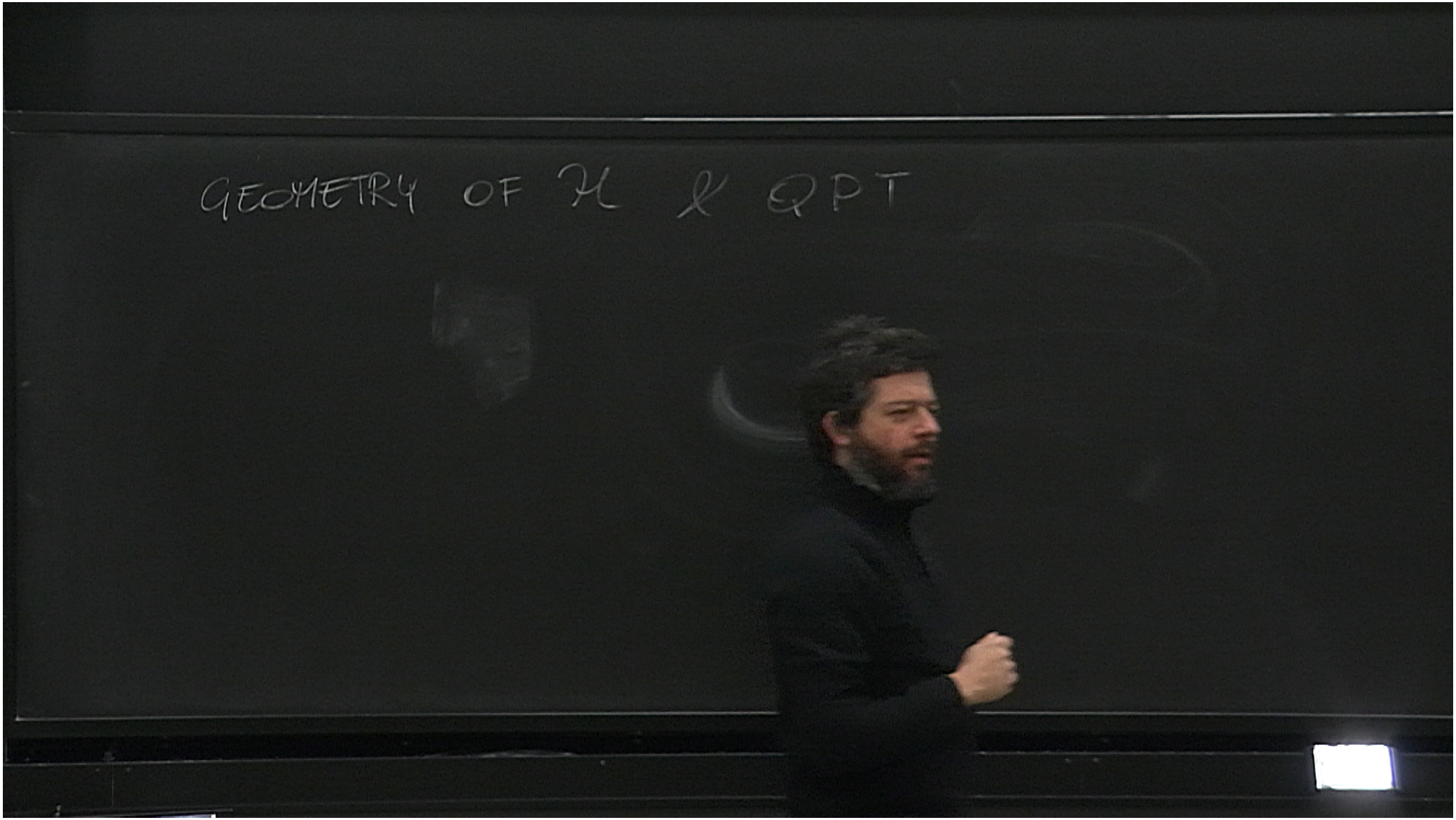
Title: PSI 2018/2019 - Condensed Matter Review - Lecture 5

Date: Jan 11, 2019 11:30 AM

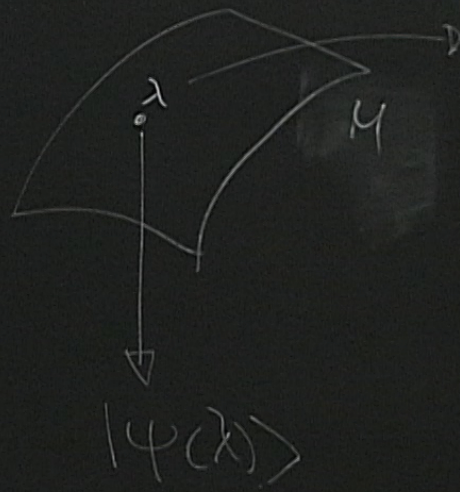
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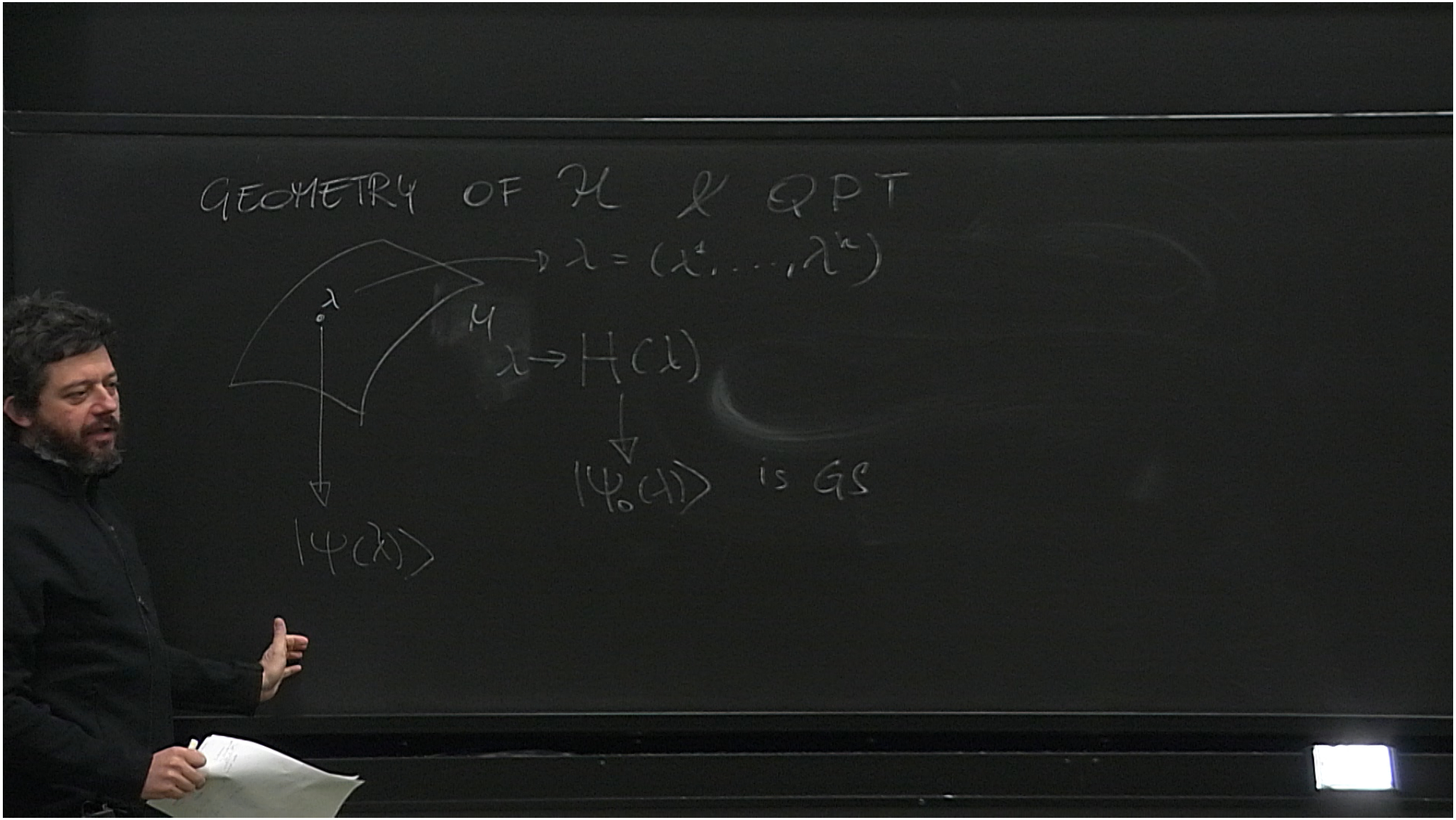
Abstract:

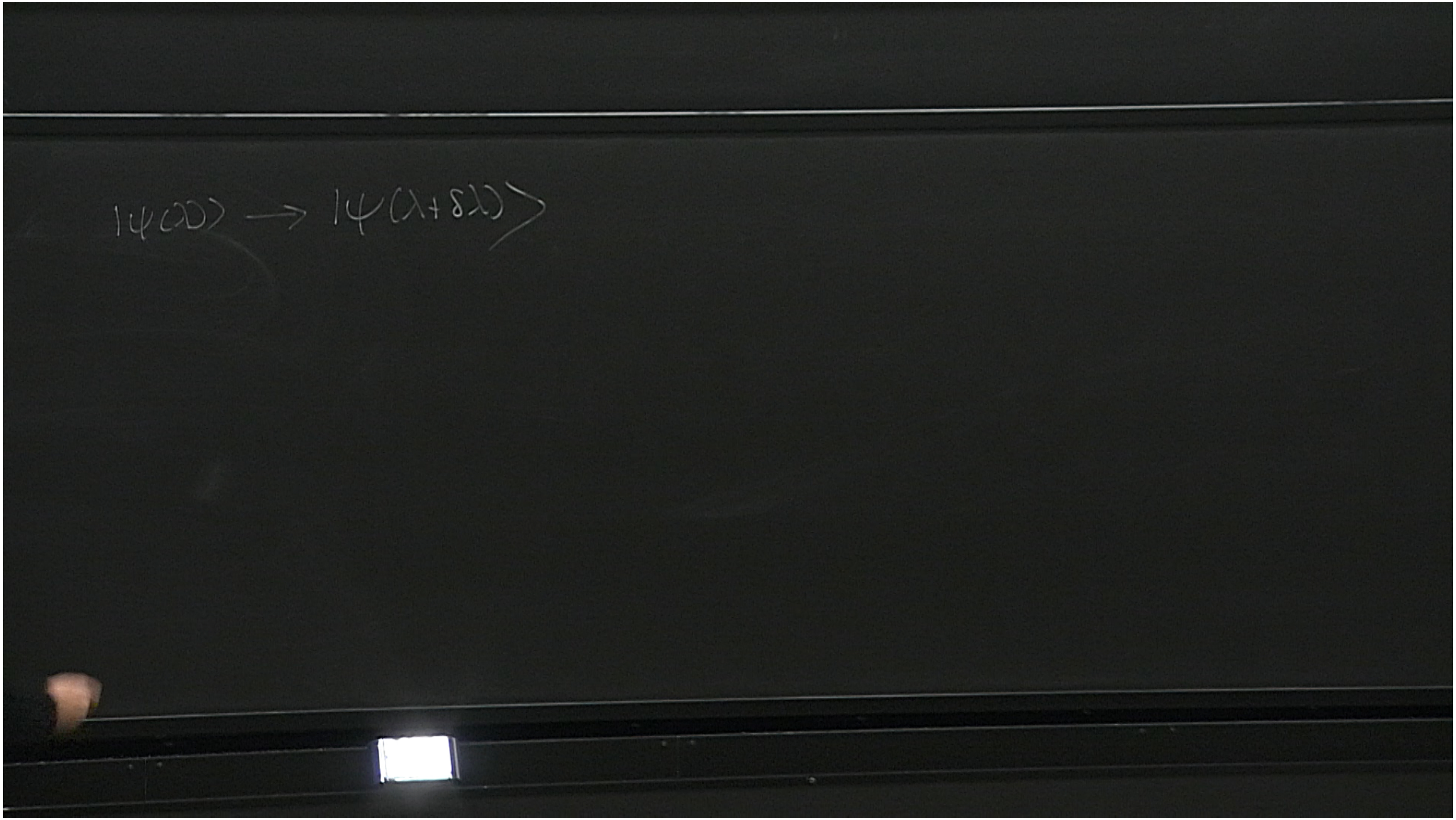

GEOMETRY OF H & QPT

GEOMETRY OF $\mathcal{H}$ & QPT

$\lambda = (\lambda^1, \ldots, \lambda^r)$




GEOMETRY OF $\mathcal{H}$ & QPT
$\lambda = (\lambda^1, \ldots, \lambda^N)$
λ
$\mathcal{M}$
$\lambda \to H(\lambda)$
$|\psi_0(\lambda)\rangle$ is GS
$|\psi(\lambda)\rangle$


|ψ(x)⟩ → |ψ(x+δx)⟩

$|\psi(\lambda)\rangle \rightarrow |\psi(\lambda+\delta\lambda)\rangle$

$d_{FS}(\psi(\lambda), \psi(\lambda+\delta\lambda)) = \cos^{-1} \gamma \simeq \sqrt{1-F^2}$

$\gamma = |\langle\psi(\lambda)|\psi(\lambda+\delta\lambda)\rangle|$

$$|\psi(\lambda)\rangle \rightarrow |\psi(\lambda+\delta\lambda)\rangle$$

$$d_{FS}(\psi(\lambda), \psi(\lambda+\delta\lambda)) = \cos^{-1}\gamma \simeq \sqrt{1-F^2}$$

$$\gamma = |\langle\psi(\lambda)|\psi(\lambda+\delta\lambda)\rangle|$$

$$F^2 = \langle\psi(\lambda)|\psi(\lambda+\delta\lambda)\rangle\langle\psi(\lambda+\delta\lambda)|\psi(\lambda)\rangle \simeq$$

$$\beta_\mu(\lambda) = -i \langle \psi(\lambda) | \partial_\mu \psi(\lambda) \rangle$$

$$\simeq \sqrt{1-F^2}$$

$$\lambda) | \psi(\lambda) \rangle \simeq \cdots$$

$$\beta_\mu(\lambda) = -i \langle \psi(\lambda) | \partial_\mu \psi(\lambda) \rangle$$

$$\gamma_{\mu\nu}(\lambda) = \langle \partial_\mu \psi(\lambda) | \partial_\nu \psi(\lambda) \rangle$$

$$\simeq \sqrt{1-F^2}$$

$$\lambda) | \psi(\lambda) \rangle \simeq \cdots$$

$$\beta_\mu(\lambda) = -i \langle \psi(\lambda) | \partial_\mu \psi(\lambda) \rangle$$

$$\gamma_{\mu\nu}(\lambda) = \langle \partial_\mu \psi(\lambda) | \partial_\nu \psi(\lambda) \rangle$$

$$Q_{\mu\nu} = \gamma_{\mu\nu} - \beta_\mu \beta_\nu$$

QUANTUM GEOMETRIC TENSOR

$$\simeq \sqrt{1 - F^2}$$

$$\lambda) |\psi(\lambda)\rangle \simeq \cdots$$

$$\beta_\mu(\lambda) = -i \langle \psi(\lambda) | \partial_\mu \psi(\lambda) \rangle$$

$$\gamma_{\mu\nu}(\lambda) = \langle \partial_\mu \psi(\lambda) | \partial_\nu \psi(\lambda) \rangle$$

$$Q_{\mu\nu} = \gamma_{\mu\nu} - \beta_\mu \beta_\nu$$

QUANTUM GEOMETRIC TENSOR

$$Re\, Q_{\mu\nu} = g_{\mu\nu} \rightarrow \text{Riemannian tensor}$$

$$Im\, Q_{\mu\nu} = i(\partial_\mu \beta_\nu - \partial_\nu \beta_\mu) = F_{\mu\nu}$$

$\simeq \sqrt{1 - F^2}$

$\rangle |\psi(\lambda)\rangle \simeq \cdots$

ℓ QPT

$(\lambda^1, \ldots, \lambda^n)$

$|\psi(\lambda)\rangle \to |\psi(\lambda+\delta\lambda)\rangle$

$d_{FS}(\psi(\lambda), \psi(\lambda+\delta\lambda)) = \cos^{-1} f \simeq \sqrt{1-F^2}$

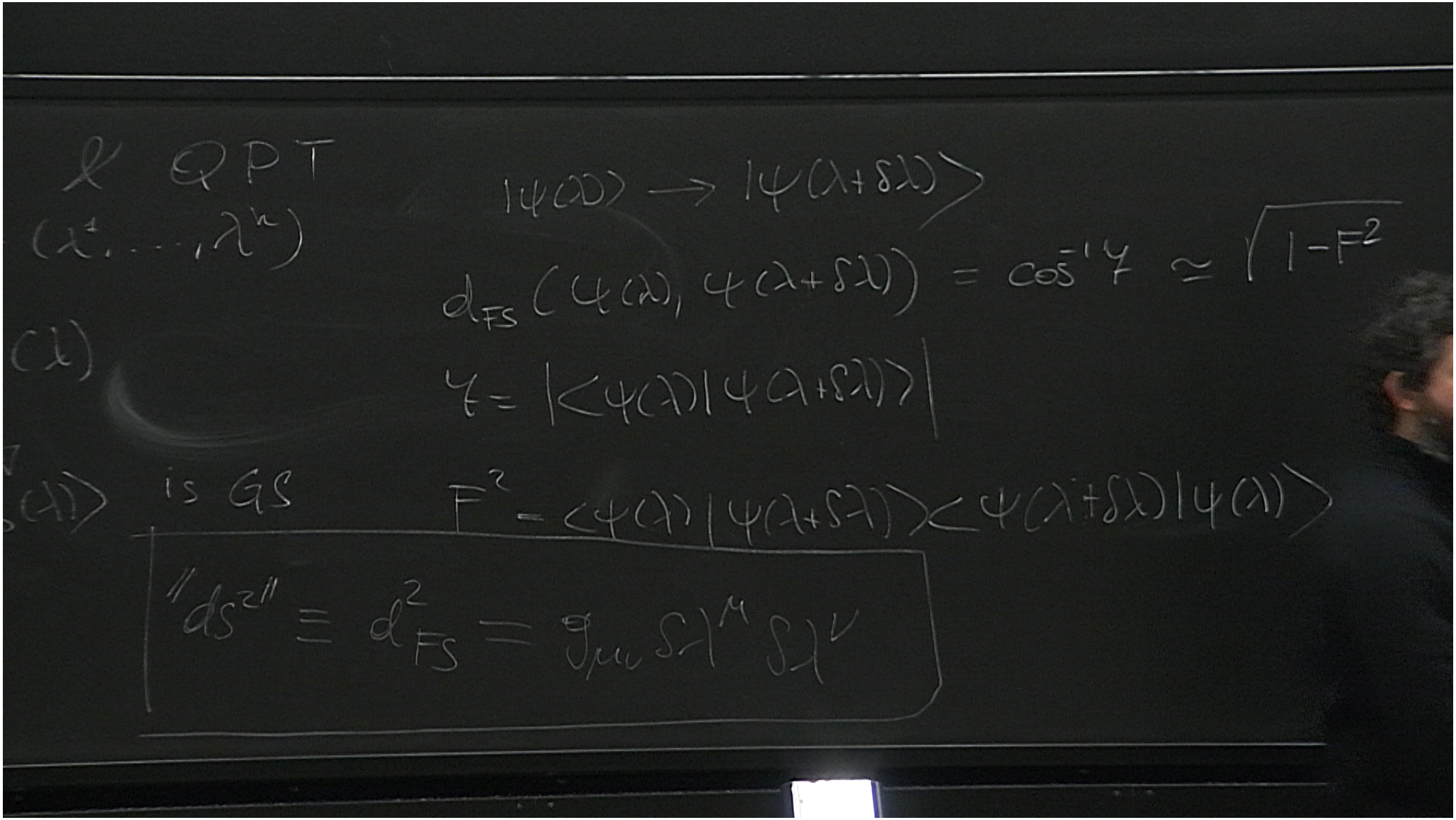
(1)

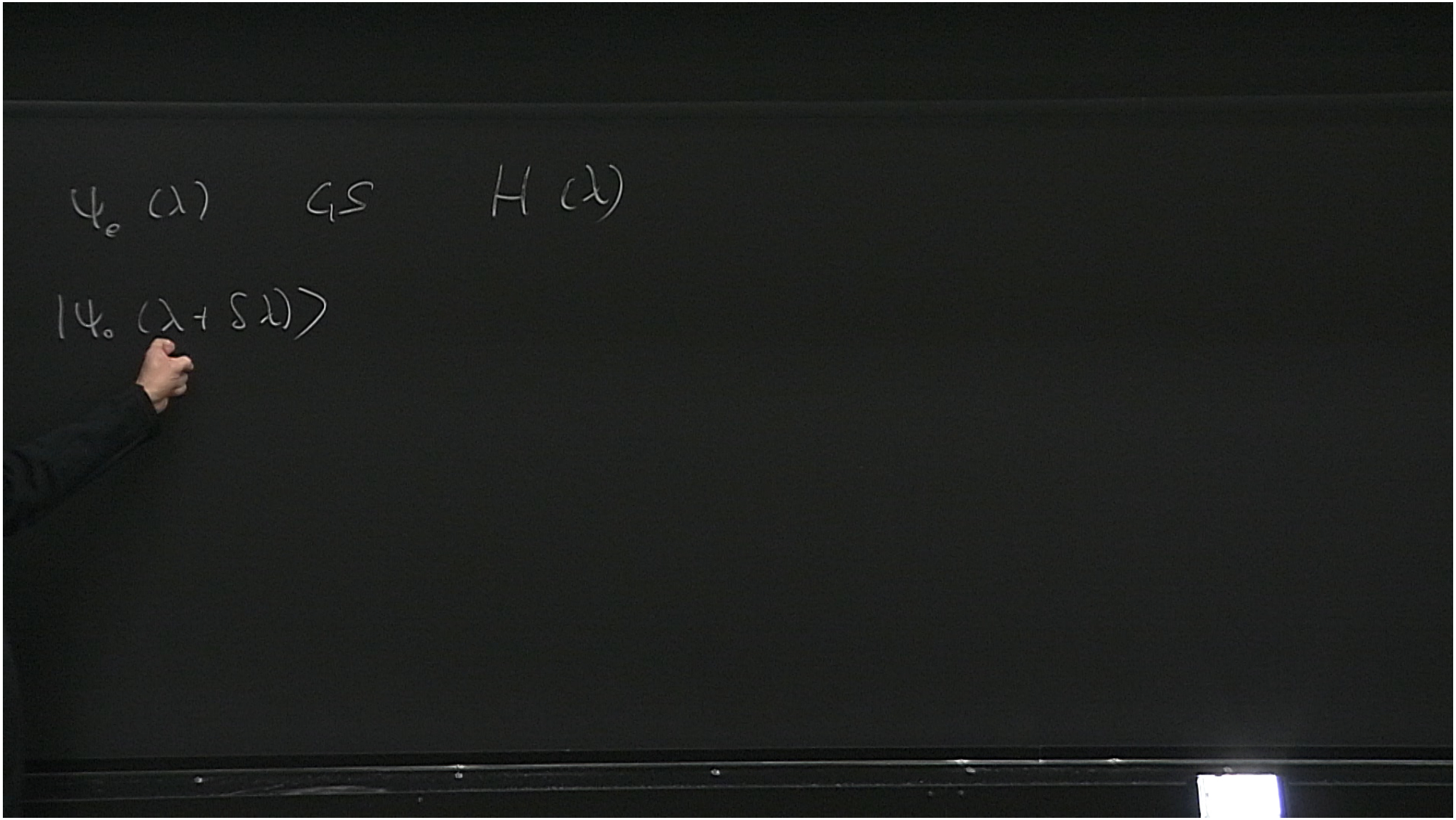
$f = |\langle\psi(\lambda)|\psi(\lambda+\delta\lambda)\rangle|$

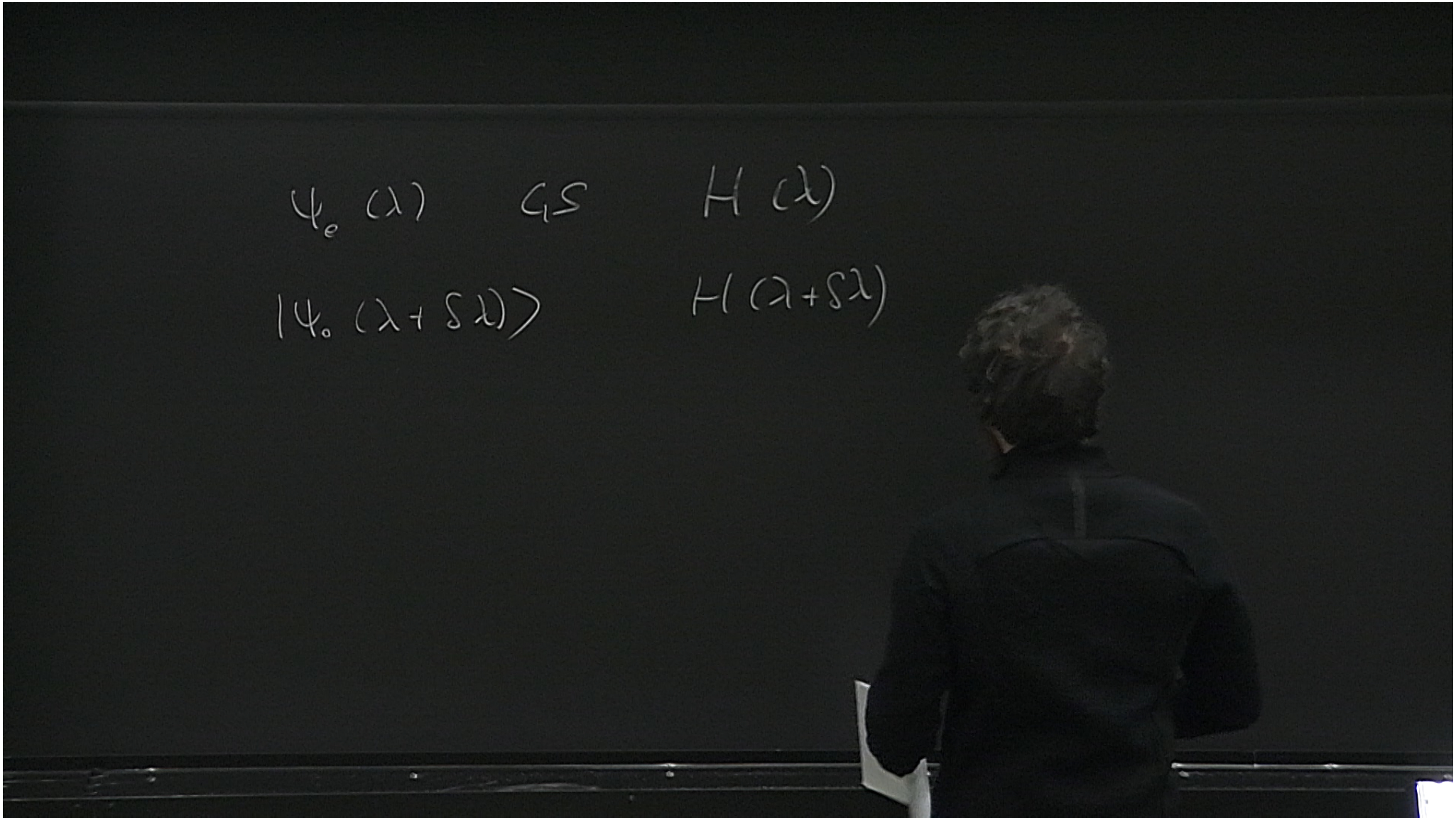
$|\psi(\lambda)\rangle$ is GS

$F^2 = \langle\psi(\lambda)|\psi(\lambda+\delta\lambda)\rangle\langle\psi(\lambda+\delta\lambda)|\psi(\lambda)\rangle \simeq \cdots$

$"ds^2" \equiv d_{FS}^2 =$


ℓ QPT
$(\lambda^1, \ldots, \lambda^n)$
$|\psi(\lambda)\rangle \to |\psi(\lambda+\delta\lambda)\rangle$
$d_{FS}(\psi(\lambda), \psi(\lambda+\delta\lambda)) = \cos^{-1} \gamma \simeq \sqrt{1-F^2}$
(1)
$\gamma = |\langle\psi(\lambda)|\psi(\lambda+\delta\lambda)\rangle|$
$|\psi(\lambda)\rangle$ is GS
$F^2 = \langle\psi(\lambda)|\psi(\lambda+\delta\lambda)\rangle\langle\psi(\lambda+\delta\lambda)|\psi(\lambda)\rangle$
$"ds^2" \equiv d_{FS}^2 = g_{\mu\nu}\delta\lambda^\mu\delta\lambda^\nu$


$\psi_0(\lambda)$ GS $H(\lambda)$
$|\psi_0(\lambda+\delta\lambda)\rangle$


$\psi_0(\lambda)$ GS $H(\lambda)$
$|\psi_0(\lambda+\delta\lambda)\rangle$ $H(\lambda+\delta\lambda)$

$\psi_0(\lambda)$ GS $H(\lambda)$

$|\psi_0(\lambda+\delta\lambda)\rangle$ $H(\lambda+\delta\lambda) = H(\lambda) + \partial_\mu H\, \delta\lambda^\mu$

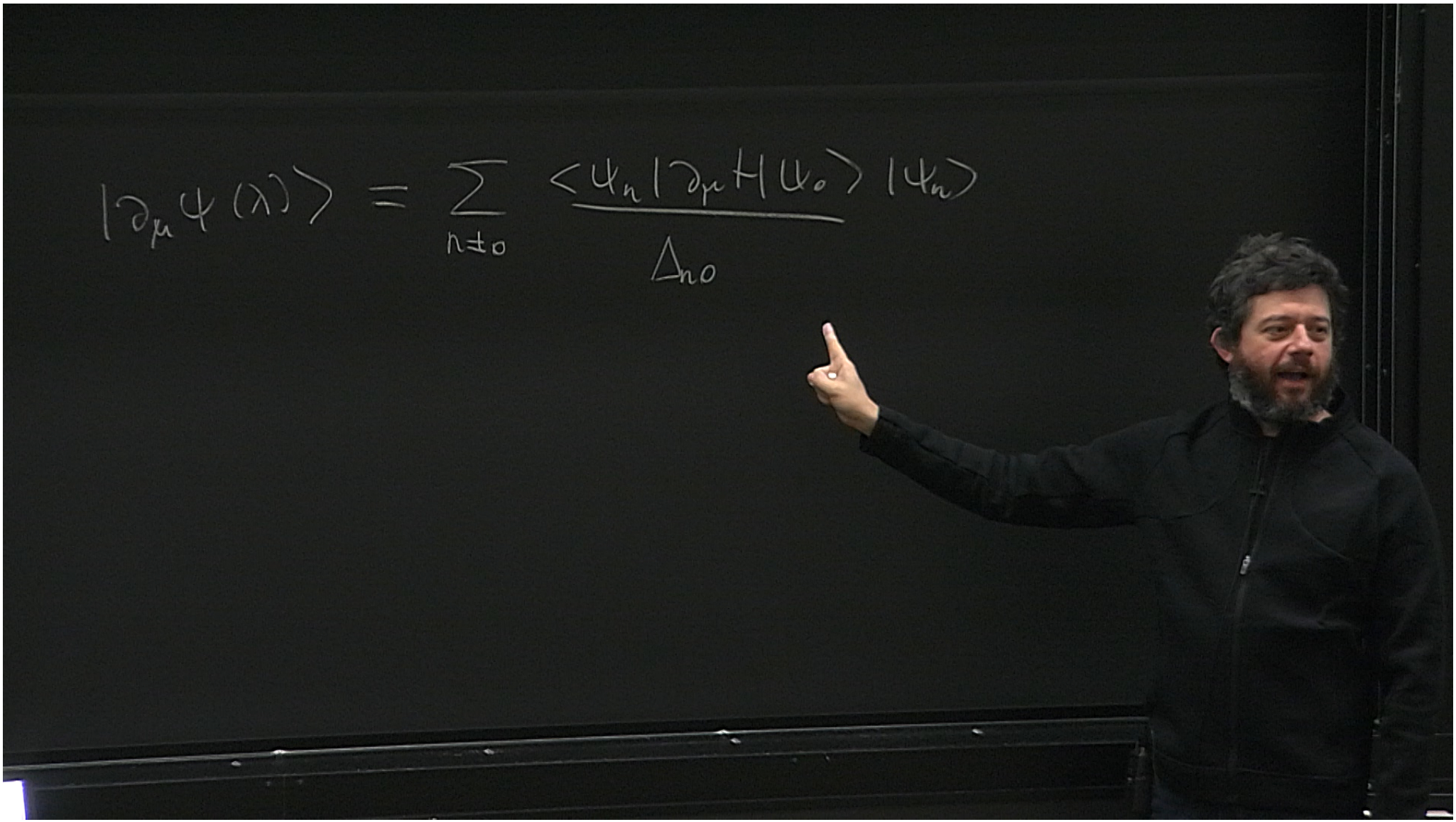
$\psi_0(\lambda)$ GS $H(\lambda)$

$|\psi_0(\lambda+\delta\lambda)\rangle$ $\quad H(\lambda+\delta\lambda) = H(\lambda) + \partial_\mu H \, \delta\lambda^\mu$

$$\| $$

$$|\psi_0(\lambda)\rangle + \sum_{n\neq 0} \frac{\langle \psi_n | \partial_\mu H \delta\lambda^\mu | \psi_0 \rangle}{\Delta_{n0}} |\psi_n\rangle$$

$$|\partial_\mu \psi(\lambda)\rangle = \sum_{n \neq 0} \frac{\langle \psi_n | \partial_\mu H | \psi_0 \rangle}{\Delta_{n0}} |\psi_n\rangle$$



$$|\partial_\mu \psi(\lambda)\rangle = \sum_{n \neq 0} \frac{\langle \psi_n | \partial_\mu H | \psi_0 \rangle}{\Delta_{n0}} |\psi_n\rangle$$

$p_\mu = 0$

$$\langle \partial_\mu \psi | \partial_\nu \psi \rangle = \sum_{n \neq 0} \frac{\langle \psi_0 | \partial_\mu H | \psi_n \rangle \langle \psi_n | \partial_\nu H | \psi_0 \rangle}{\Delta_{n0}^2}$$

$$|\partial_\mu \psi(\lambda)\rangle = \sum_{n \neq 0} \frac{\langle \psi_n | \partial_\mu H | \psi_0 \rangle}{\Delta_{n0}} |\psi_n\rangle$$

$p_\mu = 0$

$$Q_{\mu\nu} = \langle \partial_\mu \psi | \partial_\nu \psi \rangle = \sum_{n \neq 0} \frac{\langle \psi_0 | \partial_\mu H | \psi_n \rangle \langle \psi_n | \partial_\nu H | \psi_0 \rangle}{\Delta_{n0}^2}$$

+ ...

Application XY

$$H(\lambda,\gamma) = -\sum_{i=-M}^{M} \left(\frac{1+\gamma}{2} \sigma_i^x \sigma_{i+1}^x + \frac{1-\gamma}{2} \sigma_i^y \sigma_{i+1}^y - \lambda \sigma_i^z \right)$$

Application XY

$$H(\lambda,\gamma) = -\sum_{i=-M}^{M} \left(\frac{1+\gamma}{2}\sigma_i^x\sigma_{i+1}^x + \frac{1-\gamma}{2}\sigma_i^y\sigma_{i+1}^y - \lambda\sigma_i^z \right)$$

$$|\psi_0(\theta)\rangle = \bigotimes_{k=1}^{M} \left(\cos\frac{\theta_k}{2}\,|0_k 0_{-k}\rangle + i\sin\frac{\theta_k}{2}\,|1_k 1_{-k}\rangle \right)$$

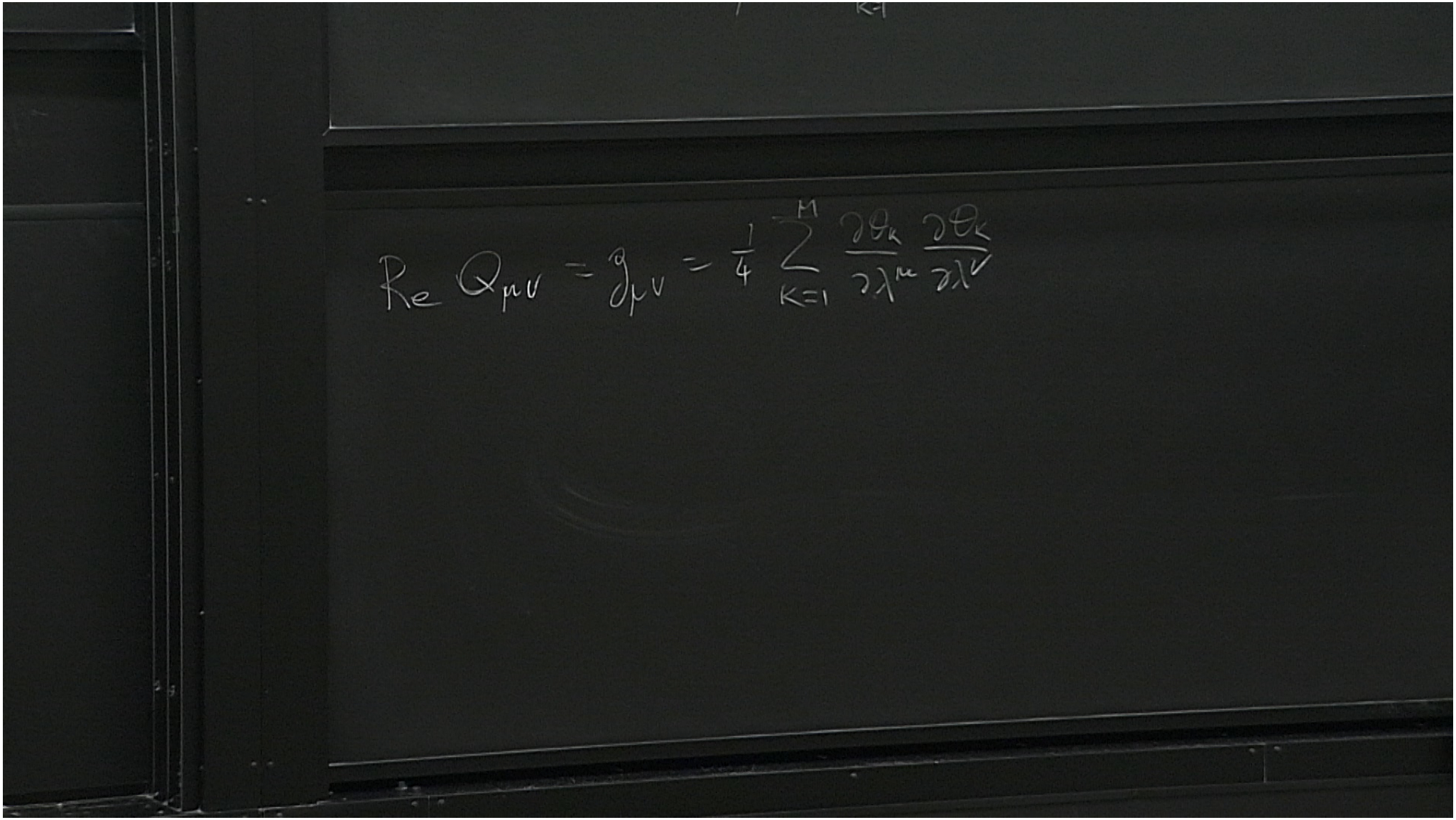
Application XY

$$H(\lambda,\gamma) = -\sum_{i=-M}^{M} \left(\frac{1+\gamma}{2}\sigma_i^x\sigma_{i+1}^x + \frac{1-\gamma}{2}\sigma_i^y\sigma_{i+1}^y - \lambda\sigma_i^z \right)$$

$$|\psi_0(\theta)\rangle = \bigotimes_{k=1}^{M} \left(\cos\frac{\theta_k}{2}|0\rangle_k|0\rangle_{-k} + i\sin\frac{\theta_k}{2}|1\rangle_k|1\rangle_{-k} \right)$$

Homework

$$\langle\psi_0(\bar{\theta})|\psi_0(\theta)\rangle = \prod_{k=1}^{M}\cos\frac{\bar{\theta}_k-\theta_k}{2}$$


$\mathrm{Re}\, Q_{\mu\nu} = g_{\mu\nu} = \frac{1}{4} \sum_{K=1}^{M} \frac{\partial \theta_K}{\partial \lambda^\mu} \frac{\partial \theta_K}{\partial \lambda^\nu}$

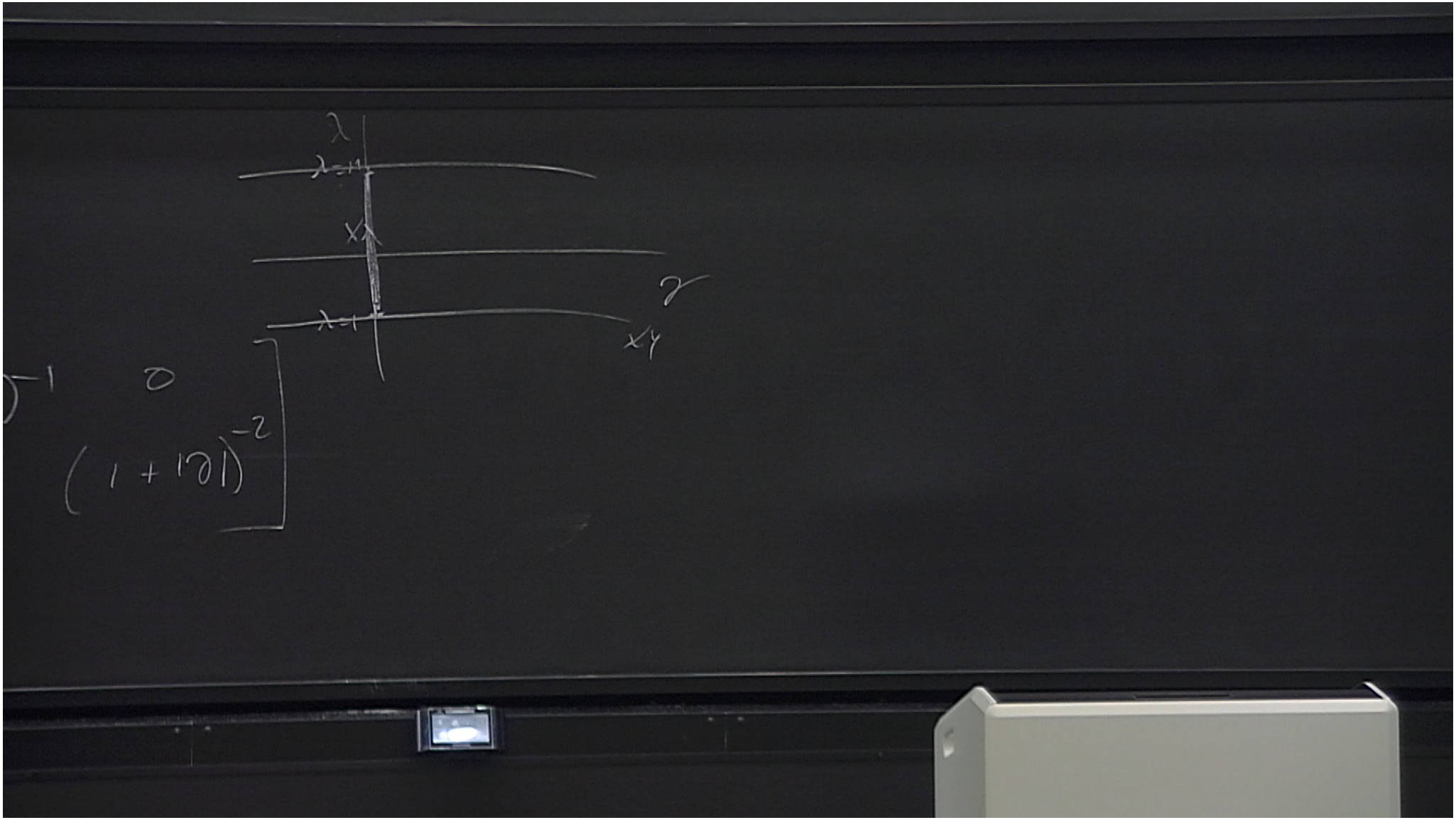
$$\mathrm{Re}\, Q_{\mu\nu} = g_{\mu\nu} = \frac{1}{4} \sum_{k=1}^{M} \frac{\partial \theta_k}{\partial \lambda^\mu} \frac{\partial \theta_k}{\partial \lambda^\nu}$$

$M \longrightarrow \infty$ th. limit $\qquad \sum_{k=1}^{M} \longrightarrow \frac{N}{2\pi} \int_0^{\pi} dx$

$$\mathrm{Re}\, Q_{\mu\nu} = g_{\mu\nu} = \frac{1}{4} \sum_{k=1}^{M} \frac{\partial \theta_k}{\partial \lambda^\mu} \frac{\partial \theta_k}{\partial \lambda^\nu}$$

$M \to \infty$ th. limit $\qquad \sum_{k=1}^{M} \to \frac{N}{2\pi} \int_0^{\pi} d\chi$

$|\lambda| \leq 1 \qquad g_{\mu\nu} = \frac{N}{16|\lambda|} \begin{bmatrix} (1-\lambda^2)^{-1} & 0 \\ 0 & (1+|\lambda|)^{-2} \end{bmatrix}$


λ
λ=+1
λ=-1
2
x4
0
(1+|2|)^-2

$$R = \text{Ricci scalar} = \begin{cases} -\frac{16}{N}\,\frac{1+|\gamma|}{|\gamma|} & |\lambda|<1 \\ \frac{16}{N}\,\frac{|\lambda|+\sqrt{\lambda^2+\gamma^2-1}}{\sqrt{\lambda^2+\gamma^2-1}} & |\lambda|>1 \end{cases}$$

$g_{\mu\nu}$ $|\phi_i\rangle$ its eigenvectors

$|\phi\rangle$ "max" eigenvector

$|g_{\mu\nu}| \leq \langle\phi| g_{\mu\nu} |\phi\rangle$

$|\phi\rangle = \phi^\mu |e_\mu\rangle$

$\delta H \equiv \sum_\mu (\partial_\mu H)\, \phi^\mu$

its eigenvectors

"max" eigenvector

$$\delta H \equiv \sum_\mu (\partial_\mu H)\, \phi^\mu = \langle \partial_\mu H | \phi \rangle$$

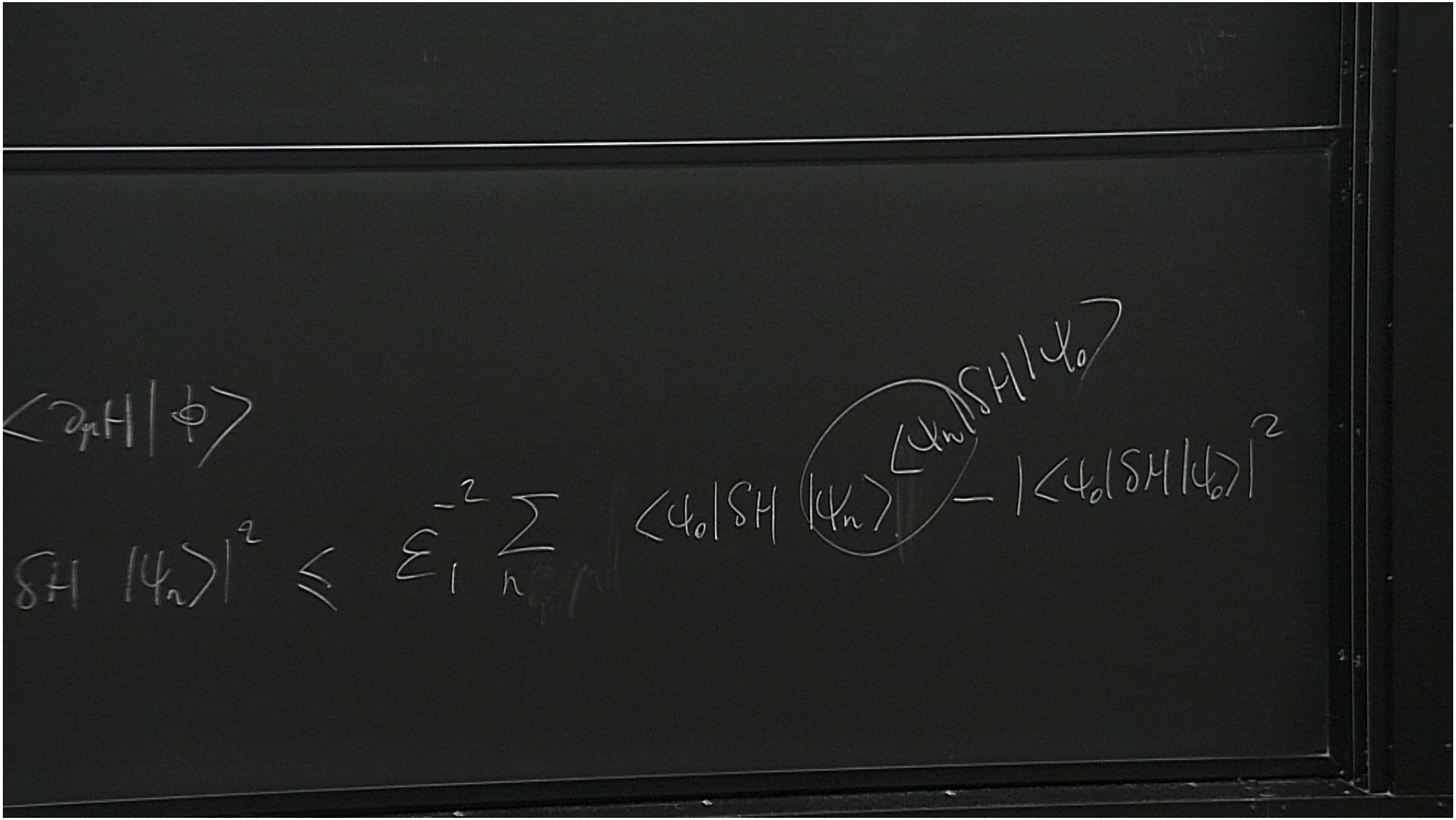
$$|g_{\mu\nu}| \leq \sum_{n>0} \Delta_{n0}^{-2} \, |\langle \psi_0 | \delta H | \psi_n \rangle|^2$$

its eigenvectors

"max" eigenvector

$$\delta H \equiv \sum_\mu (\partial_\mu H)\, \phi^\mu = \langle \partial_\mu H | \phi \rangle$$

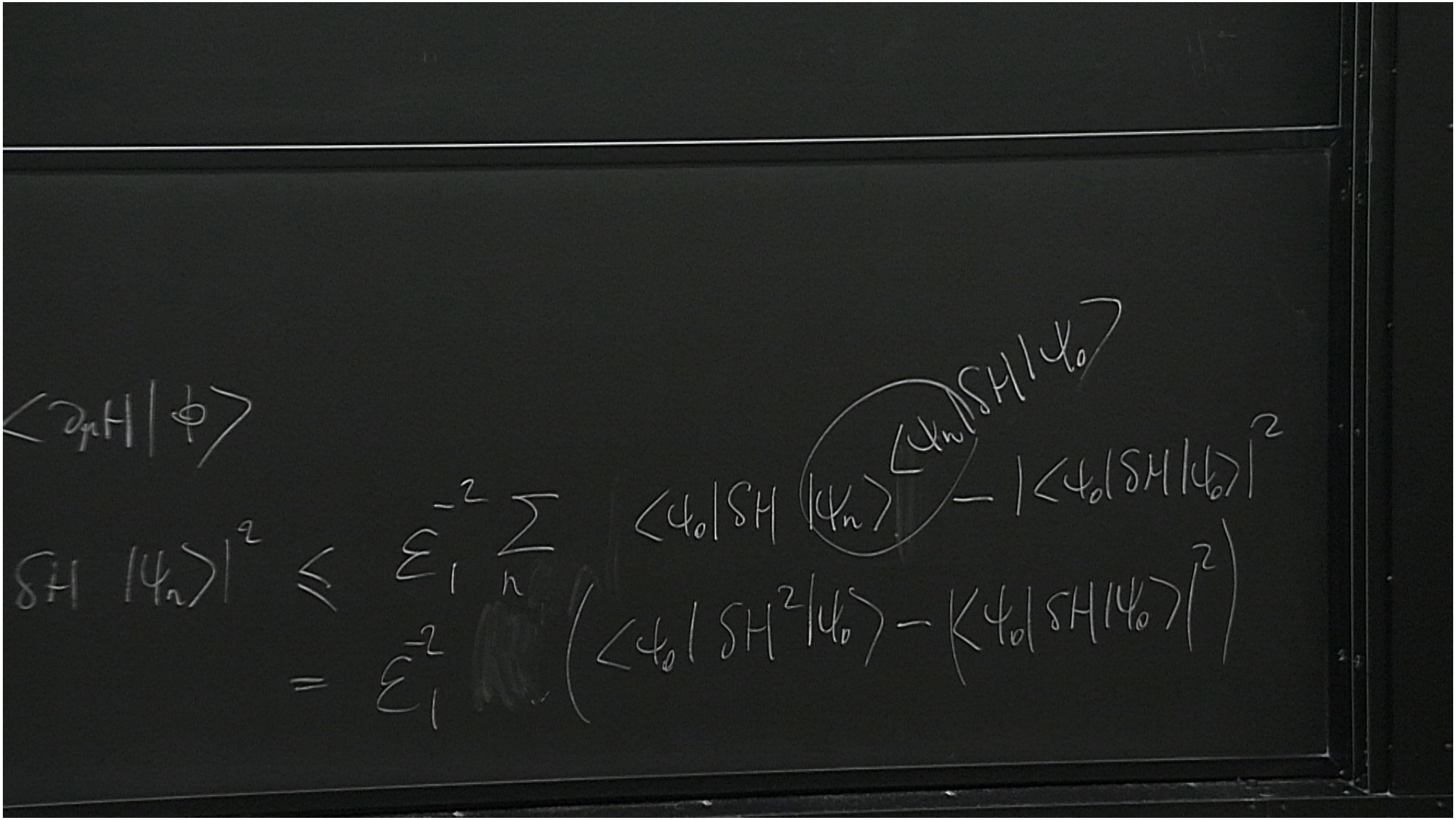
$$|g_{\mu\nu}| \leq \sum_{n>0} \Delta_{n0}^{-2} \, |\langle \psi_0 | \delta H | \psi_n \rangle|^2 \leq \varepsilon_1^{-2} \sum_n \langle \psi_0 | \delta H | \psi_n \rangle \langle \psi_n | \delta H | \psi_0 \rangle - |\langle \psi_0 | \delta H | \psi_0 \rangle|^2$$

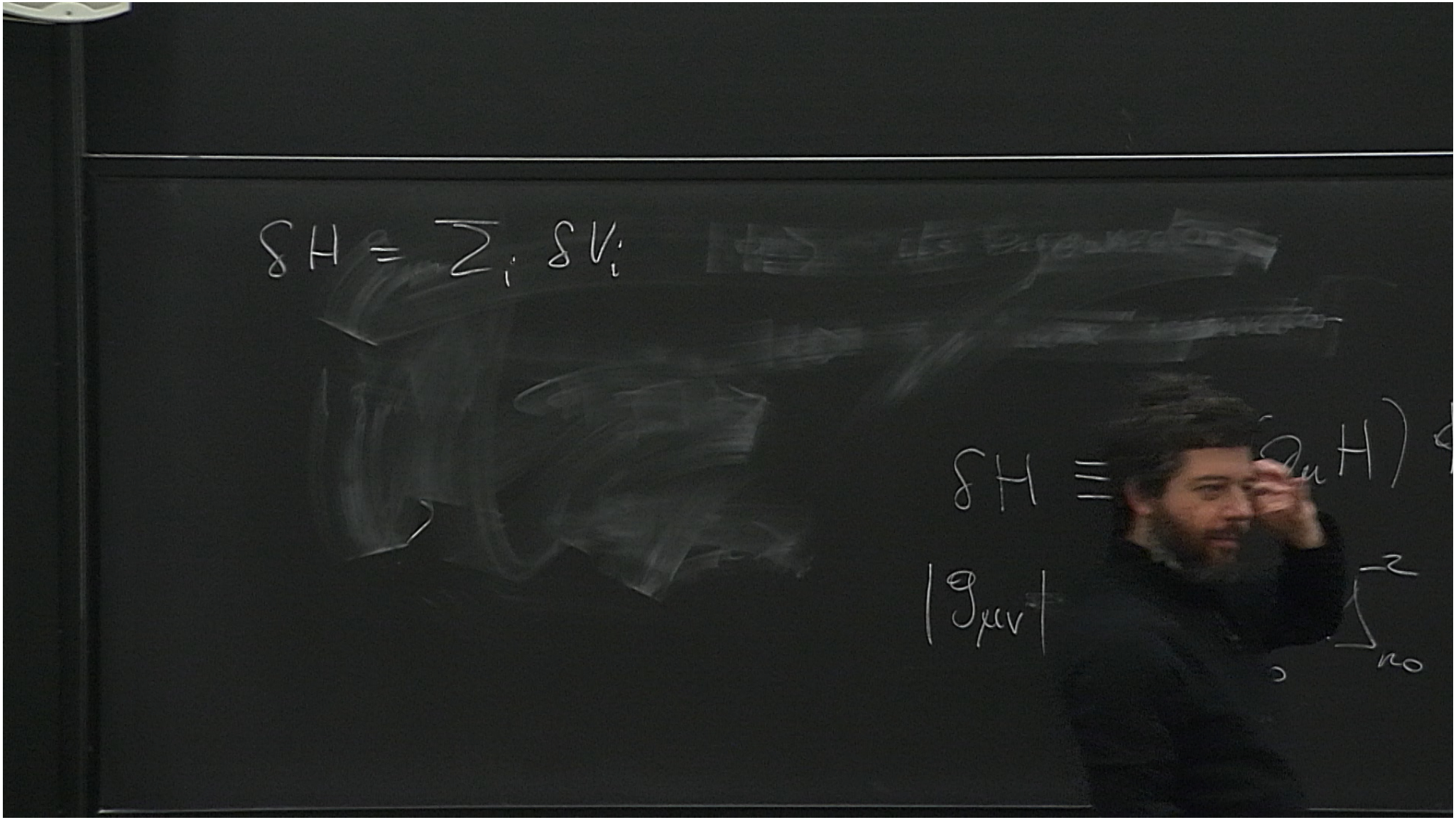

$\langle \partial_n H | \phi \rangle$
$|\delta H |\psi_n\rangle|^2 \leq \varepsilon_1^{-2} \sum_n$
$\langle \psi_0 | \delta H | \psi_n \rangle \langle \psi_n | \delta H | \psi_0 \rangle - |\langle \psi_0 | \delta H | \psi_0 \rangle|^2$

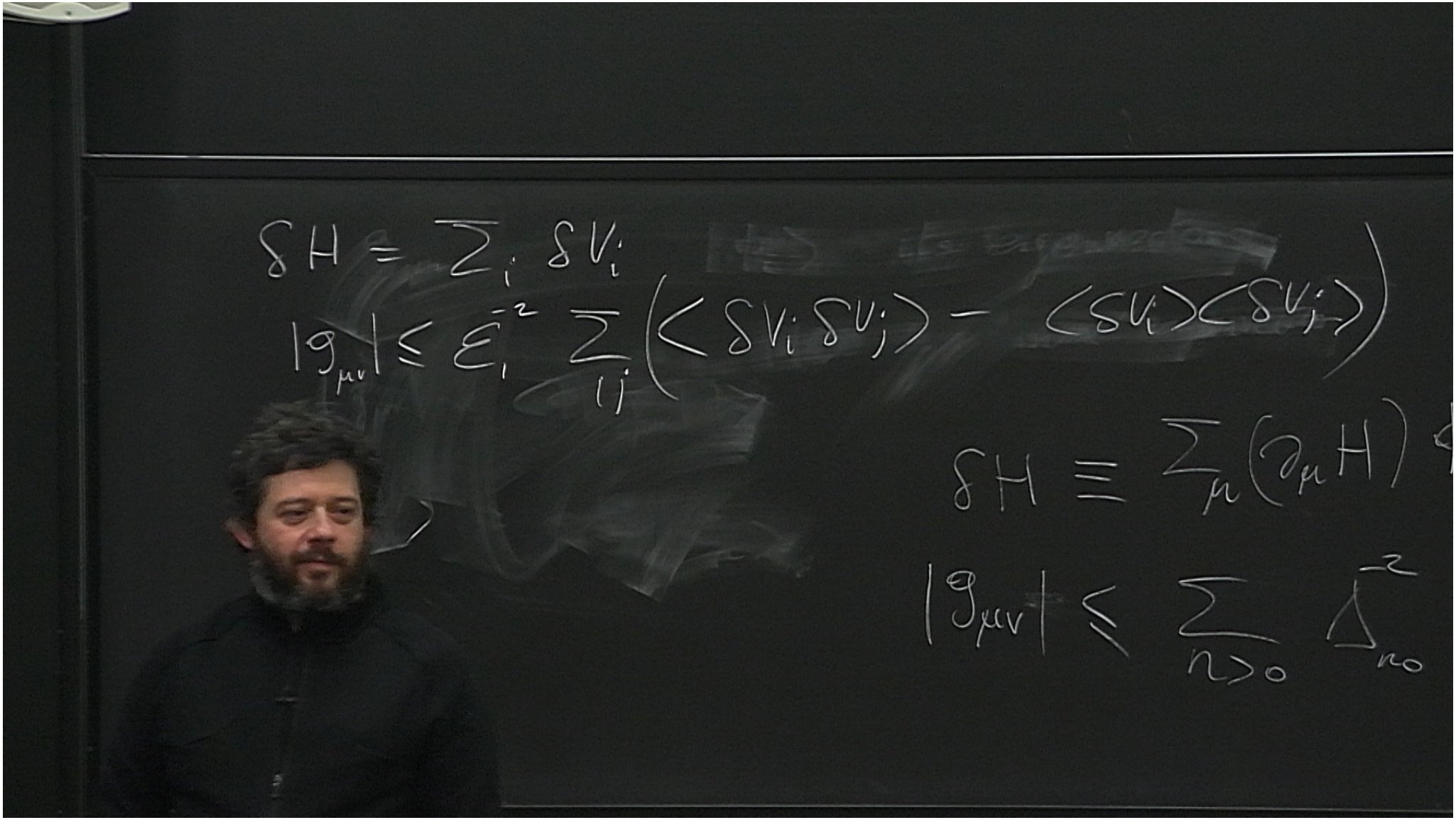
$$\langle \partial_n H | \phi \rangle$$

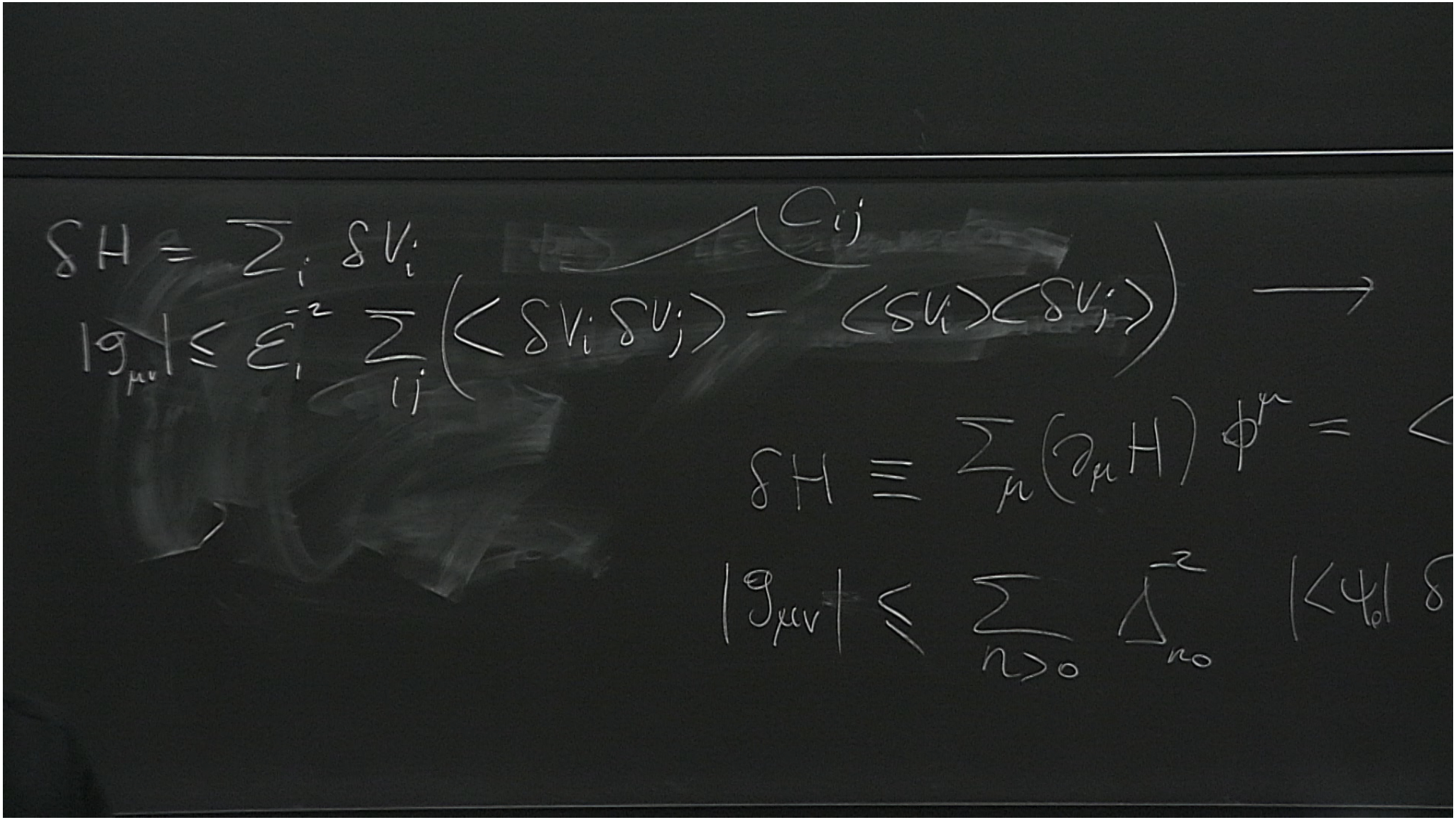
$$|\delta H |\psi_n\rangle|^2 \leq \varepsilon_1^{-2} \sum_n \langle \psi_0 | \delta H |\psi_n\rangle \langle \psi_n | \delta H | \psi_0 \rangle - |\langle \psi_0 | \delta H | \psi_0 \rangle|^2$$

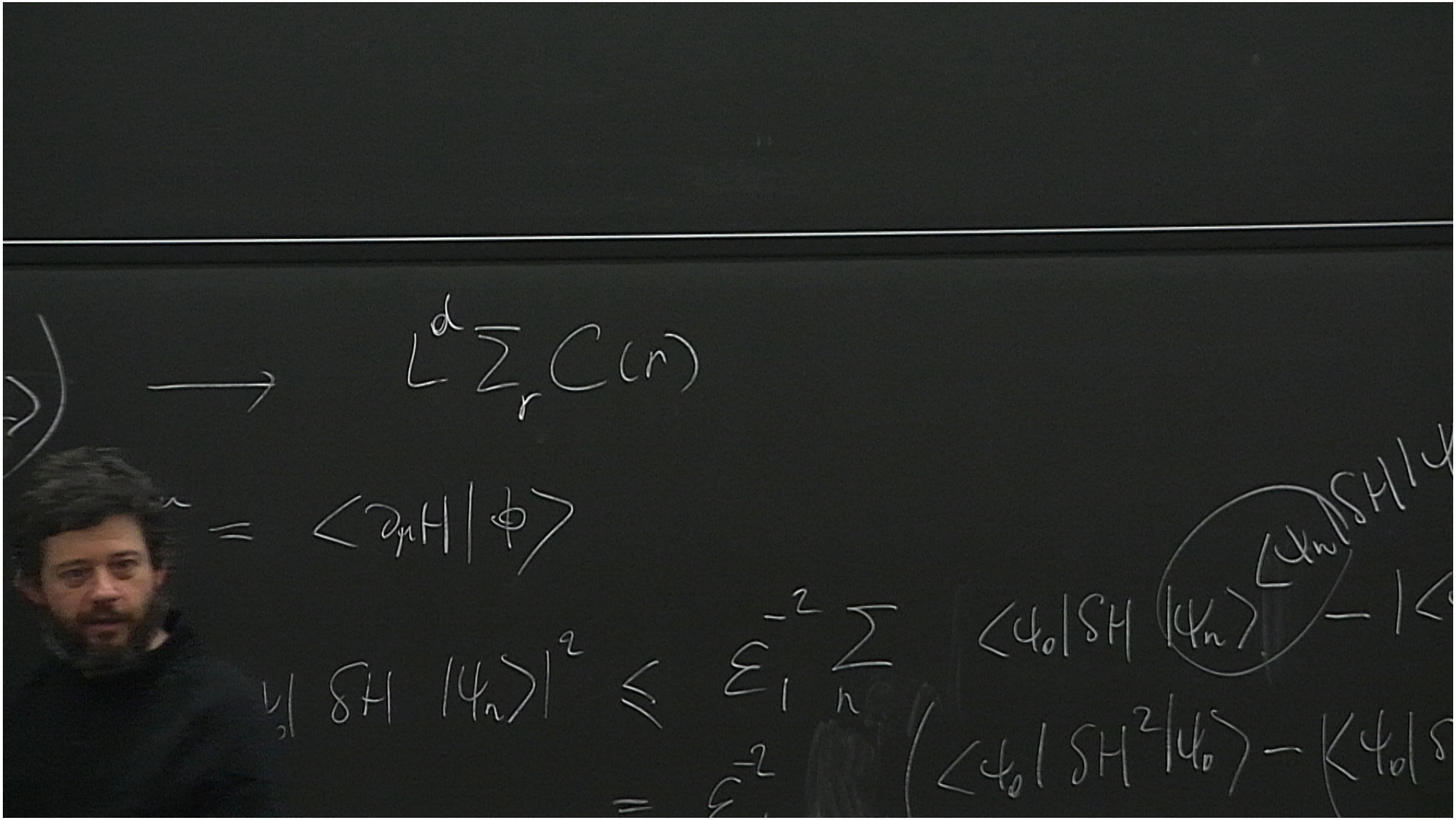
$$= \varepsilon_1^{-2} \sum_n \left(\langle \psi_0 | \delta H^2 | \psi_0 \rangle - |\langle \psi_0 | \delta H | \psi_0 \rangle|^2 \right)$$

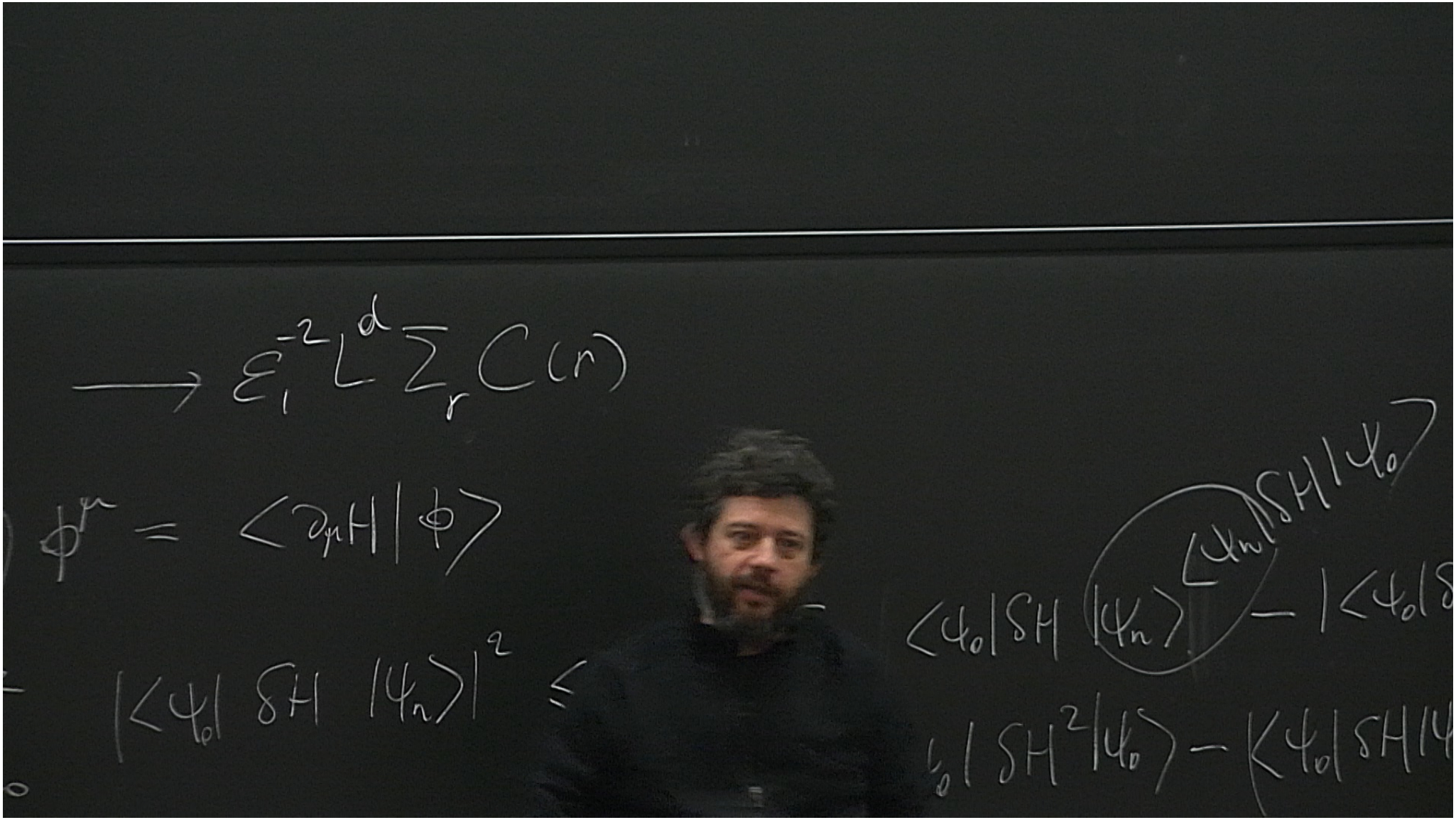

$\langle \partial_n H | \phi \rangle$
$|\delta H |\psi_n\rangle|^2 \leq \varepsilon_1^{-2} \sum_n$
$\langle \psi_0 | \delta H | \psi_n \rangle \langle \psi_n | \delta H | \psi_0 \rangle - |\langle \psi_0 | \delta H | \psi_0 \rangle|^2$
$= \varepsilon_1^{-2} \left(\langle \psi_0 | \delta H^2 | \psi_0 \rangle - |\langle \psi_0 | \delta H | \psi_0 \rangle|^2 \right)$

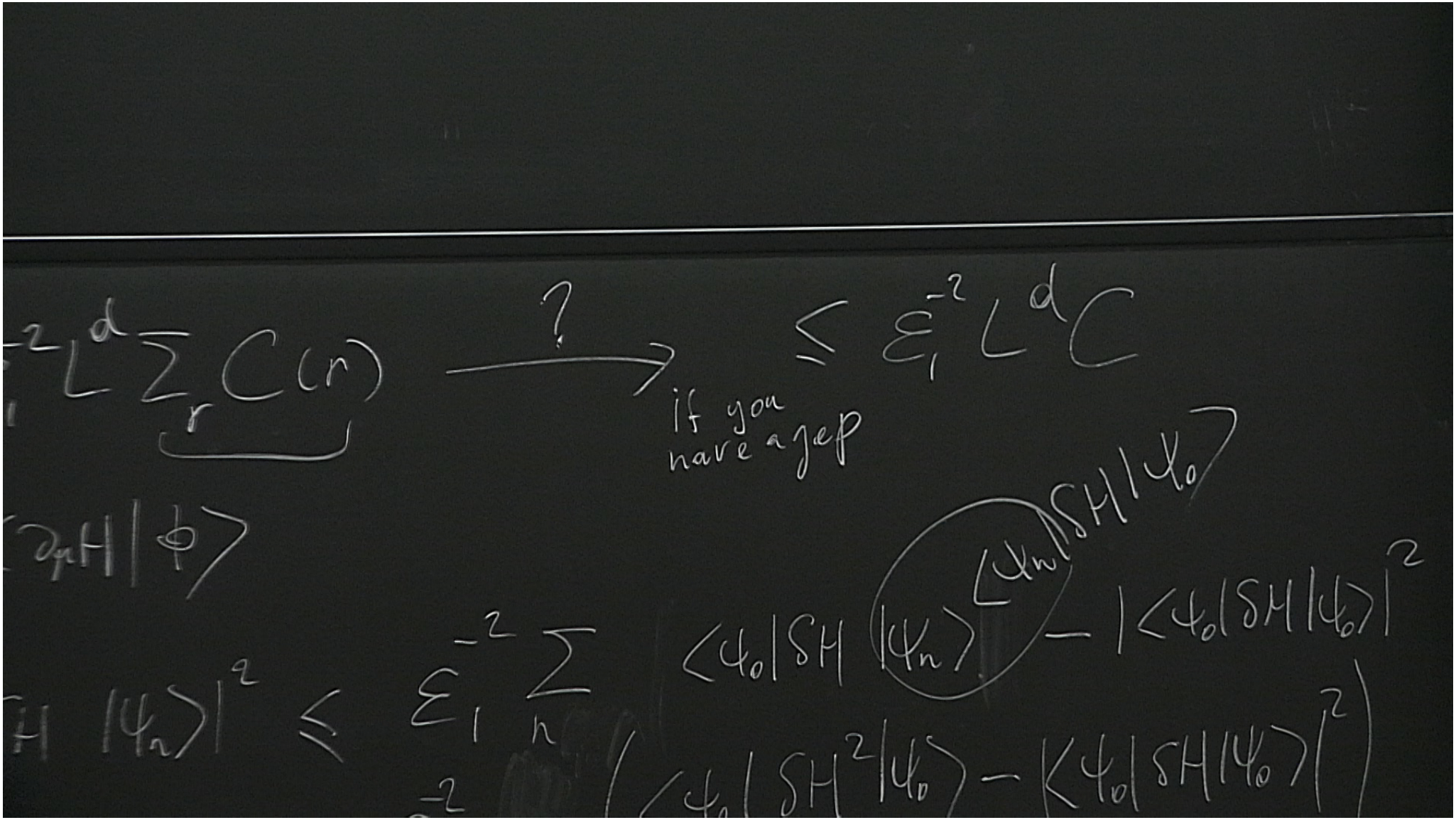

$\delta H = \sum_i \delta V_i$
$\delta H \equiv$
$|g_{\mu\nu}|$


$\delta H = \sum_i \delta V_i$
$|g_{\mu\nu}| \leq \varepsilon_i^{-2} \sum_{ij} \left(\langle \delta V_i \delta V_j \rangle - \langle \delta V_i \rangle \langle \delta V_j \rangle \right)$
$\delta H \equiv \sum_\mu (\partial_\mu H)$
$|g_{\mu\nu}| \leq \sum_{n>0} \Delta_{n0}^{-2}$


$\delta H = \sum_i \delta V_i$
c_{ij}
$|g_{\mu\nu}| \leq \varepsilon_i^{-2} \sum_{ij} \left(\langle \delta V_i \delta V_j \rangle - \langle \delta V_i \rangle \langle \delta V_j \rangle \right) \longrightarrow$
$\delta H \equiv \sum_\mu (\partial_\mu H) \phi^\mu =$
$|g_{\mu\nu}| \leq \sum_{n>0} \Delta_{n0}^{-2} |\langle \psi_0 | \delta$


$\longrightarrow L^d \Sigma_r C(r)$
$= \langle \partial_n H | \phi \rangle$
$|\langle \psi_n | \delta H | \psi_n \rangle|^2 \leq \varepsilon_1^{-2} \sum_n$
$\langle \psi_0 | \delta H | \psi_n \rangle \langle \psi_n | \delta H |$
$(\langle \psi_0 | \delta H^2 | \psi_0 \rangle -$


$\longrightarrow \varepsilon_1^{-2} L^d \Sigma_r C(r)$
$\phi^n = \langle \partial_n H | \phi \rangle$
$|\langle \psi_0 | \delta H | \psi_n \rangle|^2 \leq$
$\langle \psi_0 | \delta H | \psi_n \rangle \langle \psi_n | \delta H | \psi_0 \rangle$
$\psi_0 | \delta H^2 | \psi_0 \rangle$


$\varepsilon^{-2} L^d \sum_r C(r)$
?
if you have a gap
$\leq \varepsilon_1^{-2} L^d C$
$\langle \partial_\mu H | \phi \rangle$
$|\langle \psi_n | \delta H | \psi_0 \rangle|^2 \leq \varepsilon_1^{-2} \sum_n$
$\langle \psi_0 | \delta H | \psi_n \rangle \langle \psi_n | \delta H | \psi_0 \rangle - |\langle \psi_0 | \delta H | \psi_0 \rangle|^2$
$\langle \psi_0 | \delta H^2 | \psi_0 \rangle - |\langle \psi_0 | \delta H | \psi_0 \rangle|^2$

