Title: PSI 2018/2019 - Condensed Matter Review - Lecture 2

Date: Jan 08, 2019 11:30 AM

URL: http://pirsa.org/19010033

Abstract:

$$F = E - TS = -kT \log Z,$$

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$$f = \lim_{N \to \infty} \frac{F}{N}$$

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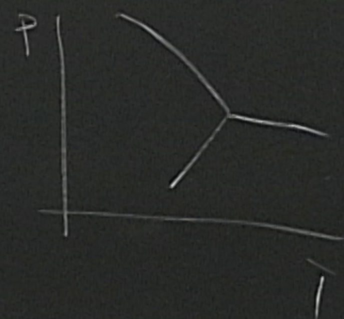
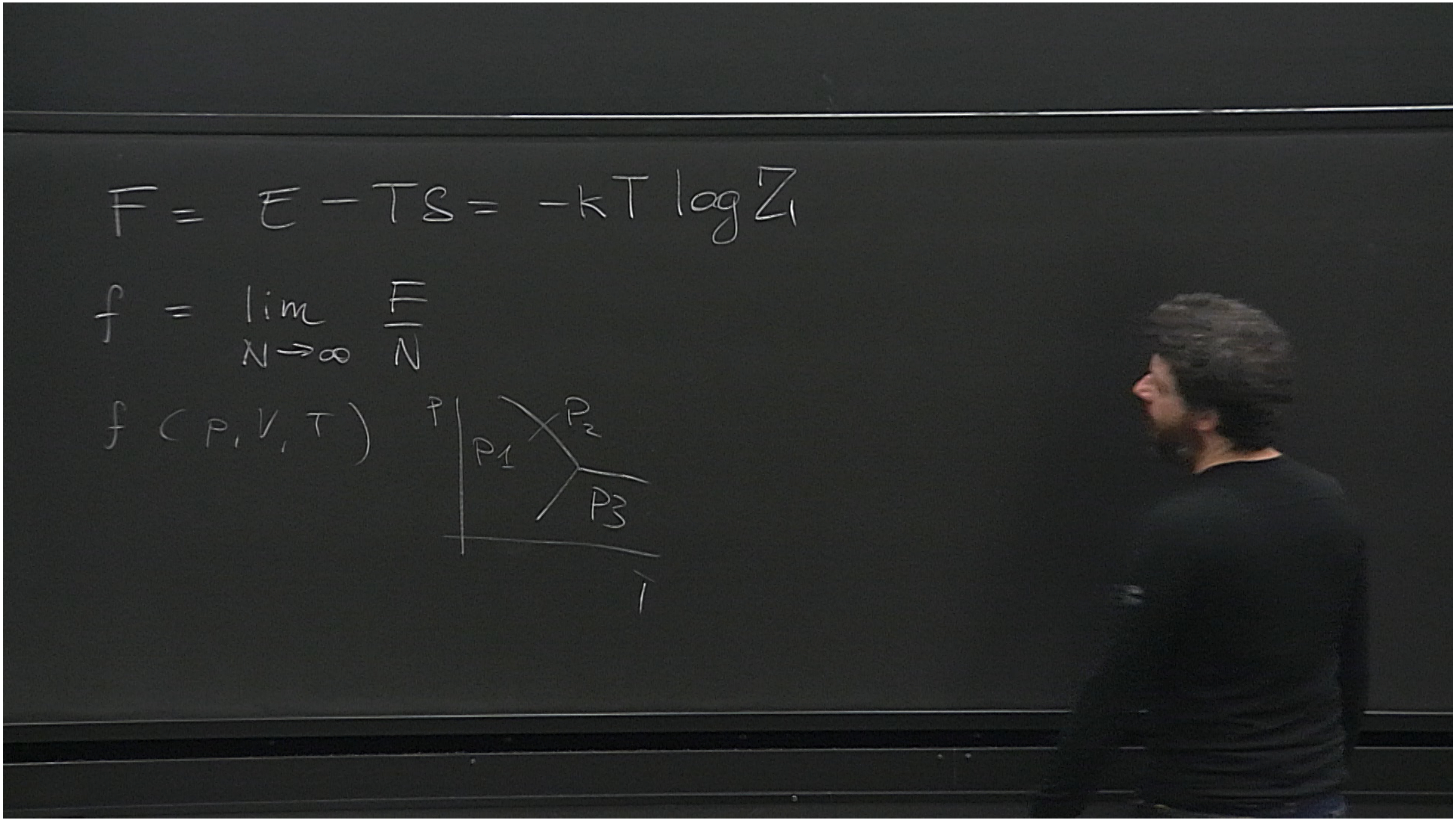
$$f(P, V, T)$$

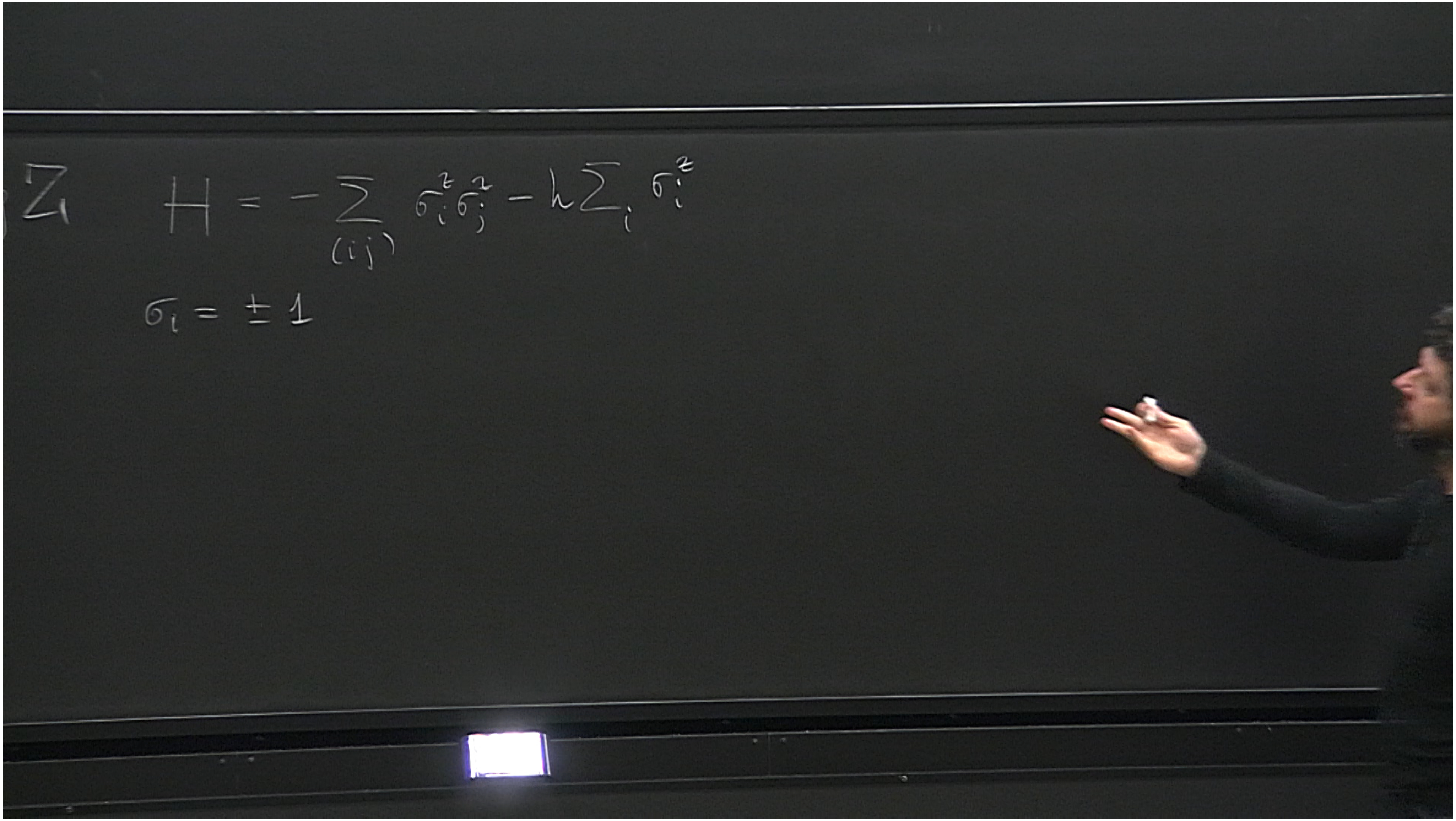




$F = E - TS = -kT \log Z_1$
$f = \lim_{N \to \infty} \frac{F}{N}$
$f(p, V, T)$
P
P1
P_2
P3
T

Z $\quad H = -\sum_{\langle ij \rangle} \sigma_i \sigma_j - h \sum_i \sigma_i$

$\sigma_i = \pm 1$


Z
$H = -\sum_{\langle ij \rangle} \sigma_i^z \sigma_j^z - h \sum_i \sigma_i^z$
$\sigma_i = \pm 1$

Z_1 $\quad H = -\sum_{(ij)} \sigma_i \sigma_j - h \sum_i \sigma_i$

$\sigma_i = \pm 1 \qquad \{\sigma_i\}$

$$\frac{\partial f}{\partial h} = \frac{1}{N}\frac{\partial F}{\partial h} = \frac{1}{N}\left(-\frac{1}{\beta}\right)\frac{\partial}{\partial h}\log Z_1$$

Z_1 $\quad H = -\sum_{(ij)} \sigma_i \sigma_j - h \sum_i \sigma_i$

$\sigma_i = \pm 1 \qquad \{\sigma_i\}$

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$$Z_1 = \sum_{\{\sigma_i\}} e^{-\beta H(\{\sigma_i\})}$$

Z_1

$$H = -\sum_{(ij)} \sigma_i \sigma_j - h \sum_i \sigma_i$$

$$\sigma_i = \pm 1 \qquad \{\sigma_i\}$$

$$\frac{\partial f}{\partial h} = \frac{1}{N}\frac{\partial F}{\partial h} = \frac{1}{N}\left(-\frac{1}{\beta}\right)\frac{\partial}{\partial h}\log Z_1 = \frac{1}{N Z_1}\sum_{\{\sigma_i\}} \overbrace{\left(\sum_i \sigma_i\right)}^{M} e^{-\beta H} = \frac{1}{N}$$

$$Z_1 = \sum_{\{\sigma_i\}} e^{-\beta H(\{\sigma_i\})}$$

$$\log Z \qquad H = -\sum_{(ij)} \sigma_i \sigma_j - h \sum_i \sigma_i$$

$$\sigma_i = \pm 1 \qquad \{\sigma_i\}$$

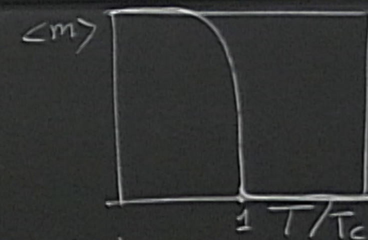
$$\frac{\partial f}{\partial h} = \frac{1}{N}\frac{\partial F}{\partial h} = \frac{1}{N}\left(-\frac{1}{\beta}\right)\frac{\partial}{\partial h}\log Z = \frac{1}{N Z}\sum_{\{\sigma_i\}} \overbrace{\left(\sum_i \sigma_i\right)}^{M} e^{-\beta H} = \frac{1}{N}\langle M \rangle = \langle m \rangle$$

$$Z = \sum_{\{\sigma_i\}} e^{-\beta H(\{\sigma_i\})}$$

$\log Z$

$$H = -\sum_{\langle ij \rangle} \sigma_i \sigma_j - h \sum_i \sigma_i$$

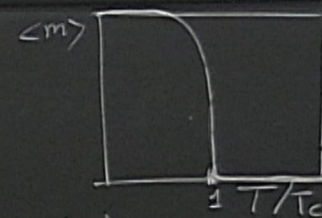
$$\sigma_i = \pm 1 \qquad \{\sigma_i\}$$



$$\frac{\partial f}{\partial h} = \frac{1}{N}\frac{\partial F}{\partial h} = \frac{1}{N}\left(-\frac{1}{\beta}\right)\frac{\partial}{\partial h}\log Z = \frac{1}{N Z}\sum_{\{\sigma_i\}} \overset{M}{\overset{\uparrow}{\left(\sum_i \sigma_i\right)}} e^{-\beta H} = \frac{1}{N}\langle M \rangle = \langle m \rangle$$

$$Z = \sum_{\{\sigma_i\}} e^{-\beta H(\{\sigma_i\})}$$

$$\ldots Z \qquad H = -\sum_{\langle ij\rangle} \sigma_i \sigma_j - h \sum_i \sigma_i$$



$$\sigma_i = \pm 1 \qquad \{\sigma_i\}$$

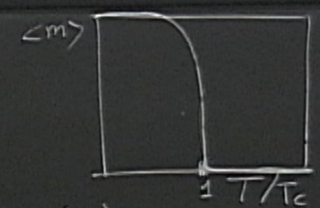
$$\left.\frac{\partial f}{\partial h}\right|_{h=0} = \frac{1}{N}\frac{\partial F}{\partial h} = \frac{1}{N}\left(-\frac{1}{\beta}\right)\frac{\partial}{\partial h}\log Z = \frac{1}{N Z}\sum_{\{\sigma_i\}} \overbrace{\left(\sum_i \sigma_i\right)}^{M} e^{-\beta H} = \frac{1}{N}\langle M\rangle = \langle m\rangle$$

$$Z = \sum_{\{\sigma_i\}} e^{-\beta H(\{\sigma_i\})}$$

$g Z$

$$H = -\sum_{\langle ij\rangle} \sigma_i \sigma_j - h \sum_i \sigma_i$$

$\sigma_i = \pm 1 \qquad \{\sigma_i\}$

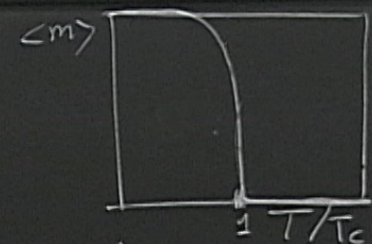


$$\left.\frac{\partial f}{\partial h}\right|_{h=0} = \frac{1}{N}\frac{\partial F}{\partial h} = \frac{1}{N}\left(-\frac{1}{\beta}\right)\frac{\partial}{\partial h}\log Z = \frac{1}{N Z}\sum_{\{\sigma_i\}} \overbrace{\left(\sum_i \sigma_i\right)}^{M} e^{-\beta H} = \frac{1}{N}\langle M\rangle = \langle m\rangle$$

$$Z = \sum_{\{\sigma_i\}} e^{-\beta H(\{\sigma_i\})}$$

$$\sum_{\langle ij\rangle} \sigma_i\sigma_j - h\sum_i \sigma_i$$

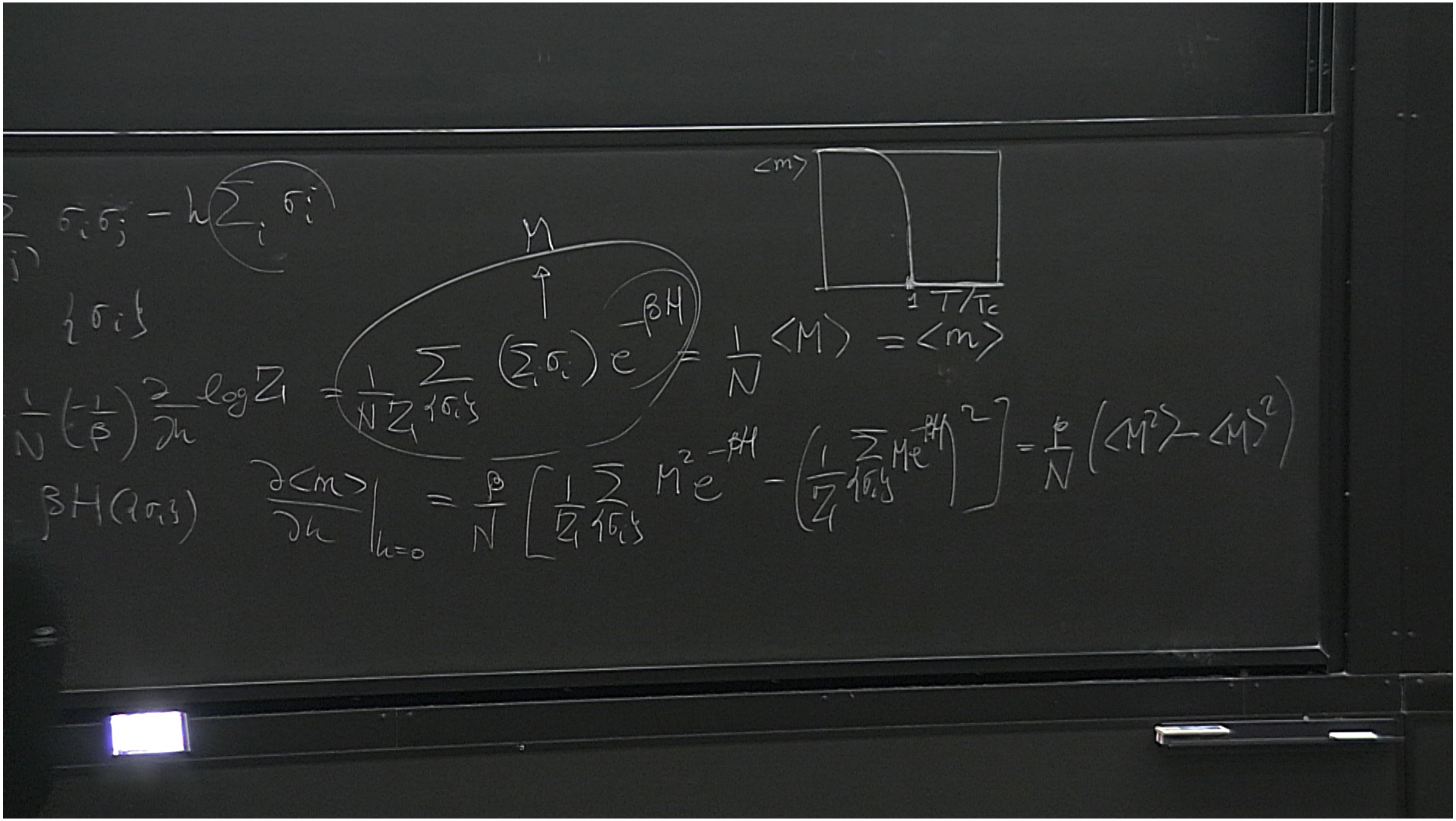
$$\{\sigma_i\}$$



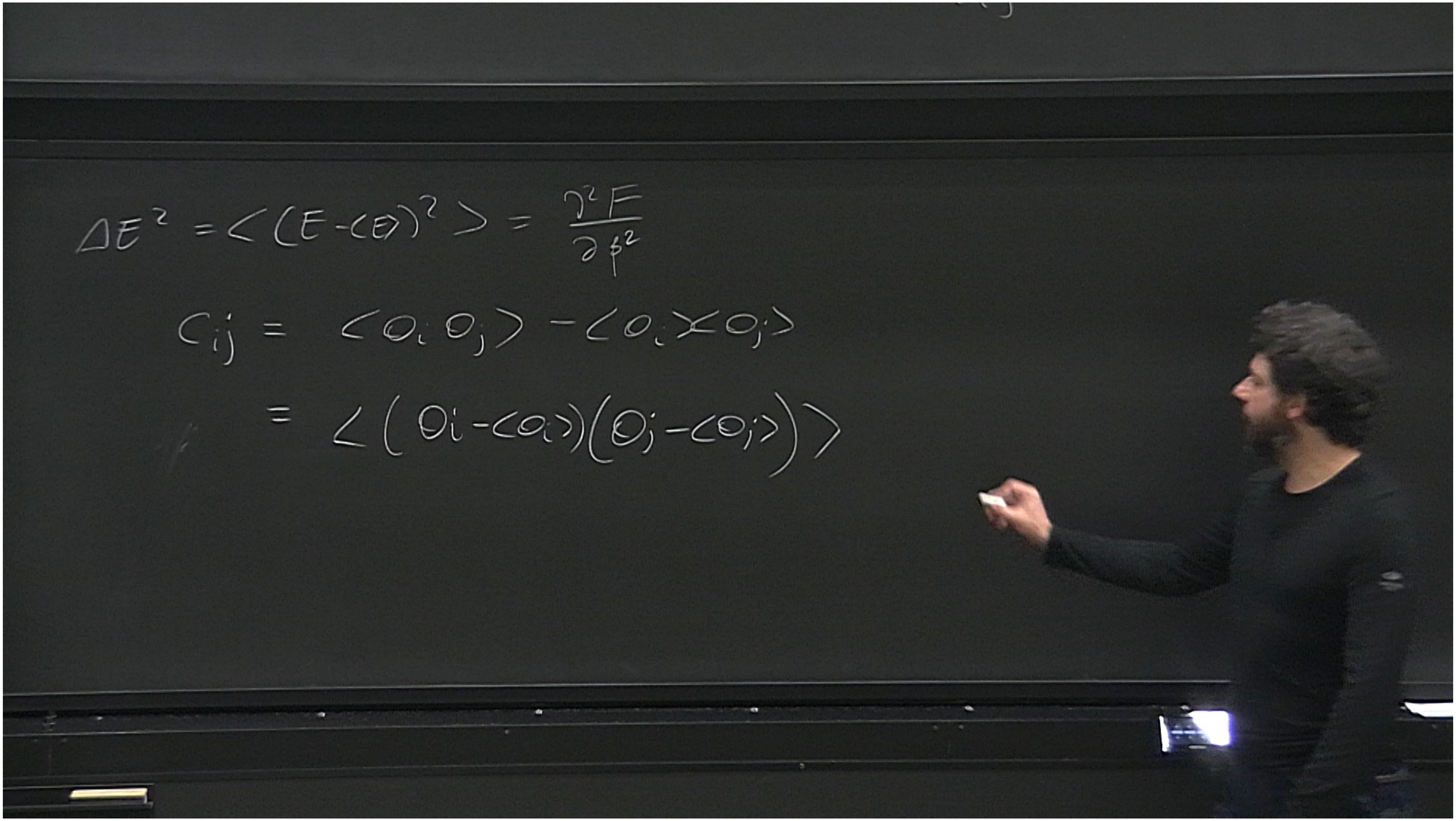
$$\frac{1}{N}\left(-\frac{1}{\beta}\right)\frac{\partial}{\partial h}\log Z_1 = \frac{1}{N Z_1}\sum_{\{\sigma_i\}} \overbrace{\left(\sum_i \sigma_i\right)}^{M} e^{-\beta H} = \frac{1}{N}\langle M\rangle = \langle m\rangle$$

$$\beta H(\{\sigma_i\})$$

$$\frac{\partial\langle m\rangle}{\partial h}\Big|_{h=0} = \frac{\beta}{N}\left[\frac{1}{Z_1}\sum_{\{\sigma_i\}} M^2 e^{-\beta H} - \left(\frac{1}{Z_1}\sum_{\{\sigma_i\}} M e^{-\beta H}\right)^2\right]$$



$\langle m \rangle$
1 T/T_c
$\sigma_i\sigma_j - h\sum_i \sigma_i$
$\{\sigma_i\}$
M
$\frac{1}{N}\left(-\frac{1}{\beta}\right)\frac{\partial}{\partial h}\log Z_1 = \frac{1}{N Z_1}\sum_{\{\sigma_i\}}\left(\sum_i\sigma_i\right)e^{-\beta H} = \frac{1}{N}\langle M\rangle = \langle m\rangle$
$\beta H(\{\sigma_i\})$
$\frac{\partial\langle m\rangle}{\partial h}\Big|_{h=0} = \frac{\beta}{N}\left[\frac{1}{Z_1}\sum_{\{\sigma_i\}} M^2 e^{-\beta H} - \left(\frac{1}{Z_1}\sum_{\{\sigma_i\}} M e^{-\beta H}\right)^2\right] = \frac{\beta}{N}\left(\langle M^2\rangle - \langle M\rangle^2\right)$


$\Delta E^2 = \langle (E - \langle E \rangle)^2 \rangle = \frac{\partial^2 F}{\partial \beta^2}$
$C_{ij} = \langle O_i O_j \rangle - \langle O_i \rangle \langle O_j \rangle$
$= \langle (O_i - \langle O_i \rangle)(O_j - \langle O_j \rangle) \rangle$

$$\Delta E^2 = \langle (E - \langle E \rangle)^2 \rangle = \frac{\partial^2 F}{\partial \beta^2}$$

$$C_{ij} = \langle O_i O_j \rangle - \langle O_i \rangle \langle O_j \rangle$$

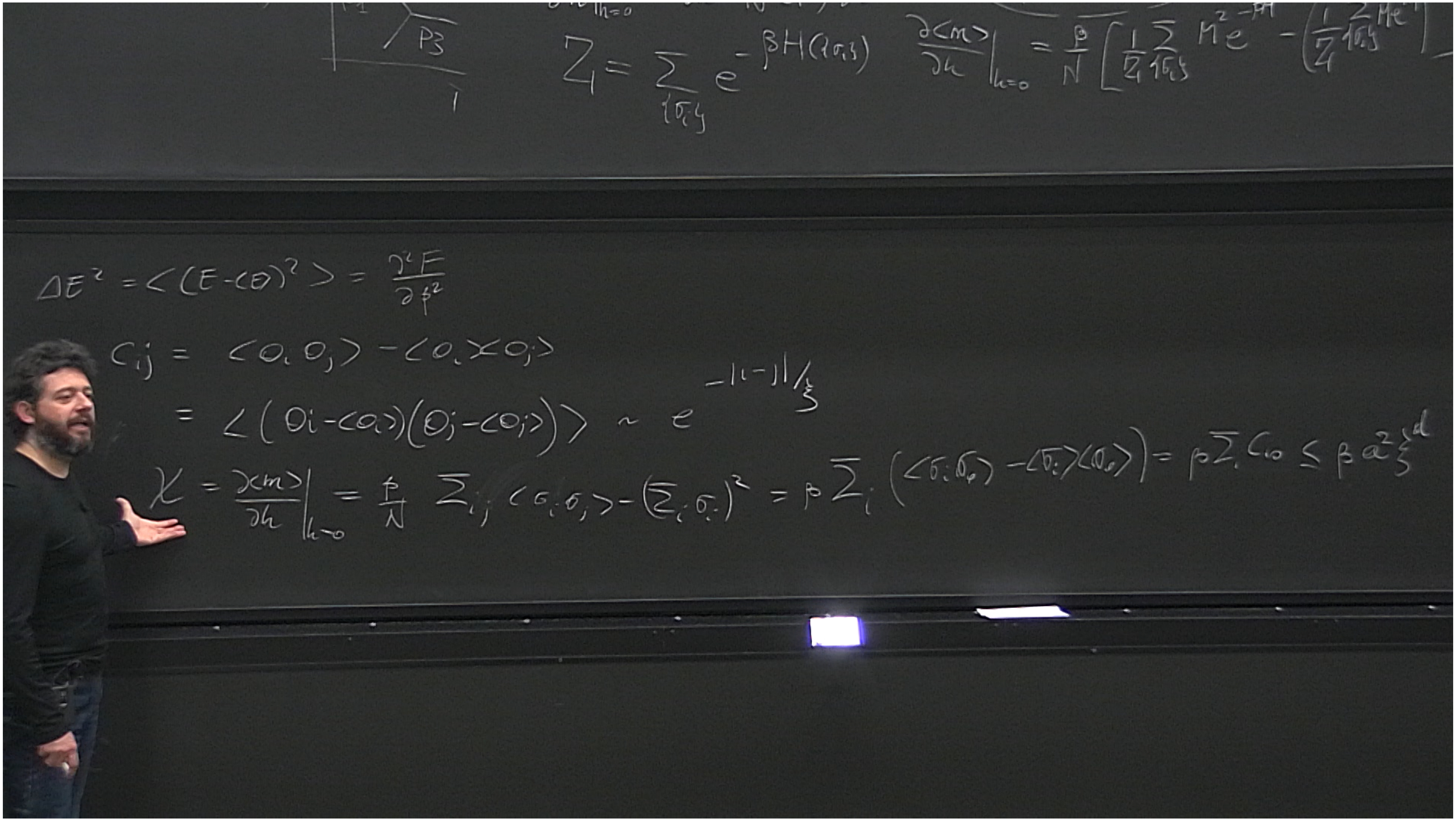
$$= \langle (O_i - \langle O_i \rangle)(O_j - \langle O_j \rangle) \rangle \sim e^{-|i-j|/\xi}$$

$$\Delta E^2 = \langle (E - \langle E \rangle)^2 \rangle = \frac{\partial^2 F}{\partial \beta^2}$$

$$C_{ij} = \langle O_i O_j \rangle - \langle O_i \rangle \langle O_j \rangle$$

$$= \langle (O_i - \langle O_i \rangle)(O_j - \langle O_j \rangle) \rangle \sim e^{-|i-j|/\xi}$$

$$\chi = \frac{\partial \langle m \rangle}{\partial h}\Big|_{h \to 0} = \frac{\beta}{N} \sum_{ij} \langle \sigma_i \sigma_j \rangle - \left(\sum_i \sigma_i\right)^2$$



$Z = \sum_{\{\sigma_i\}} e^{-\beta H(\{\sigma_i\})}$
$\frac{\partial \langle m \rangle}{\partial h}\Big|_{h=0} = \frac{\beta}{N}\Big[\frac{1}{Z}\sum_{\{\sigma_i\}} M^2 e^{-\beta H} - \Big(\frac{1}{Z}\sum_{\{\sigma_i\}} M e^{-\beta H}\Big)^2\Big]$
$\Delta E^2 = \langle (E - \langle E \rangle)^2 \rangle = \frac{\partial^2 F}{\partial \beta^2}$
$C_{ij} = \langle O_i O_j \rangle - \langle O_i \rangle \langle O_j \rangle$
$= \langle (O_i - \langle O_i \rangle)(O_j - \langle O_j \rangle) \rangle \sim e^{-|i-j|/\xi}$
$\chi = \frac{\partial \langle m \rangle}{\partial h}\Big|_{h=0} = \frac{\beta}{N} \sum_{i,j} \langle \sigma_i \sigma_j \rangle - \big(\sum_i \sigma_i\big)^2 = \beta \sum_i \big(\langle \sigma_i \sigma_0 \rangle - \langle \sigma_i \rangle \langle \sigma_0 \rangle\big) = \beta \sum_i C_{i0} \leq \beta a^2 \xi^d$

$$Z = \sum_{\{\sigma_i\}} e^{-\beta H(\{\sigma_i\})} \qquad \frac{\partial \langle m \rangle}{\partial h}\Big|_{h=0} = \frac{\beta}{N}\left[\frac{1}{Z}\sum_{\{\sigma_i\}} M^2 e^{-\beta H} - \left(\frac{1}{Z}\sum_{\{\sigma_i\}} M e^{-\beta H}\right)\right.$$

$$\Delta E^2 = \langle (E - \langle E \rangle)^2 \rangle = \frac{\partial^2 F}{\partial \beta^2}$$

$$C_{ij} = \langle O_i O_j \rangle - \langle O_i \rangle \langle O_j \rangle$$

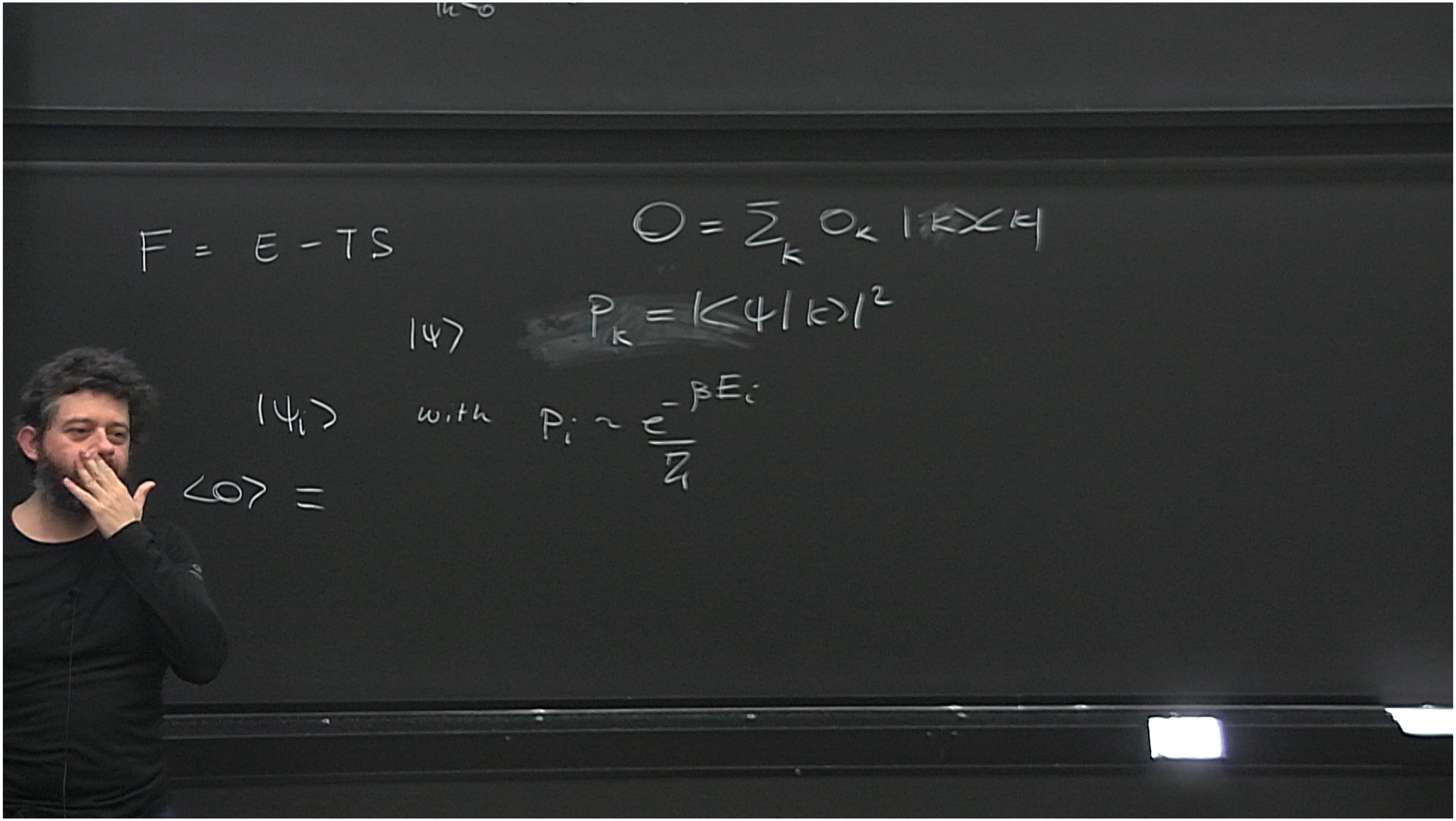
$$= \langle (O_i - \langle O_i \rangle)(O_j - \langle O_j \rangle) \rangle \sim e^{-|i-j|/\xi} \xrightarrow{\xi \to \infty} |T - T_c|^{-\gamma}$$

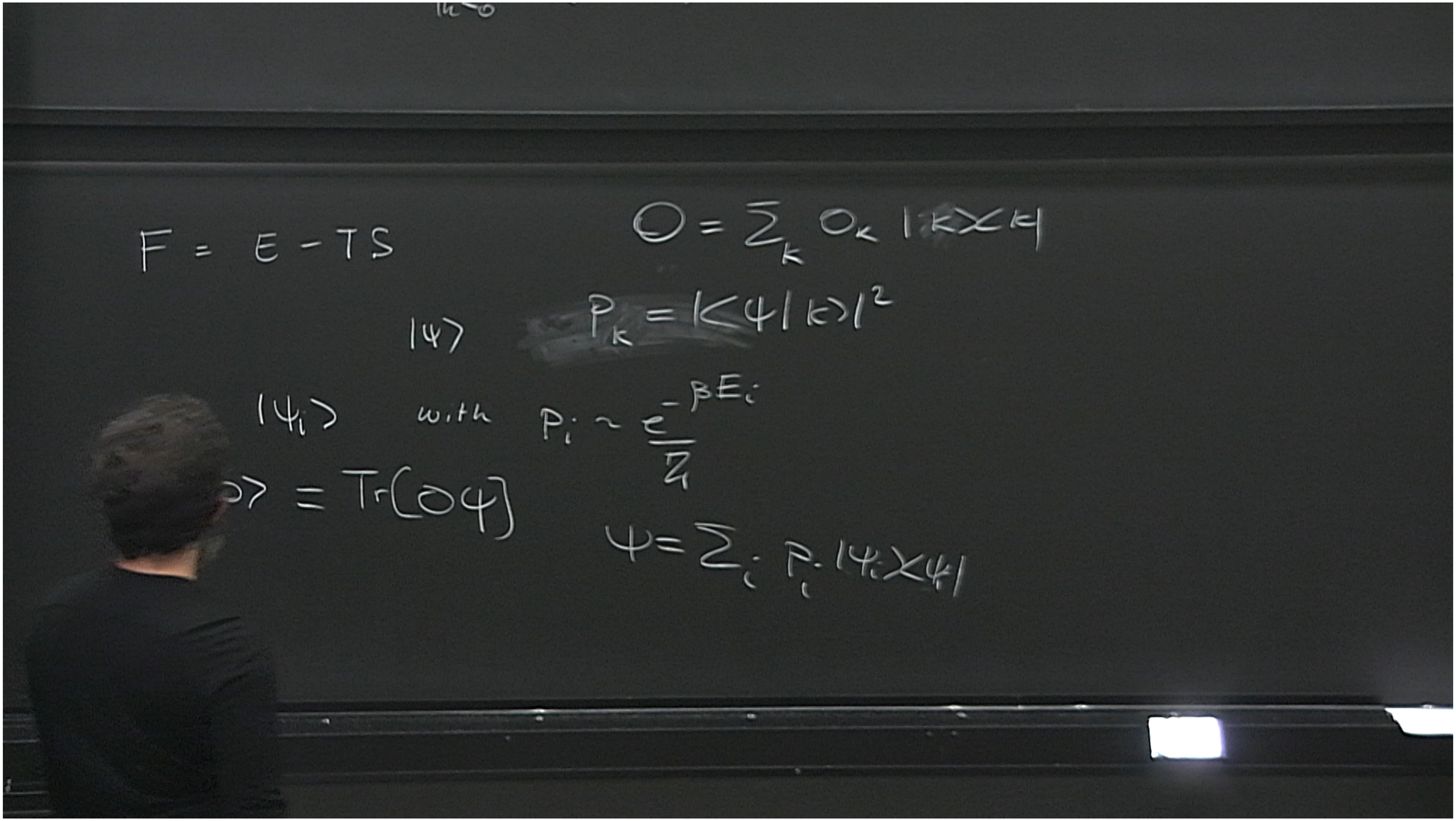
$$\chi = \frac{\partial \langle m \rangle}{\partial h}\Big|_{h=0} = \frac{\beta}{N} \sum_{ij} \langle \sigma_i \sigma_j \rangle - \left(\sum_i \sigma_i\right)^2 = \beta \sum_i \left(\langle \sigma_i \sigma_0 \rangle - \langle \sigma_i \rangle \langle \sigma_0 \rangle\right) = \beta \sum_i C_{i0} \leq \beta a^2 \xi^d$$

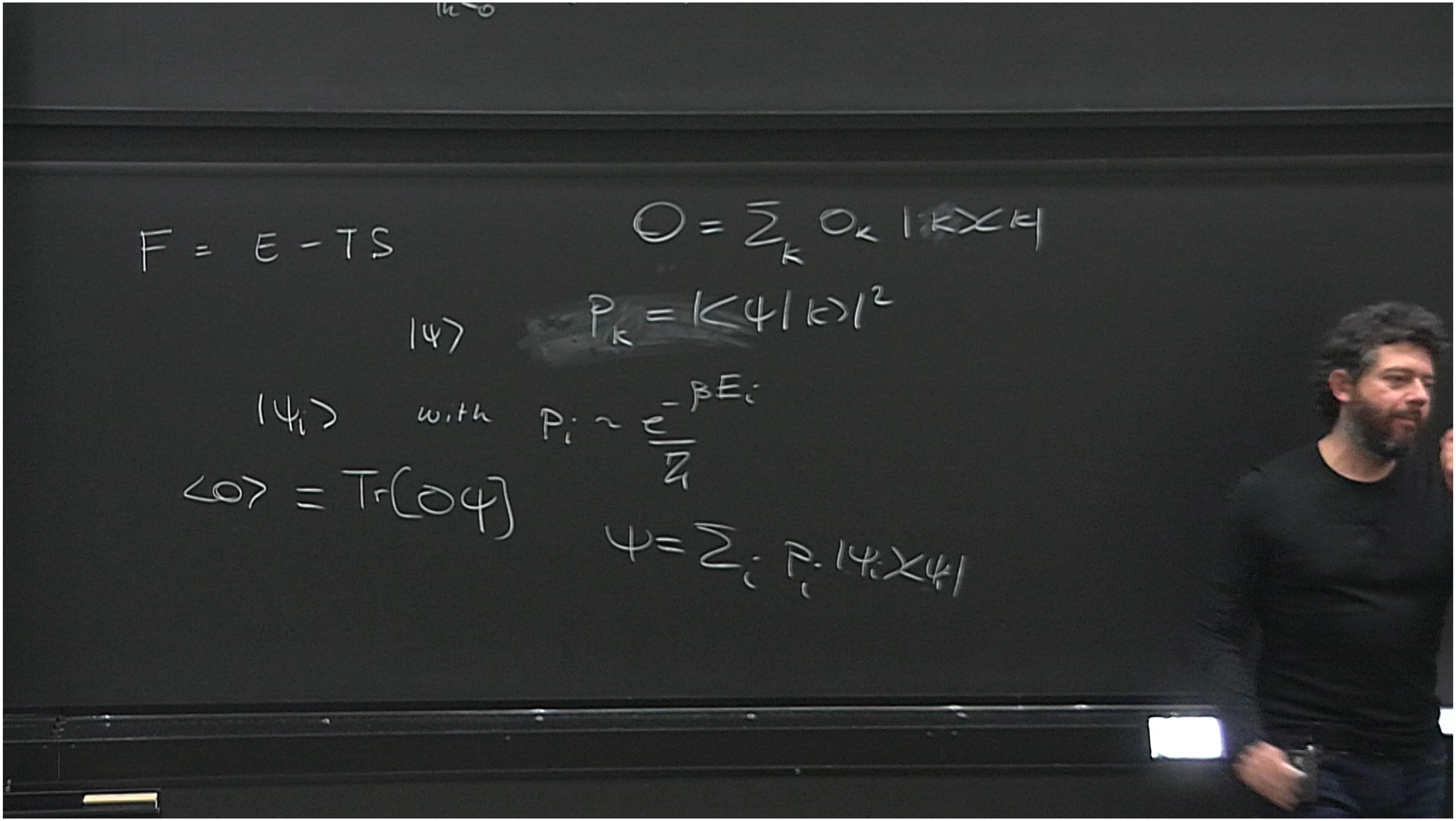
$$\xi \sim |T - T_c|^{-\nu}$$

$$|T - T_c|^{-\gamma}$$

$$\rangle - \langle O_i \rangle \langle O_0 \rangle \Big) = \beta \sum_i C_{i0} \leq \beta a^2 \xi^d$$


$F = E - TS$
$O = \sum_k O_k |k\rangle\langle k|$
$|\psi\rangle$
$P_k = |\langle\psi|k\rangle|^2$
$|\psi_i\rangle$ with $P_i \sim \frac{e^{-\beta E_i}}{Z}$
$\langle O\rangle =$


$F = E - TS$
$O = \sum_k O_k |k\rangle\langle k|$
$|\psi\rangle$
$P_k = |\langle\psi|k\rangle|^2$
$|\psi_i\rangle$ with $P_i \sim \frac{e^{-\beta E_i}}{Z}$
$\rangle = \mathrm{Tr}[O\psi]$
$\psi = \sum_i P_i |\psi_i\rangle\langle\psi_i|$


F = E - TS
O = Σ_k O_k |k⟩⟨k|
P_k = |⟨ψ|k⟩|^2
|ψ⟩
|ψ_i⟩ with P_i ~ e^{-βE_i} / Z
⟨O⟩ = Tr[Oψ]
ψ = Σ_i P_i |ψ_i⟩⟨ψ_i|

$$F = E - TS$$

$$O = \sum_k O_k |k\rangle\langle k|$$

$|\psi\rangle$ $\quad P_k = |\langle\psi|k\rangle|^2$

$|\psi_i\rangle$ with $P_i \sim \frac{e^{-\beta E_i}}{Z}$ $\quad T \to 0$ we are in GS

$$\langle O\rangle = \mathrm{Tr}[O\psi]$$

$$\psi = \sum_i P_i |\psi_i\rangle\langle\psi_i|$$

$F = E - \cancel{TS}$

$\downarrow$

$\mathcal{E}_{GS}$ $\quad |\psi\rangle$

$O = \sum_k O_k |k\rangle\langle k|$

$P_k = |\langle\psi|k\rangle|^2$

Q. Phase

$|\psi_i\rangle$ with $P_i \sim \frac{e^{-\beta E_i}}{Z}$

$T \to 0$ we are in GS

$\langle O\rangle = \mathrm{Tr}[O\psi]$

$\psi = \sum_i P_i |\psi_i\rangle\langle\psi_i|$

$\langle H \rangle$

Q. Phases are domains of analicity of $\mathcal{E}$

$\lambda = (\lambda^1, \dots, \lambda^n)$

$$H(\lambda) = \sum_X \Phi_X(\lambda)$$

we are in GS

$< K|$

Q. Phases are domains of analiticity of $\mathcal{E}$

$$\lambda = (\lambda^1, \dots, \lambda^n)$$

$$H(g) = -g \sum \hat{\sigma}_i^x - J \sum_{(ij)} \hat{\sigma}_i^z \sigma_j^z$$

$$H(\lambda) = \sum_X \Phi_X(\lambda)$$

we are in GS

Q. Phases are domains of analicity of $\mathcal{E}$

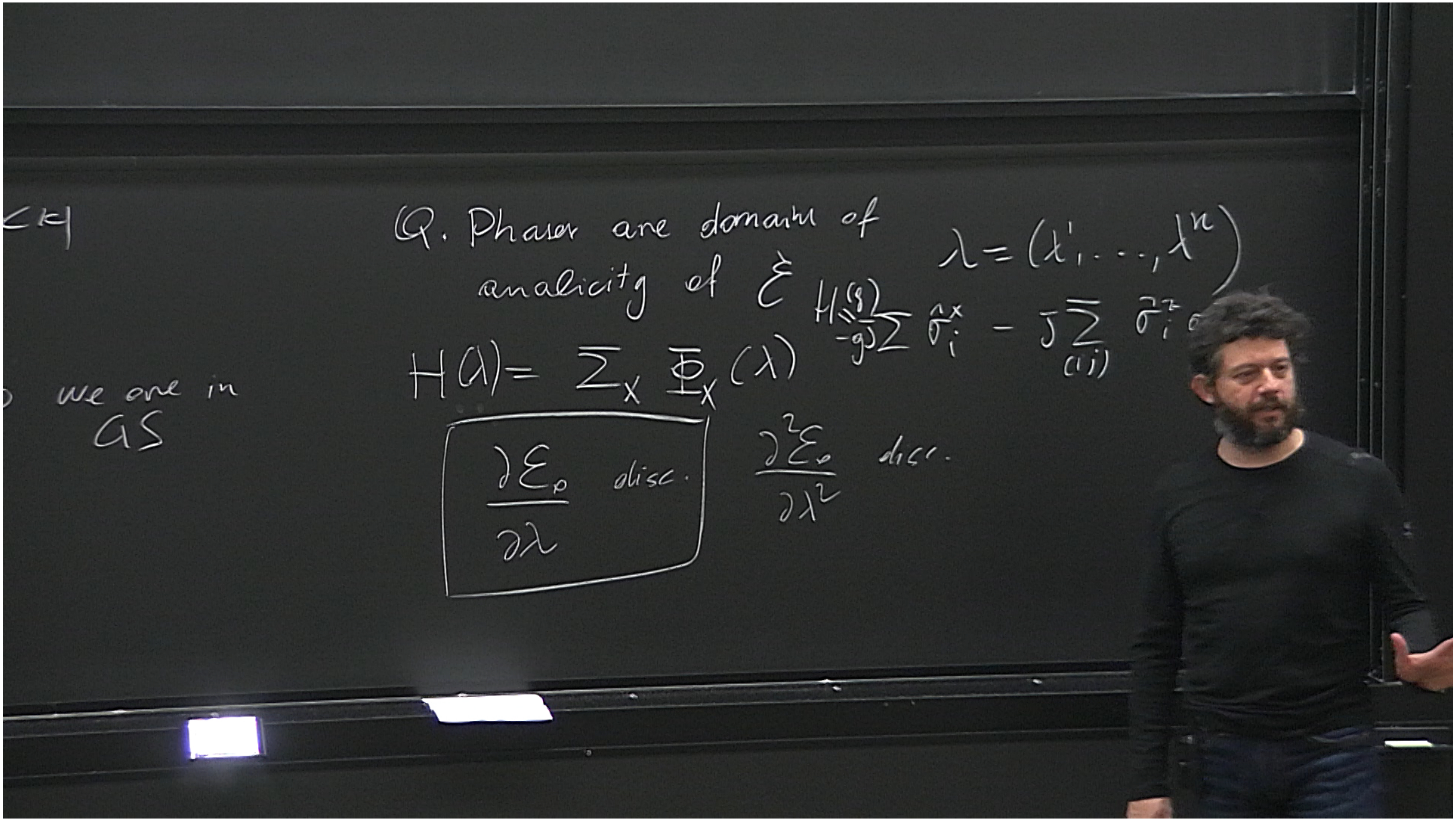
$\lambda = (\lambda^1, \ldots, \lambda^n)$

$H(g) = -g \sum \hat{\sigma}_i^x - J \sum_{(ij)} \hat{\sigma}_i^z \sigma_j^z$

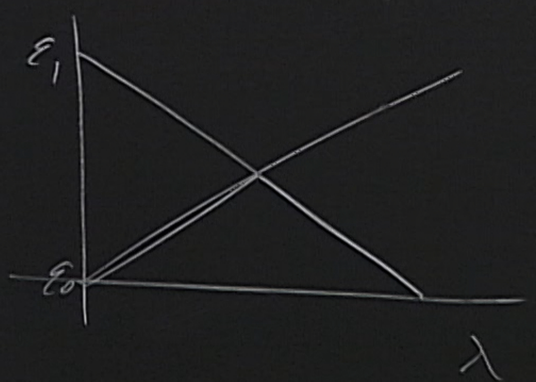
$H(\lambda) = \sum_X \Phi_X(\lambda)$

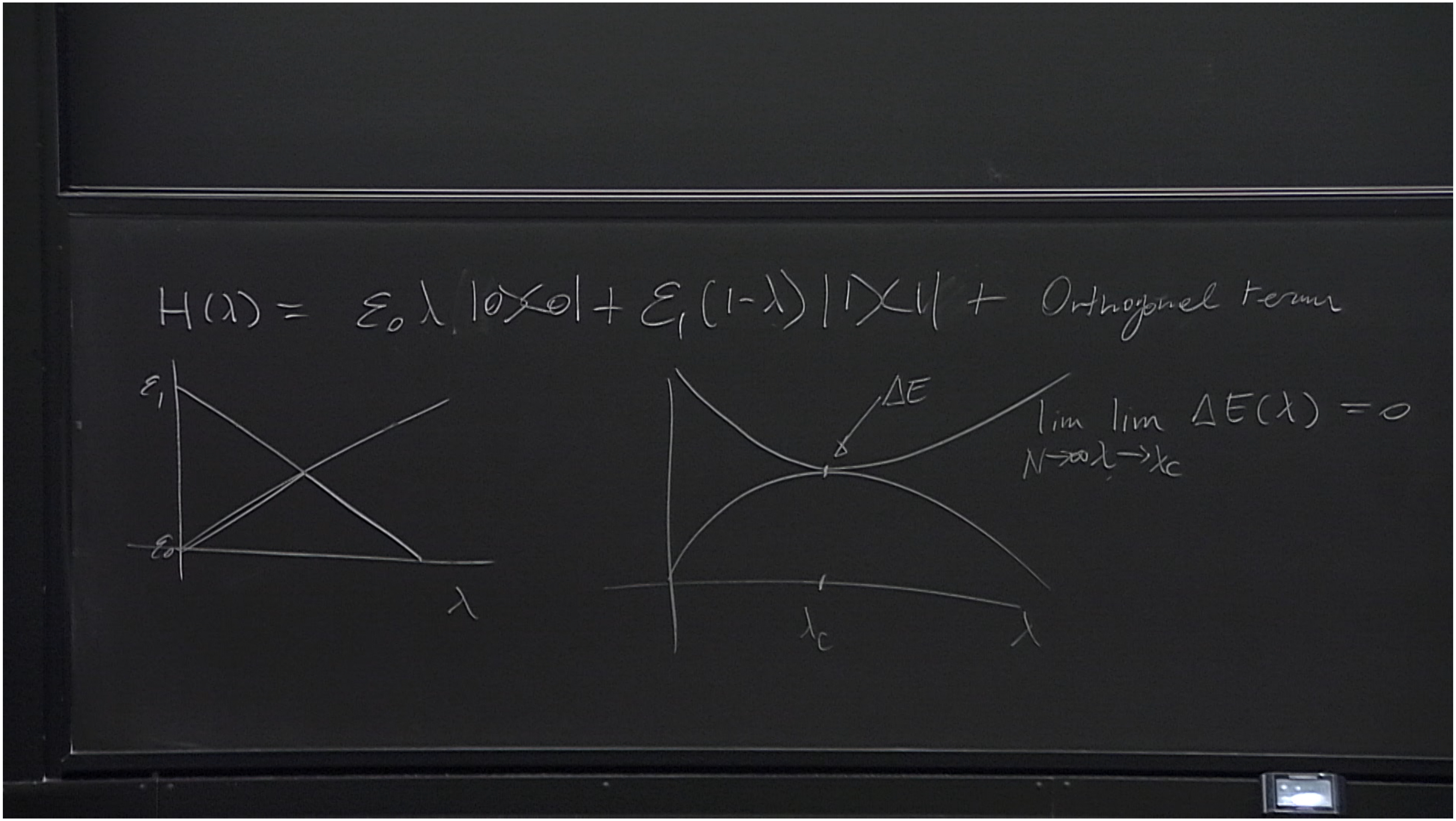
we are in GS

$\frac{\partial \mathcal{E}_0}{\partial \lambda}$ disc. $\frac{\partial^2 \mathcal{E}_0}{\partial \lambda^2}$ disc.


Q. Phases are domain of
analicity of $\mathcal{E}$
$\lambda = (\lambda^1, \dots, \lambda^n)$
$H(g) = -g \sum \hat{\sigma}_i^x - J \sum_{(ij)} \hat{\sigma}_i^z$
$H(\lambda) = \sum_X \Phi_X(\lambda)$
$\frac{\partial \mathcal{E}_0}{\partial \lambda}$ disc.
$\frac{\partial^2 \mathcal{E}_0}{\partial \lambda^2}$ disc.
we are in
GS

$$H(\lambda) = \varepsilon_0 \lambda |0\rangle\langle 0| + \varepsilon_1 (1-\lambda) |1\rangle\langle 1| + \text{Orthogonal terms}$$




$H(\lambda) = \varepsilon_0 \lambda |0\rangle\langle 0| + \varepsilon_1 (1-\lambda) |1\rangle\langle 1| + \text{Orthogonal term}$
ε_1
ε_0
λ
ΔE
λ_c
λ
$\lim_{N\to\infty} \lim_{\lambda \to \lambda_c} \Delta E(\lambda) = 0$

λ_c λ

$$C_{ij} \sim e^{-|i-j|/\xi}$$

$$\xi \sim |\lambda - \lambda_c|^{-\nu}$$

$$\boxed{\Delta E^{-1} = \xi^{z} \sim |\lambda - \lambda_c|^{-\nu z}}$$