Title: PSI 2018/2019 - Condensed Matter Review - Lecture 8

Date: Jan 16, 2019 11:30 AM

URL: http://pirsa.org/19010039

Abstract:

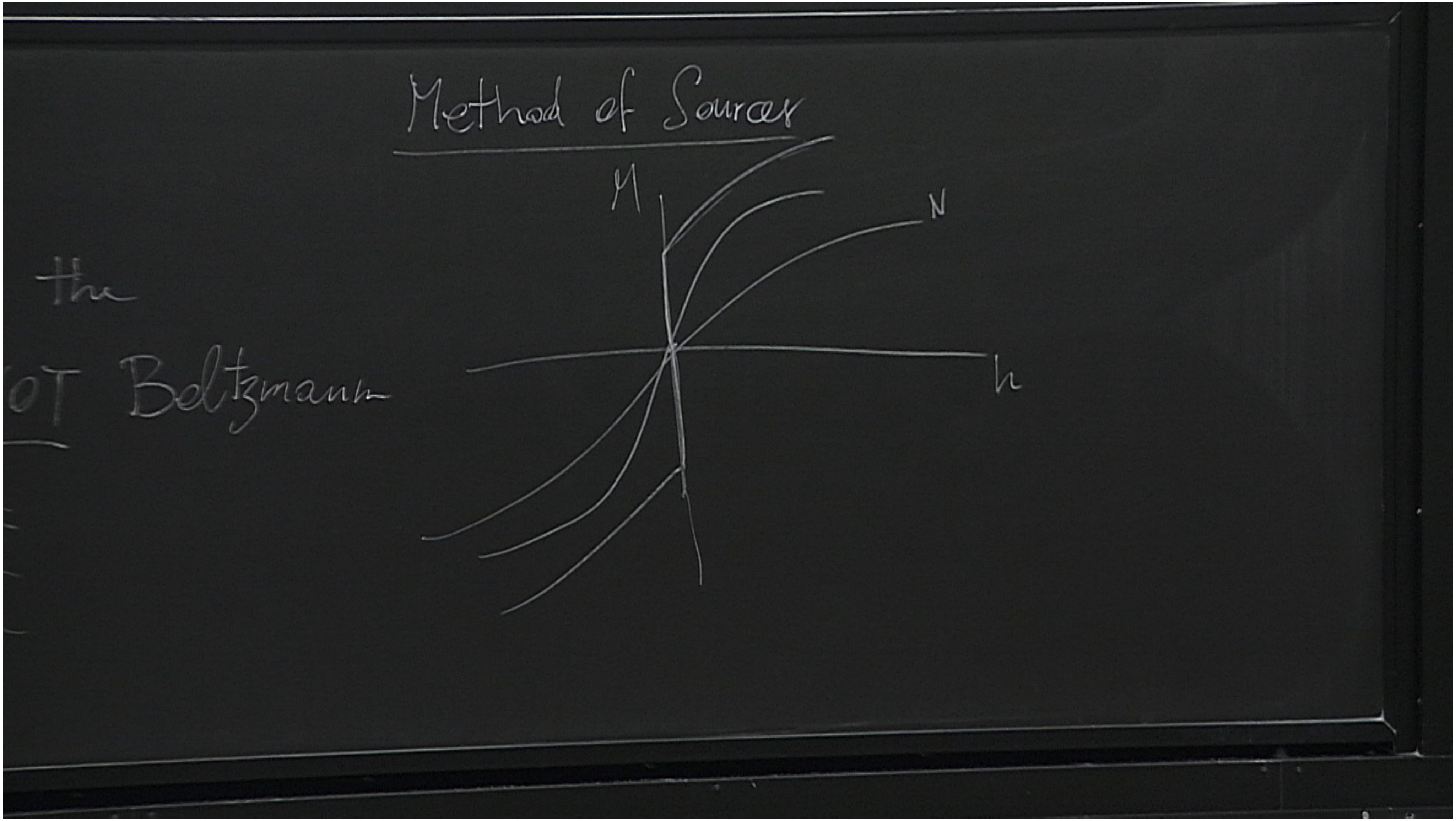
$$H = -J \sum_{\langle ij \rangle} s_i s_j - h \sum_i s_i$$

$$\{s_i\} \rightarrow \{-s_i\}$$

$$P(\{s_i\}) = \frac{1}{Z} e^{-\beta H(\{s_i\})}$$

$$M = \langle s_i \rangle = \sum_{\{s_i\}} P(\{s_i\})\, s_i = 0\,!$$

In TDL the P is NOT Boltzmann


Method of Sources
M
N
h
the
Boltzmann

$= \frac{1}{Z} e^{-\beta H(\{S_i\})}$

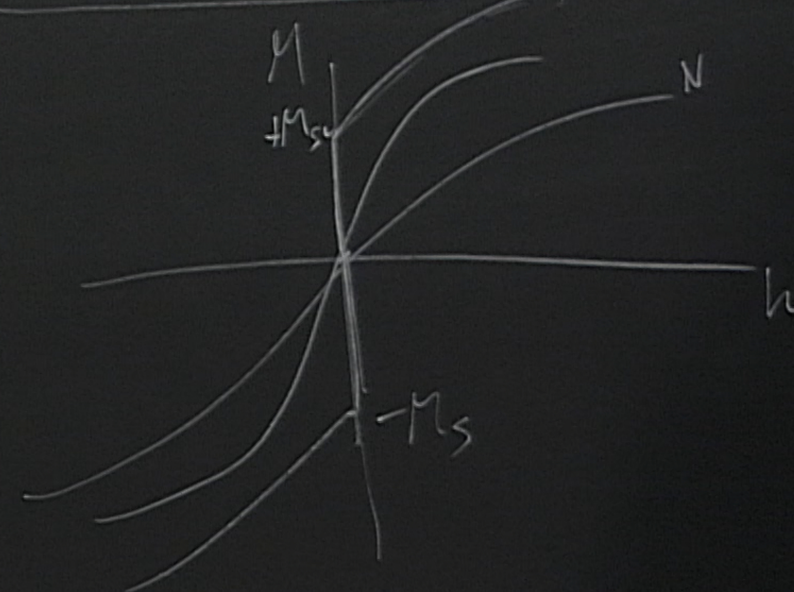
$= 0$!

In TDL the

P is NOT Boltzmann

$M = \frac{1}{N}\frac{\partial F}{\partial h}$ $\quad \lim_{h \to 0} \lim_{N \to \infty} M$

Method of Sources



$$H = -J\sum_{\langle ij\rangle} S_i S_j - h\sum_i S_i$$

$$\{S_i\} \to \{-S_i\}$$

$$P(\{S_i\}) = \frac{1}{Z} e^{-\beta H(\{S_i\})}$$

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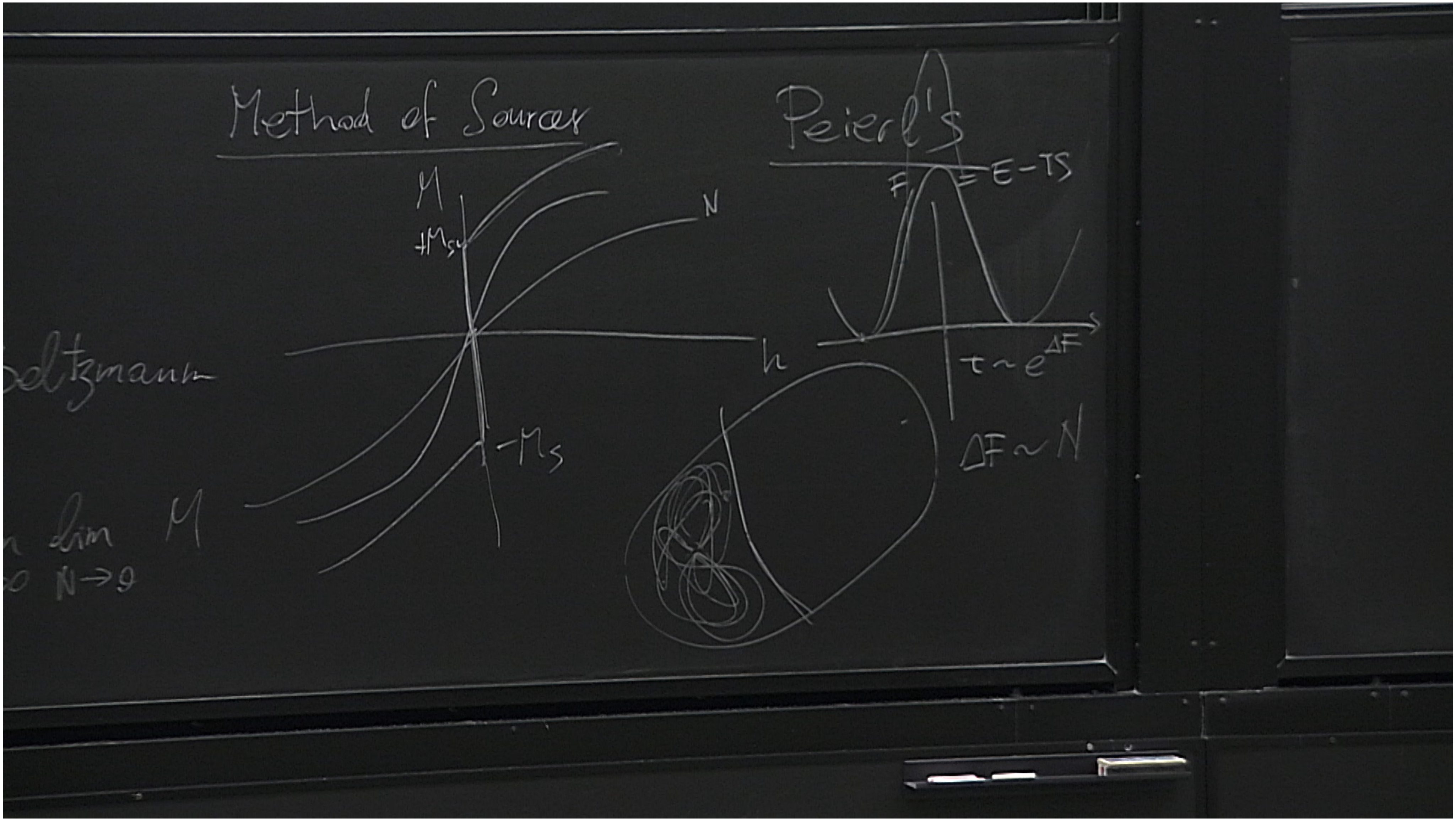
Ergodicity Breaking

In TD

P is

$M = \frac{1}{N}$

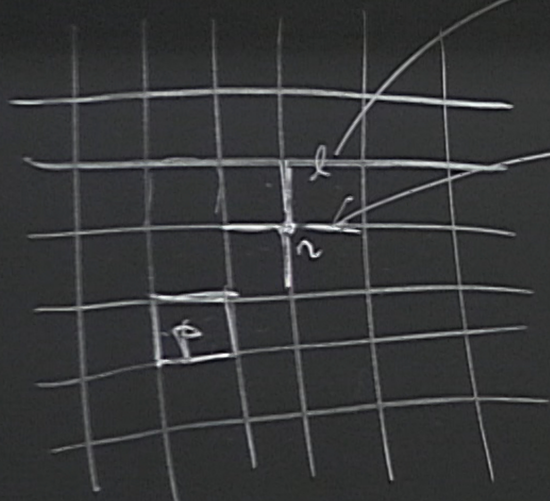

Method of Sources
M
+Ms
N
h
-Ms
Peierl's
F = E − TS
S
t ~ e^ΔF
ΔF ~ N
oltzmann
n lim M
N → ∞

QCD $\longrightarrow$ Lattice Gauge Theory

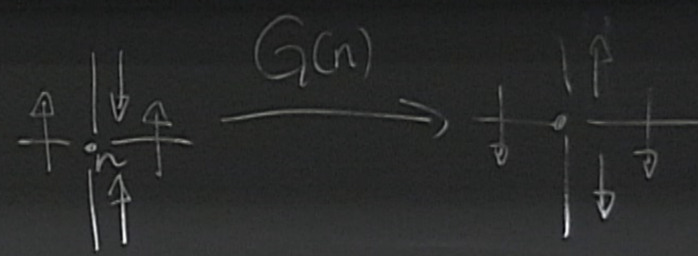
$\mathbb{Z}_{12}$ symmetry	$\mathbb{Z}_{12}$ Gauge Symmetry
Global	local

Wegner

$S_\ell = \pm 1$



$\ell_n \in n$



$\mathbb{Z}_2$ lattice gauge theory

$$H = -J \sum_p B_p$$

$$B_p = \underbrace{SSSS}_{\text{around } p}$$

$$H = -J\sum_p ssss - h'\sum_\ell s_\ell \qquad h \equiv \beta h'$$

$$\langle s_\ell \rangle_h = \frac{1}{Z} \sum_{\{s_\ell\}} s_\ell \, e^{-\beta H(\{s_\ell\})}$$

$$= \frac{1}{Z} \sum_{\{s_\ell\}} s_\ell \exp\left[\beta J \sum_p ssss + h \sum_\ell s_\ell\right]$$

A gauge symm. CANNOT be spontaneously broken

(local)

ELITZUR'S THEOREM

$$\{S_e\} \xrightarrow{G(n)} \{S'_e\}$$

$$S(e_n) \longrightarrow S'(e_n) = -S(e_n)$$

H

$\langle S_e \rangle$

A gauge (local) symm. CANNOT be spontaneously broken

ELITZUR'S THEOREM

$$\{S_\ell\} \xrightarrow{G(n)} \{S'_\ell\}$$

$$S(\ell_n) \longrightarrow S'(\ell_n) = -S(\ell_n)$$

$$h\sum_\ell S_\ell = h\sum_\ell S'_\ell + h\sum_\ell \delta S_\ell$$

$$\sim -2S(\ell_n)$$

$$H = -J\sum_p S$$

$$\langle S_\ell \rangle_h = \frac{1}{Z}$$

$$= \frac{1}{Z}\sum_{\{S_\ell\}}$$

around 1

$$H = -J\sum_p ssss - h'\sum_\ell s_\ell \qquad h \equiv \beta h'$$

$$\langle s_\ell \rangle_h = \frac{1}{Z}\sum_{\{s_\ell\}} s_\ell\, e^{-\beta H(\{s_\ell\})}$$

$$= \frac{1}{Z}\sum_{\{s_\ell\}} -s_\ell\, \exp\Big[\beta J \sum_p s's's's' + h\sum_\ell s'_\ell - h\sum_{\{\ell_n\}} \delta s(\ell_n)\Big]$$

$$= -s(\ell_n)$$

$$+ h\sum_\ell \delta s_\ell$$

$$\sim -2s(\ell_n)$$

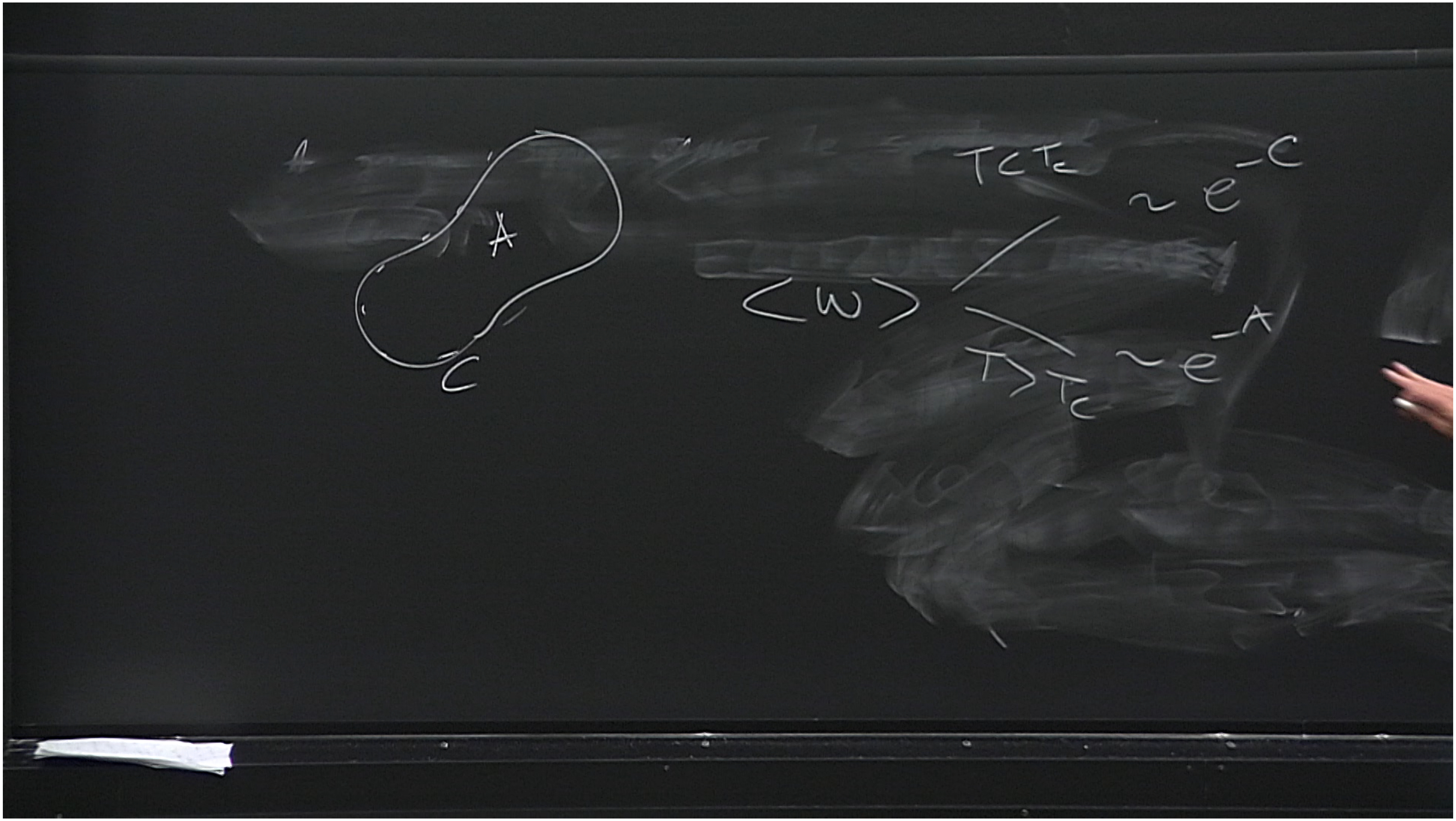
$$\left|\langle s_e\rangle_h - \langle -s_e\rangle_h\right| = \Big|\Big\langle -s_e \big(\underbrace{e^{-h\sum_{\{e_n\}} \delta S}}_{\le e^{4dh}} - 1\big)\Big\rangle_h\Big|$$

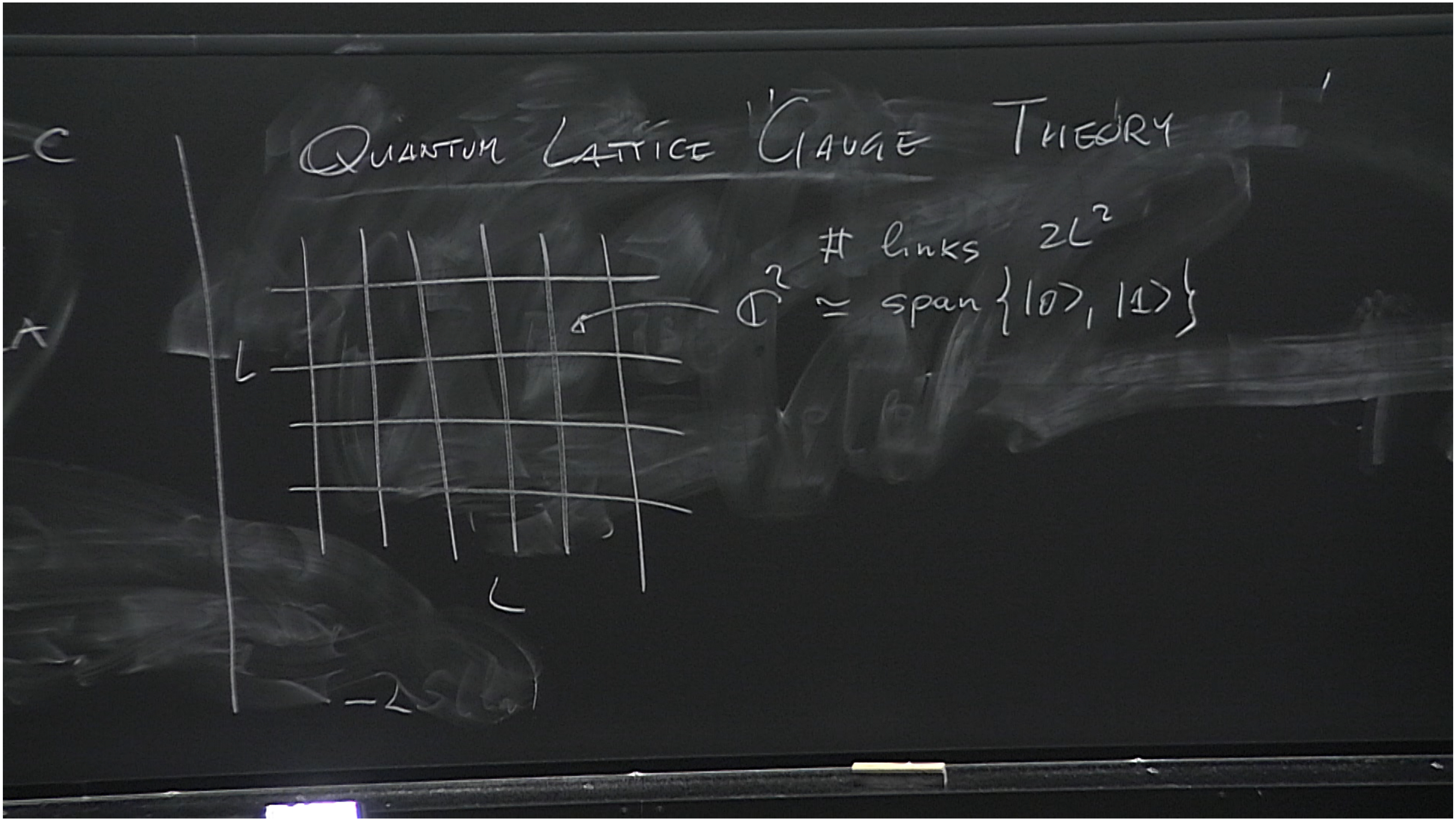
$$\le \left(e^{4dh}-1\right)\left|\langle -s_e\rangle_h\right|$$

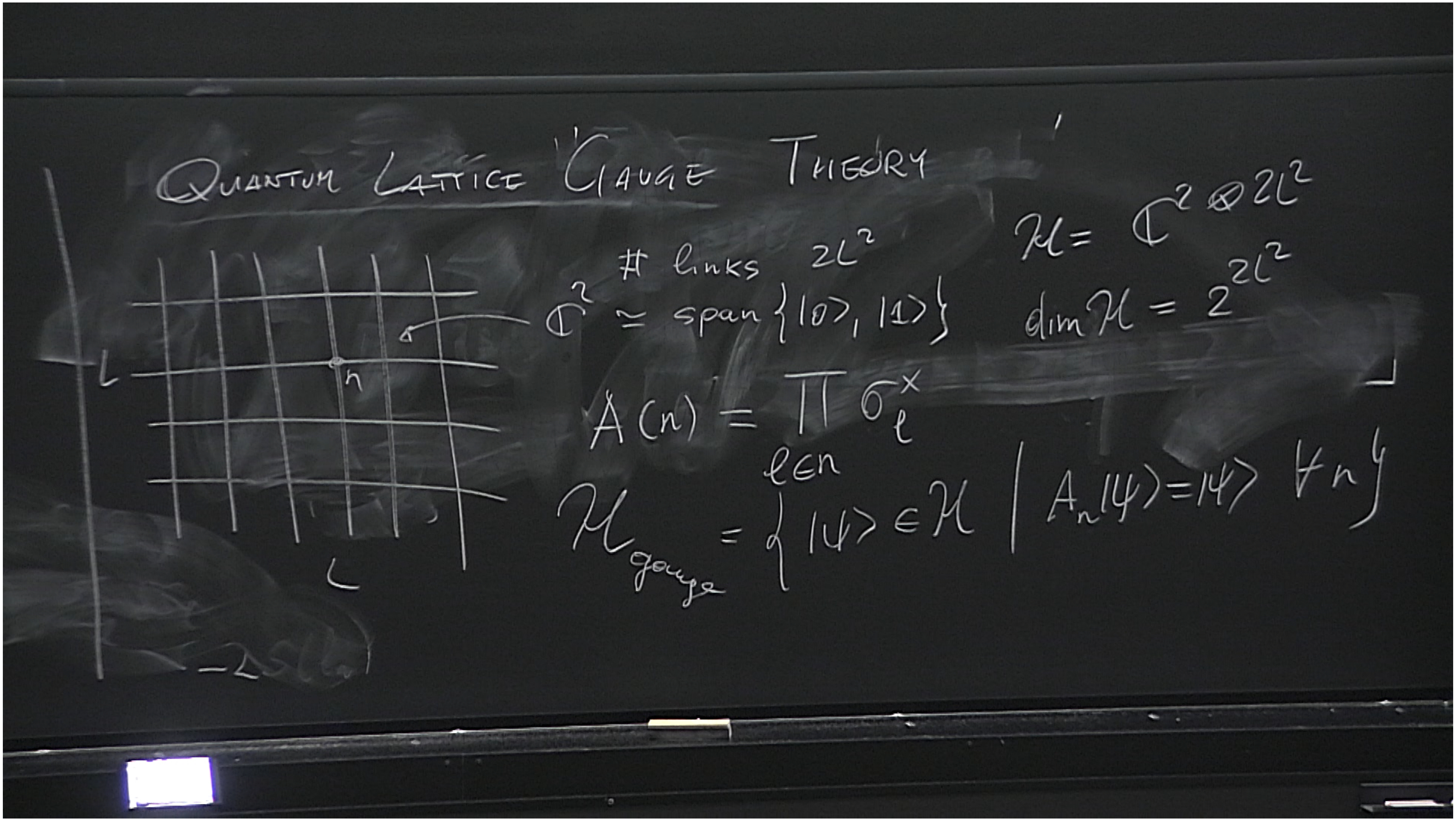
$\| \quad h \to 0$

0

$h \to 0$


A
C
T<Tc
~ e^-C
<W>
T>Tc
~ e^-A


QUANTUM LATTICE GAUGE THEORY
links $2L^2$
$\mathbb{C}^2 \simeq \mathrm{span}\{|0\rangle, |1\rangle\}$
L
L


QUANTUM LATTICE GAUGE THEORY
links $2L^2$
$\mathbb{C}^2 \simeq \mathrm{span}\{|0\rangle, |1\rangle\}$
$\mathcal{H} = \mathbb{C}^2 \otimes 2L^2$
$\dim \mathcal{H} = 2^{2L^2}$
$A(n) = \prod_{\ell \in n} \sigma_\ell^x$
$\mathcal{H}_{gauge} = \{ |\psi\rangle \in \mathcal{H} \mid A_n|\psi\rangle = |\psi\rangle \ \forall n \}$
L
n
L

$$A_n P_n|\chi\rangle = A_n \frac{1+A_n}{2}|\chi\rangle = P_n|\chi\rangle$$

$$P_n^2 \overset{?}{=} P_n \qquad P_n^2 = \left(\frac{1+A_n}{2}\right)^2 = \frac{1}{4}\left(1+2A_n+1\right)$$

$$P_k \cdots P_{n'} P_n |\chi\rangle \qquad = \frac{1+A_n}{2} = P_n$$

Gauge Theory

\# links $2L^2$

$\mathcal{H} = \mathbb{C}^{2 \otimes 2L^2}$

$\mathbb{C}^2 \simeq \text{span}\{|0\rangle, |1\rangle\}$

$\dim \mathcal{H} = 2^{2L^2}$

$$A(n) = \prod_{\ell \in n} \sigma^x_\ell$$

$$\mathcal{H}_{gauge} = \{ |\psi\rangle \in \mathcal{H} \mid \underbrace{A_n|\psi\rangle = |\psi\rangle}\ \forall n \}$$

$$\dim \mathcal{H}_{gauge} = 2^{L^2+1}$$