Title: PSI 2018/2019 - Condensed Matter Review - Lecture 4

Date: Jan 10, 2019 11:30 AM

URL: http://pirsa.org/19010035

Abstract:

$$H(\lambda) = \sum_X \Phi_X(\lambda)$$

GS $|\psi_0(\lambda)\rangle$

param. ferro

g_c g

GS unique

$\updownarrow$

Has same symm. of H

gap

$$\langle \sigma_i \sigma_j \rangle \sim e^{-|i-j|/\xi}$$

$\Delta E \to 0$

GS is not unique

Breaks symm. $T|\psi_0^1\rangle = |\psi_0^2\rangle$

$$\langle \sigma^i \sigma^j \rangle - \langle \sigma^i \rangle \langle \sigma^j \rangle = f(g)$$ long-range order

$$|\uparrow\uparrow\uparrow\uparrow\rangle + |\downarrow\downarrow\downarrow\downarrow\rangle$$

$$\langle \sigma^z \rangle \neq 0$$

$$H(\lambda) = \sum_X \Phi_X(\lambda)$$

GS $|\Psi_0(\lambda)\rangle$

param. ferro

$\mathbb{Z}_2$ symm. g_c $\mathbb{Z}_2$ broken g

GS unique

$\updownarrow$

Has same symm. of H

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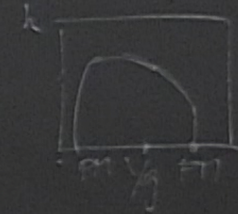
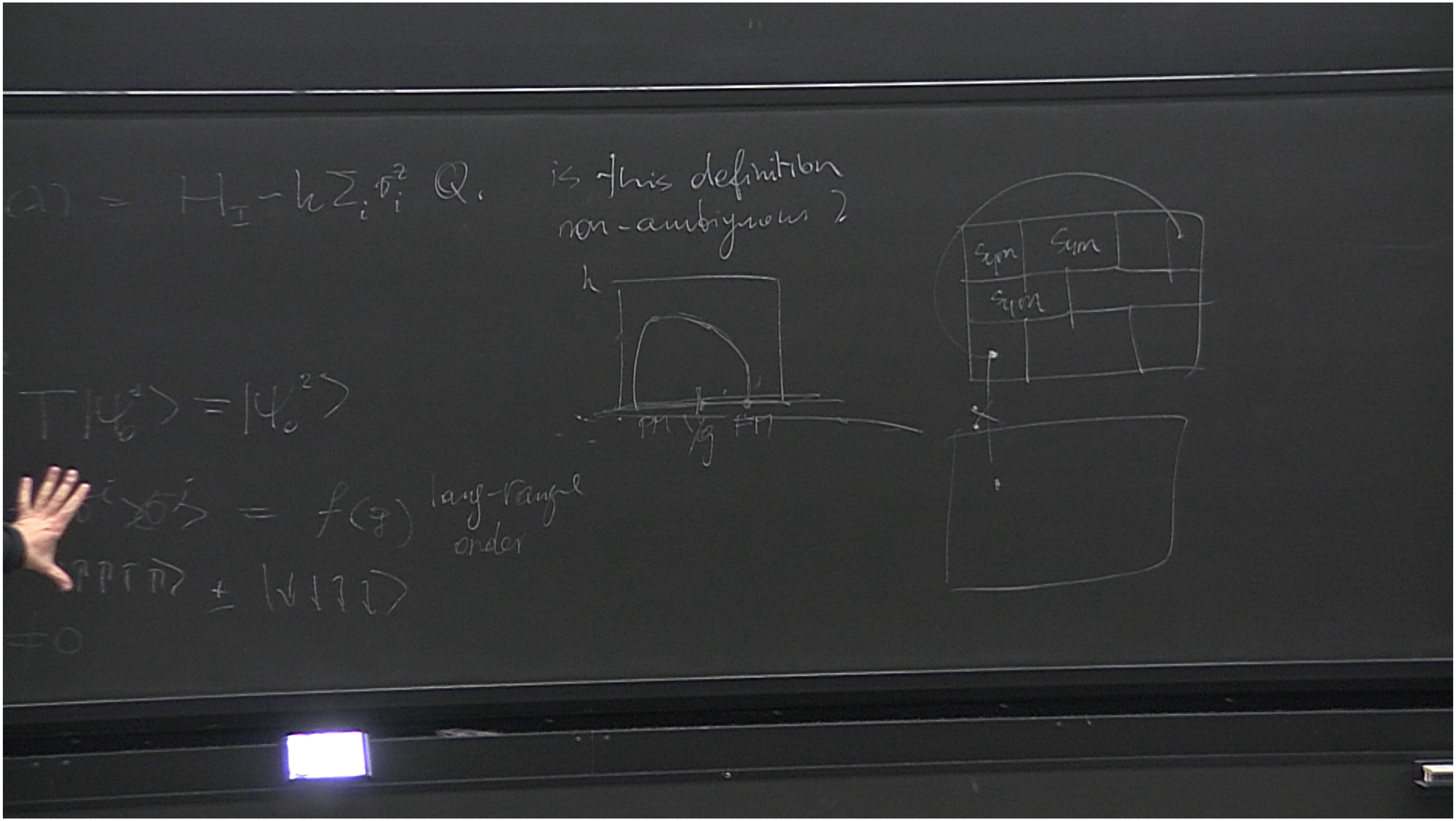
$$|\uparrow\uparrow\uparrow\uparrow\rangle \pm |\downarrow\downarrow\downarrow\downarrow\rangle$$

$$\langle \sigma^z \rangle \neq 0$$

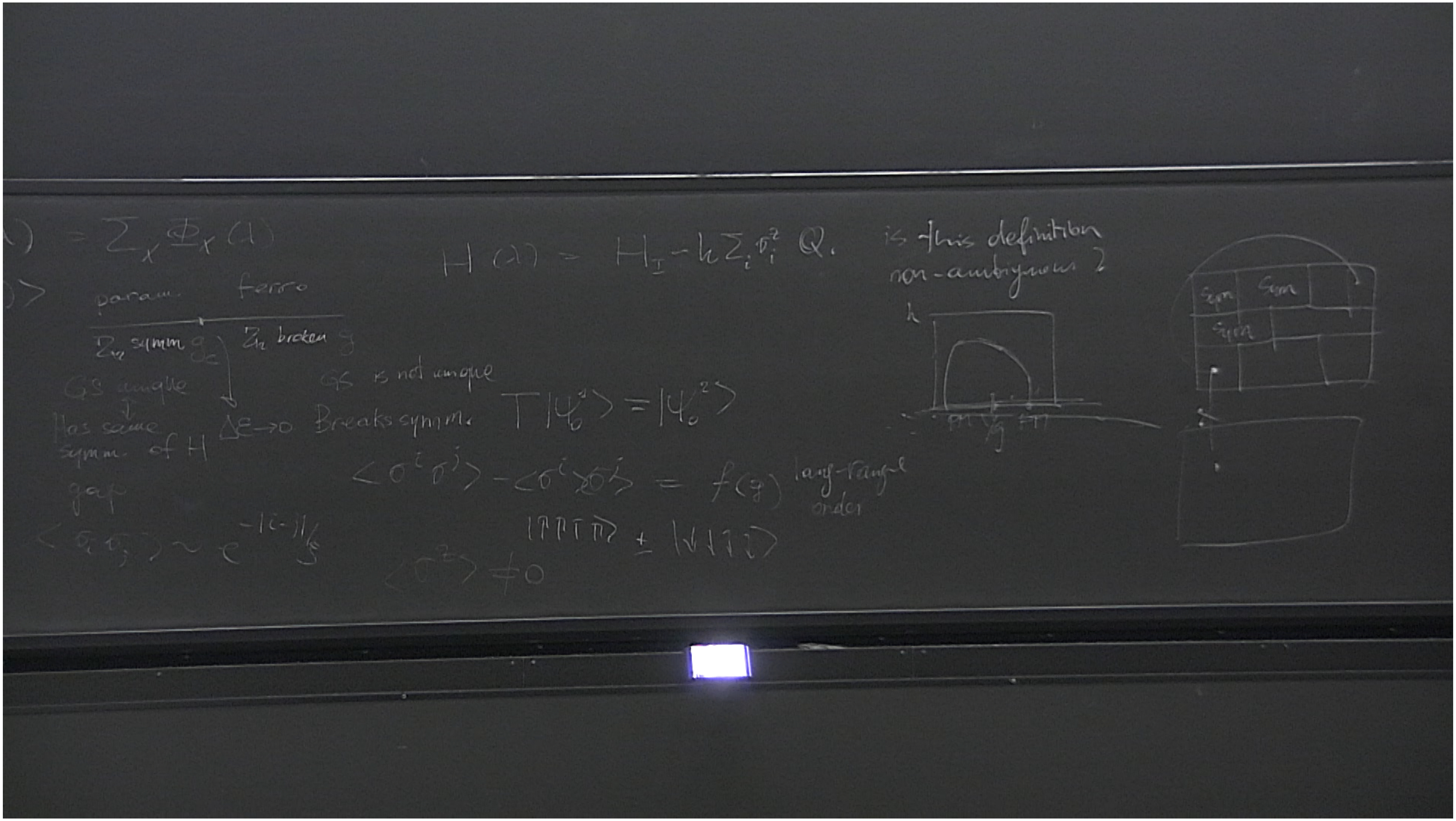
$$H(\lambda) = H_I - h \sum_i \sigma_i^z \quad Q.$$

is this definition non-ambiguous?





is this definition
non-ambiguous ?
h
PM
FM
long-range
order
Sym
Sym
Sym



H(λ) = H_I − h Σ_i σ_i^z Q.
is this definition
non-ambiguous ?
param.
ferro
Z_2 symm
Z_2 broken
GS unique
GS is not unique
Has same
symm. of H
ΔE → 0
Breaks symm.
T|Ψ_0^1⟩ = |Ψ_0^2⟩
gap
⟨σ^i σ^j⟩ − ⟨σ^i⟩⟨σ^j⟩ = f(g)
long-range
order
|↑↑↑↑↑⟩ ± |↓↓↓↓⟩
⟨σ^z⟩ ≠ 0
sym

What is the distance between Q. states?

p_x $\quad D(p_x, q_x) = \frac{1}{2} \sum_x |p_x - q_x|$ Tr

q_x $\quad D(p_x, p_z) \leq D(p_x, p_y) + D(p_y, p_z)$

$\langle \sigma^i \sigma^j \rangle - \langle \sigma^i \rangle \langle \sigma^j \rangle = f(g)$ long-range order

$\sim e^{-|i-j|/\xi}$

$|\uparrow\uparrow\uparrow\uparrow\uparrow\rangle \pm |\downarrow\downarrow\downarrow\downarrow\rangle$

$\langle \sigma^z \rangle \neq 0$

...stance between Q. states?

$D(p_x, q_x) = \frac{1}{2} \sum_x |p_x - q_x|$

Trace distance $= \max_S \left| \sum_{x \in S} p_x - \sum_{x \in S} q_x \right|$

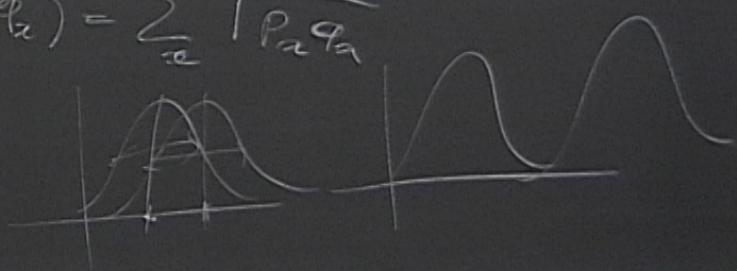
$|p+q| \leq \Delta p + \Delta q$

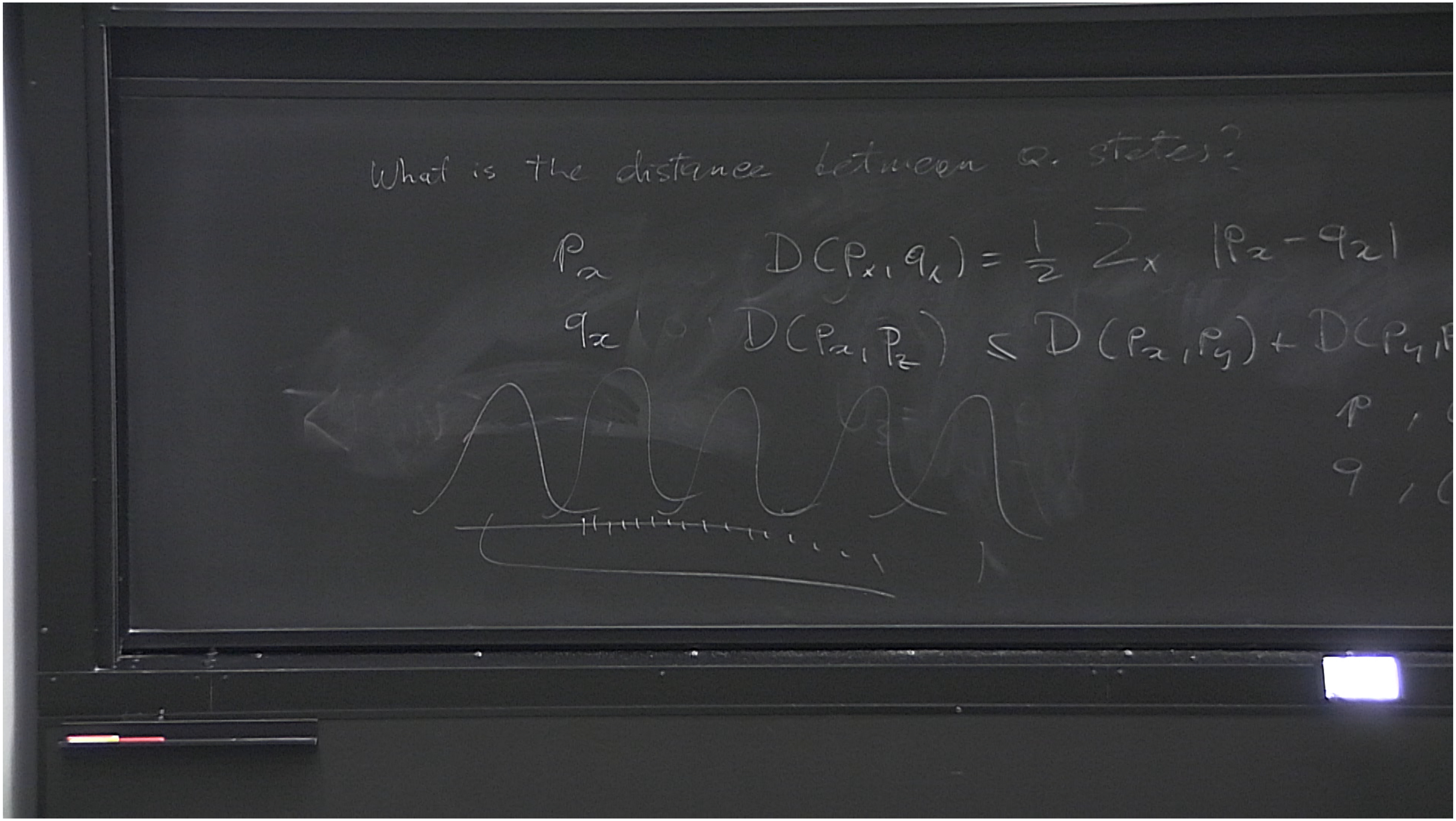
$D(p_x, p_z) \leq D(p_x, p_y) + D(p_y, p_z)$

$F(p_x, q_x) = \sum_x \sqrt{p_x q_x}$

$p \, , (1-p)$ n trials

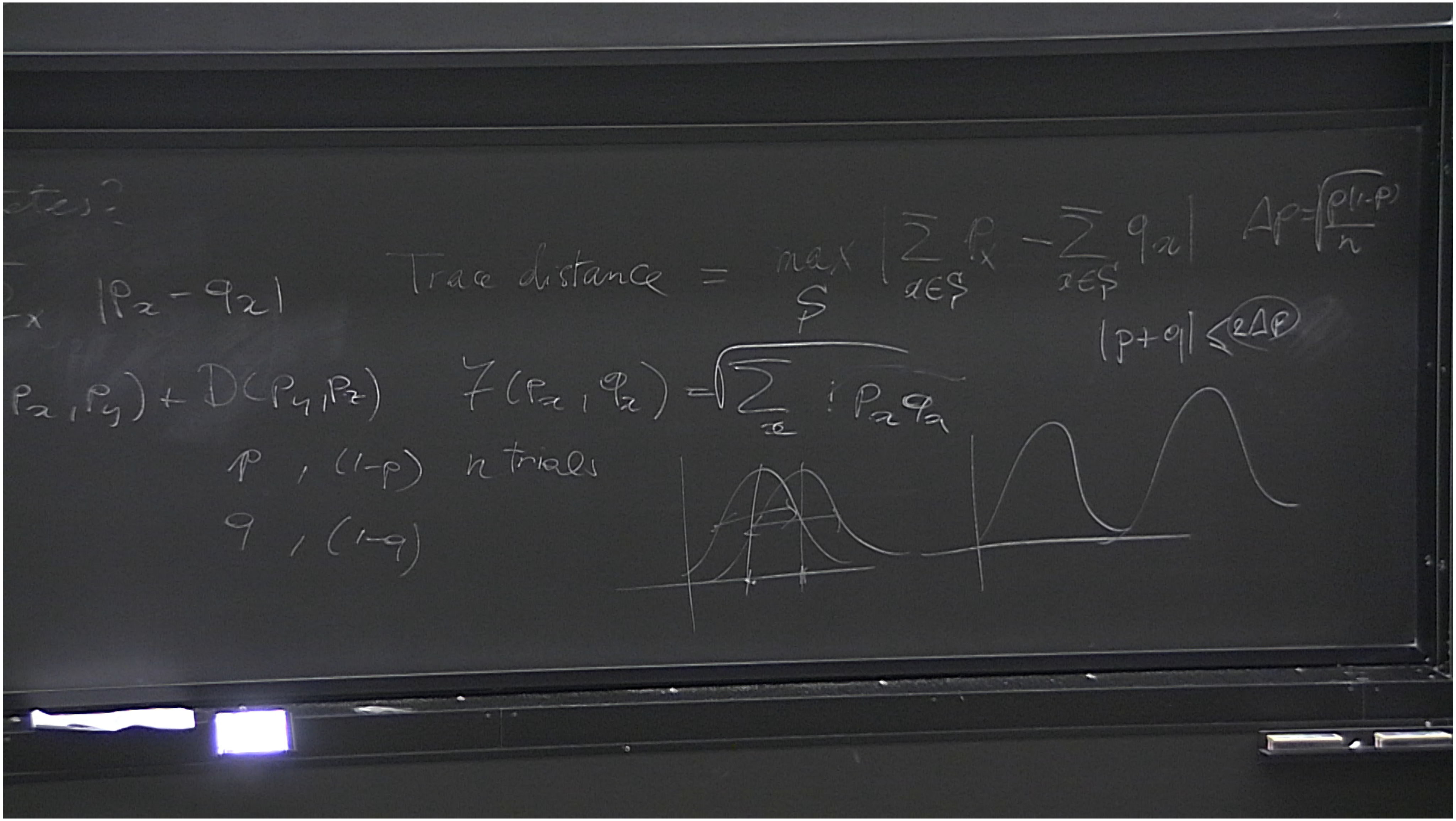
$q \, , (1-q)$



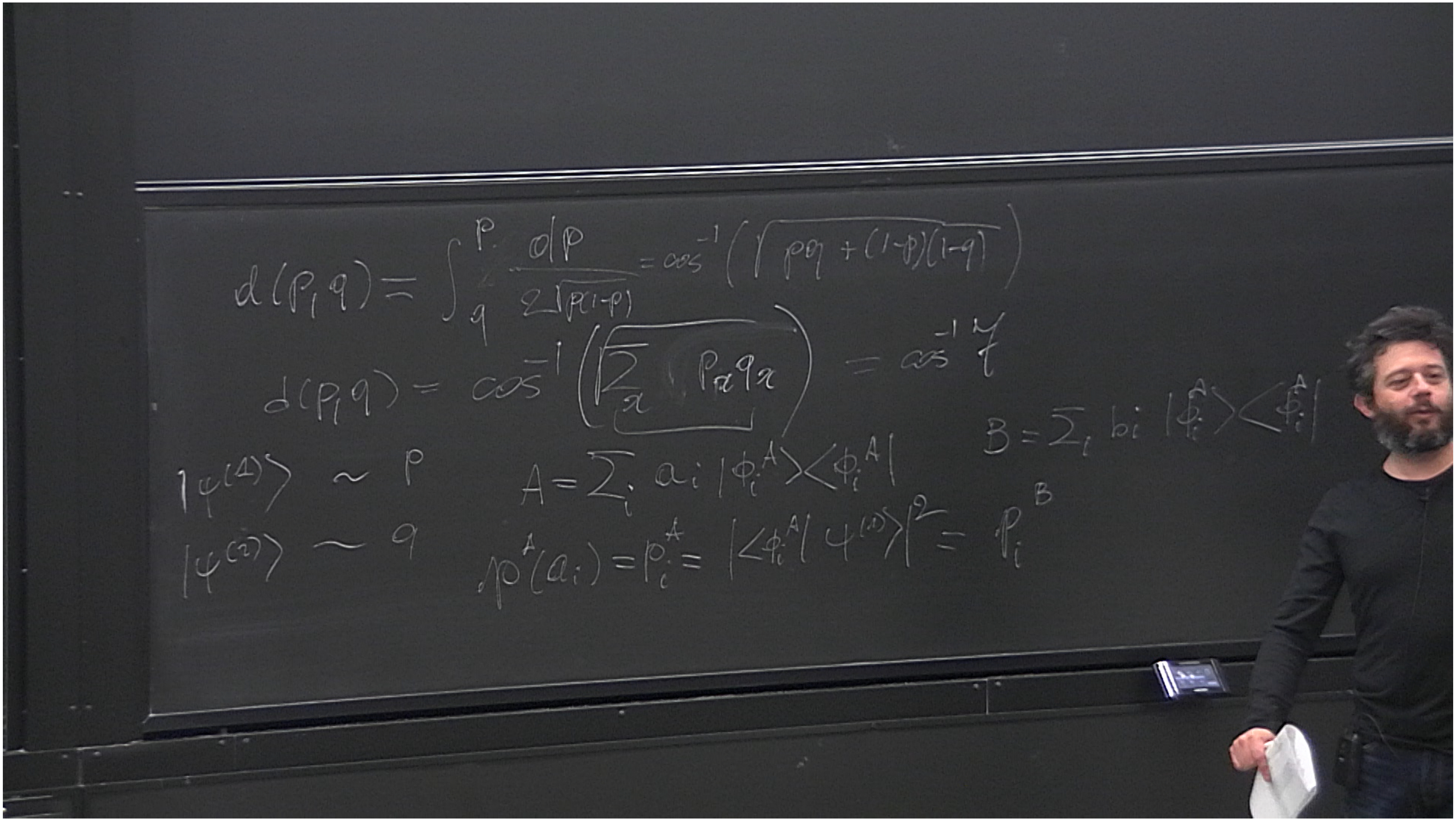

What is the distance between Q. states?
p_x
$D(p_x, q_x) = \frac{1}{2} \sum_x |p_x - q_x|$
q_x
$D(p_x, p_z) \le D(p_x, p_y) + D(p_y, p$
p ,
q ,

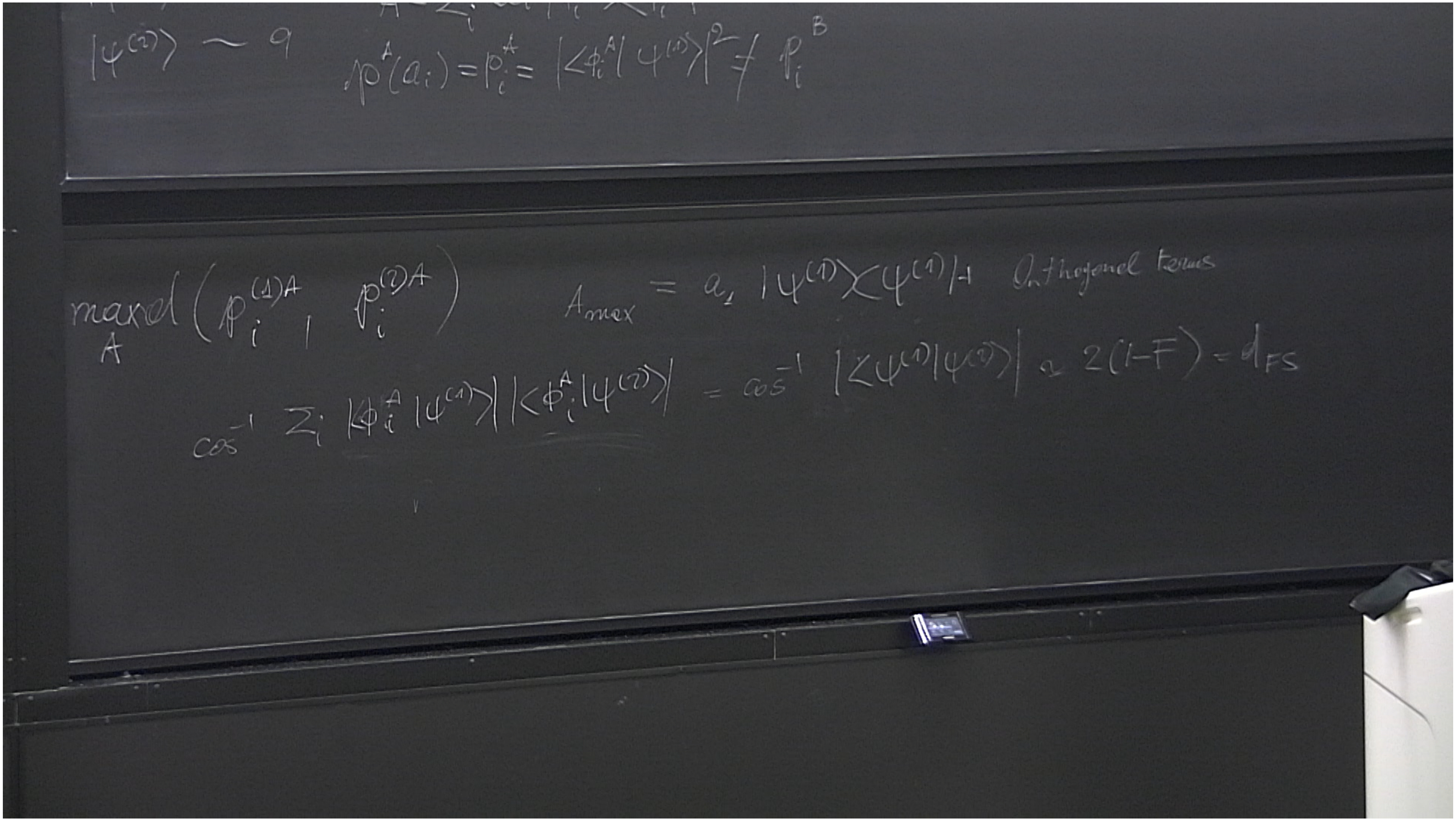
$$d(p,q) = \int_q^p \frac{dp}{2\sqrt{p(1-p)}} = \cos^{-1}\left(\sqrt{pq + (1-p)(1-q)}\right)$$

$$d(p,q) = \cos^{-1}\left(\sum_x \sqrt{p_x q_x}\right) = \cos^{-1} F$$

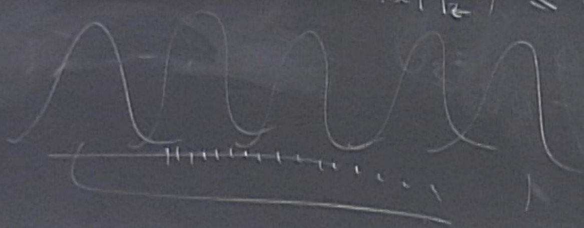


Trace distance = max_S | Σ_{x∈S} p_x − Σ_{x∈S} q_x |
|p_x − q_x|
Δp = √(p(1−p)/n)
|p+q| ≤ 2Δp
D(p_x, p_y) + D(p_y, p_z)
F(p_x, q_x) = √(Σ_x p_x q_x)
p , (1−p) n trials
q , (1−q)


$d(p,q) = \int_q^p \frac{dp}{2\sqrt{p(1-p)}} = \cos^{-1}\left(\sqrt{pq} + \sqrt{(1-p)(1-q)}\right)$
$d(p,q) = \cos^{-1}\left(\sqrt{\sum_x p_x q_x}\right) = \cos^{-1} \gamma$
$B = \sum_i b_i |\phi_i^A\rangle\langle\phi_i^A|$
$|\psi^{(1)}\rangle \sim p$
$A = \sum_i a_i |\phi_i^A\rangle\langle\phi_i^A|$
$|\psi^{(2)}\rangle \sim q$
$p^A(a_i) = p_i^A = |\langle\phi_i^A|\psi^{(1)}\rangle|^2 = p_i^B$

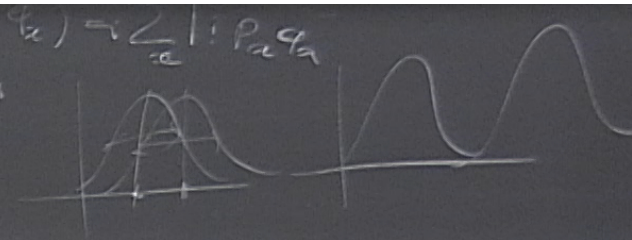

$|\psi^{(2)}\rangle \sim 9$
$p^A(a_i) = p_i^A = |\langle\phi_i^A|\psi^{(1)}\rangle|^2 \neq p_i^B$
$\max_A d\left(p_i^{(1)A},\ p_i^{(2)A}\right)$
$A_{max} = a_1 |\psi^{(1)}\rangle\langle\psi^{(1)}| +$ Orthogonal terms
$\cos^{-1} \sum_i |\langle\phi_i^A|\psi^{(1)}\rangle| |\langle\phi_i^A|\psi^{(2)}\rangle| = \cos^{-1} |\langle\psi^{(1)}|\psi^{(2)}\rangle| \approx 2(1-F) = d_{FS}$

$D(p_x,p_z) \le D(p_x,p_y) + D(p_y,p_z)$ $\quad F(p_x,q_x) = \sum_x \sqrt{p_x q_x}$



$p\ ,\ (1-p)$ n trials

$q\ ,\ (1-q)$



$$\|M\|_p = \left(\mathrm{tr}\,(M^\dagger M)^{\frac{p}{2}}\right)^{\frac{1}{p}} = \left(\mathrm{tr}\,|M|^p\right)^{1/p}$$

$$\sqrt{M^\dagger M} = |M|$$

$p=1 \qquad \|M\|_1 = \mathrm{tr}\,|M|$

M Hermitean

$p=2 \qquad \|M\|_2 = \sqrt{\mathrm{tr}\,M^\dagger M} = \sqrt{\mathrm{tr}\,M^2}$

$$\frac{1}{2}\|\rho-\sigma\|_1 = \frac{1}{2}\mathrm{tr}\,|\rho-\sigma| = \max_P |\mathrm{tr}(P\rho) - \mathrm{tr}(P\sigma)| = D(\rho,\sigma)$$

Theorem