Title: PSI 2018/2019 - Condensed Matter Review - Lecture 10

Date: Jan 18, 2019 11:30 AM

URL: http://pirsa.org/19010041

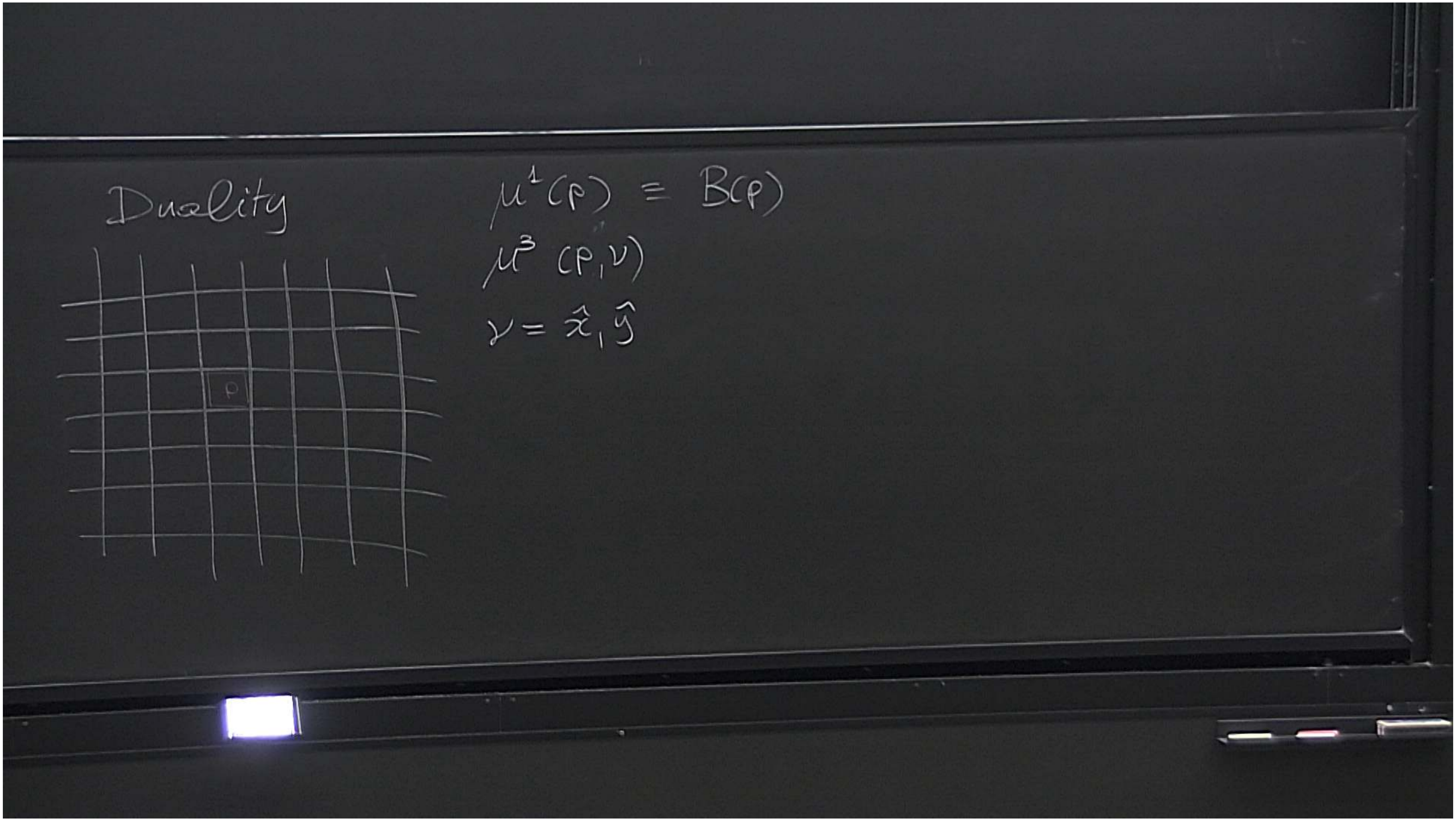
Abstract:

$$H = -J \sum_p B_p - g \sum_\ell \sigma^x_\ell$$

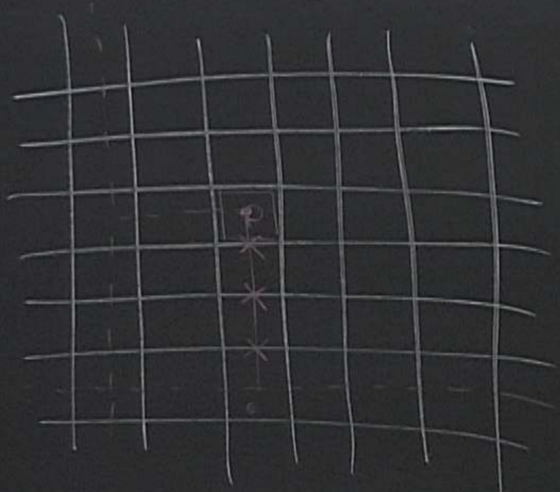
$\mathcal{H}_{gauge}$ large g PARAMAGNET

large J "TOPOLOGICAL" PHASE

Gap ; Degeneracy $\begin{cases} \text{Topological } 4^g \\ \underline{\text{Robust}} \end{cases}$


Duality
$\mu^1(p) = B(p)$
$\mu^3(p, \nu)$
$\nu = \hat{x}, \hat{y}$
p

Duality

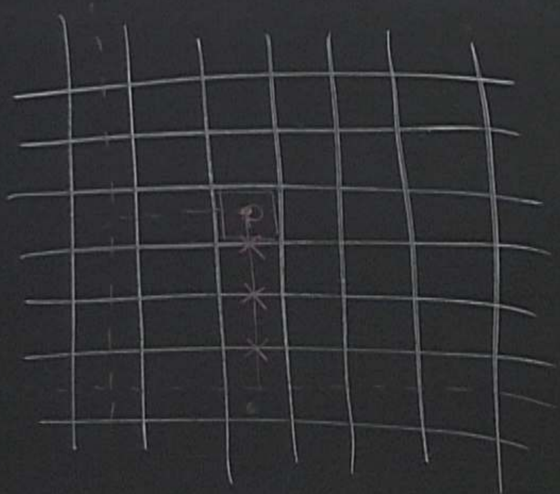


$$\mu^1(p) \equiv B(p)$$

$$\mu^3(p,\nu) = \prod_{\ell<p} \sigma^x_{\ell,\nu}$$

$$\nu = \hat{x}, \hat{y}$$

Duality



$$\mu^1(p) \equiv B(p)$$

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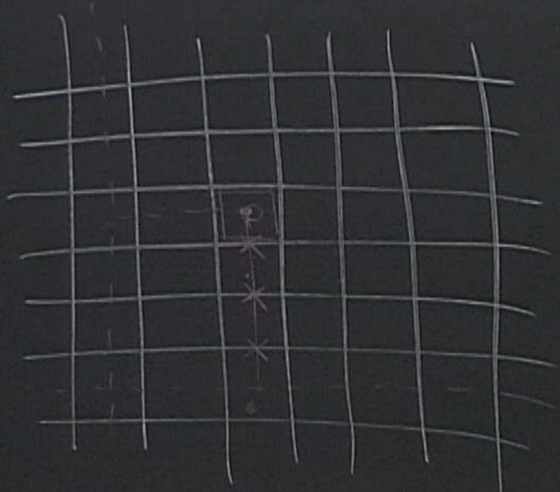
$$\nu = \hat{x}, \hat{y}$$

$$\sigma^x_{l,\nu} - \mu^3(p-\nu,\nu)\,\mu^3(p,\nu)$$

Duality



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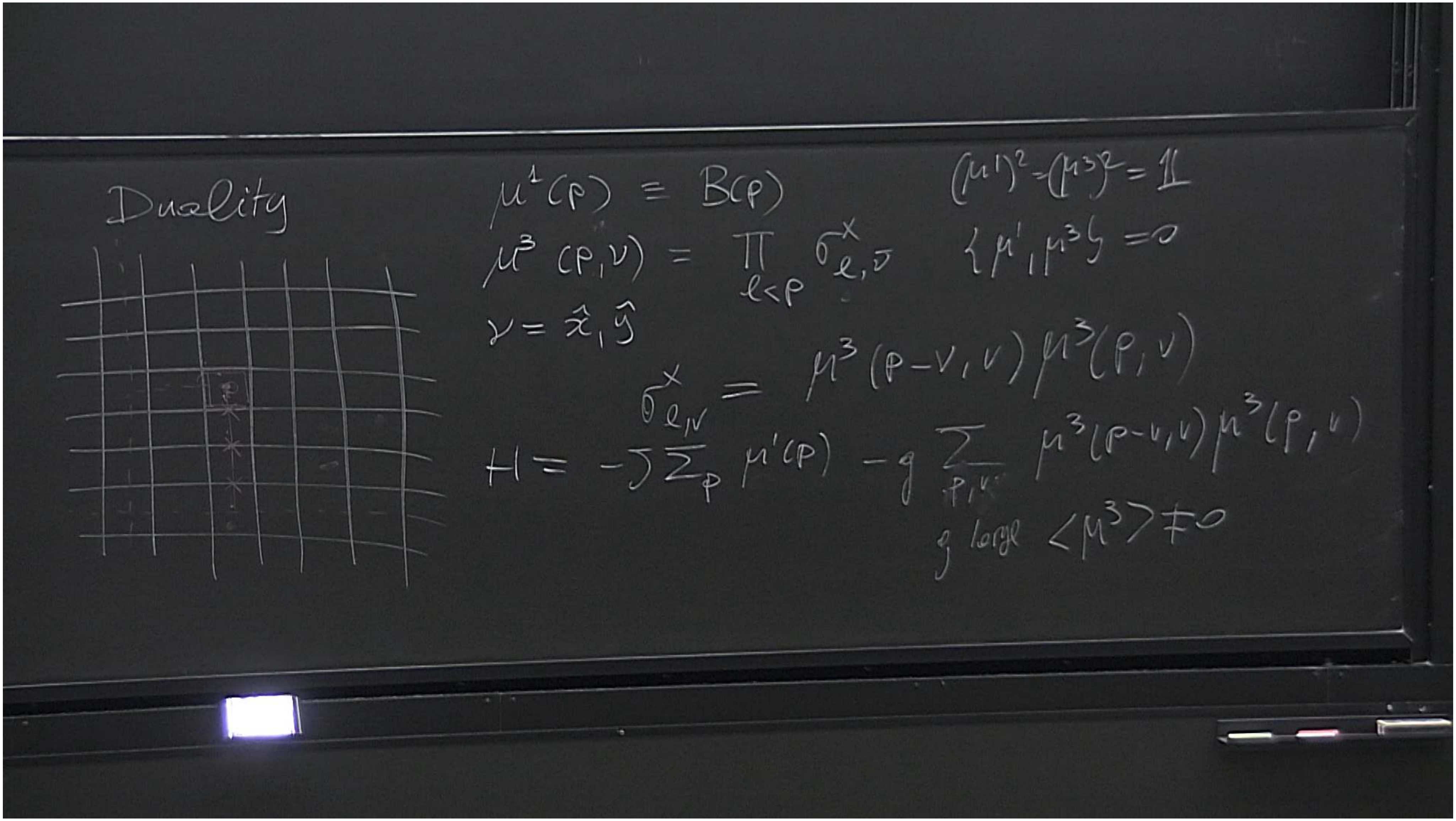
$$H = -J\sum_p \mu^1(p) - g\sum_{p,\nu} \mu^3(p-\nu,\nu)\,\mu^3(p,\nu)$$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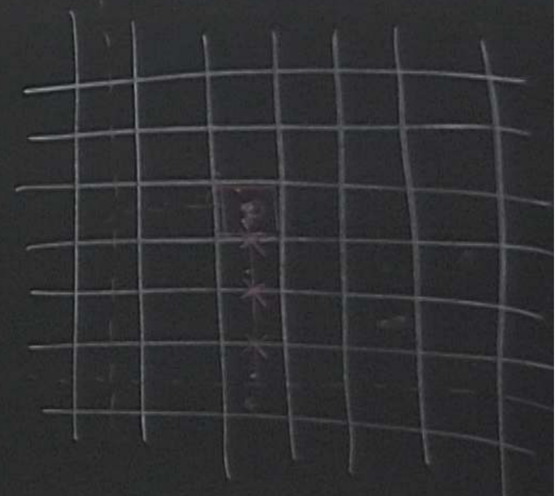
$(\mu^1)^2 = (\mu^3)^2 = \mathbb{1}$
$\{\mu^1, \mu^3\} = 0$
$\sigma^x_{l,\nu} = \mu^3(p-\nu,\nu)\,\mu^3(p,\nu)$
$H = -J\sum_p \mu^1(p) - g\sum_{p,\nu} \mu^3(p-\nu,\nu)\,\mu^3(p,\nu)$
g large $\langle \mu^3 \rangle \neq 0$

$$H = -J\sum_p B_p - g\sum_\ell \sigma^x_\ell$$

Duality



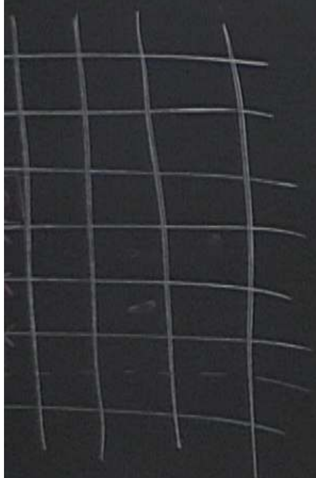
$\mathcal{H}_{gauge}$

large g PARAMAGNET $\langle \prod_{\ell \in P} \hat{\sigma}_{\ell,v} \rangle \neq 0$

large J "TOPOLOGICAL" PHASE

Gap ; Degeneracy $\begin{cases} \text{Topological } 4^g \\ \text{Robust} \end{cases}$

ity



$$\mu^1(p) = B(p)$$

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$$\nu = \hat{x}, \hat{y}$$

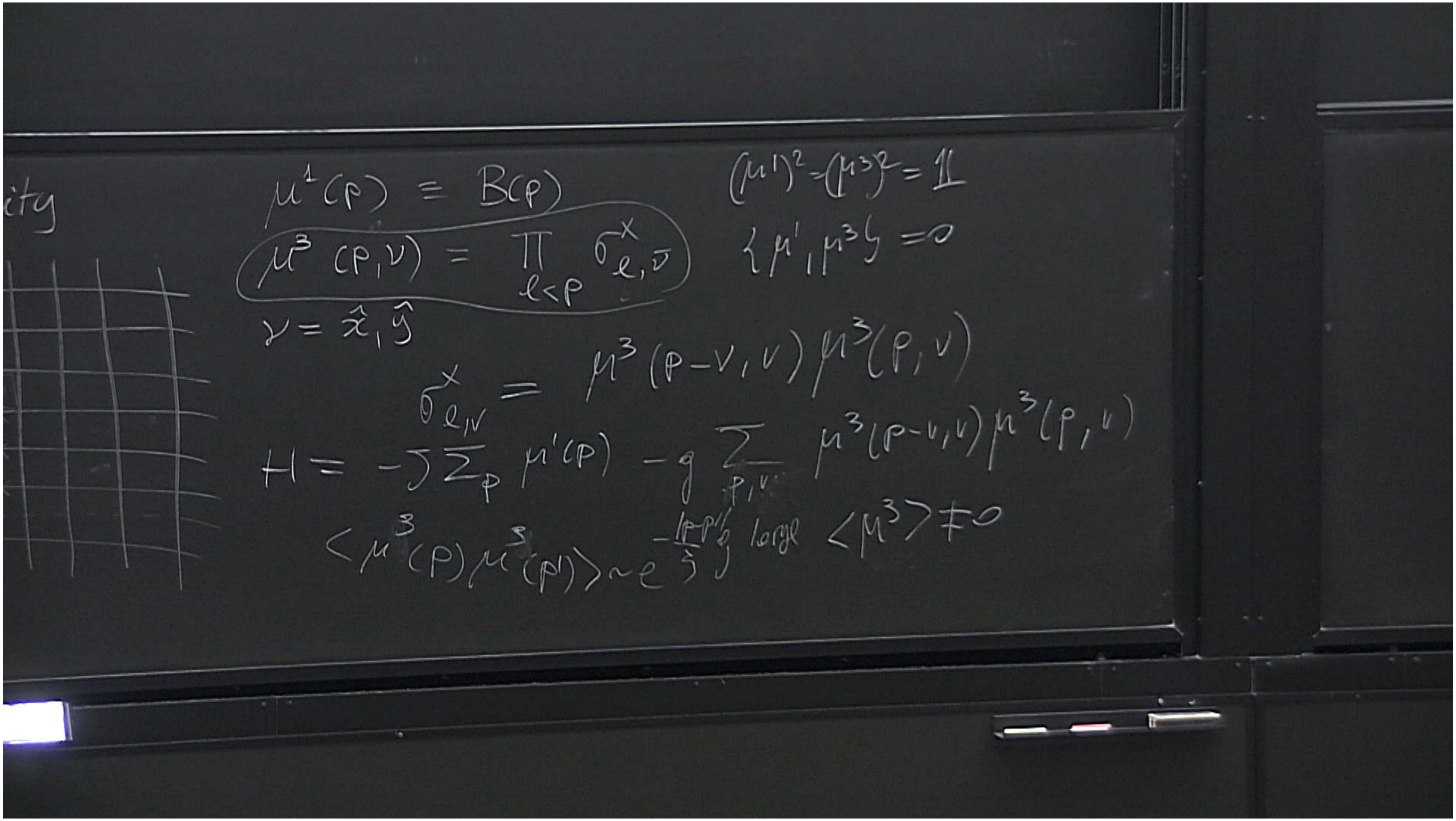
$$(\mu^1)^2 = (\mu^3)^2 = \mathbb{1}$$

$$\{\mu^1, \mu^3\} = 0$$

$$\sigma^x_{\ell,\nu} = \mu^3(p-\nu,\nu)\,\mu^3(p,\nu)$$

$$H = -J\sum_p \mu^1(p) - g\sum_{p,\nu} \mu^3(p-\nu,\nu)\,\mu^3(p,\nu)$$

$$\langle \mu^1(p)\,\mu^1(p')\rangle \sim e^{-|p-p'|/\xi} \quad \text{large} \quad \langle \mu^3\rangle \neq 0$$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$H = -J \sum_p \mu^1(p) - g \sum_{p,\nu} \mu^3(p-\nu,\nu)\,\mu^3(p,\nu)$
$\langle \mu^3(p)\,\mu^3(p') \rangle \sim e^{-|p-p'|/\xi}$
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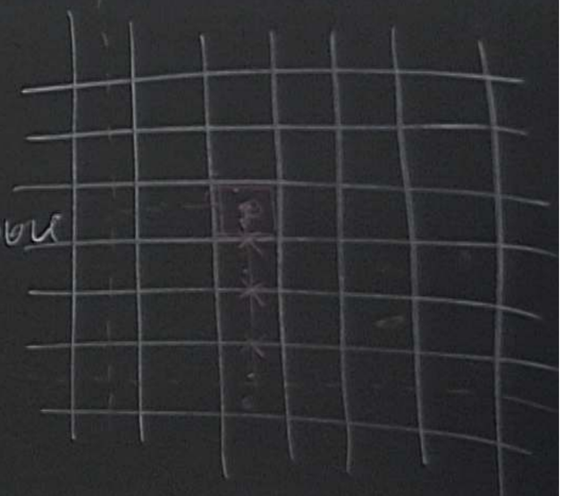
$$H = -J\sum_p B_p - g\sum_\ell \sigma^x_\ell$$

Duality



$\mathcal{H}_{gauge}$

large g PARAMAGNET $\langle \prod_{\ell \in P} \hat\sigma_{\ell,v} \rangle \neq 0$

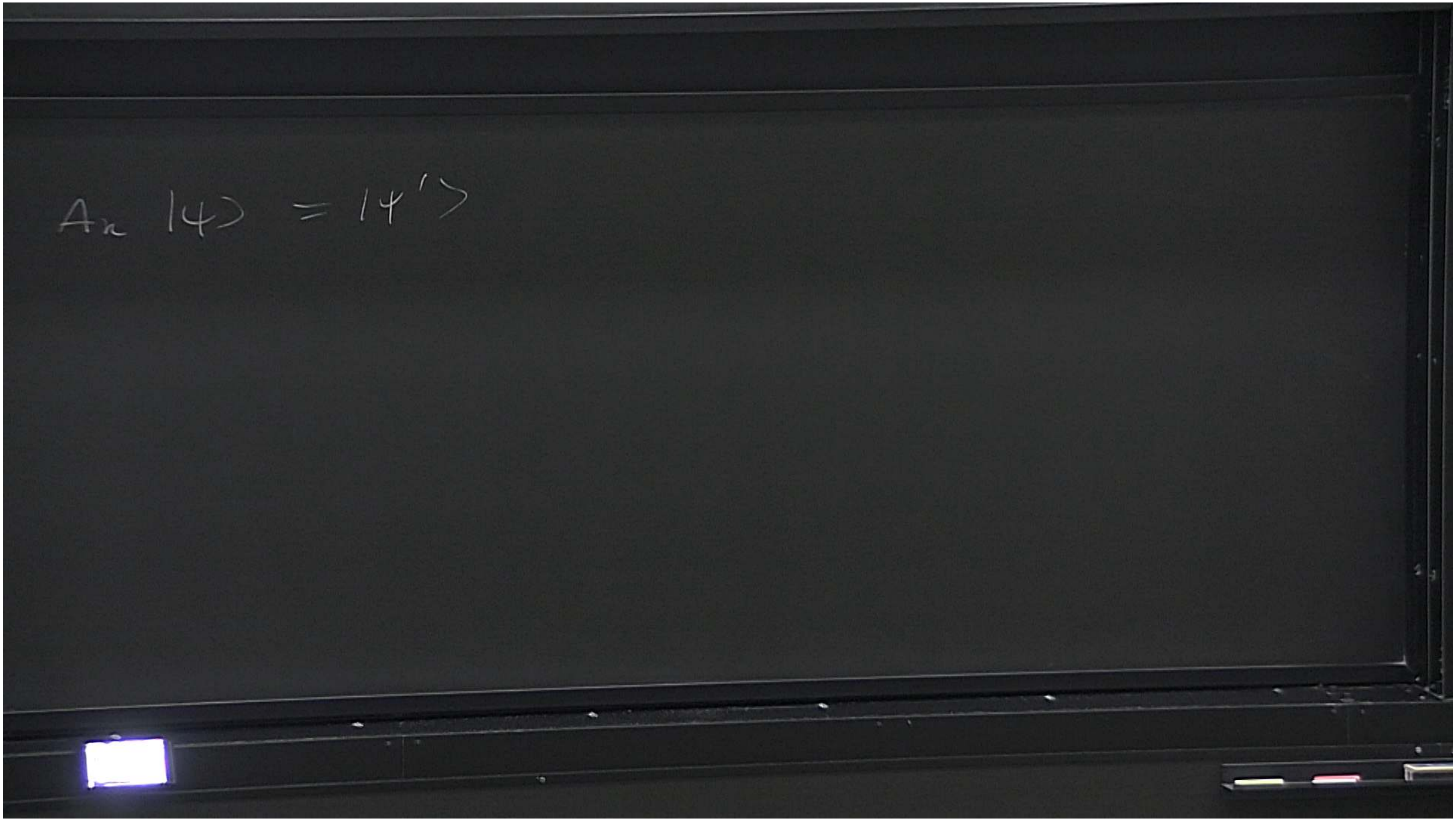
Z_2 vortex condensation

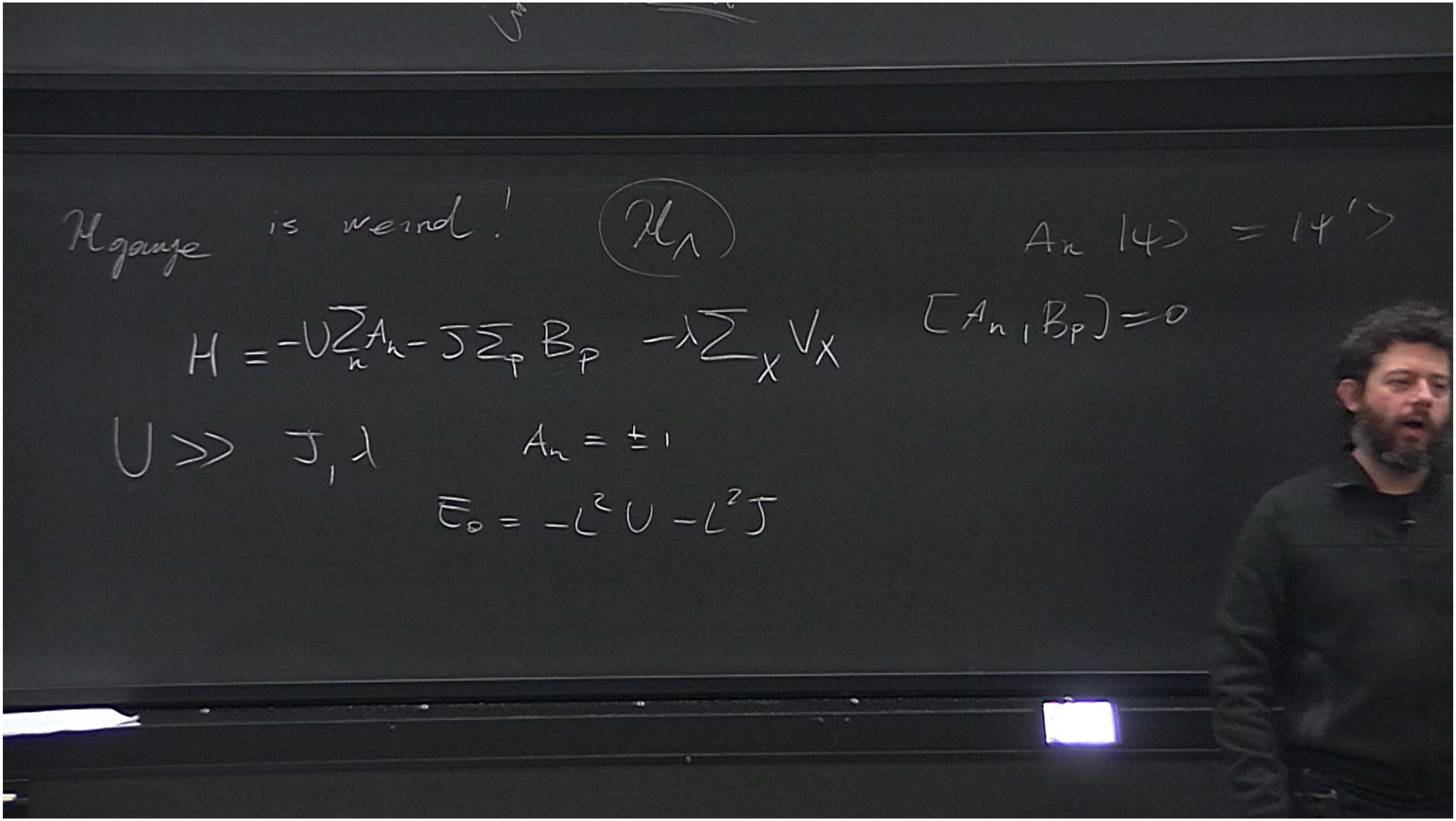
large J "TOPOLOGICAL" PHASE

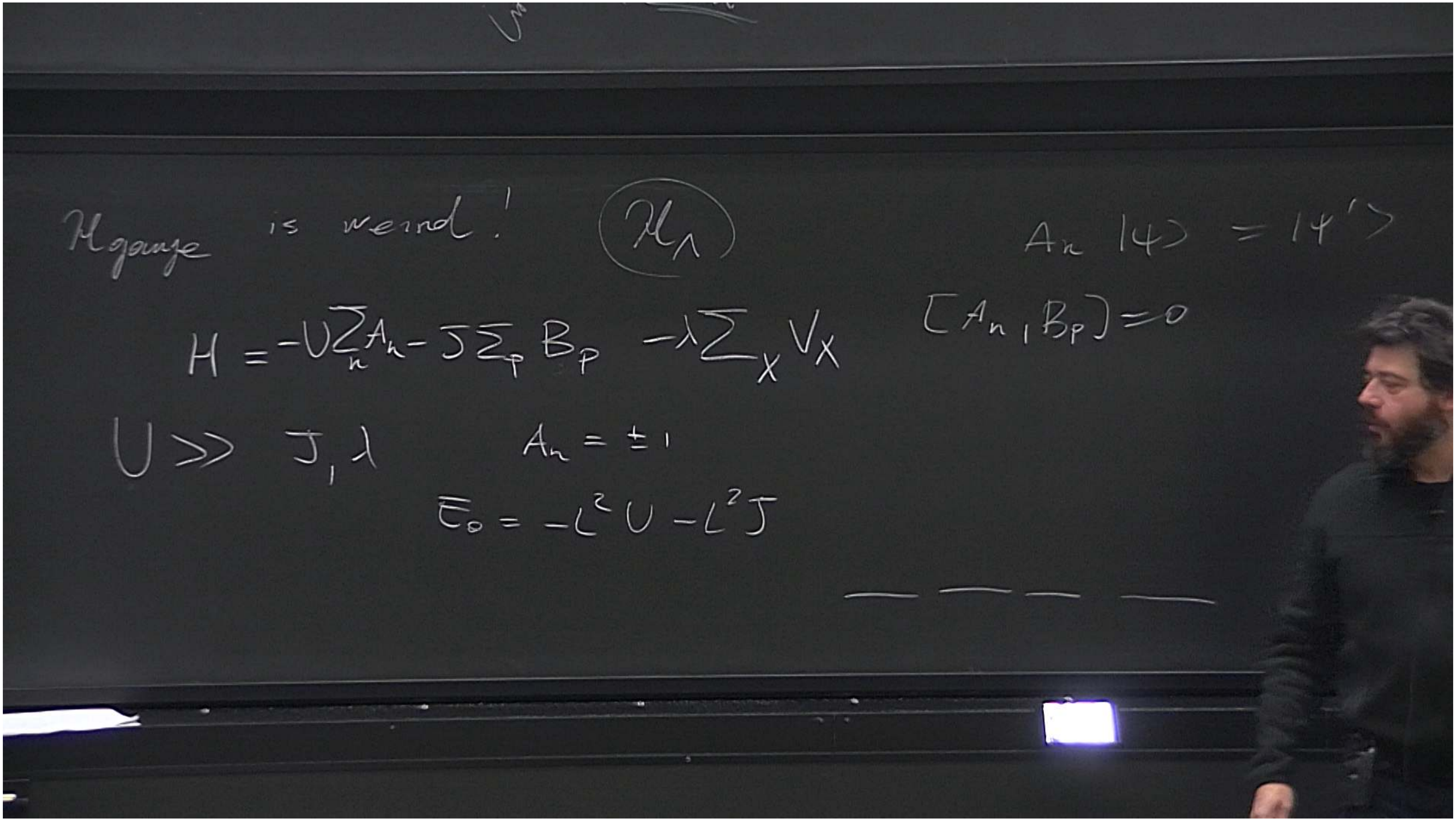
Gap; Degeneracy $\left\{ \begin{array}{l} \text{Topological } \xi \\ \text{Robust} \end{array} \right.$

$\langle \sigma^x_i \sigma^x_j \rangle \sim e^{-|i-j|/\xi}$

$\mathcal{H}_{gauge}$ is weird!


$A_x |\psi\rangle = |\psi'\rangle$


$\mathcal{H}_{gauge}$ is weird!
$\mathcal{H}_\Lambda$
$A_n |\psi\rangle = |\psi'\rangle$
$H = -U\sum_n A_n - J\sum_p B_p - \lambda\sum_X V_X$
$[A_n, B_p] = 0$
$U \gg J, \lambda$
$A_n = \pm 1$
$E_0 = -L^2 U - L^2 J$



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$\mathcal{H}_{gauge}$ is weird! $\mathcal{H}_\Lambda$

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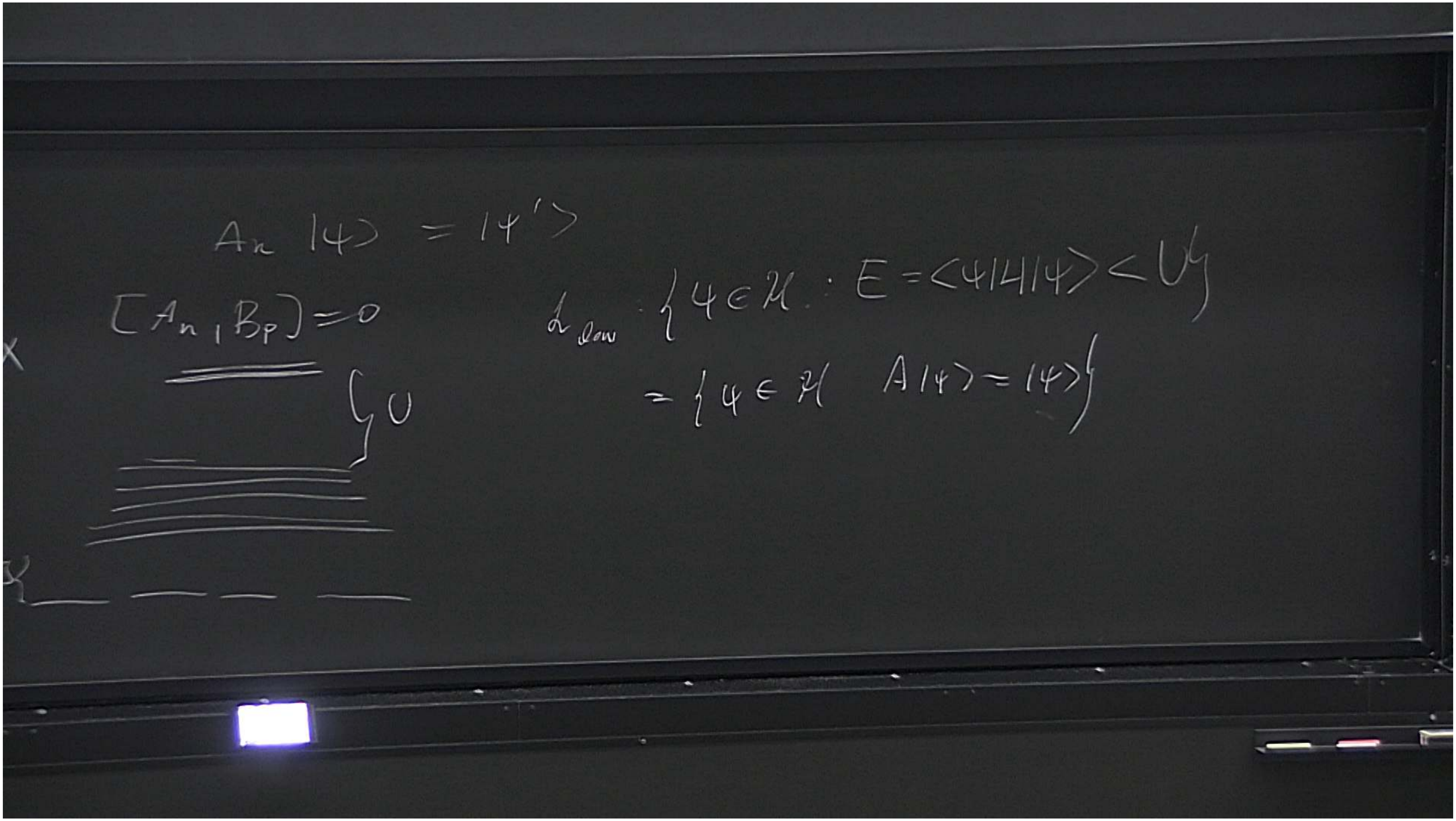
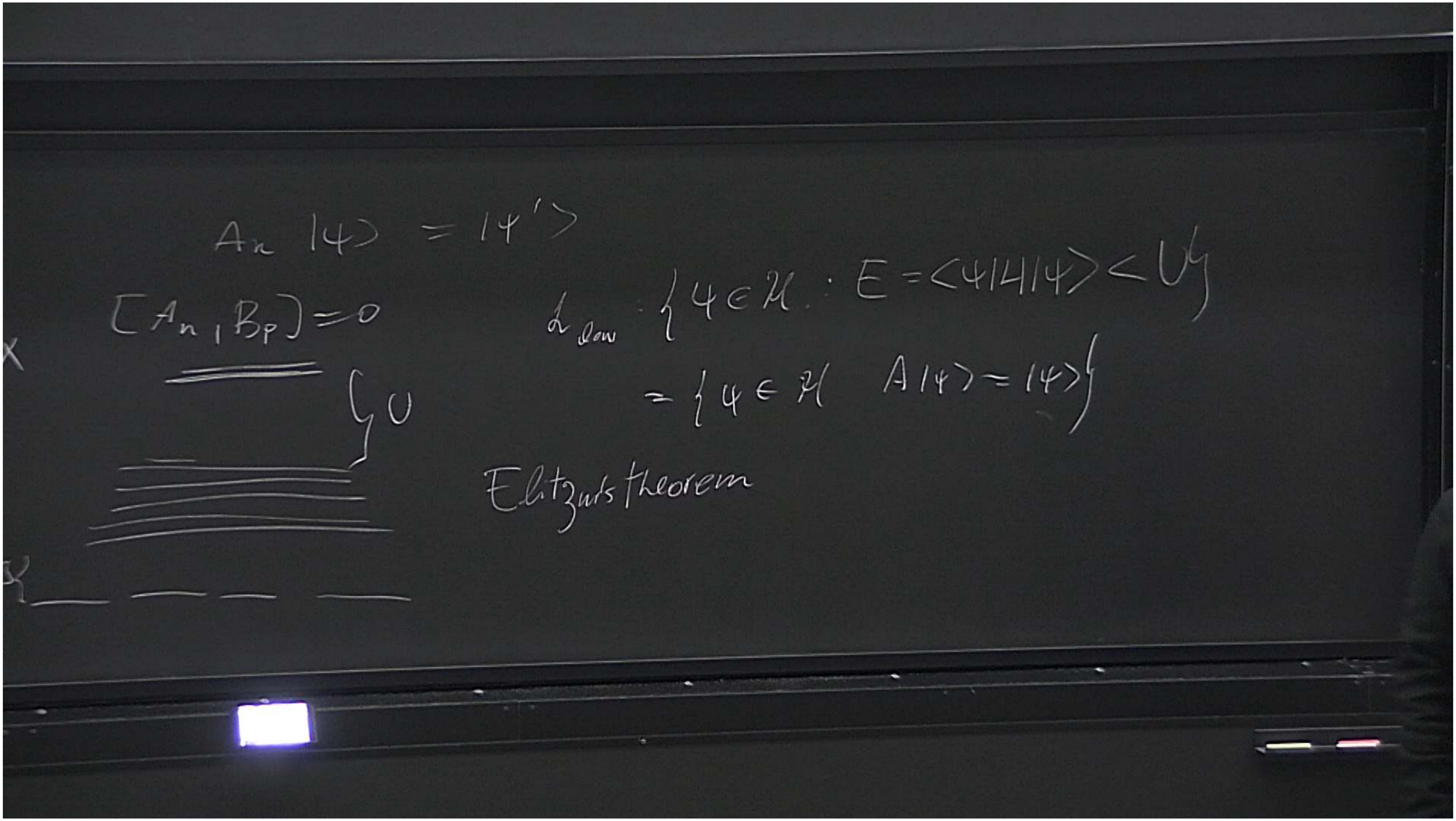
$$H = -U\sum_n A_n - J\sum_p B_p - \lambda \sum_X V_X$$

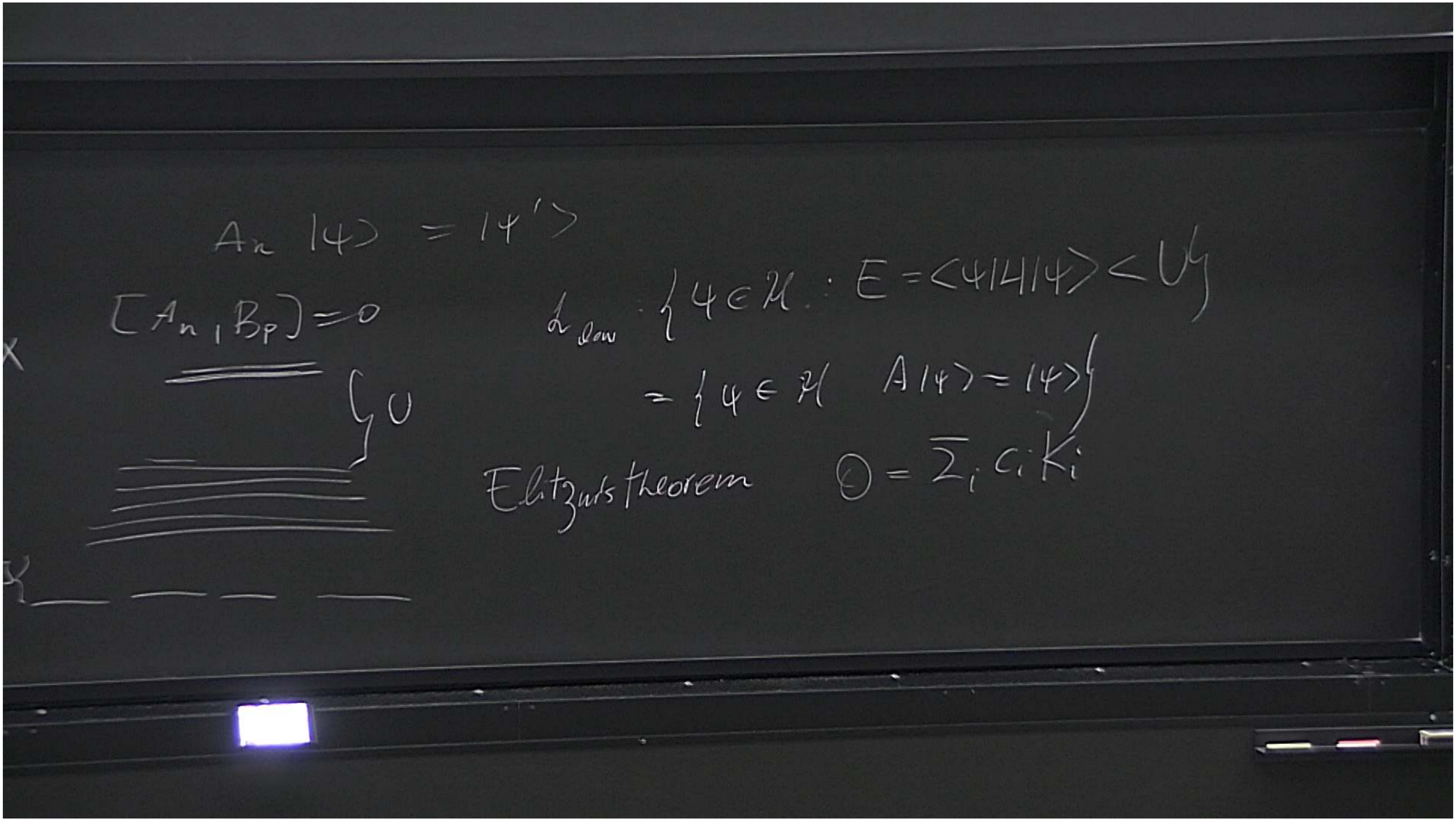
$[A_n, B_p] = 0$

$U \gg J, \lambda$ $\qquad A_n = \pm 1$

$$E_0 = -L^2 U - L^2 J$$


$A_n |\psi\rangle = |\psi'\rangle$
$[A_n, B_p] = 0$
U
$\mathcal{L}_{low} : \{\psi \in \mathcal{H} : E = \langle \psi | H | \psi \rangle < U\}$
$= \{\psi \in \mathcal{H} \;\; A|\psi\rangle = |\psi\rangle\}$


A_x |ψ> = |ψ'>
[A_n , B_p] = 0
U
L_low : {ψ ∈ H : E = <ψ|H|ψ> < U}
= {ψ ∈ H A|ψ> = |ψ>}
Elitzur's theorem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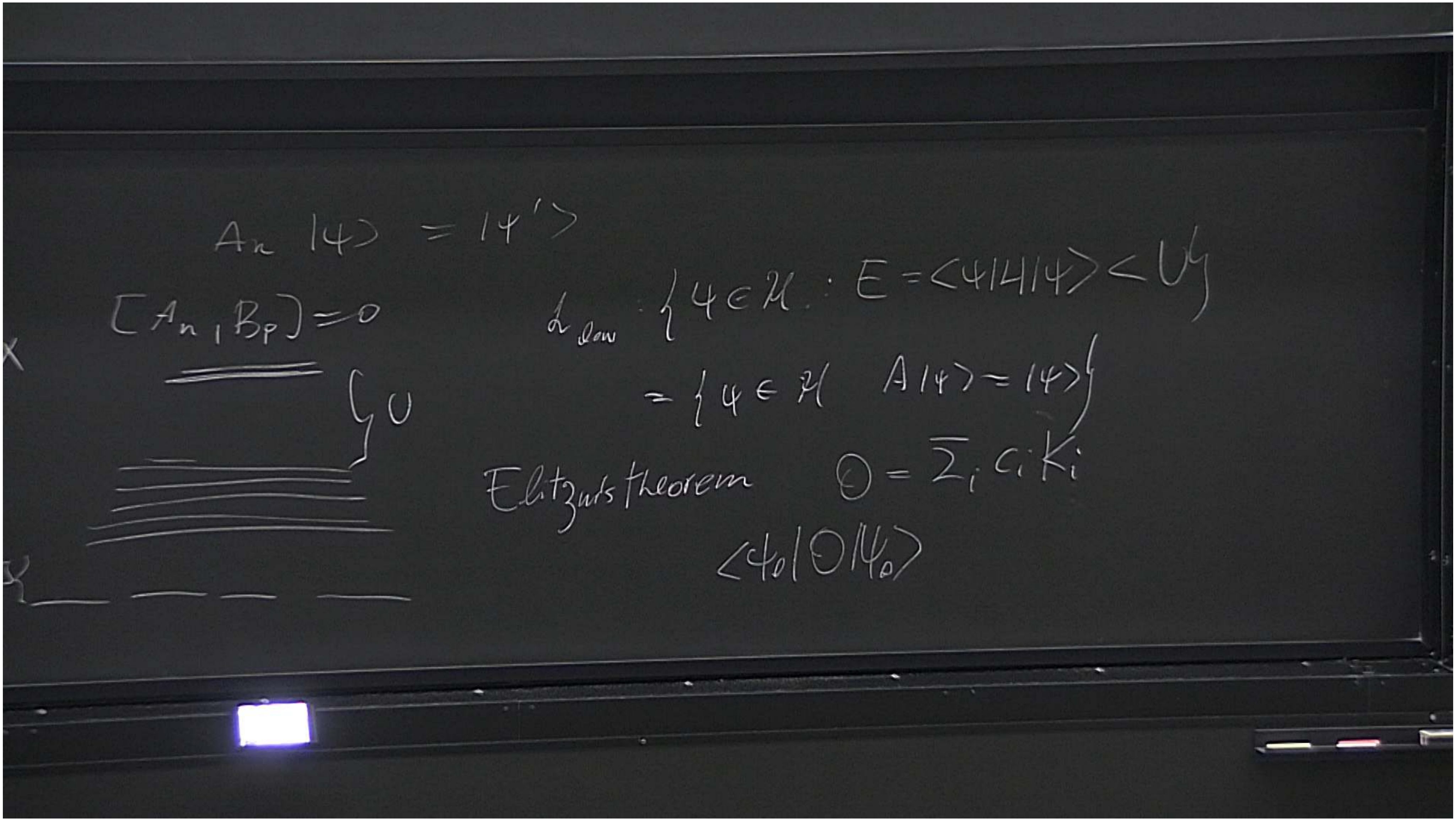
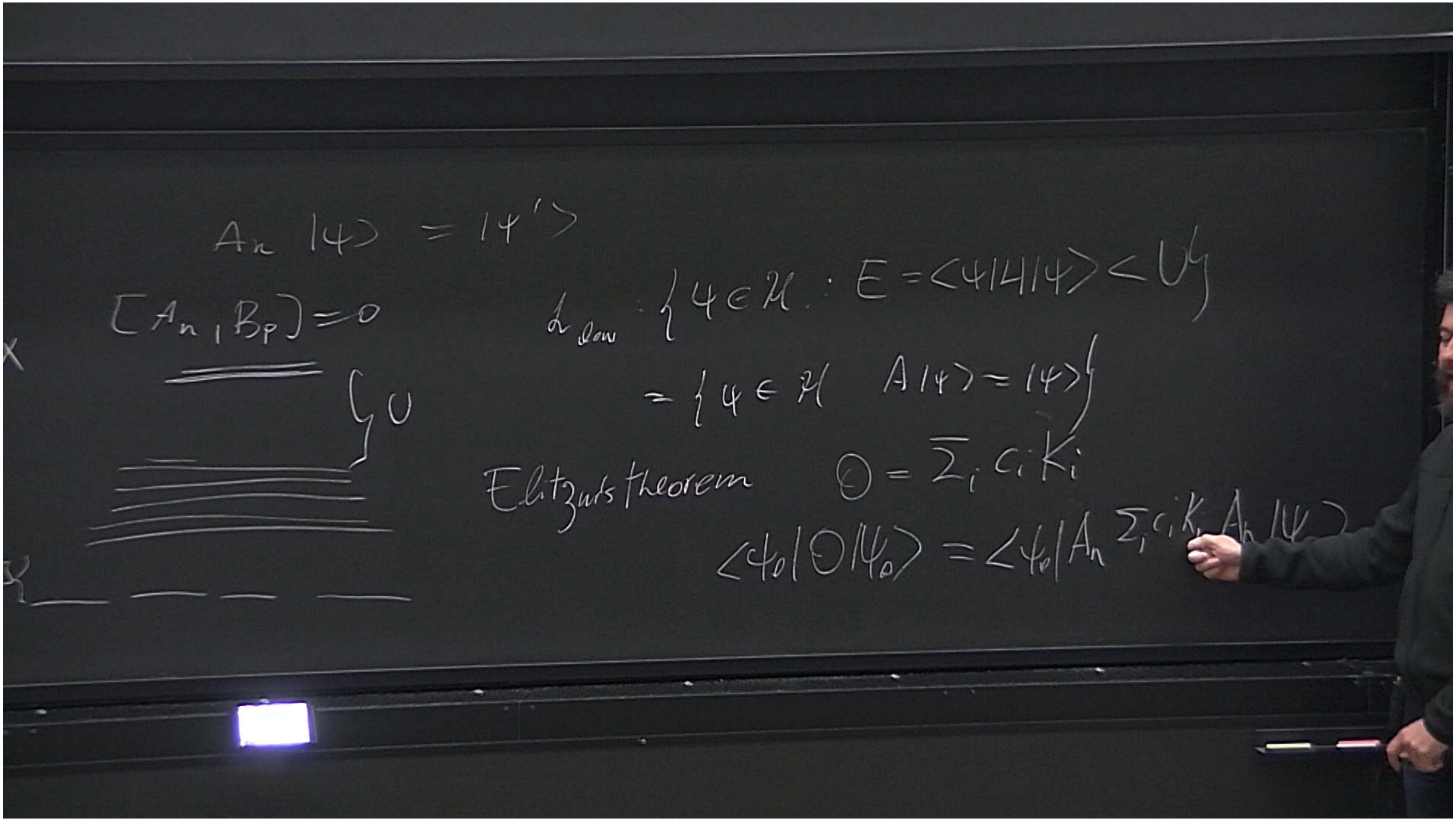
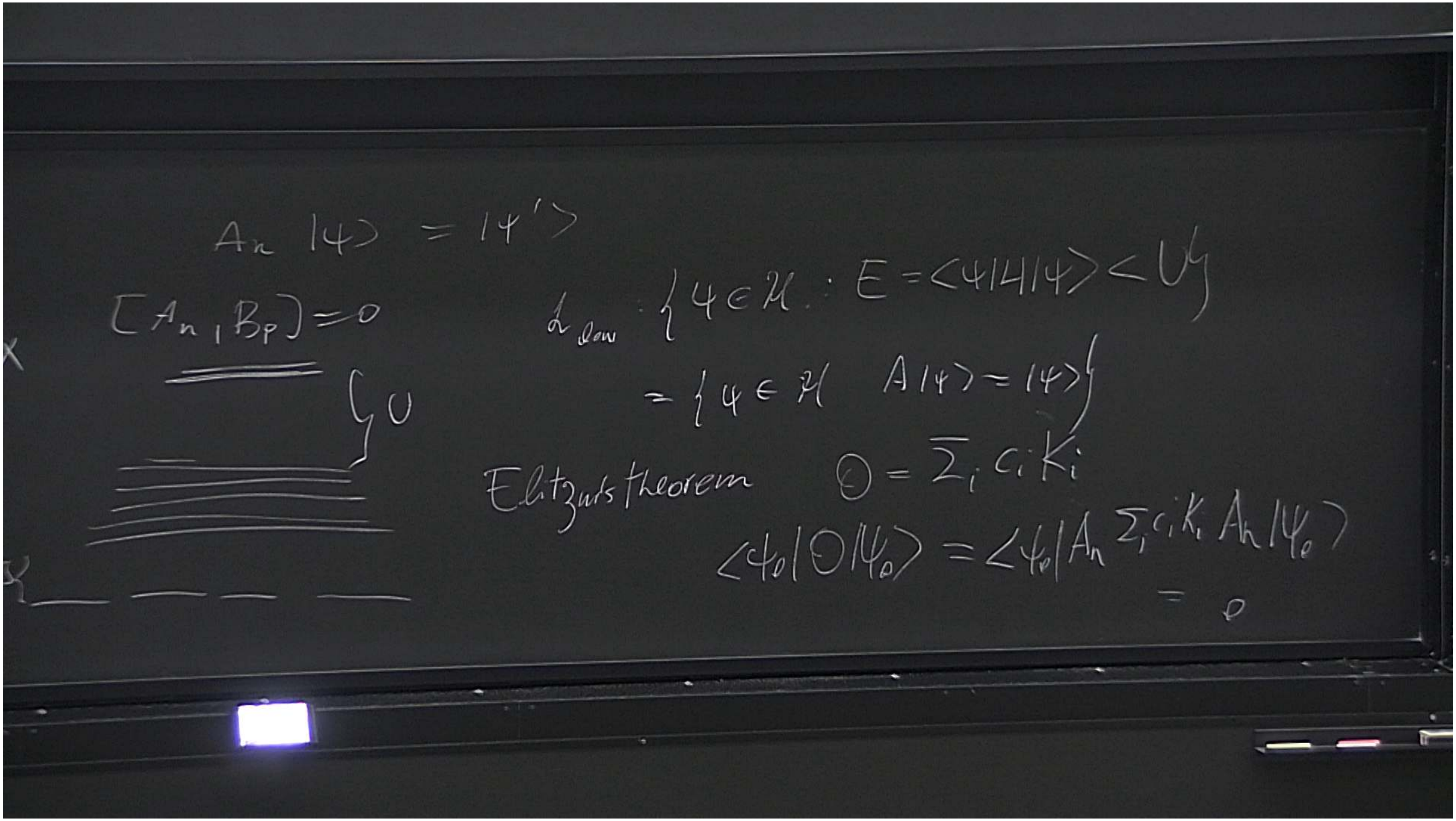
Elitzur's theorem
$O = \Sigma_i c_i K_i$


$A_n |\psi\rangle = |\psi'\rangle$
$[A_n, B_p] = 0$
U
$\mathcal{L}_{low} \{\psi \in \mathcal{H} : E = \langle\psi|H|\psi\rangle < U\}$
$= \{\psi \in \mathcal{H} \quad A|\psi\rangle = |\psi\rangle\}$
Elitzur's theorem $\quad \mathcal{O} = \sum_i c_i K_i$
$\langle\psi_0|\mathcal{O}|\psi_0\rangle$



A_n |ψ> = |ψ'>
[A_n, B_p] = 0
U
Elitzur's theorem
ℒ_low {ψ ∈ ℋ : E = <ψ|H|ψ> < U}
= {ψ ∈ ℋ A|ψ> = |ψ>}
Θ = Σ_i c_i K_i
<ψ_0|Θ|ψ_0> = <ψ_0|A_n Σ_i c_i K_i A_n|ψ>



$A_n |\psi\rangle = |\psi'\rangle$
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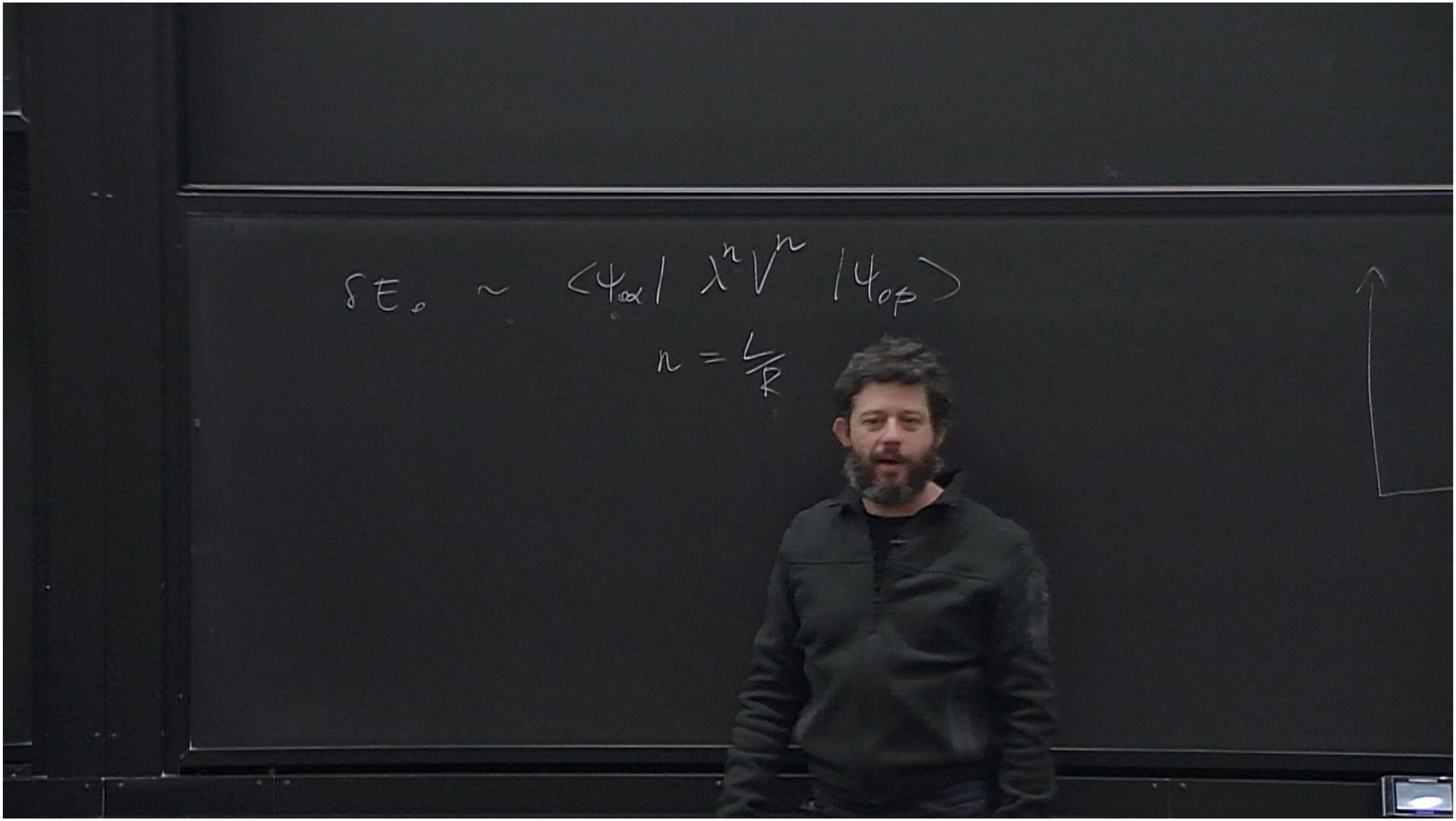
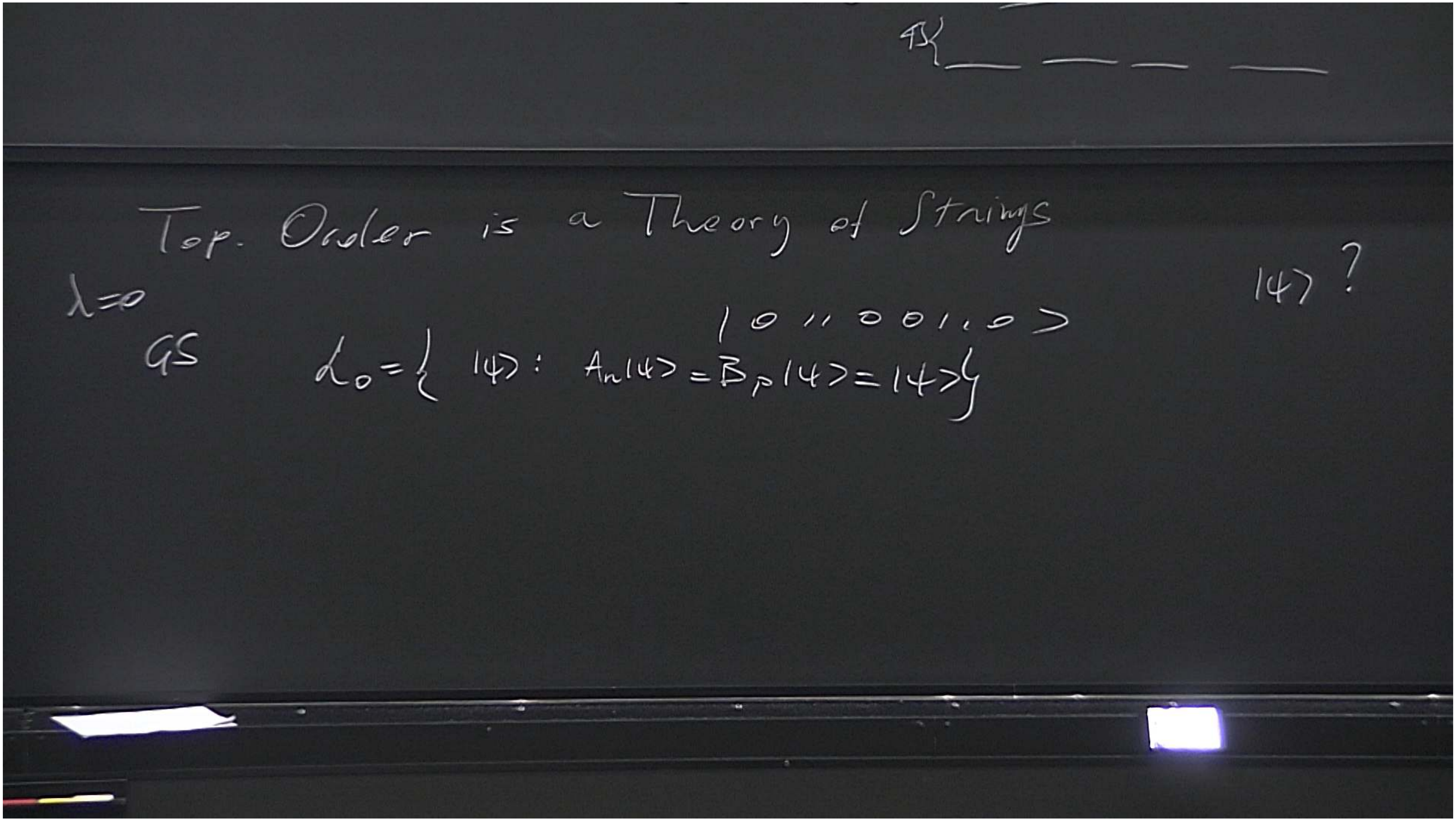
$\mathcal{L}_{low} : \{\psi \in \mathcal{H} : E = \langle\psi|H|\psi\rangle < U\}$
$= \{\psi \in \mathcal{H} \quad A|\psi\rangle = |\psi\rangle\}$
Elitzurs theorem
$O = \sum_i c_i K_i$
$\langle\psi_0|O|\psi_0\rangle = \langle\psi_0|A_n \sum_i c_i K_i A_n|\psi_0\rangle$
$= 0$


$\delta E_0 \sim \langle \Psi_{0\alpha} | \lambda^n V^n | \Psi_{0\beta} \rangle$
$n = L/R$

$$\delta E_0 \sim \langle \psi_{0\alpha} | \lambda^n V^n | \psi_{0\beta} \rangle$$

$$n = L/R$$

$$\sim \left(\frac{\lambda}{J}\right)^{L/R} \sim e^{-L/\chi}$$


Top. Order is a Theory of Strings
λ=0
GS
|ψ⟩ ?
L₀ = { |ψ⟩ : Aₙ|ψ⟩ = Bₚ|ψ⟩ = |ψ⟩ }

$$\langle\psi_0|\mathcal{O}|\psi_0\rangle = \langle\psi_0|A_n \cdots A_n|\psi_0\rangle$$
$$= 0$$

$|\psi\rangle$?

$$|0\rangle = |0\rangle^{\otimes L^2}$$

$$\langle\psi_0|O|\psi_0\rangle = \langle\psi_0|A_n Z_i \cdots A_n|\psi_0\rangle$$
$$= 0$$

$|\psi\rangle$?

$$|0\rangle = |0\rangle^{\otimes L^2} \qquad B_p|0\rangle = |0\rangle \quad \forall p$$

$$\langle\Psi_0|\mathcal{O}|\Psi_0\rangle = \langle\Psi_0|A_n \sum_i c_i X_i A_n|\Psi_0\rangle$$

$$= 0$$

$|\psi\rangle$?

$$|0\rangle = |0\rangle^{\otimes L^2} \qquad B_p|0\rangle = |0\rangle \quad \forall p$$

$$|\psi\rangle \in \mathcal{L}_0$$

$$|\psi\rangle = P|0\rangle = N\prod_n \frac{\mathbb{1}+A_n}{2}|0\rangle$$

$$\langle \psi_0 | \mathcal{O} | \psi_0 \rangle = \langle \psi_0 | A_n \, Z_1 \ldots X_1 \, A_n | \psi_0 \rangle$$

$$= 0$$

$|\psi\rangle$?

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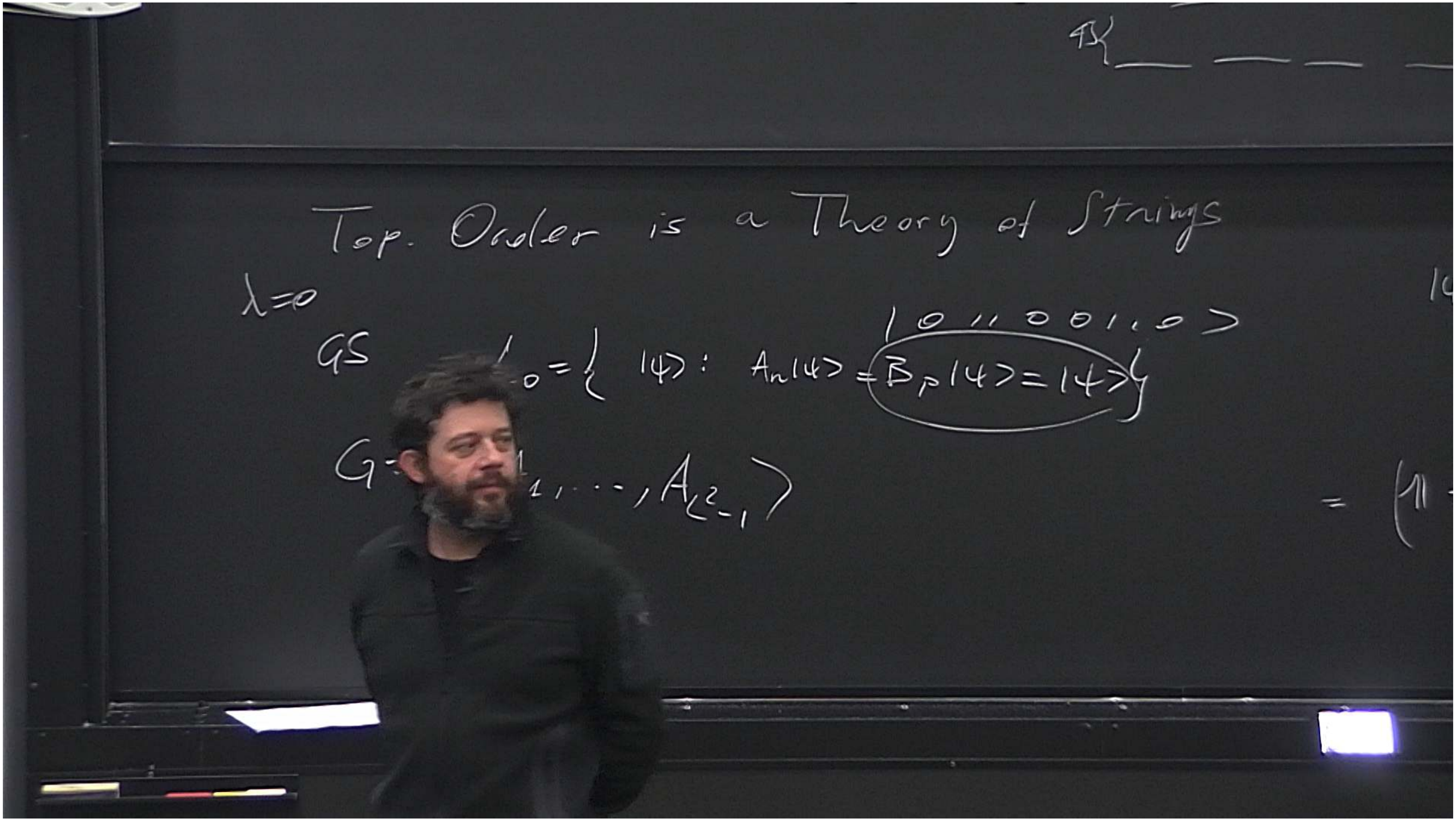
$$|\psi\rangle \in \mathcal{L}_0$$

$$|\psi\rangle = P|0\rangle = N \prod_n \frac{\mathbb{1} + A_n}{2} |0\rangle$$

$$= \Big(\mathbb{1} + \sum_n A_n + \sum_{nm} A_n A_m + \sum_{nmr} A_n A_m A_r + \cdots \Big) |0\rangle$$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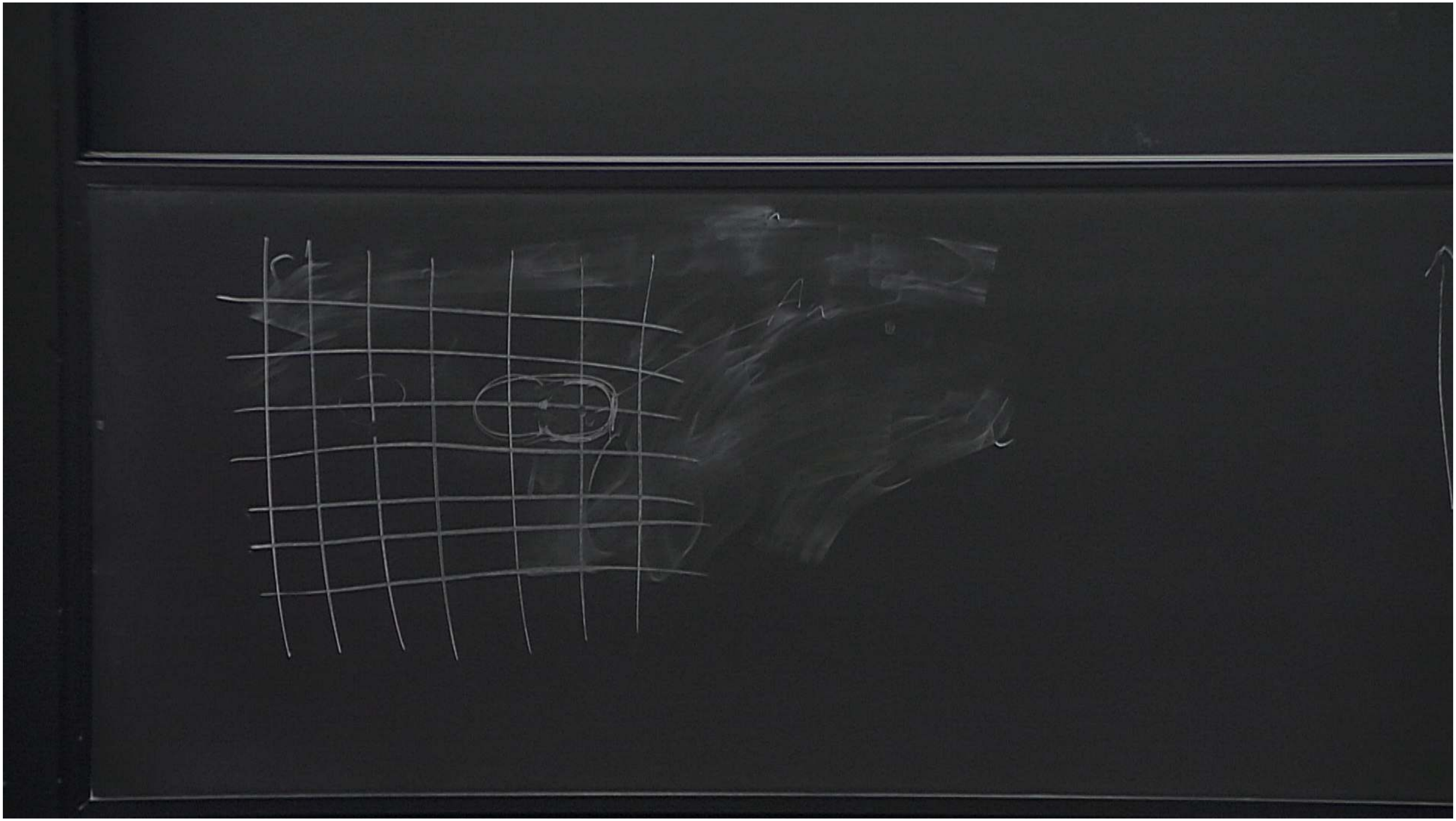
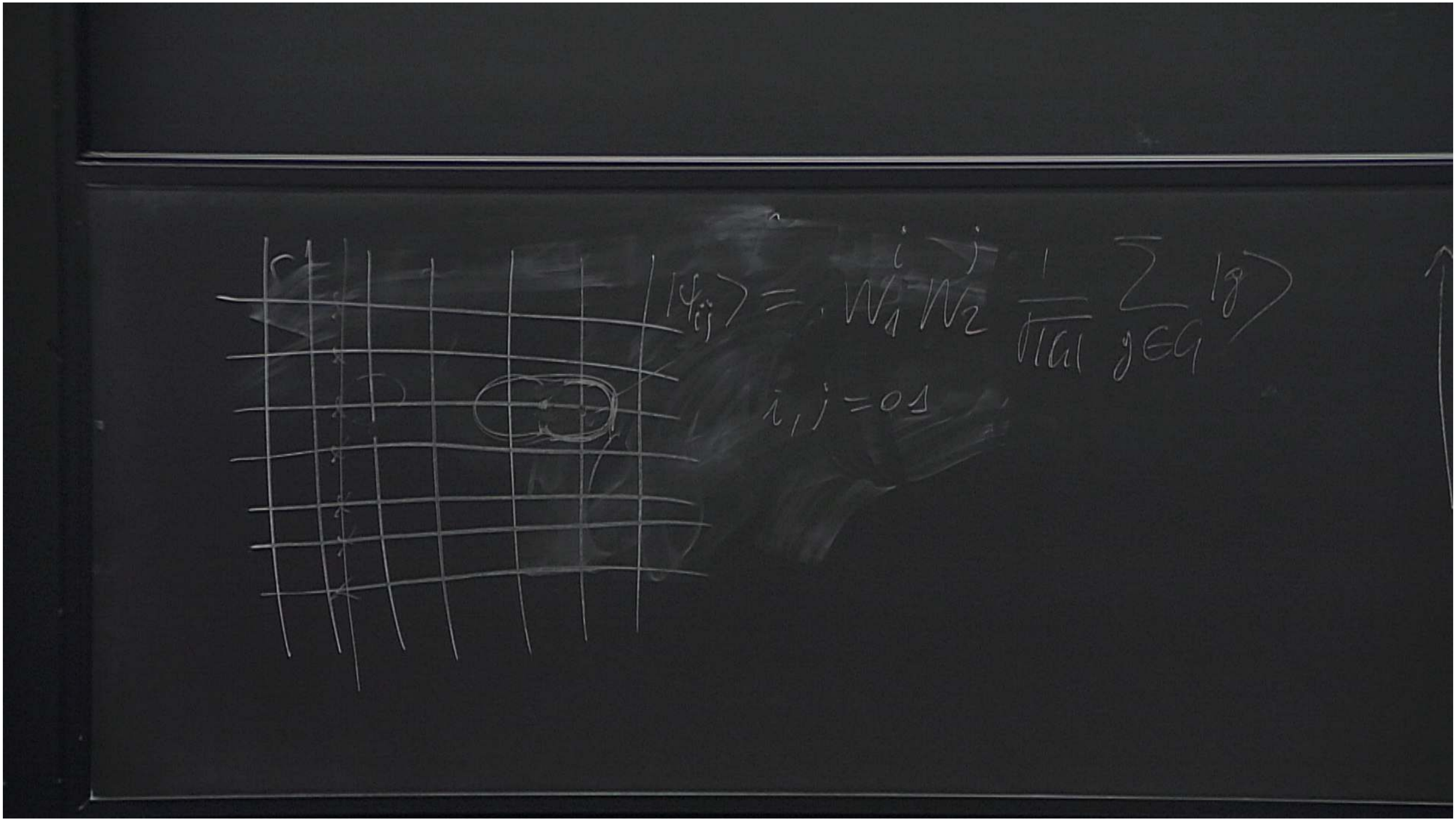
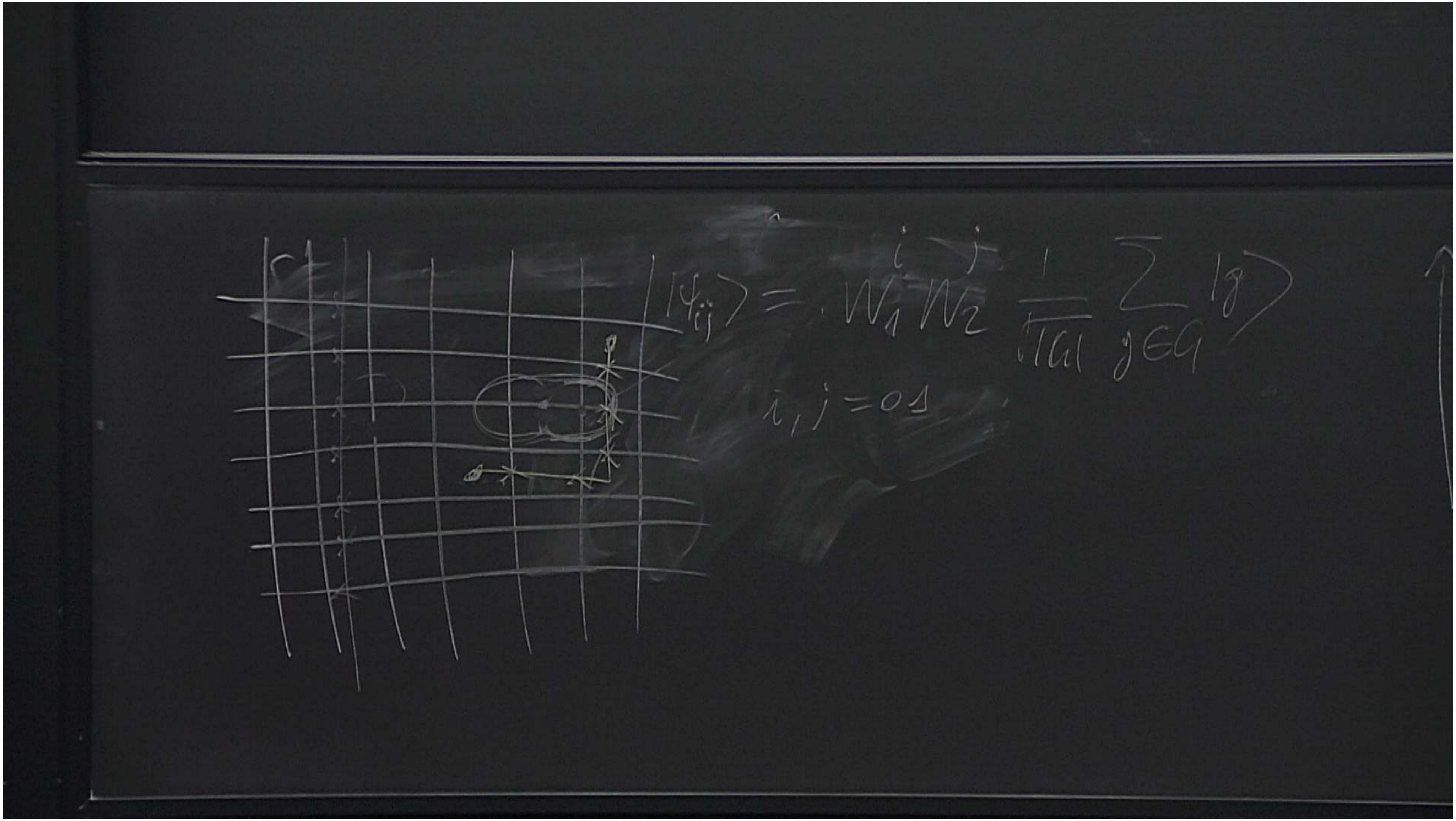
λ=0
GS
|Ψ⟩ : A_n|Ψ⟩ = B_p|Ψ⟩ = |Ψ⟩




$|\psi_{ij}\rangle = W_1^i W_2^j \frac{1}{\sqrt{|G|}} \sum_{g \in G} |g\rangle$
$i, j = 0\,1$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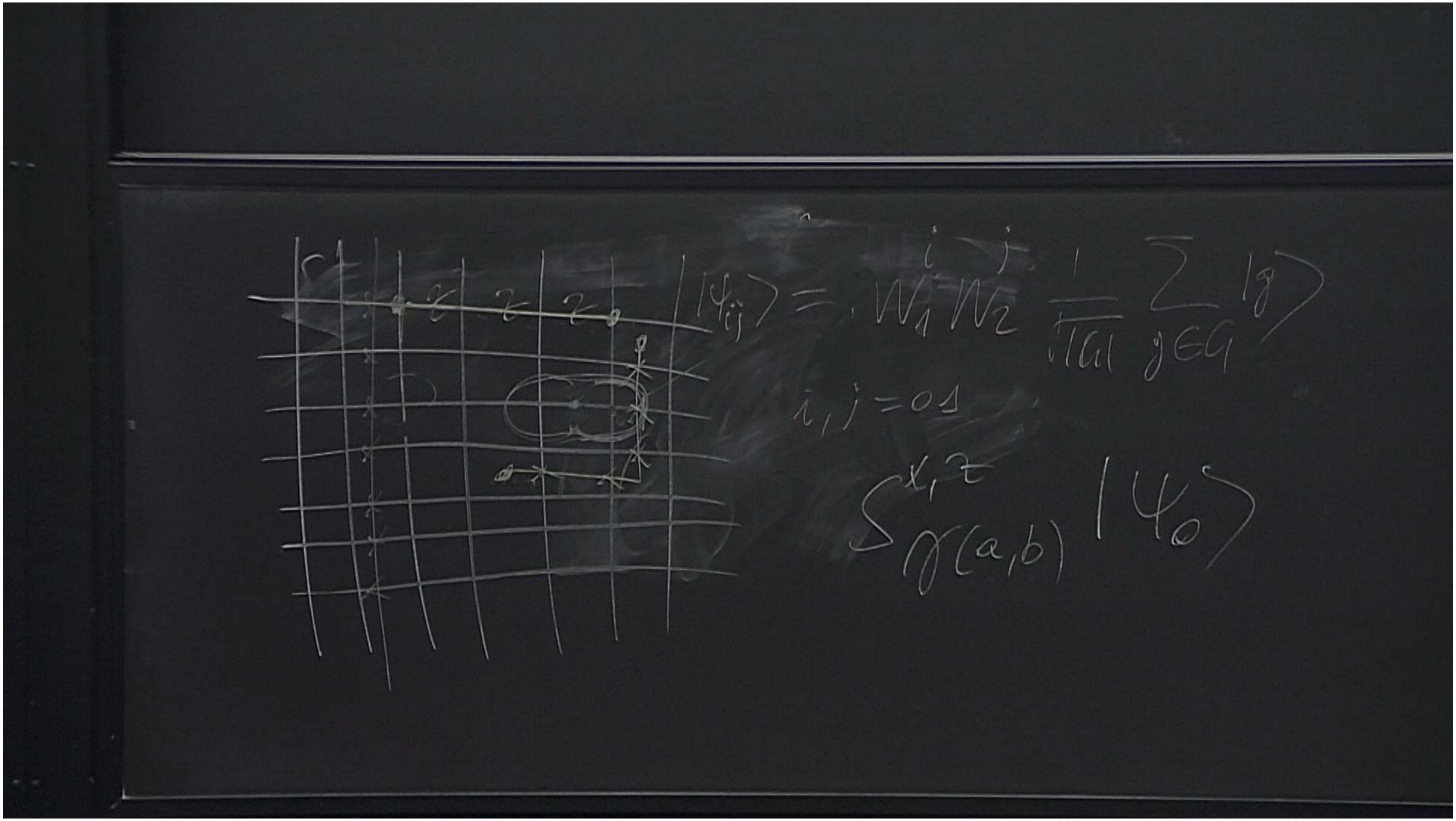
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$S^{x,z}_{\gamma(a,b)} |\psi_0\rangle$