

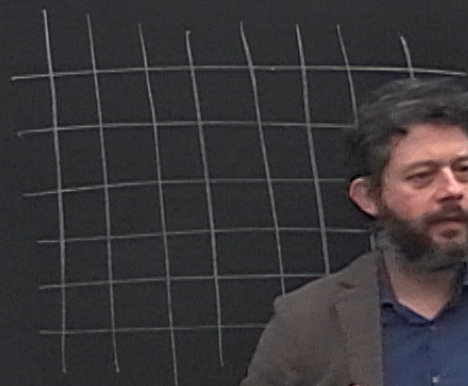
Title: PSI 2018/2019 - Condensed Matter Review - Lecture 9

Date: Jan 17, 2019 11:30 AM

URL: <http://pirsa.org/19010040>

Abstract:

Z_2 Quantum Lattice Gauge Theory

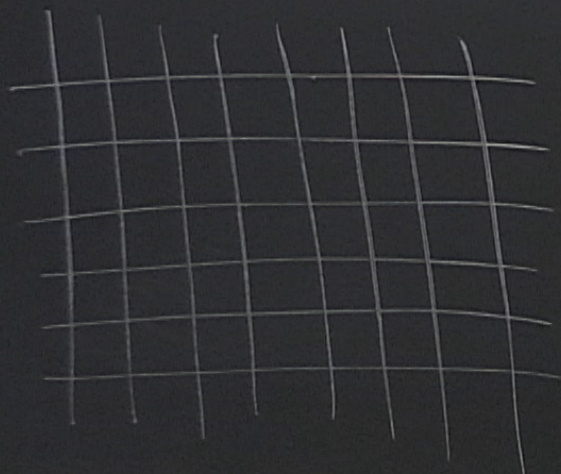


spins $-\frac{1}{2}$ on the bonds $A_n = \prod_{l \in n} \sigma_l^x$

$$\mathcal{H}_{\text{tot}} = \mathbb{C}^2 \otimes 2L^2$$

$$\mathcal{H}_{\text{gauge}} = \{ | \psi \rangle \in \mathcal{H}_{\text{tot}} : A_n | \psi \rangle = | \psi \rangle \forall n \}$$

Z_2 Quantum Lattice Gauge Theory



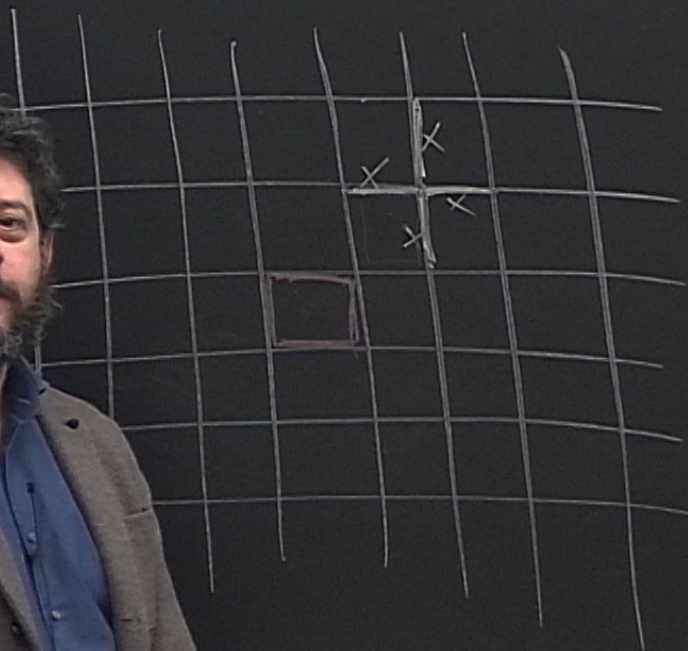
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$$\mathcal{H}_{\text{TOT}} = \mathbb{C}^2 \otimes 2L^2$$

$$\mathcal{H}_{\text{gauge}} = \{ |\psi\rangle \in \mathcal{H}_{\text{TOT}} : A_n |\psi\rangle = |\psi\rangle \forall n\} = \mathbb{P} \mathcal{H}_{\text{TOT}}$$

$$P = \prod_n \frac{1 + A_n}{2} \quad \dim \mathcal{H}_{\text{gauge}} = 2^{L^2 + 1}$$

Z_2 Quantum Lattice Gauge Theory



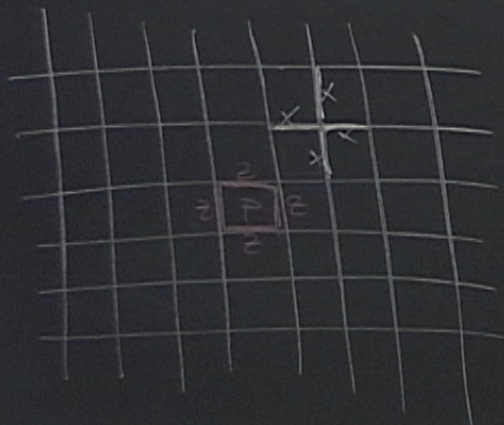
spins $-\frac{1}{2}$ on the bonds

$$\mathcal{H}_{\text{Tot}} = \mathbb{C}^2 \otimes 2L^2$$

$$\mathcal{H}_{\text{gauge}} = \{ |4\rangle \in \mathcal{H}_{\text{Tot}} \}$$

$$E = \prod_n \frac{1 + A_n}{2}$$

Z_2 Quantum Lattice Gauge Theory



spins $-\frac{1}{2}$ on the bonds

$$\mathcal{H}_{\text{TOT}} = \mathbb{C}^2 \otimes 2L^2$$

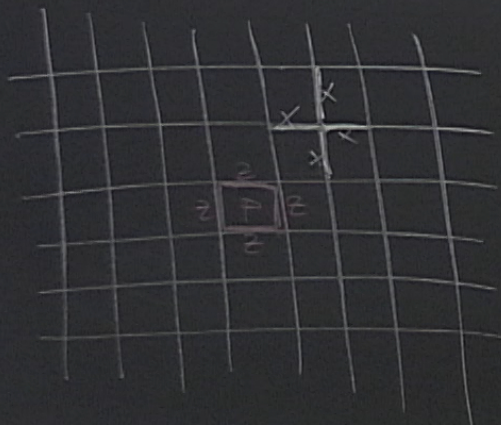
$$\left\{ \begin{array}{l} A_n = \prod_{l \in n} \sigma_l^x \\ B_p = \prod_{l \in p} \sigma_l^z \end{array} \right.$$

$$[A_n, B_p] = 0 \quad \forall n, p$$

$$\mathcal{H}_{\text{gauge}} = \{ |\psi\rangle \in \mathcal{H}_{\text{TOT}} : A_n |\psi\rangle = |\psi\rangle \quad \forall n \} = \mathbb{P} \mathcal{H}_{\text{TOT}}$$

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Z_2 Quantum Lattice Gauge Theory



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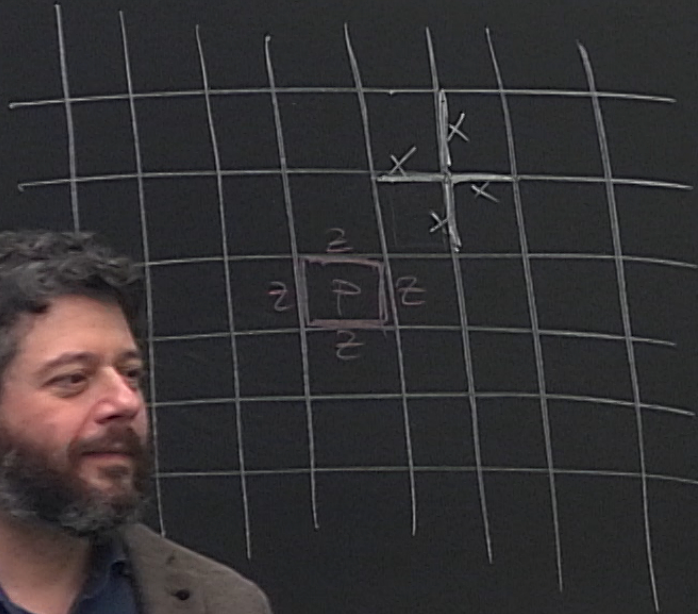
$$\mathcal{H}_{\text{gauge}} = \{ |\psi\rangle \in \mathcal{H}_{\text{tot}} : A_n |\psi\rangle = |\psi\rangle \forall n \} = \mathbb{P} \mathcal{H}_{\text{tot}}$$

$$P = \prod_n \frac{1 + A_n}{2} \quad \dim \mathcal{H}_{\text{gauge}} = 2^{L^2 + 1}$$

$$[A_n, B_p] = 0 \quad \forall n, p$$

$$B_p = \pm 1$$

Z_2 Quantum Lattice Gauge Theory



spins $-\frac{1}{2}$ on the bonds

$$\mathcal{H}_{\text{TOT}} = \mathbb{C}^2 \otimes 2L^2$$

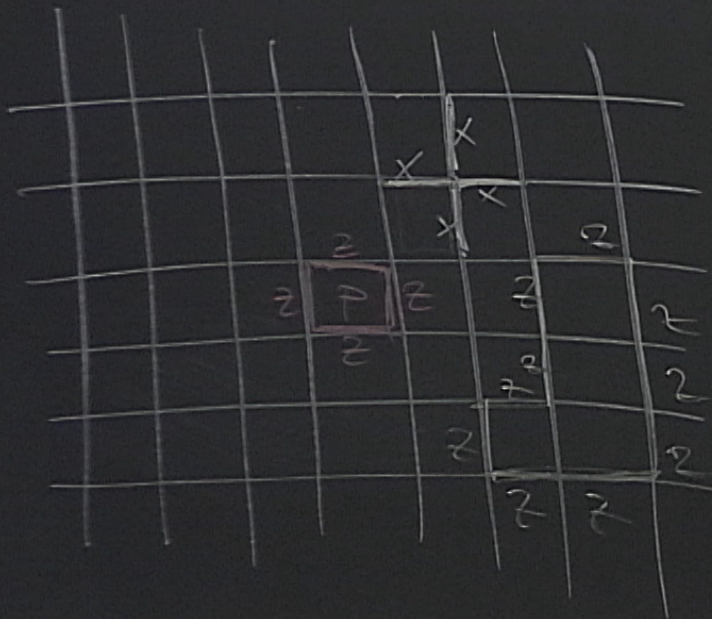
$$\mathcal{H}_{\text{gauge}} = \{ |4\rangle \in \mathcal{H}_{\text{TOT}} : A$$

$$E = \prod_n \frac{1 + A_n}{2}$$

$\dim \mathcal{H}_g$



Z_2 Quantum Lattice Gauge Theory



spins $-\frac{1}{2}$ on the bonds

$$\mathcal{H}_{\text{Tot}} = \mathbb{C}^2 \otimes 2L^2$$

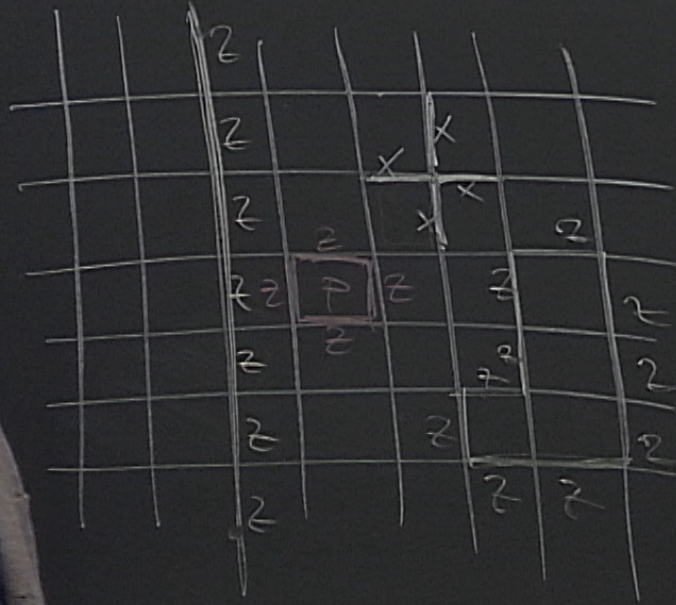
$$\left\{ \begin{array}{l} A_n = \prod_{l \in \text{link}} \\ B_p = \end{array} \right.$$

$$\mathcal{H}_{\text{gauge}} = \{ |4\rangle \in \mathcal{H}_{\text{Tot}} : A_n |4\rangle = 1 \}$$

$$E = \prod_n \frac{1 + A_n}{2} \quad \dim \mathcal{H}_{\text{gauge}} =$$



Z_2 Quantum Lattice Gauge Theory

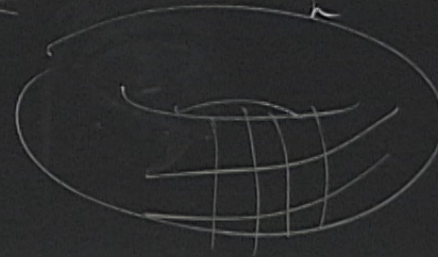


spins $-\frac{1}{2}$ on the bonds

$$\mathcal{H}_{\text{Tot}} = \mathbb{C}^2 \otimes 2L^2$$

$$\mathcal{H}_{\text{gauge}} = \{ |4\rangle \in \mathcal{H}_{\text{Tot}} \}$$

$$E = \prod_n \frac{1 + A_n}{2} \quad \dim$$



$\mathbb{Z}_2$ Quantum Lattice Gauge Theory



spins $-\frac{1}{2}$ on the bonds

$$\mathcal{H}_{\text{tot}} = \mathbb{C}^2 \otimes 2L^2$$

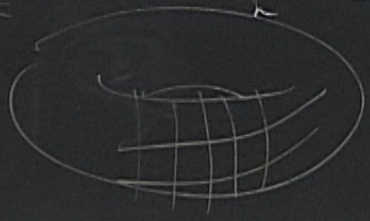
$$\left\{ \begin{array}{l} A_n = \prod_{l \in n} \sigma_l^x \\ B_p = \prod_{l \in p} \sigma_l^z \end{array} \right.$$

$$\mathcal{H}_{\text{gauge}} = \{ |\psi\rangle \in \mathcal{H}_{\text{tot}} : A_n |\psi\rangle = |\psi\rangle \forall n \} = \mathbb{P} \mathcal{H}_{\text{tot}}$$

dim span

$$E = \prod_n \frac{1 + A_n}{2}$$

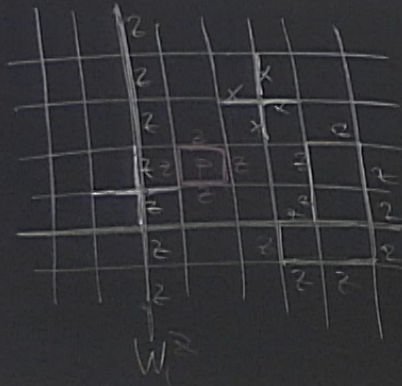
$$\dim \mathcal{H}_{\text{gauge}} = 2^{L^2 + 1}$$



$$[A_n, B_p] =$$

$$B_p =$$

$\mathbb{Z}_2$ Quantum Lattice Gauge Theory

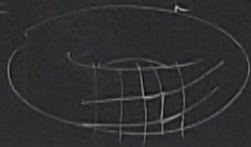


spins $-\frac{1}{2}$ on the bonds

$$\mathcal{H}_{\text{TOT}} = \mathbb{C}^2 \otimes 2^{L^2}$$

$$\mathcal{H}_{\text{gauge}} = \{ | \psi \rangle \in \mathcal{H}_{\text{TOT}} : A_n | \psi \rangle = | \psi \rangle \forall n \} = \mathcal{P} \mathcal{H}_{\text{TOT}}$$

$$\mathcal{P} = \prod_n \frac{1 + A_n}{2} \quad \dim \mathcal{H}_{\text{gauge}} = 2^{L^2 + 1}$$



$$\left\{ \begin{array}{l} A_n = \prod_{l \in n} \sigma_l^x \\ B_p = \prod_{l \in p} \sigma_l^z \end{array} \right.$$

$$[A_n, B_p] = 0 \quad \forall n, p$$

$$B_p = \pm 1; \quad \prod_p B_p = 1$$

$L^2 - 1$ indep. quantum numbers

$$\text{span} \{ | B_1, B_2, \dots, B_{L^2-1}, W, N \rangle \}$$

$$[A_n, B_p] = 0 \quad \forall n, p$$

$$B_p = \pm 1; \quad \prod_p B_p = 1$$

$L^2 - 1$ indep. quantum numbers

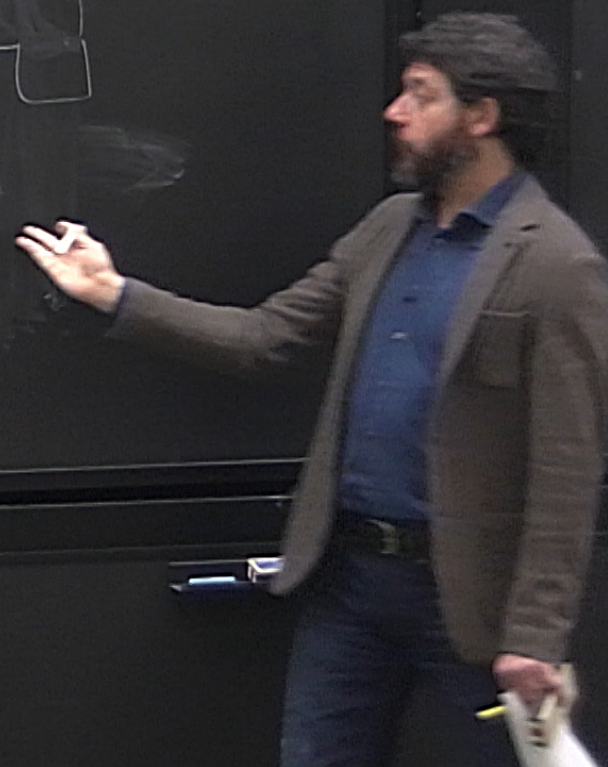
$$\prod_{l \in n} \sigma_l^x$$

$$\prod_{l \in p} \sigma_l^z$$

$$|\psi\rangle \in \mathcal{H}_{\text{tot}}$$

$$= 2^{L+1}$$

$$\text{span} \{ |B_1, B_2, \dots, B_{L-1}, W_1, W_2, \dots, W_L\rangle \}$$



$$H_{Z_2} = -J \sum_p B_p - g \sum_e \sigma_e^x$$

$$g \gg J$$

paramagnet. GS unique
 Δ gap

"disordered" phase

$$g \ll J$$

?

$$H_{Z_2} = -J \sum_p B_p - g \sum_e \sigma_e^x$$

$g \gg J$ paramagnet. GS unique "disordered" phase
 Δ gap

$g \ll J$ GS = no fluxes $E_0 = -N^2 J$

$$\begin{aligned} &|W_1=+1, W_2=+1\rangle \\ &|W_1=+1, W_2=-1\rangle \\ &|W_1=-1, W_2=+1\rangle \\ &|W_1=-1, W_2=-1\rangle \end{aligned}$$

$$H_{Z_2} = -J \sum_p B_p - g \sum_e \sigma_e^x$$

$g \gg J$ paramagnet. GS unique Δ gap "disordered" phase

$g \ll J$ GS $\equiv$ no fluxes $E_0 = -N^2 J$
 $E_0 + 2J$ $|B_i = -1 \dots\rangle$

$|W_1 = +1, W_2 = +1\rangle$
 $|W_1 = +1, W_2 = -1\rangle$
 $|W_1 = -1, W_2 = +1\rangle$
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$$H_{Z_2} = -J \sum_p B_p - g \sum_e \sigma_e^x$$

$g \gg J$ paramagnet. GS unique Δ gap "disordered" phase

$g \ll J$ GS = no fluxes $E_0 = -N^2 J$
 $E_0 + 4J$ | $B_i = -1$ $B_j = +1$ $|W_1 = +1, W_2 = +1\rangle$
 $|W_1 = +1, W_2 = -1\rangle$
 $|W_1 = -1, W_2 = +1\rangle$
 $|W_1 = -1, W_2 = -1\rangle$

$\mathcal{H}_{\text{gauge}} = \{ (A) \in \mathcal{H}_{\text{gauge}} \mid (A) = \sum_{i=1}^L \alpha_i B_i \}$
 $E = \frac{1}{2} \sum_{i=1}^L \frac{1 + A_i}{2}$ $\dim \mathcal{H}_{\text{gauge}} = 2^{L+1}$ $= \text{span} \{ |B_1, B_2, \dots, B_{L-1}\rangle |W_1, W_L\rangle \}$



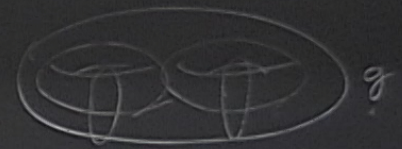
$$H_{Z_2} = -J \sum_p B_p - g \sum_e \sigma_e^x$$

$g \gg J$ paramagnet. GS unique Δ gap "disordered" phase $4J \ll E_0$

$g \ll J$ GS = no fluxes $E_0 = -N^2 J$
 $E_0 + 4J \quad |B_i = -1 \quad B_j = -1\rangle$

$$\begin{aligned}
 &|W_1 = 1, W_2 = 1\rangle \\
 &|W_1 = 1, W_2 = -1\rangle \\
 &|W_1 = -1, W_2 = 1\rangle \\
 &|W_1 = -1, W_2 = -1\rangle
 \end{aligned}$$

$$\dim d_0 = 4^g$$



$\mathcal{H}_{\text{gauge}} = \{ (A) \in \mathcal{H}_{\text{gauge}} \mid \dots \} = \text{span} \{ |B_1, B_2, \dots, B_{L-1}, W_1, W_L\rangle \}$
 $E = \frac{1}{2} \sum_p (1 + A_p)$ $\dim \mathcal{H}_{\text{gauge}} = 2^{L+1}$

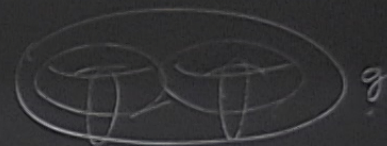

$$H_{Z_2} = -J \sum_p B_p - g \sum_e \sigma_e^x$$

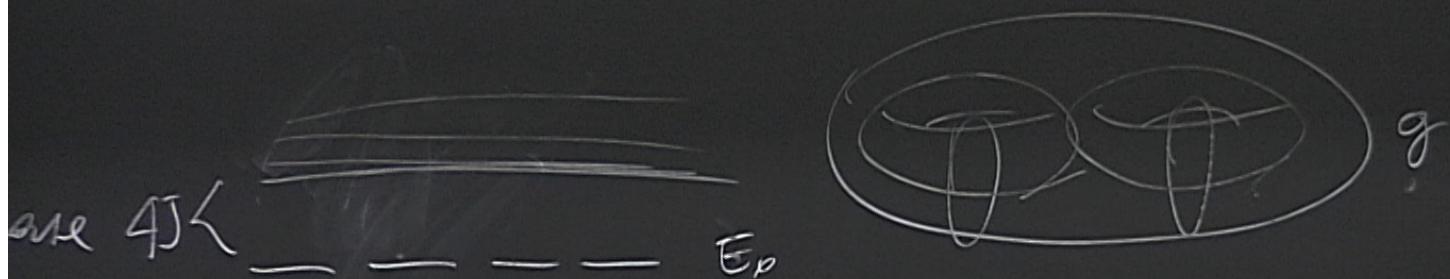
$g \gg J$ paramagnet. GS unique Δ gap "disordered" phase $4J \ll \dots$

$g \ll J$ GS = no fluxes $E_0 = -N^2 J$
 $E_0 + 4J \mid B_i = -1 \ B_j = +1 \rangle$

$$\begin{aligned}
 &|W_1 = +1 \ W_2 = +1\rangle \\
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 &|W_1 = -1 \ W_2 = -1\rangle
 \end{aligned}$$

$$\dim \mathcal{L}_0 = 4^g \\
 \psi_0 \in \mathcal{L}_0 \quad \sigma^z$$





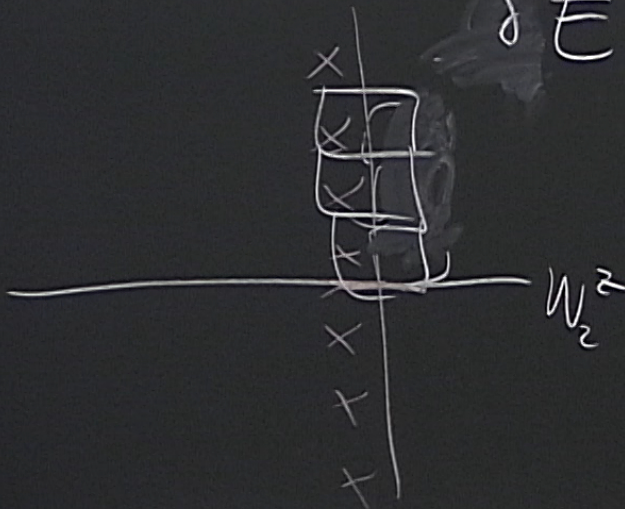
$$\dim \mathcal{L}_0 = 4^g$$

$$\psi_0 \in \mathcal{L}_0 \quad \langle \psi_0 | \sigma_x^2 | \psi_0 \rangle = \langle \psi_0 | A_n \sigma_x^2 A_n | \psi_0 \rangle$$

$$= \langle \psi_0 | \sigma_x^2 A_n A_n | \psi_0 \rangle$$

$$H_{Z_2} \rightarrow H_{Z_2} + \lambda \sum_i V_i$$

δE splitting in L_0 ?



$$\langle \psi_{\alpha} | \lambda V$$

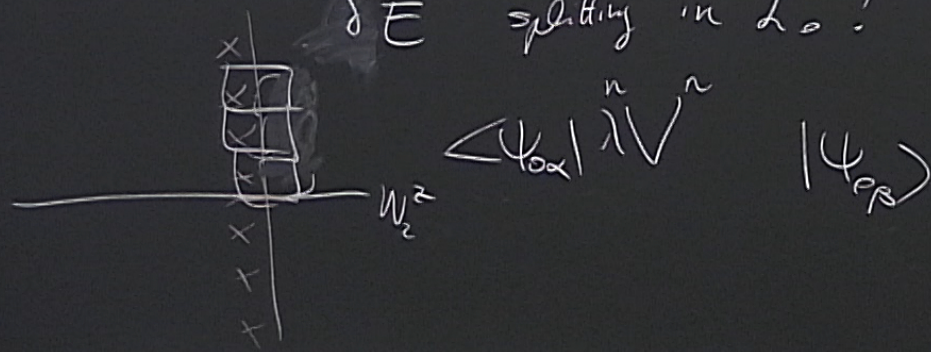
$$| \psi_{\beta} \rangle$$

$$B_i = -1 \quad B_j = +1$$

$$\begin{aligned} &|N_1 = -1 \quad N_2 = +1\rangle \\ &|N_1 = +1 \quad N_2 = -1\rangle \end{aligned}$$

$$H_{Z_2} \rightarrow H_{Z_2} + \lambda \sum_i V_i$$

δE splitting in L_0 ?



$$nR = L$$

$$n = \frac{L}{R} \text{ -th order}$$

$$\delta E \sim \left(\frac{\lambda}{J} \right)^{L/R} \sim e^{-L/R}$$

$$g \ll J \quad GS \equiv \text{no fluxes} \quad E_0 = -N^2 J$$

$$E_0 + 4J \quad | B_i = -1 \quad B_j = -1 \rangle$$

$$|W_1=+1 \quad W_2=+1\rangle$$

$$|W_1=+1 \quad W_2=-1\rangle$$

$$|W_1=-1 \quad W_2=+1\rangle$$

$$|W_1=-1 \quad W_2=-1\rangle$$

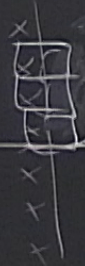
$$\dim \mathcal{L}_0 = 4^8$$

$$\psi_0 \in \mathcal{L}_0 \quad \langle \psi_0 | \sigma_i^z | \psi_0 \rangle = \langle \psi_0 | A \sigma_i^z A^{-1} | \psi_0 \rangle$$

$$= \langle \psi_0 | \sigma_i^z | \psi_0 \rangle$$

$$H_{Z_2} \rightarrow H_{Z_2} + \lambda \sum_i V_i$$

δE splitting in $\mathcal{L}_0$?



$$\langle \psi_{0\alpha} | \lambda V^n | \psi_{0\beta} \rangle$$

$$nR = L$$

$$n = \frac{L}{R} \text{ -th order}$$

$$\delta E \sim \left(\frac{\lambda}{J} \right)^{L/R} \sim e^{-L/R}$$

$$\langle \psi_0 | \sigma_i \cdot \sigma_j | \psi_0 \rangle$$

$$\langle \psi_0 | \sigma_i^z \sigma_j^z | \psi_0 \rangle \sim e^{-\frac{|i-j|}{3}}$$