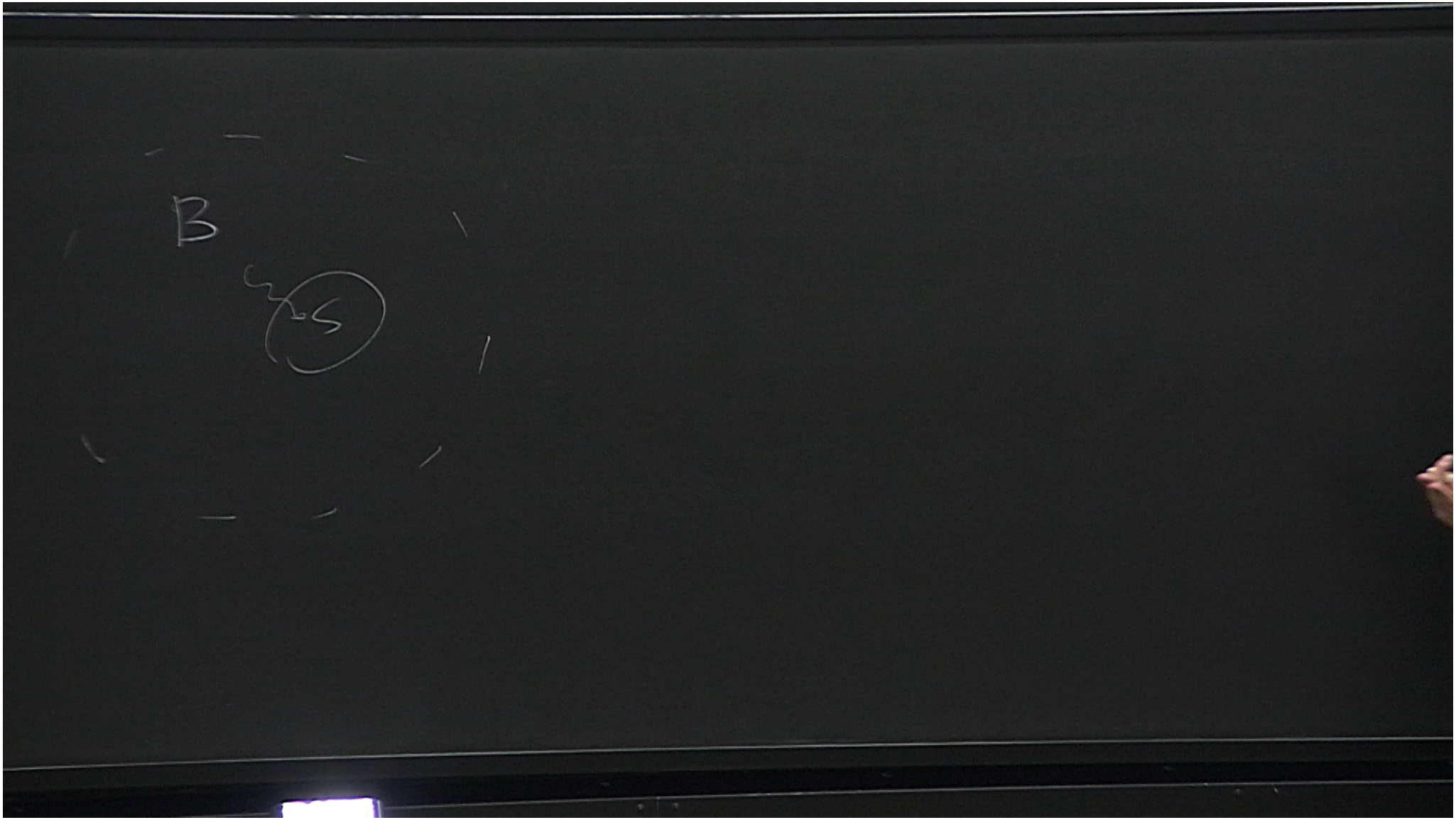
Title: PSI 2018/2019 - Condensed Matter Review - Lecture 12

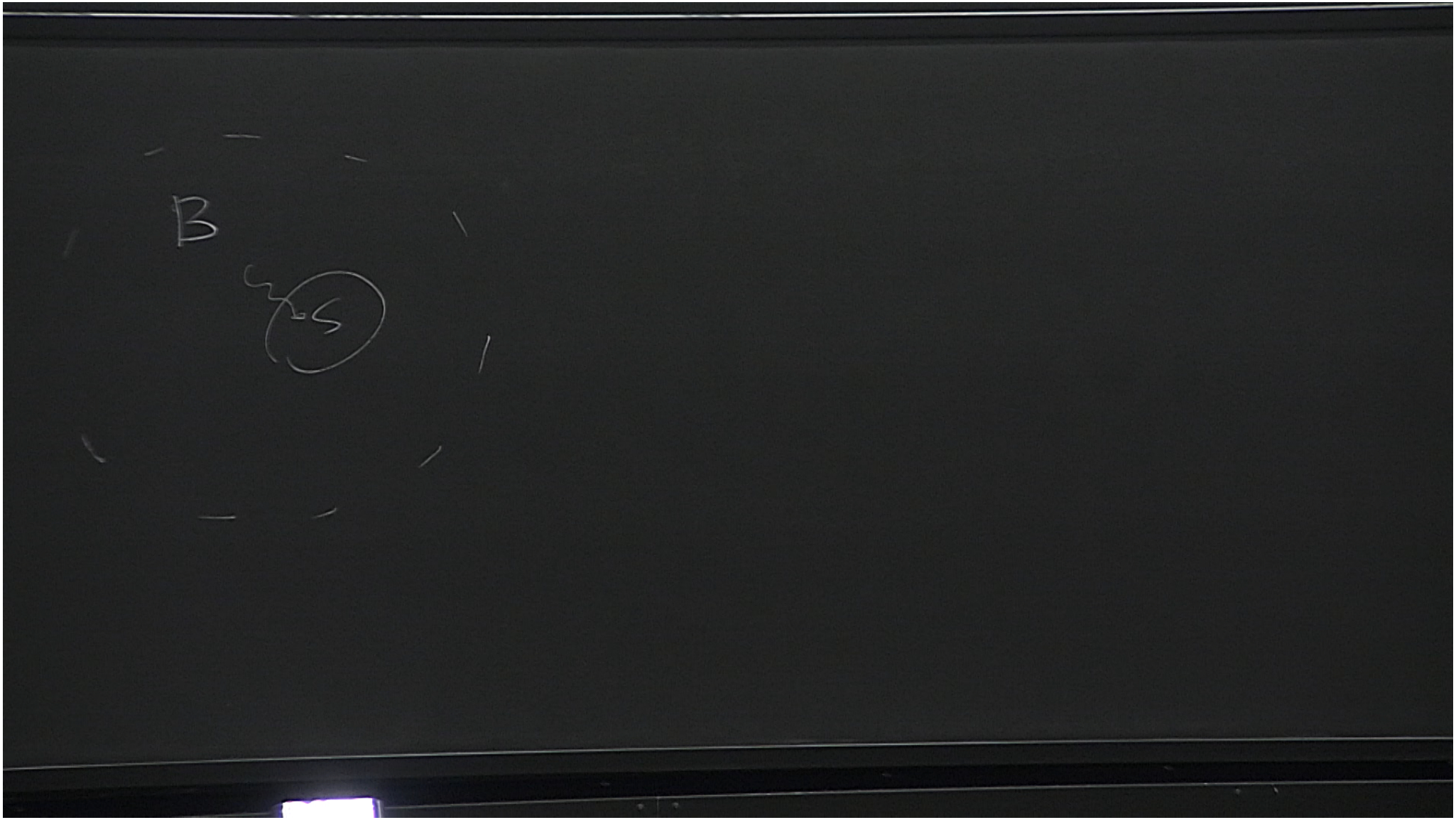
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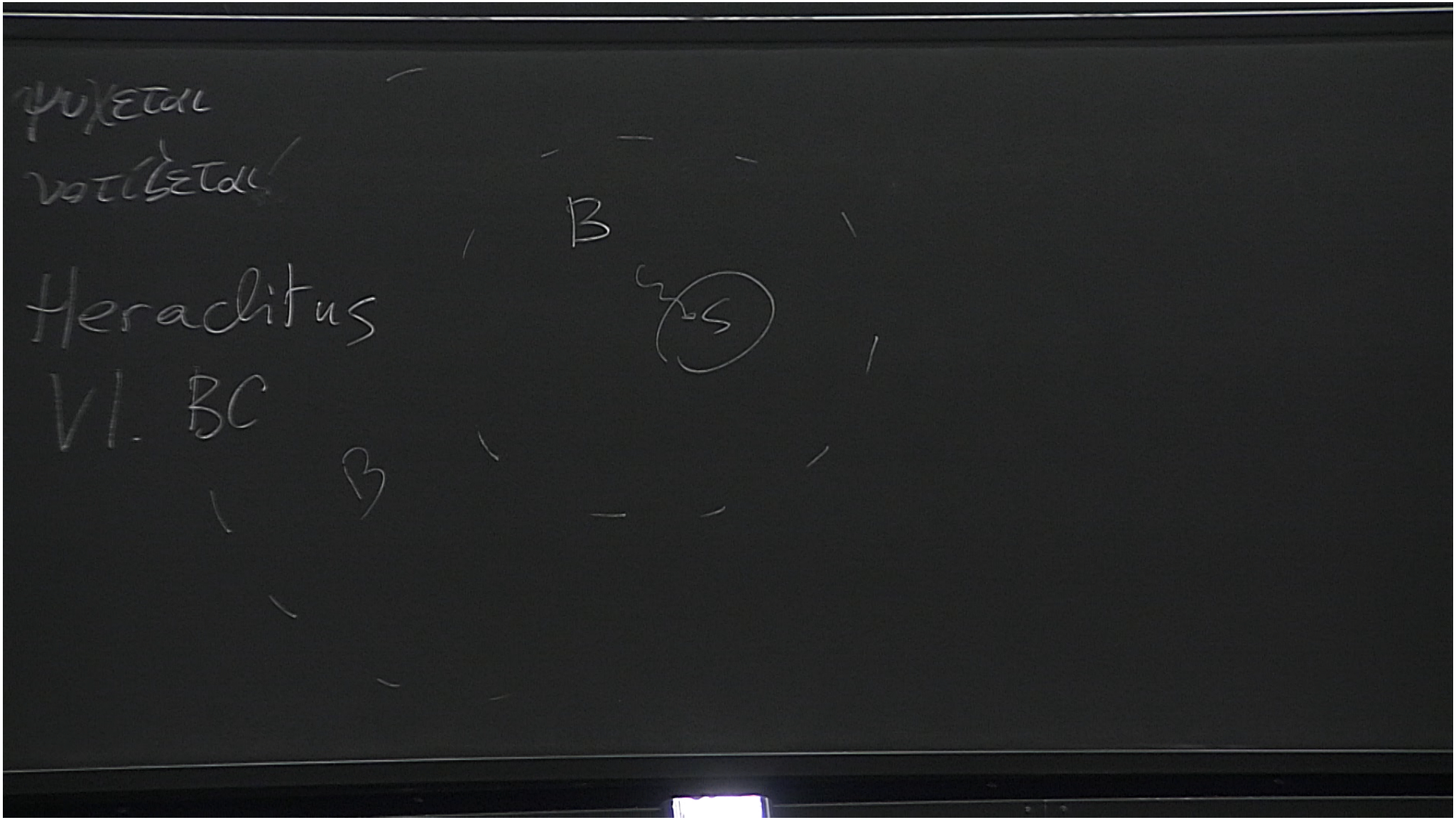
URL: http://pirsa.org/19010043

Abstract:


τὰ ψυχρὰ θέρεται, θερμὸν ψύχεται
ὑγρὸν αὐαίνεται, καρφαλέον νοτίζεται
Heraclitus
VI. BC


B
S


B
S


ψυχεται
νοτιζεται
Heraclitus
VI. BC
B
B
S

$$H = \sum_n E_n \Pi_n \qquad \Pi_n = |E_n\rangle\langle E_n|$$

$$\phi_t = \mathcal{U}_t \phi = U_t \phi U_t^\dagger \qquad U_t = e^{-iHt}$$

$$\hat{A}_t = \mathcal{U}_t \hat{A} = U_t^\dagger \hat{A} U_t$$

$$A = \mathrm{Tr}[\hat{A}\phi]$$

$$A_t = \mathrm{Tr}[\hat{A}_t \phi] = \mathrm{Tr}[\hat{A}\phi_t]$$

Equilibration

$\phi \longrightarrow \phi_t \longrightarrow \phi_{eq}$ steady

$[\phi, H] = 0$

$\phi = \sum_n p_n \Pi_n$ steady states

$\lim_{t\to\infty} \phi_t = \phi_{eq}$

$$\lim_{t\to\infty} \|\phi_t - \phi_{eq}\| = \lim_{t\to\infty} \|U_t \phi - U_t \phi_{eq}\| = \lim_{t\to\infty} \|U_t(\phi - \phi_{eq})\| =$$

$A_t = \mathrm{Tr}\left[\hat{A}_t \phi\right] = \mathrm{Tr}\left[\hat{A}\, \phi_t\right]$

Equilibration

$\phi \longrightarrow \phi_t \longrightarrow \phi_{eq}$ steady $\quad [\phi, H] = 0$

$\phi = \sum_n p_n \Pi_n$ steady states

$\lim_{t\to\infty} \phi_t = \phi_{eq}$

$0 = \lim_{t\to\infty} \left\| \phi_t - \phi_{eq} \right\| = \lim_{t\to\infty} \left\| \mathcal{U}_t \phi - \mathcal{U}_t \phi_{eq} \right\| = \lim_{t\to\infty} \left\| \mathcal{U}_t (\phi - \phi_{eq}) \right\| = \left\| \phi - \phi_{eq} \right\|$

$A = \mathrm{Tr}[A\phi]$

$A_t = \mathrm{Tr}[\hat{A}_t\phi] = \mathrm{Tr}[\hat{A}\phi_t]$

Equilibration

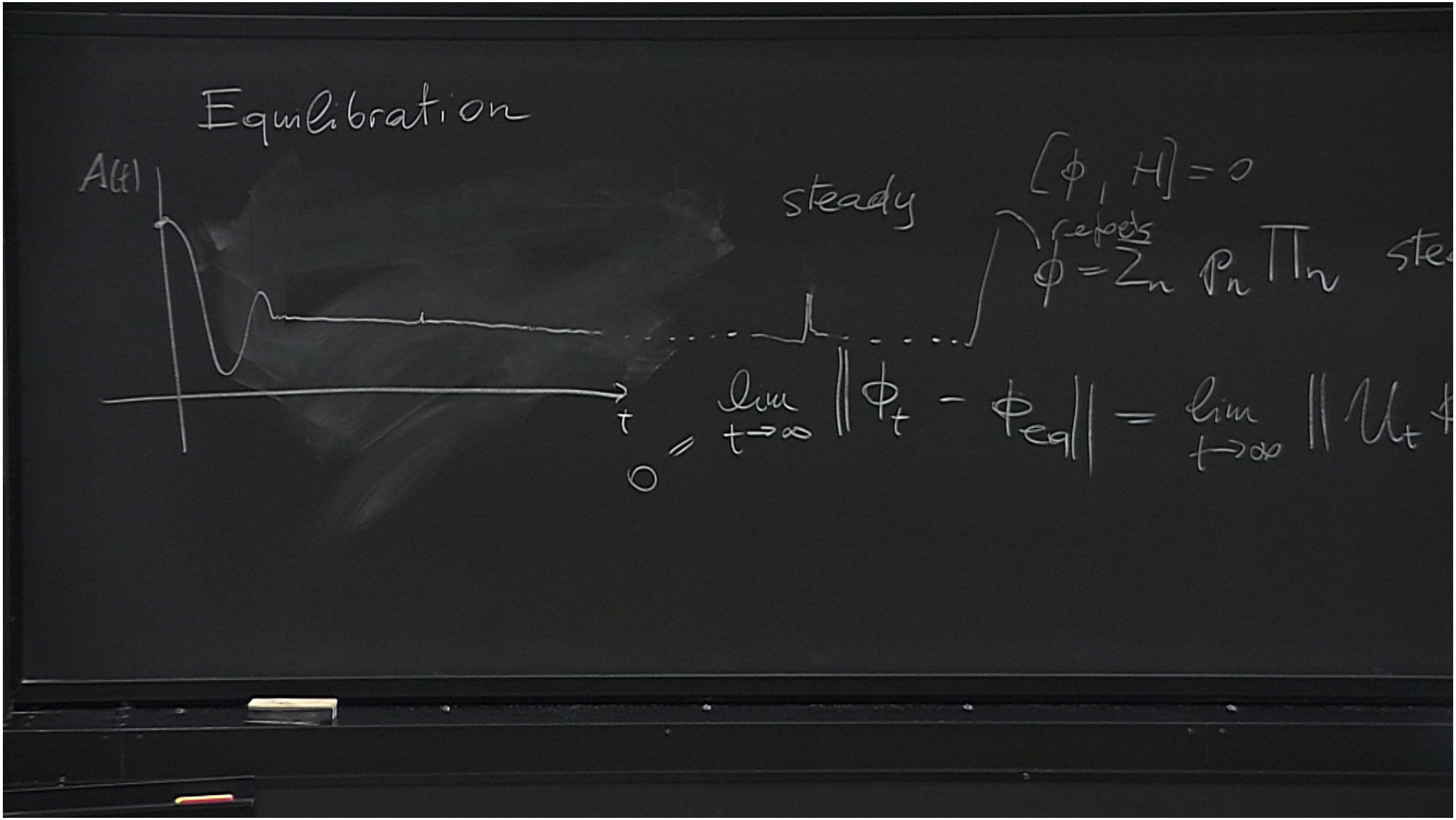
$\phi \longrightarrow \phi_t \longrightarrow \phi_{eq}$ steady $\quad [\phi, H] = 0$

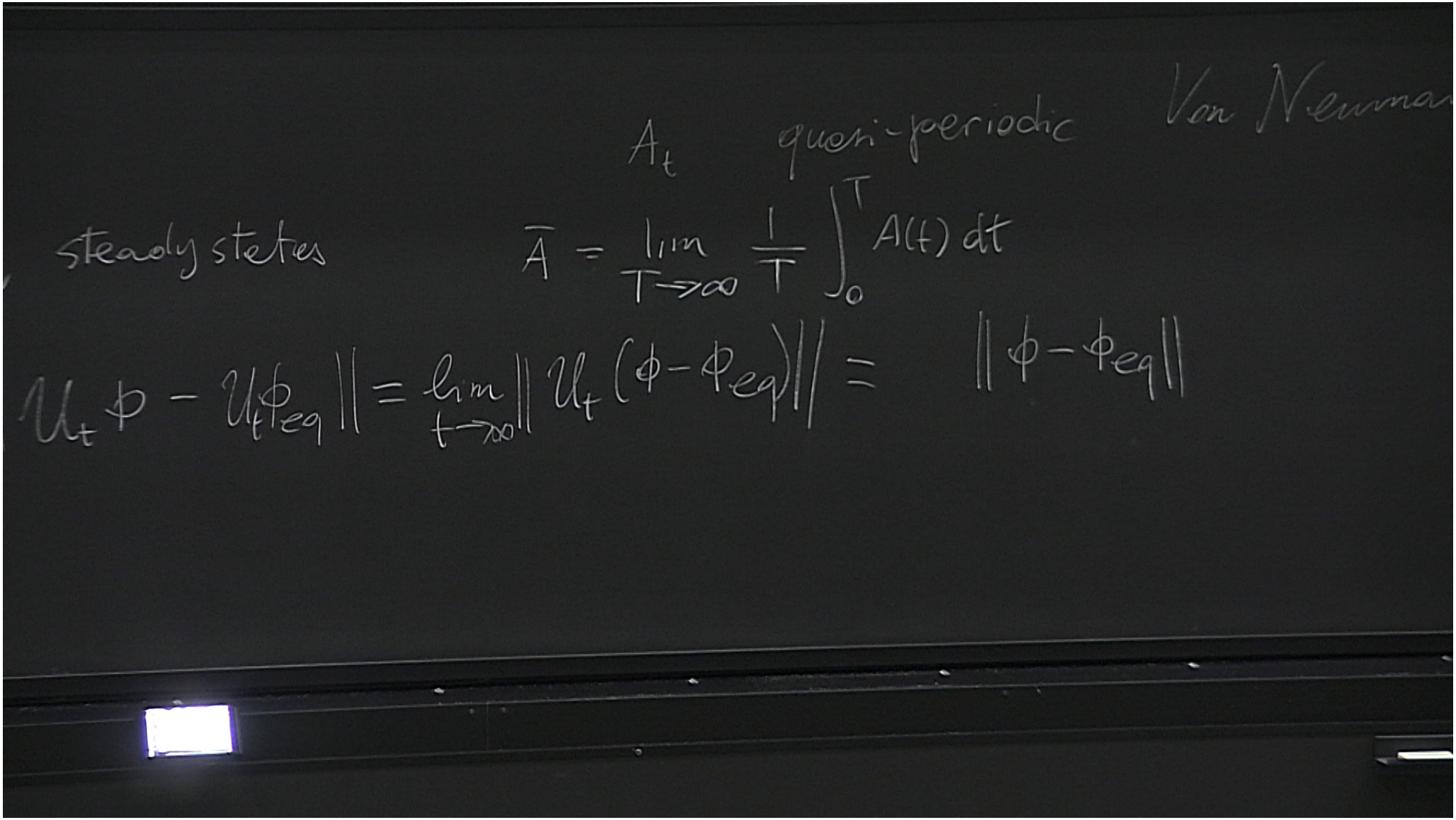
$\phi = \sum_n p_n \Pi_n$ steady states

A_t quasi-periodic Von Neumann

$\lim_{t\to\infty} \phi_t = \phi_{eq}$

$0 = \lim_{t\to\infty} \|\phi_t - \phi_{eq}\| = \lim_{t\to\infty} \|U_t\phi - U_t\phi_{eq}\| = \lim_{t\to\infty} \|U_t(\phi - \phi_{eq})\| = \|\phi - \phi_{eq}\|$


Equilibration
A(t)
t
steady
$[\phi, H] = 0$
repeats
$\phi = \sum_n p_n \Pi_n$
$\lim_{t\to\infty} \|\phi_t - \phi_{eq}\| = \lim_{t\to\infty} \|\mathcal{U}_t$
0


A_t quasi-periodic Von Neumann
steady states $\bar{A} = \lim_{T\to\infty} \frac{1}{T} \int_0^T A(t)\,dt$
$\| U_t \phi - U_t \phi_{eq} \| = \lim_{t\to\infty} \| U_t (\phi - \phi_{eq}) \| = \| \phi - \phi_{eq} \|$

A_t quasi-periodic Von Neumann

$$\bar{A} = \lim_{T\to\infty} \frac{1}{T} \int_0^T A(t)\,dt = \mathrm{Tr}\left[\hat{A}\,\bar{\phi}\right]$$

$$\lim_{t\to\infty} \| \mathcal{U}_t(\phi - \phi_{eq}) \| = \| \phi - \phi_{eq} \|$$

$$\phi_t = \sum_{n,m} \phi_{nm} e^{-i(E_n - E_m)t} |E_n\rangle\langle E_m|$$

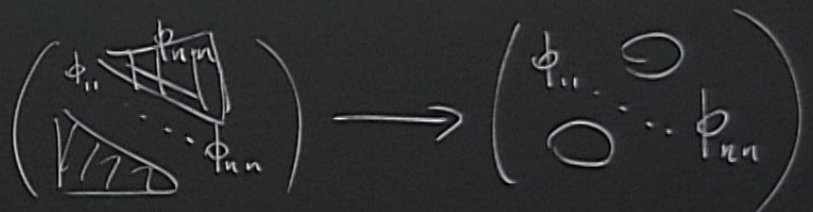
$$\bar{\phi} = \sum_{n,m} \bar{\phi}_{nm} e^{\overline{-i(E_n - E_m)t}} |E_n\rangle\langle E_m|$$

$$\phi_t = \sum_{nm} \phi_{nm} e^{-i(E_n - E_m)t} |E_n\rangle\langle E_m|$$

$$\bar{\phi} = \overline{\sum_{nm} \phi_{nm} e^{-i(E_n - E_m)t} |E_n\rangle\langle E_m|}$$

$$= \sum_n \phi_{nn} \overbrace{|E_n\rangle\langle E_n|}^{\Pi_n}$$

$$= \sum_n \Pi_n \phi \Pi_n$$



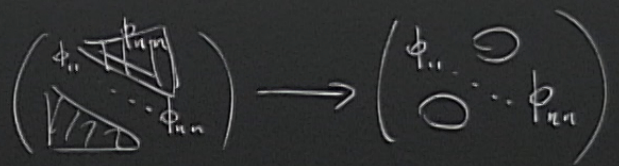
Completely dephased state

$$\phi_t = \sum_{nm} \phi_{nm} e^{-i(E_n - E_m)t} |E_n\rangle\langle E_m|$$

$$\bar{\phi} = \overline{\sum_{nm} \phi_{nm} e^{-i(E_n - E_m)t} |E_n\rangle\langle E_m|}$$

$$= \sum_n \phi_{nn} \overline{|E_n\rangle\langle E_n|}$$

$$= \sum_n \Pi_n \phi \Pi_n$$



Completely dephased state in the basis of H

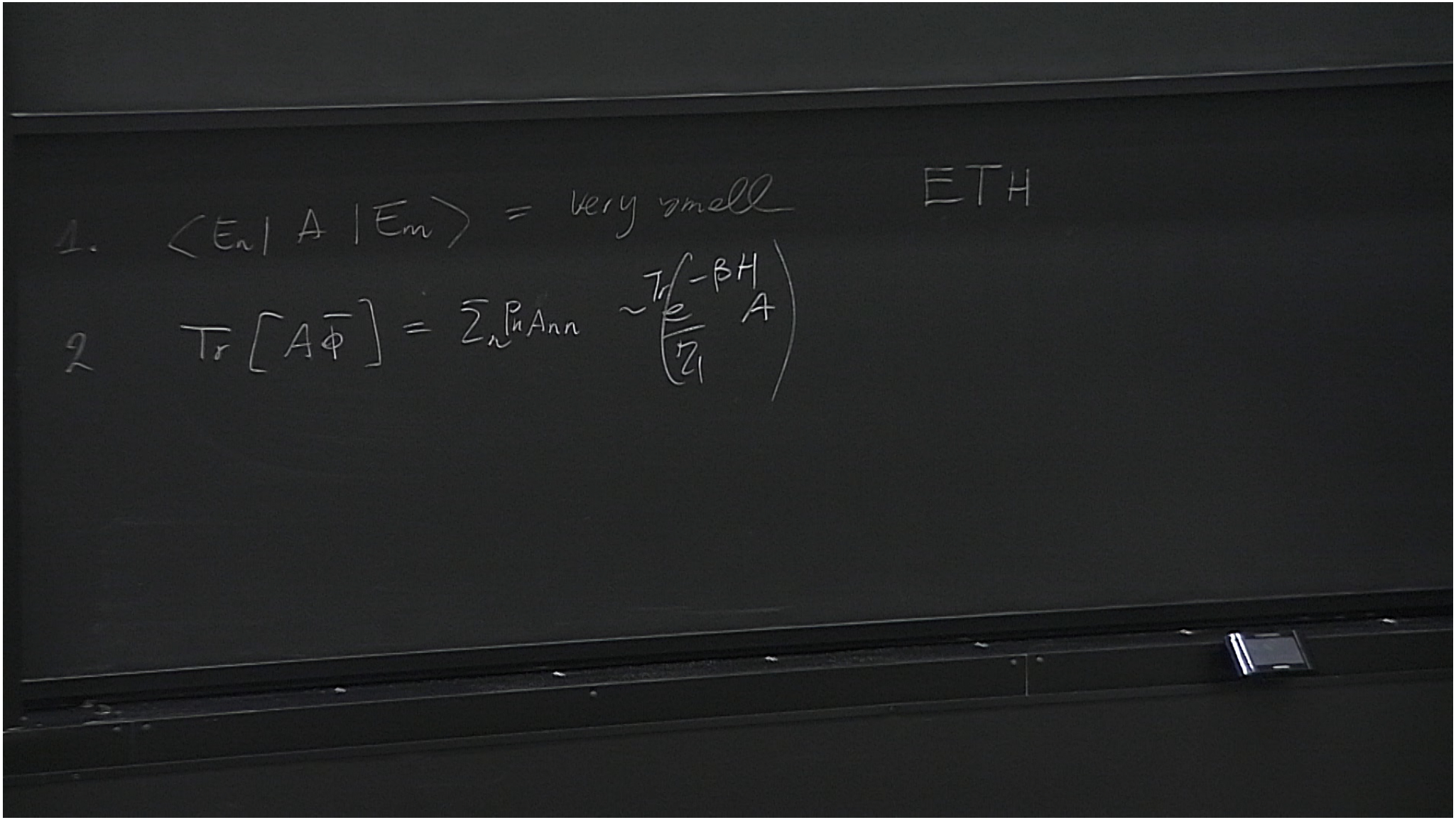
$$\bar{A} = tr(\hat{A}\bar{\phi}) = \sum_n p_n A_{nn}$$

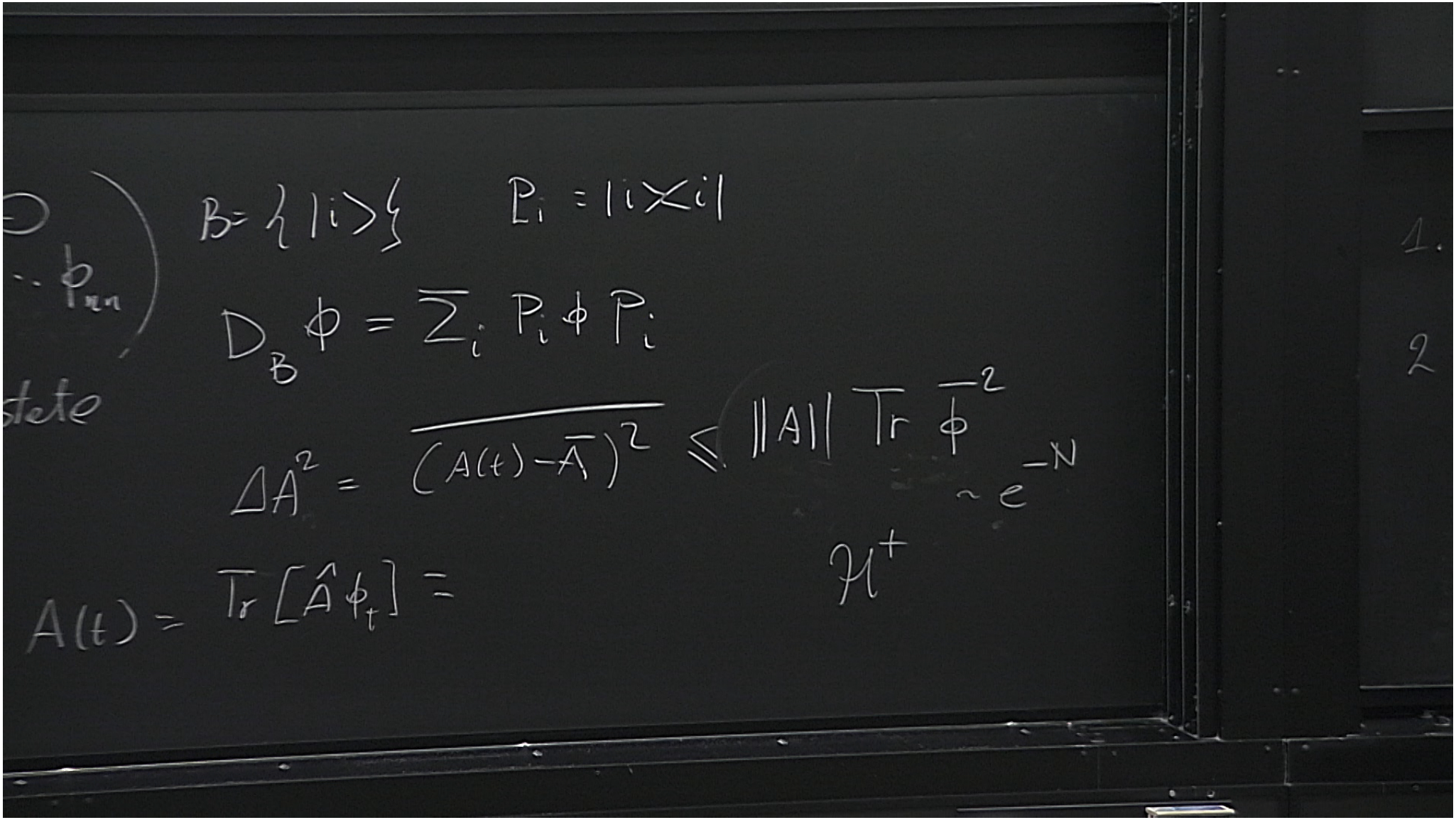
$\mathcal{B} = \{|i\rangle\}$ $\quad P_i = |i\rangle\langle i|$

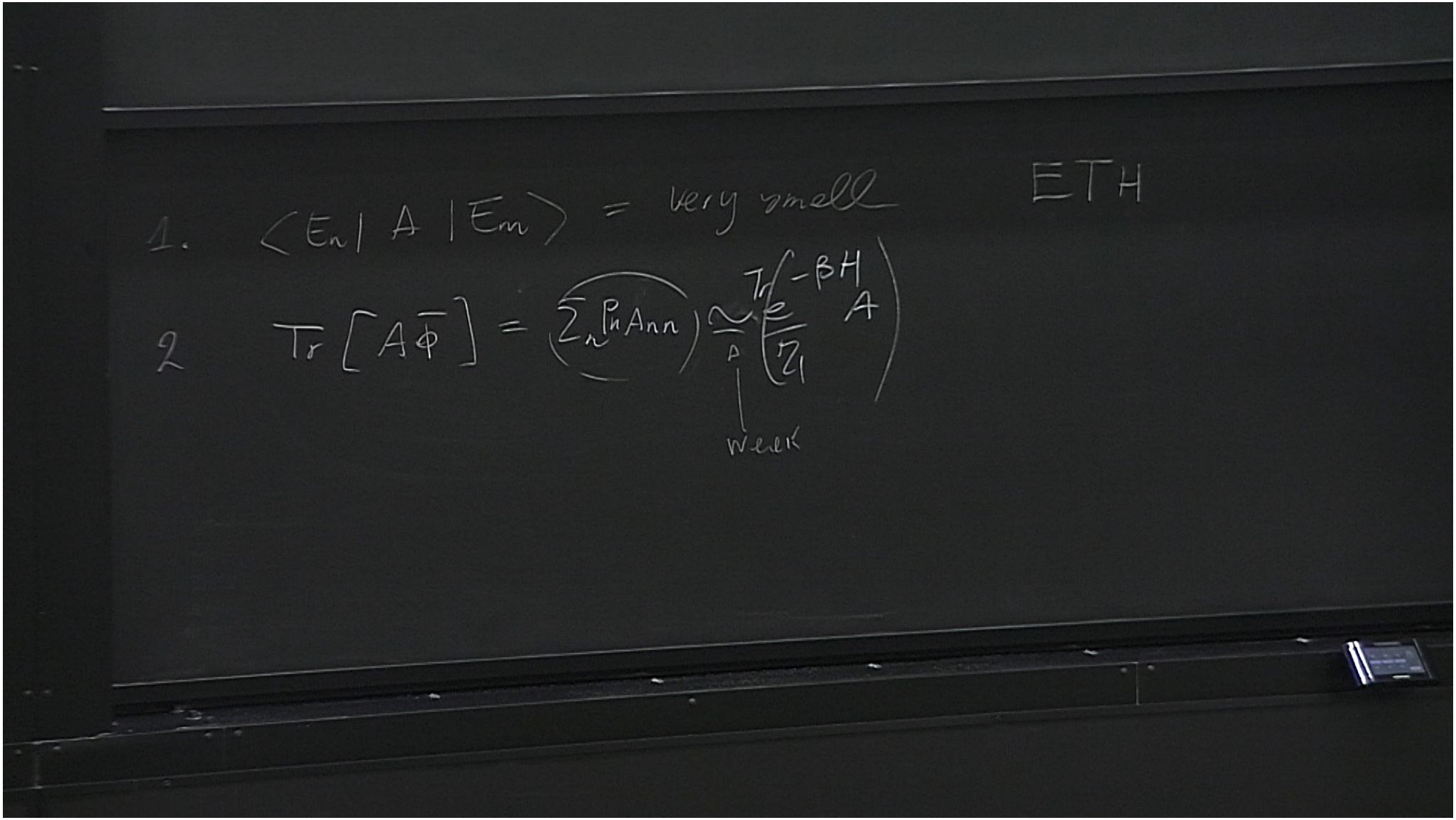
$$D_{\mathcal{B}}\phi = \sum_i P_i \phi P_i$$

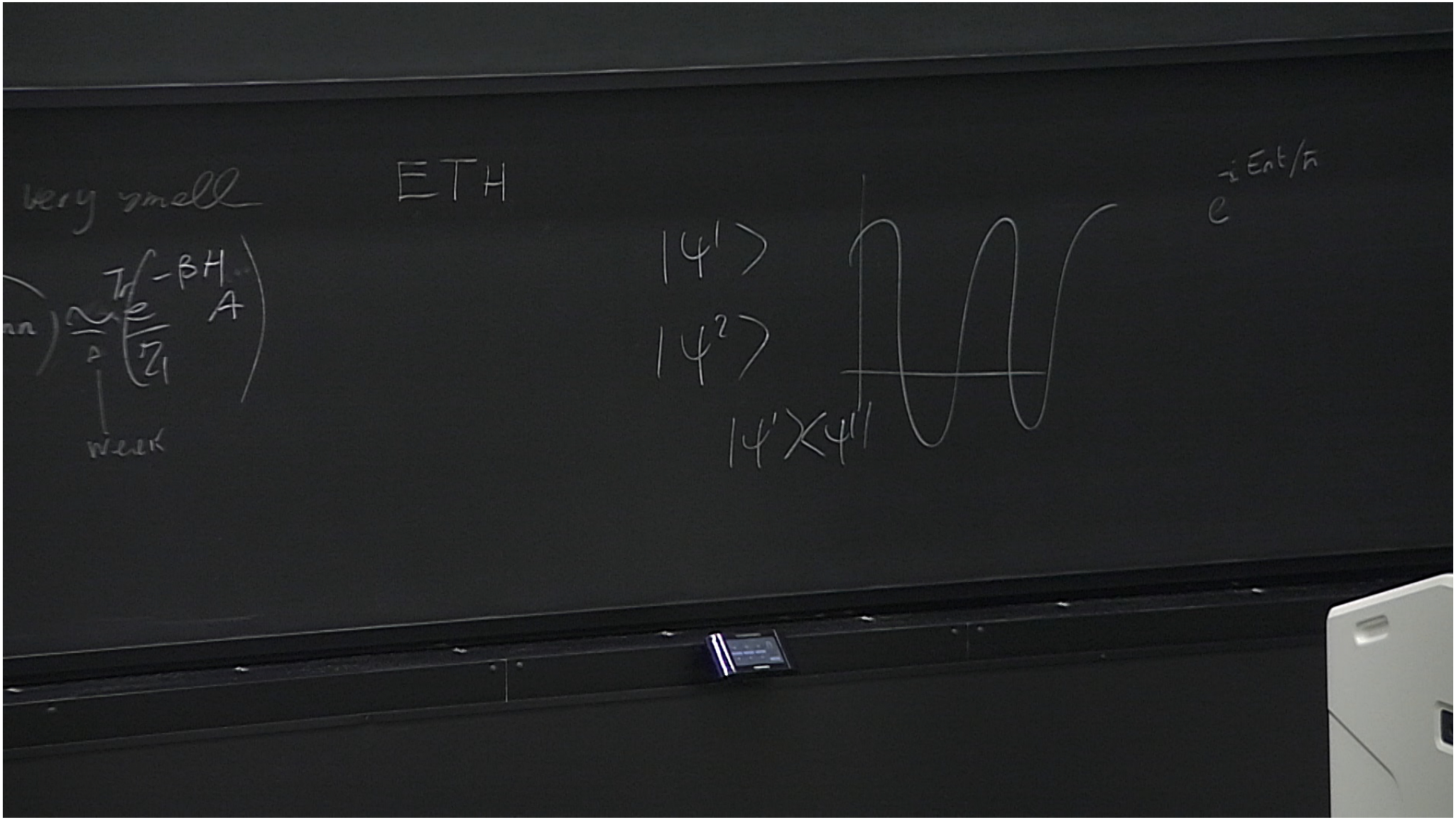
$$\Delta A^2 = \overline{(A(t) - \bar{A})^2}$$

$$A(t) = Tr[\hat{A}\phi_t]$$


1. ⟨En| A |Em⟩ = very small
ETH
2 Tr[AΦ] = Σn Pn Ann ~ Tr(e^(-βH)/Z A)


$B = \{|i\rangle\}$ $P_i = |i\rangle\langle i|$
$D_B \phi = \sum_i P_i \phi P_i$
$\Delta A^2 = \overline{(A(t) - \bar{A})^2} \leq \|A\| \operatorname{Tr} \bar{\phi}^{-2}$
$\sim e^{-N}$
$A(t) = \operatorname{Tr}[\hat{A} \phi_t] =$
$\mathcal{H}^+$
1.
2


ETH
1. ⟨En| A |Em⟩ = very small
2 Tr[AΦ] = (Σn PnAnn) ≃ Tr(e^-βH A / Z)
A
Weak


very small
ETH
$|4'\rangle$
$|4^2\rangle$
$|4'\rangle\times|4''\rangle$
$e^{-iEnt/\hbar}$
$\mathrm{Tr}(e^{-\beta H} A)$
weak