

Title: PSI 2018/2019 - Condensed Matter Review - Lecture 7

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Abstract:

CLUSTERING OF CORRELATIONS

$|\phi\rangle$ unique GS $\Rightarrow \exists$ finite
gap Δ above GS correlation length

LR Bounds $\| [A(t), B] \| \leq 2\|A\|\|B\| (e^{\mu\sigma t} - 1) e^{-\mu d}$

HW

① LR Bounds $\| [A(t), B] \| \leq 2\|A\|\|B\| (e^{\mu\omega t} - 1) e^{-\mu d}$

$$\left\{ \begin{array}{l} \hat{f}(\omega) \equiv F(f(t)) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt \\ f(t) \equiv F^{-1}(\hat{f}(\omega)) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{f}(\omega) e^{i\omega t} d\omega \end{array} \right. \quad \hat{f}(\omega) \equiv f(t)$$

① LR Bounds $\| [A(t), B] \| \leq 2 \|A\| \|B\| (e^{\mu \nu t} - 1) e^{-\mu d}$

$$\hat{f}(\omega) \equiv F(f(t)) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt \quad \hat{f}(\omega) \equiv f(t)$$

$$f(t) \equiv F^{-1}(\hat{f}(\omega)) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{f}(\omega) e^{i\omega t} d\omega$$

$$f * g(x) = \int_{-\infty}^{\infty} f(x-t) g(t) dt$$

$$\begin{aligned} F(f * g) &= F[f] \cdot F[g] \\ F(f \cdot g) &= \frac{1}{2\pi} F[f] * F[g] \end{aligned}$$

$$-1)e^{-\mu d}$$

$$= f(t)$$

$$\begin{aligned} &= F(t) \cdot F(g) \\ &= \frac{1}{2\pi} F(t) * F(g) \end{aligned}$$

(HW)

FACT 1

LEMMA 1

LEMMA 2

$$\begin{cases} e^{-\alpha t^2} \Leftrightarrow \sqrt{\frac{\pi}{\alpha}} e^{-\frac{\omega^2}{4\alpha}} \\ \Theta(\omega) \Leftrightarrow \pi \delta(t) - \frac{i}{t} \equiv \hat{\Theta}(t) \end{cases}$$

$$\frac{1}{2\pi} \int_{-\infty}^{+\infty} \hat{\Theta}(t) e^{-iEt - \alpha t^2} dt =$$

$$\int_{-a}^a \frac{e^x - 1}{x} dx = O(e^a)$$

$$k \frac{2\alpha}{\Delta} e^{-\frac{\Delta^2}{4\alpha}}$$

$$O(e^{-\frac{\Delta^2}{4\alpha}})$$

$$E \geq \Delta$$

$$E \leq \Delta$$

006)

$$\langle 0 | A B | 0 \rangle$$

$$\boxed{\langle 0 | A | 0 \rangle = \langle 0 | B | 0 \rangle = 0}$$

$$\langle 0 | [A(t), B] | 0 \rangle \stackrel{HW}{=} \sum_{i \geq 1} (e^{-iE_i t} A_{0i} B_{i0} + e^{iE_i t} B_{0i} A_{i0})$$

$$006) \quad \langle 0 | A B | 0 \rangle = \sum_{i \geq 1} A_{0i} B_{i0}$$

$$\boxed{\langle 0 | A | 0 \rangle = \langle 0 | B | 0 \rangle = 0}$$

$$\langle 0 | [A(t), B] | 0 \rangle \stackrel{\text{HW}}{=} \sum_{i \geq 1} \left(\underbrace{e^{-iE_i t} A_{0i} B_{i0}}_{\text{positive freq.}} + \underbrace{e^{iE_i t} B_{0i} A_{i0}}_{\text{negative freq.}} \right)$$

$$\langle 0 | AB | 0 \rangle = F^{-1}[\Theta(\omega) \cdot F[\langle 0 | A(t), B | 0 \rangle]] \Big|_{t=0} = \frac{1}{2}$$

$$\langle 0 | A B | 0 \rangle = F^{-1} [\Theta(\omega) F[\langle 0 | A(t), B | 0 \rangle]] \Big|_{t=0} = \frac{1}{2\pi} \hat{\Theta}(\omega)$$

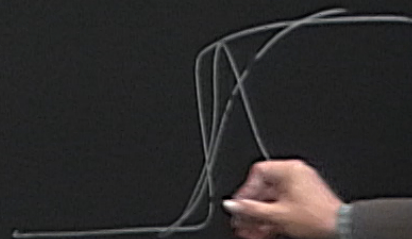
$$\begin{aligned}
 \langle 0 | AB | 0 \rangle &= F^{-1} \left[\Theta(\omega) \cdot F[\langle 0 | [A(t), B] | 0 \rangle] \right] \Big|_{t=0} = \frac{1}{2\pi} \hat{\Theta}(t) \\
 &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \langle 0 | [A(t), B] | 0 \rangle \hat{\Theta}(-t) dt
 \end{aligned}$$

$$\begin{aligned}
 \left. \int_{-\infty}^{\infty} \langle 0 | [A(t), B] | 0 \rangle dt \right|_{t=0} &= \frac{1}{2\pi} \hat{\Theta}(t) * \langle 0 | [A(t), B] | 0 \rangle \Big|_{t=0} \\
 \int_{-\infty}^{\infty} \langle 0 | [A(t), B] | 0 \rangle \hat{\Theta}(-t) e^{-\alpha t^2} dt
 \end{aligned}$$

$$\begin{aligned} \rangle &= \mathcal{F}^{-1} \left[\Theta(\omega) \cdot \mathcal{F} \left[\langle 0 | [A(t), B] | 0 \rangle \right] \right] \Big|_{t=0} = \frac{1}{2\pi} \hat{\Theta}(t) * \langle 0 | [A(t), B] | 0 \rangle \\ &\approx \frac{1}{2\pi} \int_{-\infty}^{\infty} \langle 0 | [A(t), B] | 0 \rangle \left(\hat{\Theta}(-t) e^{-\alpha t^2} \right) dt \end{aligned}$$

$\uparrow$
 α large?

$$\boxed{\mathcal{F}(\hat{\Theta}(t) e^{-\alpha t^2}) = \frac{1}{2\pi} \Theta(\omega) * \sqrt{\frac{\pi}{\alpha}} e^{-\frac{\omega^2}{4\alpha}}}$$



$$\begin{array}{c}
 |t=\infty \qquad \qquad \qquad |t=0 \\
 \langle O|AB|O \rangle = I_F + \langle O|AB|O \rangle - I_F \\
 \qquad \qquad \qquad \downarrow \qquad \qquad \qquad \downarrow \\
 \text{LR + Gaussian} \qquad \qquad \delta_F \text{ error} \\
 \text{Tail} \qquad \qquad \qquad \text{on the} \\
 \qquad \qquad \qquad \text{filter}
 \end{array}$$

PART I I_F is small

$$\int_{-\infty}^{+\infty} = \int_{|t| < cd} + \int_{|t| > cd}$$

LR Bands

$$|t| < cd : \left| \frac{1}{\sigma} \right| < cd$$

$$|t| < cd : \left| \frac{1}{\sigma} \int_{|t| < cd} \langle \phi | [A(t), B] | \phi \rangle \hat{G}(-t) e^{-\alpha t^2} dt \right| \leq \frac{1}{2} \int_{|t| < cd}$$

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} \langle 0 | [A(t), B] | 0 \rangle \hat{Q}(t) e^{-\alpha t^2} dt = \sum_{|n|} A_{i0} B_{j0} \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-\alpha t^2} dt$$

$$\hat{O}(-t) e^{-at^2} dt = \sum_{|i| \gg 1} A_{i0} B_{i0} \frac{L}{2\pi} \int_{-\infty}^{\infty} e^{-iE_i t} \hat{O}(-t) e^{-at^2} dt$$

$$- \sum_{|i| \gg 1} B_{i0} A_{i0} \frac{L}{2\pi} \int_{-\infty}^{\infty} e^{-i(-E_i)t} \hat{O}(-t) e^{-at^2} dt$$

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} \langle 0 | [A(t), B] | 0 \rangle \Theta(t) e^{-\Delta t^2} dt = \sum_{|i| \geq 1} A_{i0} B_{i0}$$

$$= \sum_{|i| \geq 1} A_{0i} B_{i0} \left(1 + O\left(e^{-\frac{\Delta^2}{4\tau}}\right) \right) - \sum_{|i| \geq 1} B_{0i} A_{i0} \frac{1}{2\pi}$$

$$\delta_F \leq \langle 0 | AB | 0 \rangle$$

$$\langle 0|AB|0\rangle = I_F + \delta_F \lesssim \|A\| \|B\| \left\{ O(e^{-\|1-vc\|md}) + O(\dots) \right\}$$

$$\delta_F \leq \|A\| \|B\| \left\{ O(e^{-(1-\nu c)\mu d}) + O(e^{-x c^2 d^2}) + O(e^{-\frac{\Delta^2}{4x}}) \right\}$$

$$x = \frac{1}{\Delta^2}$$

$$(1-\nu c)\mu d = x c^2 d^2 = \frac{\Delta^2}{4x} = x \Delta d$$

$$\langle 0|AB|0\rangle = I_F + \delta_F \leq \|A\| \|B\| \left\{ O(e^{-(1-\nu c)\mu d}) + O(\dots) \right\}$$

$$d = \frac{\Delta}{a} \frac{1}{4\alpha}$$

$$c = 2\alpha$$

$$\alpha = \frac{1}{2v + \Delta/\mu}$$

$$(1-\nu c)\mu d = \alpha c^2 d^2 = \frac{\Delta^2}{4\alpha} = \pi \Delta d$$

$$= \|A\| \|B\| O(e^{-d^{1/3}})$$