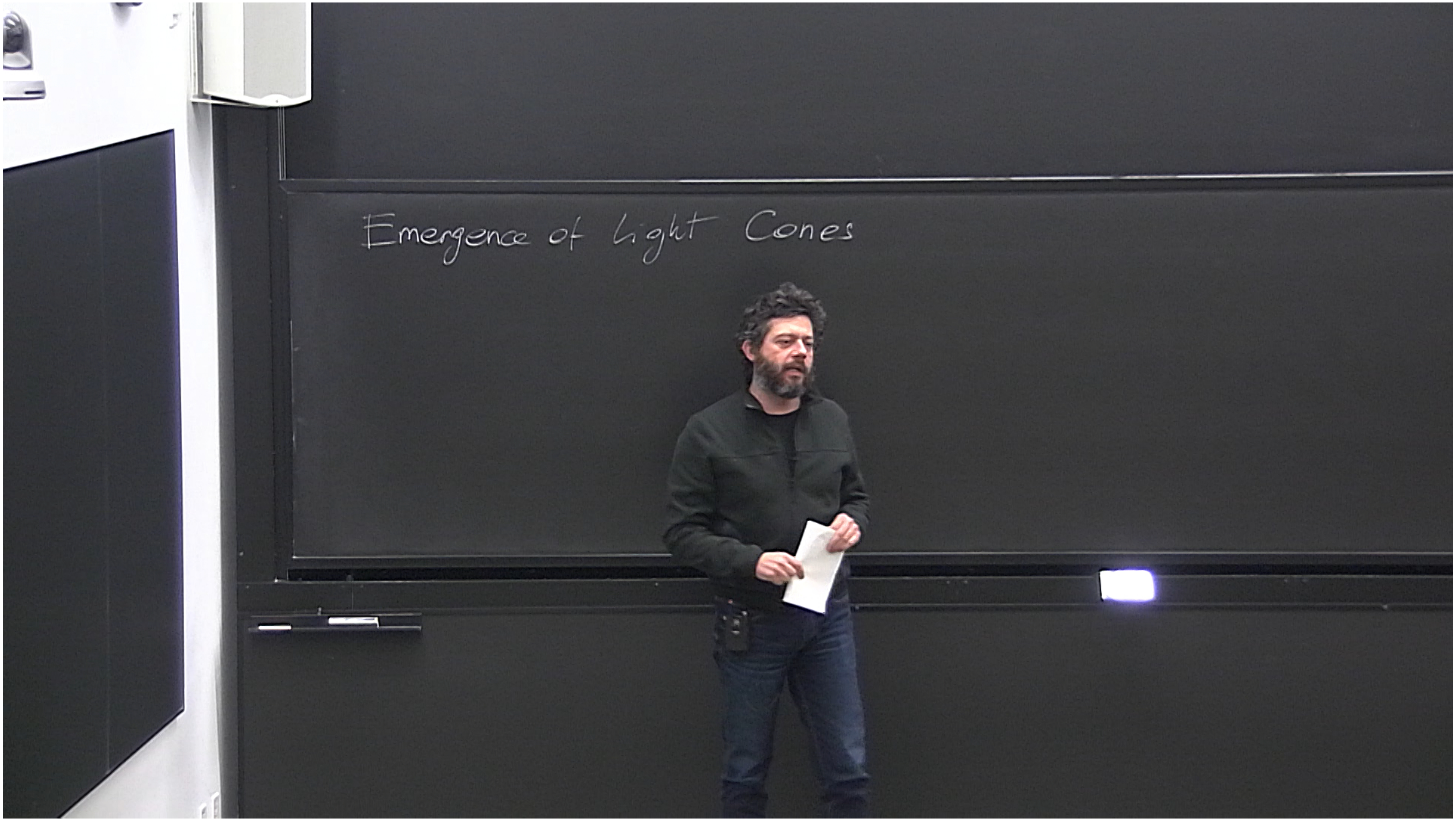
Title: PSI 2018/2019 - Condensed Matter Review - Lecture 6

Date: Jan 14, 2019 11:30 AM

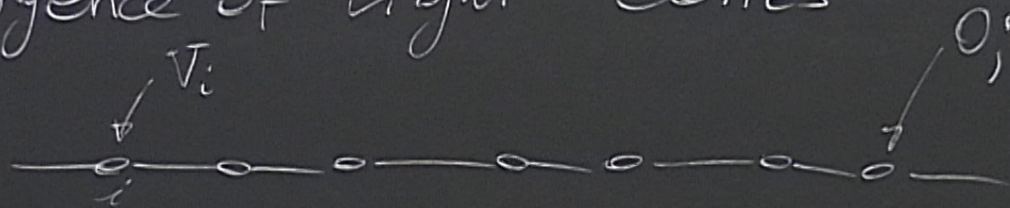
URL: http://pirsa.org/19010037

Abstract:


Emergence of Light Cones

Emergence of Light Cones

$O_j(t) = \langle\psi(t)|O_j|\psi$

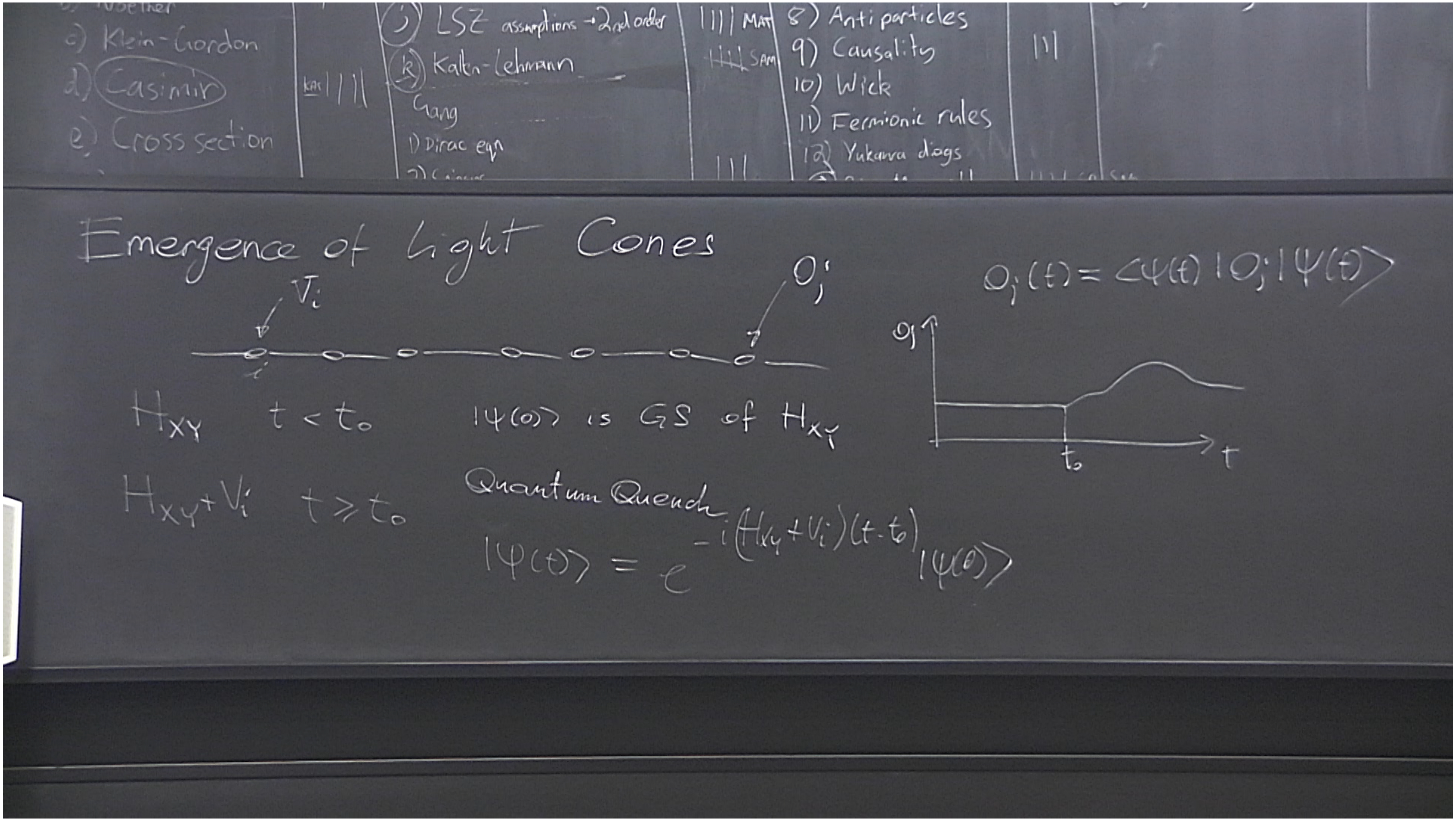


H_{XY} $\quad t < t_0 \quad$ $|\psi(0)\rangle$ is GS of H_{XY}

$H_{XY} + V_i$ $\quad t \geq t_0$

Quantum Quench

$$|\psi(t)\rangle = e^{-i(H_{XY}+V_i)(t-t_0)}|\psi(0)\rangle$$

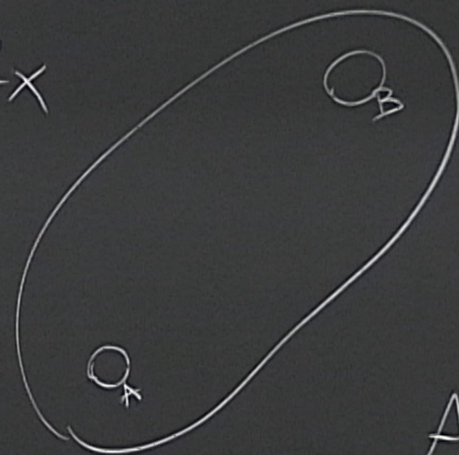

c) Klein-Gordon
d) Casimir
e) Cross section
LSZ assumptions → 2nd order
k) Kallen-Lehmann
Gang
1) Dirac eqn
8) Antiparticles
9) Causality
10) Wick
11) Fermionic rules
12) Yukawa diags
Emergence of Light Cones
V_i
O_j
$O_j(t) = \langle\Psi(t)|O_j|\Psi(t)\rangle$
H_{XY} $t < t_0$ $|\Psi(0)\rangle$ is GS of H_{XY}
$H_{XY} + V_i$ $t \geq t_0$
Quantum Quench
$|\Psi(t)\rangle = e^{-i(H_{XY}+V_i)(t-t_0)}|\Psi(0)\rangle$
t_0
t

$$H = \sum_{X \subset \Lambda} \Phi_X$$

$$|X| \leq \sigma$$

$$\operatorname{diam} X \leq R$$

$$\|\Phi_X\| < h$$



$$\hat{A}$$

$$[A, B] = 0$$

$$U = e^{-iHt}$$

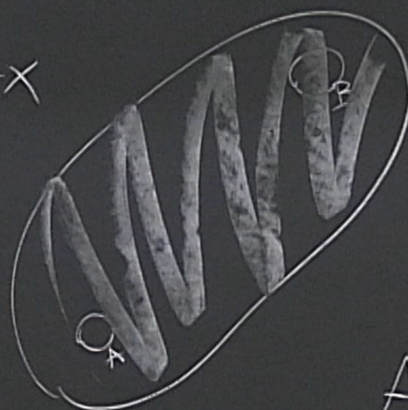
$$A(t) = U^{\dagger} A U$$

$$H = \sum_{X \subset \Lambda} \Phi_X$$

$$|X| \le \sigma$$

$$\operatorname{diam} X \le R$$

$$\|\Phi_X\| < h$$



$$\hat{A}$$

$$[A, B] = 0 \xrightarrow{?} [A(t), B]$$

$$U = e^{-iHt}$$

$$A(t) = U^\dagger A U = e^{iHt} A e^{-iHt}$$

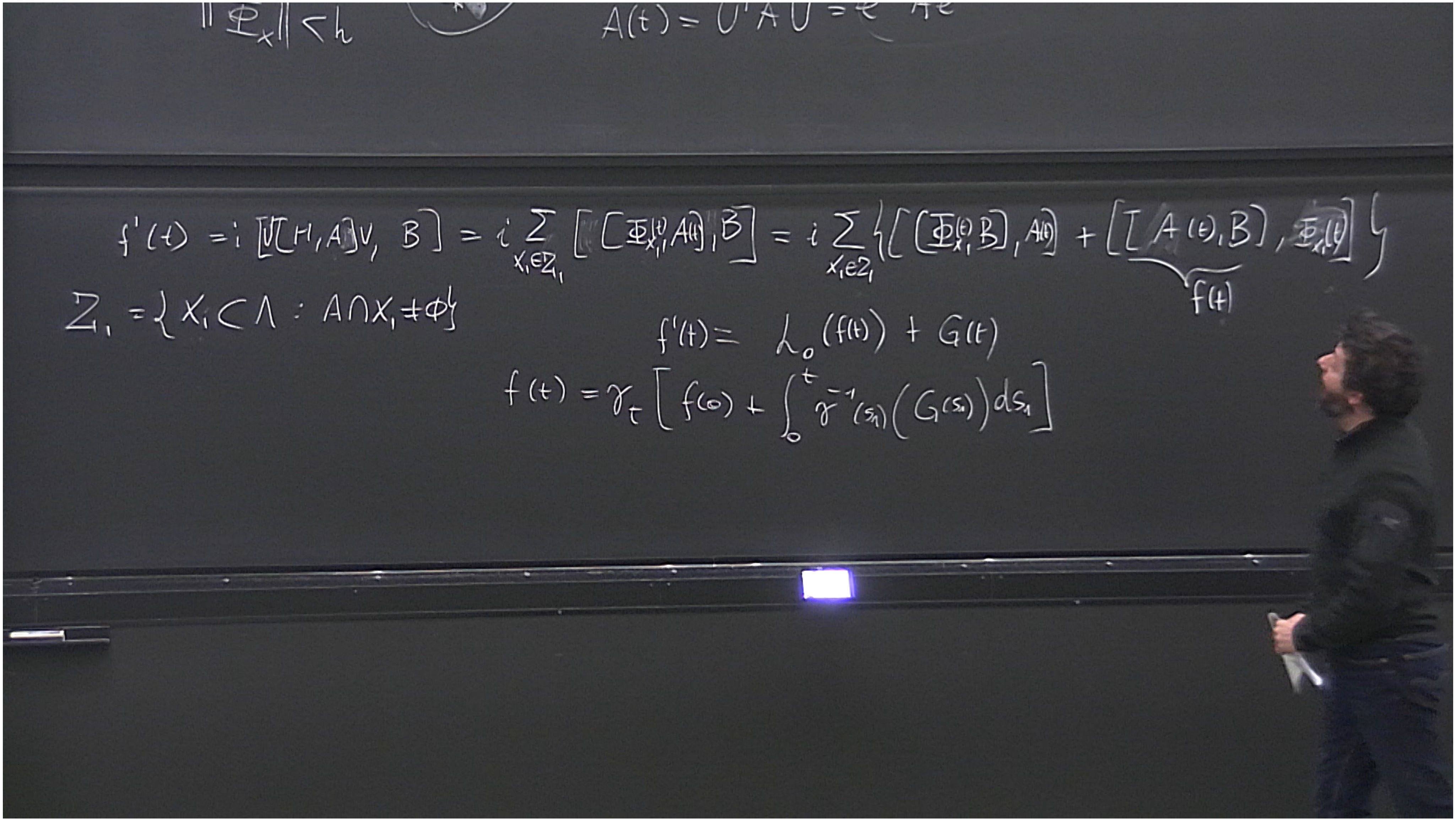
A

$[A, B] = 0 \xrightarrow{?} [A(t), B] \neq 0 \qquad \| [A(t), B] \|$ is small

$[A(t), B] = f(t)$

$$U = e^{-iHt}$$

$$A(t) = U^{\dagger} A U = e^{iHt} A e^{-iHt}$$



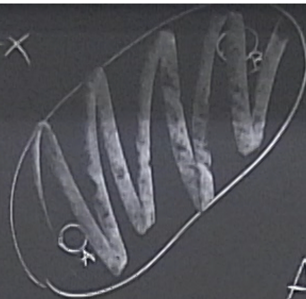
$\|\Phi_x\| < h$
$A(t) = U^* A U = e \; A e$
$f'(t) = i\,[U[H,A]U, B] = i \sum_{X_1 \in Z_1} [[\Phi_{X_1}(t), A(t)], B] = i \sum_{X_1 \in Z_1} \{ [[\Phi_{X_1}(t), B], A(t)] + [\underbrace{[A(t), B]}_{f(t)}, \Phi_{X_1}(t)] \}$
$Z_1 = \{ X_1 \subset \Lambda : A \cap X_1 \neq \phi \}$
$f'(t) = \mathcal{L}_0(f(t)) + G(t)$
$f(t) = \gamma_t \left[f(0) + \int_0^t \gamma^{-1}(s_1)\big(G(s_1)\big)\, ds_1 \right]$

$H = \sum_{X\subset\Lambda} \Phi_X$

$|X| \le \sigma$

$\mathrm{diam}\, X \le R$

$\|\Phi_X\| < h$



A

$[A,B] = 0 \xrightarrow{?} [A(t), B] \neq 0$ $\quad \|[A(t),B]\|$ is

$\quad\quad\quad\quad\quad\quad\quad \shortparallel\ f(t)$

$U = e^{-iHt}$

$A(t) = U^\dagger A U = e^{iHt} A e^{-iHt}$

$$f'(t) = i\,[U[H,A]U,\ B] = i\sum_{X_1\in Z_1}\left[[\Phi_{X_1}(t), A(t)], B\right] = i\sum_{X_1\in Z_1}\left\{\left[[\Phi_{X_1}(t), B], A(t)\right] + \left[\underbrace{[A(t),B]}_{f(t)}, \Phi_{X_1}(t)\right]\right\}$$

$Z_1 = \{X_1 \subset \Lambda : A \cap X_1 \neq \emptyset\}$

$$f'(t) = \mathcal{L}_0(f(t)) + G(t)$$

$$f(t) = \gamma_t\left[f(0) + \int_0^t \gamma^{-1}(s_1)\big(G(s_1)\big)\,ds_1\right]$$

$$\|f(t)\| \le \|f(0)\| + \int_0^t \|G(s_1)\|\,ds_1$$

$$\|f(t)\| = \|[A,B]\| + \int_0^t \sum_{X_1} 2\|A\| \, \underbrace{\|[\Phi_{X_1}(s_1), B]\|}_{f_1(s_1)} \, ds_1 \leq \|f(0)\| + \int_0^t \sum_{X_1} 2\|A\| \sum_{X_2} \|f_1(0)\| + \sum_{X_1} \int_0^t 2\|A\| \sum_{X_2} 2\|\Phi_{X_1}\| \int_0^{s_1} \|[\Phi_{X_2}(s_2), B]\| \, ds_2$$

$$f_1'(s_1) = i \sum_{X_2 \in Z_{12}} \left[[\Phi_{X_1}(s_1), \Phi_{X_2}], B \right] = i \sum_{X_2} \left\{ \underbrace{[[\Phi_{X_1}(s_1), B], \phi_{X_2}(s_1)]}_{f_1(s_1)} + [[\Phi_{X_2}(s_1), B], \Phi_{X_1}(s_1)] \right\}$$

$$\|f_1(s_1)\| \leq \|f_1(0)\| + \sum_{X_2} \int_0^{s_1} 2\|\Phi_{X_1}\| \, \|[\Phi_{X_2}(s_2), B]\| \, ds_2$$

$$f_1(s_1) = i\sum_{X_2\in Z_{12}} \left[\left[\Phi_{X_1}^{(s_1)}, \Phi_{X_2}\right], B\right] = i \sum_{X_2} \left\{ \underbrace{\left[\left[\Phi_{X_1}^{(s_1)}, B\right], \phi_{X_2}^{(s)}\right]}_{f_1(s)} + \left[\left[\Phi_{X_2}^{(s_1)}, B\right], \Phi_{X_1}^{(s_1)}\right] \right\}$$

$$\|f_2(s_2)\| \le \|f_1(0)\| + \sum_{X_2}\int_0^{s_2} 2\|\Phi_{X_1}\| \left\|\left[\Phi_{X_2}^{(s_2)}, B\right]\right\| ds_2$$

$$\Big\|\underbrace{\left[\Phi_{X_k}^{(s_k)}, B\right]}_{f_k}\Big\| \le \|f_k(0)\| + \int_0^{s_k} ds_{k+1}\, 2\|\Phi_k\| \sum_{X_{k+1}\in Z_{k+1}} \left\|\left[B, \Phi_{X_{k+1}}\right]\right\|$$

$$\|f(t)\| \le \sum_{n=0}^{\infty} \sum_{X_1, X_2, \dots X_n} 2\|A\|\,\|B\|\, 2^n h^n \int_0^t \int_0^{s_1} \cdots \int_0^{s_{n-1}} \dots \, dt\, ds_1 \cdots ds_n$$

$\mathrm{diam}\,X \leq R$

$\|\Phi_X\| < h$

$U = e^{\ldots}$

$A(t) = U^\dagger A U = e^{iHt} A e^{-iHt}$

$$f'(t) = i\,[U[H,A]U, B] = i \sum_{X_1 \in Z_1} \left[[\Phi_{X_1}(t), A(t)], B \right] = i \sum_{X_1 \in Z_1} \left\{ \left[[\Phi_{X_1}(t), B], A(t) \right] + \left[\underbrace{[A(t), B]}_{f(t)}, \Phi_{X_1}(t) \right] \right\}$$

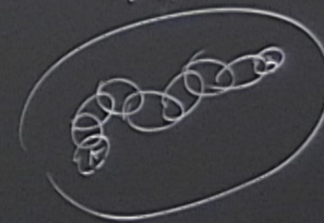
$$Z_1 = \{ X_1 \subset \Lambda : A \cap X_1 \neq \emptyset \}$$

$$Z_{1_2} = \{ X_2 \subset \Lambda : X_2 \cap X_1 \neq \emptyset, X_1 \in Z_1 \}$$

$$f'(t) = \mathcal{L}_0(f(t)) + G(t)$$

$$f(t) = \gamma_t \left[f(0) + \int_0^t \gamma^{-1}(s_1)\big(G(s_1)\big)\, ds_1 \right]$$

$$\|f(t)\| \leq \|f(0)\| + \int_0^t \|G(s_1)\|\, ds_1$$



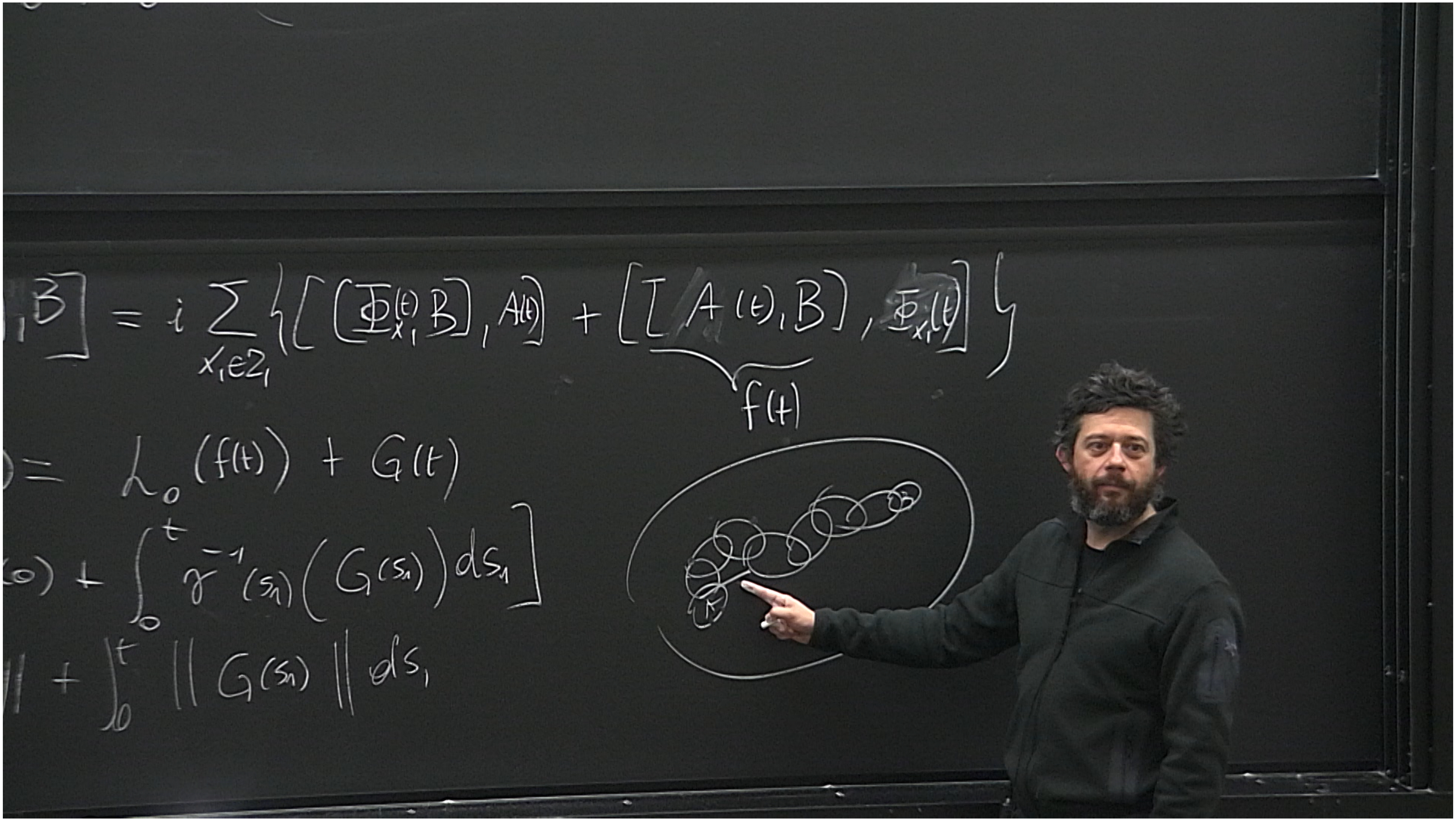
$2\|A\|\,\|B\|$

$$\|f(t)\| \leq \sum_{n=0}^{\infty} S_n \frac{2^n h^n t^n}{n!}$$

d is graph distance between A, B

$nR < d$

$nR \geq d$


$, B] = i \sum_{x_1 \in Z_1} \{ [[\Phi_{x_1}(t), B], A(t)] + [\underbrace{[A(t), B]}_{f(t)}, \Phi_{x_1}(t)] \}$
$) = \mathcal{L}_0 (f(t)) + G(t)$
$(0) + \int_0^t \gamma^{-1}(s_1) (G(s_1)) ds_1]$
$\| + \int_0^t \| G(s_1) \| ds_1$

$$\| f(t) \| \leq \sum_{n=0}^{\infty} \underbrace{\sum_{x_1, x_2, \ldots x_n}}_{S_n} 2\|A\| \, \|B\| \, 2^n h^n \int_0 \int_0 \cdots \int_0 dt \, ds \ldots ds_n$$

$$\frac{1}{n!} \underbrace{\int_0^t \cdots \int_0^t}_{n}$$

$$\| f(t) \| \leq 2\|A\|\|B\| \sum_{n=0}^{\infty} \frac{2^n h^n t^n \sigma^n}{n!} e^{\alpha(nR - d)} = 2\|A\|\|B\| e^{-\alpha d} e^{2ht\sigma e^{\alpha R}} = 2\|A\|\|B\| e^{\alpha\left(\frac{2ht\sigma e^{\alpha R}}{\alpha} - d\right)}$$

d is graph distance between A, B

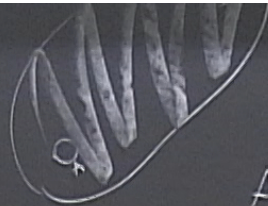
$nR < d \quad S_n = 0 \quad \sigma^n$

$nR \geq d \quad S_n \leq$ # paths with n steps $< \sigma^n e^{\alpha(nR - d)}$

$|X| \leq \sigma$

$\mathrm{diam}\, X \leq R$

$\|\Phi_X\| < h$



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$$\|f(t)\| \leq \|f(0)\| + \int_0^t \|G(s_1)\|\, ds_1$$

$$\|[A(t),B]\| \leq C e^{-\alpha(v_{LR} t - d)}$$

$$\frac{2h\sigma e^{\alpha R}}{\alpha}\, t, \qquad \frac{2h\sigma e^{\alpha R}}{\alpha} = v_{LR}$$

f_x

$$\|f(t)\| \le \sum_{n=0}^{\infty} \underbrace{\sum_{X_1, X_2, \dots X_n}}_{S_n} 2\|A\|\,\|B\|\, 2^n h^n \underbrace{\int_0^t \int_0^{s_1} \cdots \int_0^{s_{n-1}}}_{\frac{1}{n!}\underbrace{\int_0^t \cdots \int_0^t}_{n}} dt\, ds_1 \cdots ds_n$$

$$\|f(t)\| \le 2\|A\|\,\|B\| \sum_{n=0}^{\infty} \frac{2^n h^n t^n \sigma^n}{n!} e^{\alpha(nR-d)} = 2\|A\|\,\|B\|\, e^{-\alpha d} e^{2ht\sigma e^{\alpha R}} = 2\|A\|\,\|B\|\, e^{\alpha\left(\frac{2ht\sigma e^{\alpha R}}{\alpha} - d\right)}$$

d is graph distance between A, B

$nR < d$ $\quad S_n = 0$ $\quad \sigma^n$

$nR \geq d$ $\quad S_n \le$ # paths with n steps $< \sigma^n e^{\alpha(nR-d)}$