Title: PSI 2018/2019 - Condensed Matter Review - Lecture 3

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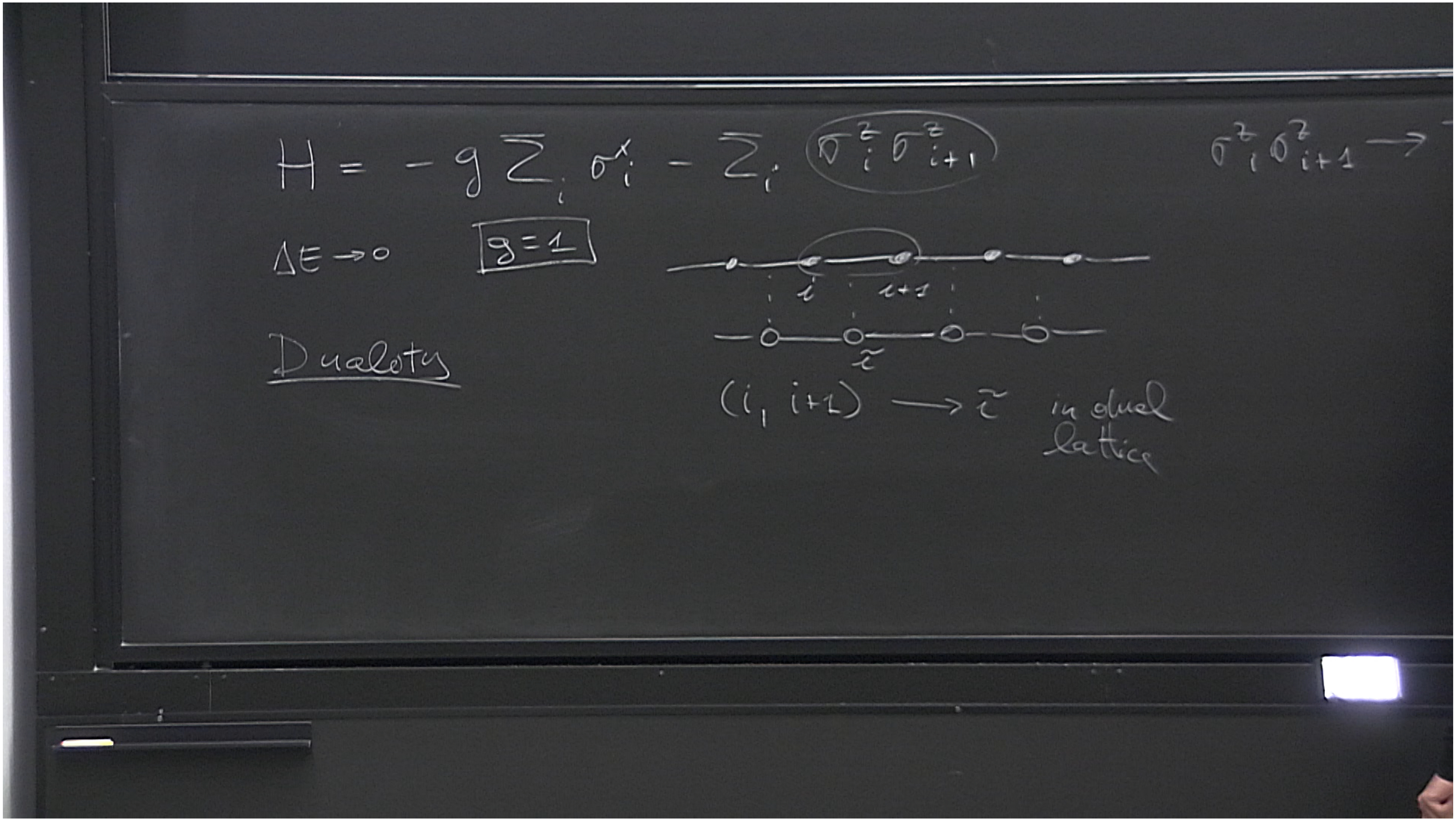
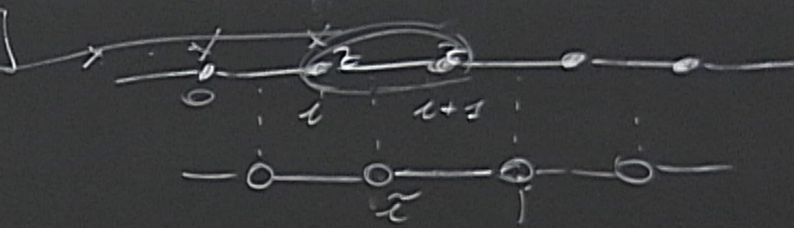
Abstract:


$H = -g \sum_i \sigma^x_i - \sum_i \sigma^z_i \sigma^z_{i+1}$
$\sigma^z_i \sigma^z_{i+1} \rightarrow$
$\Delta E \rightarrow 0$
$\boxed{g = 1}$
Duality
i
$i+1$
$\tilde{\imath}$
$(i, i+1) \longrightarrow \tilde{\imath}$ in dual lattice

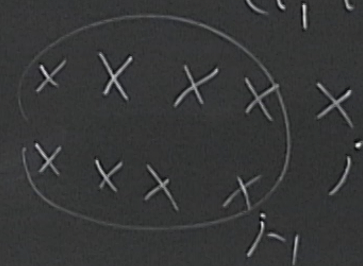
$$H = -g\sum_i \sigma^x_i - \sum_i \sigma^z_i \sigma^z_{i+1}$$

$\Delta E \to 0$ $\boxed{g=1}$



Duality

$(i, i+1) \longrightarrow \tilde{i}$ in dual lattice

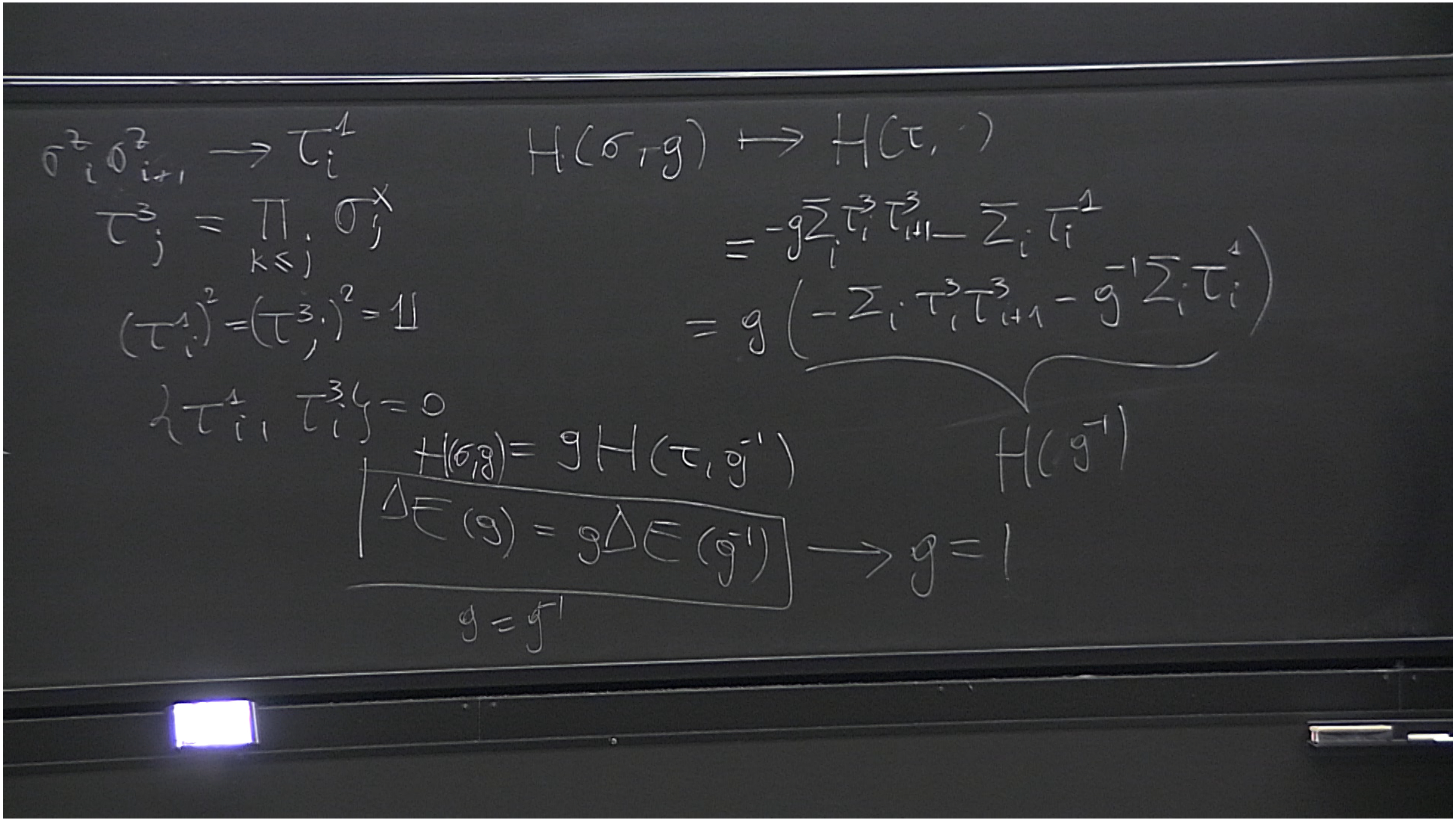


$\sigma^z_i \sigma^z_{i+1} \longrightarrow$

$\tau^3_j = \prod_{k \leq}$

$(\tau^1_i)^2 = (\tau$

$\{\tau^1_i,$


$\sigma^z_i \sigma^z_{i+1} \to \tau^1_i$
$\tau^3_j = \prod_{k \leq j} \sigma^x_j$
$(\tau^1_i)^2 = (\tau^3_j)^2 = 1$
$\{\tau^1_i, \tau^3_i\} = 0$
$H(\sigma, g) \mapsto H(\tau, \cdot)$
$= -g \sum_i \tau^3_i \tau^3_{i+1} - \sum_i \tau^1_i$
$= g \underbrace{\left(-\sum_i \tau^3_i \tau^3_{i+1} - g^{-1} \sum_i \tau^1_i\right)}_{H(g^{-1})}$
$H(\sigma, g) = g H(\tau, g^{-1})$
$\boxed{\Delta E(g) = g \Delta E(g^{-1})} \longrightarrow g = 1$
$g = g^{-1}$

Exact Diagonalization of XY model

$$H = -\sum_{n=1}^{N-1}\left[J_x\,\sigma_n^x\sigma_{n+1}^x + J_y\,\sigma_n^y\sigma_{n+1}^y\right] - h\sum_{n=1}^{N}\sigma_n^z$$

$\uparrow$ on site i $\longrightarrow$ presence of (spinless) fermion @ n

$\downarrow$ $\longrightarrow$ absence " " " "

$$E(g) = g\Delta E(g^{-1}) \longrightarrow g = 1$$

$$g = g^{-1}$$

Jordan-Wigner Transformation

σ^z_n $\quad c_n, c^\dagger_n$ $\quad \{c_m, c^\dagger_n\} = \delta_{mn}$

$$\begin{cases} \sigma^z_n = 2c^\dagger_n c_n - 1 \equiv 2\rho_n - 1 \\ \sigma^-_n = \frac{1}{2}(\sigma^x_n - i\sigma^y_n) = c_n e^{i\pi \sum_{j=1}^{n-1} \rho_j} \end{cases}$$

$$\{c_m, c_n\} = \{c^\dagger_m, c^\dagger_n\} = 0$$

$$N_f = \sum_n^N c^\dagger_n c_n$$

$$H = \sum_{n=1}^{N-1} \left[-(J_x + J_y)\left(C_n^{+} C_{n+1} + C_{n+1}^{+} C_n\right) + (J_x - J_y)\left(C_{n+1}^{+} C_n^{+} + C_n C_{n+1}\right) \right]$$

$$- (-1)^{N_f} \left[-(J_x + J_y)\left(C_N^{+} C_1 + C_1^{+} C_N\right) + (J_x - J_y)\left(C_1^{+} C_N^{+} + C_N C_1\right) \right.$$

$$- \sum_{n=1}^{N} h \left(2 C_n^{+} C_n - 1\right)$$

$$J_y)\left(C^{\dagger}_{n+1}C^{\dagger}_{n} + C_n C_{n+1}\right)\Big]$$

$$(J_x - J_y)\left(C^{\dagger}_{1}C^{\dagger}_{N} + C_N C_1\right)$$

$$c_n \longrightarrow c_k = \sum_{n=1}^{N} \frac{c_n\, e^{-ik\underbrace{na}_{x}}}{\sqrt{N}}$$

$$k_m = -\frac{\pi}{a},\ -\frac{\pi}{a} + \frac{2\pi n}{Na} \rightarrow (2m+1),\ \dots,\ +\frac{\pi}{a} \qquad m = 1, \dots, N$$

N_f odd

$$H = \sum_{k>0} \begin{pmatrix} c_k^{+} & c_{-k} \end{pmatrix} H_k \begin{pmatrix} c_k \\ c_{-k}^{+} \end{pmatrix}$$

$$H_k = 2 \begin{pmatrix} -(J_x + J_y)\cos ka - h & i\,(J_x - J_y)\sin ka \\ -i\,(J_x - J_y)\sin ka & (J_x + J_y)\cos ka + h \end{pmatrix}$$

$$\left\{\begin{array}{l} d_k^\dagger = \sin\theta_k c_k + i\cos\theta_k c_{-k}^\dagger \\ d_{-k}^\dagger = \sin\theta_k c_{-k} - i\cos\theta_k c_k^\dagger \end{array}\right.$$

$$\tan 2\theta_k = \frac{-(J_x - J_y)\sin ka}{(J_x + J_y)\cos ka + h}$$

$$H = \sum_{k>0} \left(c_k^\dagger \quad c_{-k} \right) H_k \begin{pmatrix} c_k \\ c_{-k}^\dagger \end{pmatrix}$$

$$H_k = 2\begin{pmatrix} -(J_x + J_y)\cos ka - h & i(J_x - J_y)\sin ka \\ -i(J_x - J_y)\sin ka & (J_x + J_y)\cos ka + h \end{pmatrix}$$

$$H = \sum_{k>0} \omega_k \left(d_k^\dagger d_k + d_{-k}^\dagger d_{-k} - 1 \right)$$

$$\begin{cases} d_k^\dagger = \mathrm{si} \\ d_{-k}^\dagger = \mathrm{si} \end{cases}$$

$$\tan 2\theta_k =$$

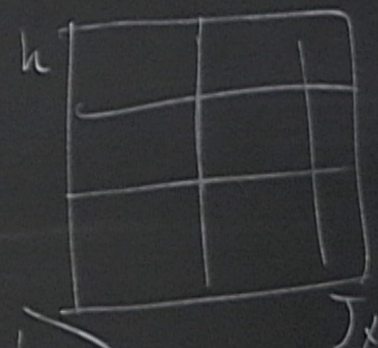
$$\omega_k = 2\left[h^2 + J_x^2 + J_y^2 + 2h(J_x + J_y)\cos ka + 2J_xJ_y\cos 2ka\right]^{1/2}$$

$d_k|GS\rangle = 0$

$d_{-k}|GS\rangle = 0$

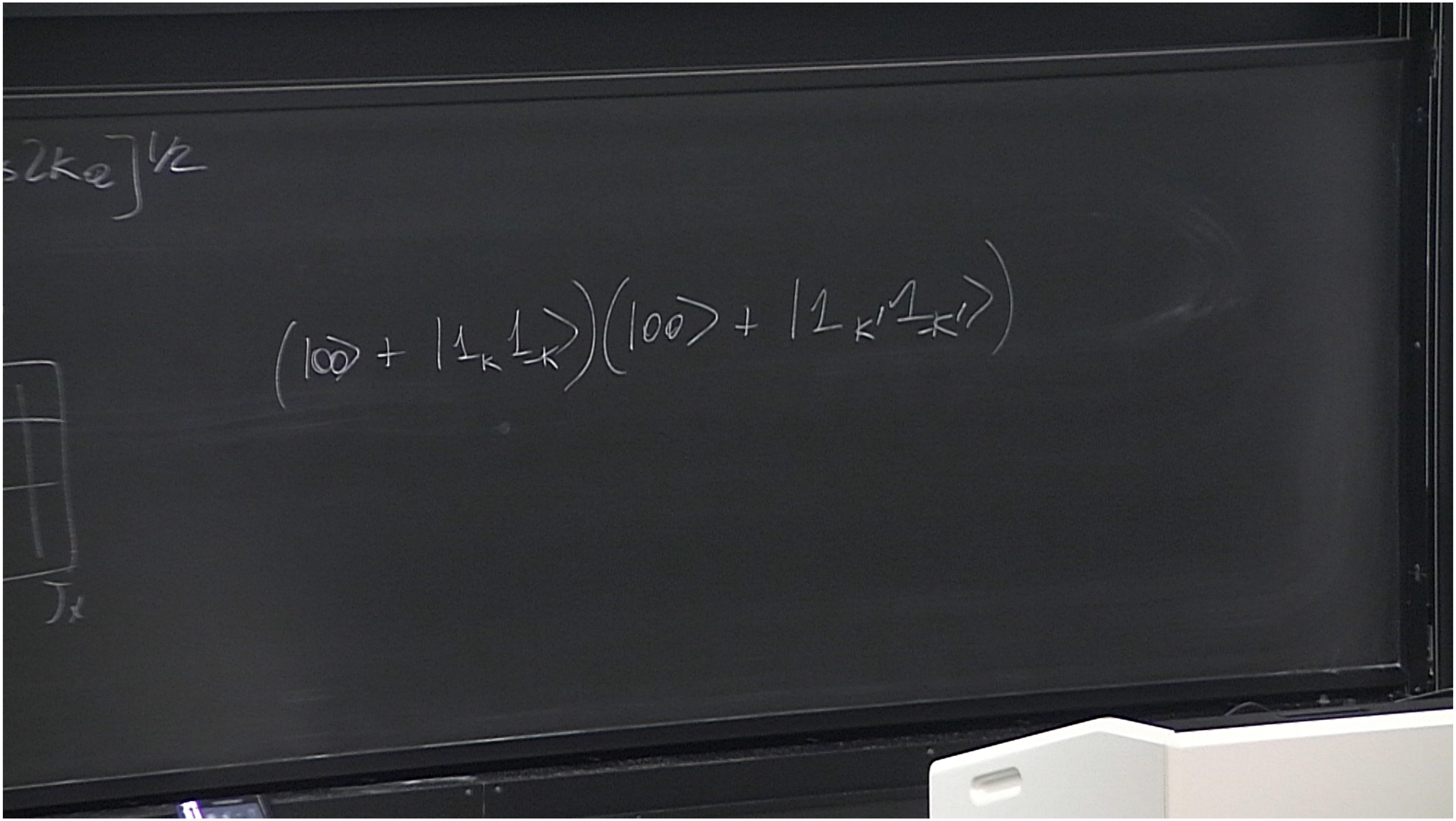
$\omega_k \to 0$? $N \to \infty$

h, J_x, J_y



$|\Phi\rangle$ the vacuum of C_k

$$|GS\rangle = \bigotimes_{k>0}\left(\cos\theta_k + i\sin\theta_k\, C_k^\dagger C_{-k}^\dagger\right)|\Phi\rangle$$


$2ka]^{1/2}$
$(|00\rangle + |1_k 1_{-k}\rangle)(|00\rangle + |1_{k'} 1_{-k'}\rangle)$
J_x