Title: PSI 2018/2019 - Foundations of Quantum Mechanics - Lecture 14

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Abstract:

10) Interpretations of Quantum Theory

1) A Map of the Madness
2) de Broglie-Bohm Theory
3) Spontaneous Collapse Theories
4) Everett/Many Worlds
5) Copenhagenish Interpretations

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10.4) Everett/Many Worlds

Axioms of Quantum Theory:

Dynamical Axioms:

1. A physical system A is described by a Hilbert Space $\mathcal{H}_A$.
2. States are vectors in the Hilbert Space: $|\psi\rangle_A \in \mathcal{H}_A$.
3. States evolve unitarily according to the Schrödinger equation:

$$i\frac{d|\psi\rangle_A}{dt} = H|\psi\rangle_A \quad \Rightarrow \quad |\psi(t)\rangle_A = U(t_0,t)|\psi(t_0)\rangle_A, \quad U^\dagger U = I_A$$

4. Composite systems are described by tensor products: $\mathcal{H}_{AB} = \mathcal{H}_A \otimes \mathcal{H}_B$

Measurement Axioms

5. Observables are described by self-adjoint operators: M s.t. $M^\dagger = M$
6. The Born probability rule for measurement outcomes: $\mathrm{Prob}(P_j) = \langle\psi|P_j|\psi\rangle_A$
7. The collapse of the wavefunction/projection postulate: $|\psi\rangle_A \to \frac{P_j|\psi\rangle_A}{\sqrt{\langle\psi|P_j|\psi\rangle_A}}$

The Church of the Larger Hilbert Space (Puritan Version)

- Everett first proposed his interpretation in 1957. H. Everett, *Rev. Mod. Phys*. 29:454–462.
- The approach described here is a mixture of:
  - Oxford Everettianism: D. Wallace, The Emergent Multiverse (OUP, 2012)
  - Zurek's ideas on decoherence: W. Zurek, arXiv:quant-ph/0306072 (2003). R. Blume-Kohout, W. Zurek Phys. Rev. A:062310 (2006)
- The basic idea is to view the dynamical axioms as unproblematic:
  - The quantum state is an ontic physical state and it evolves unitarily in time. The entire universe obeys these rules.
- In orthodox interpretation this comes into conflict with the measurement axioms:
  - so Everett proposed we simply discard the measurement axioms.
  - They are to be derived as effective/emergent rules that an observer who is a quantum subsystem would use.

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Everettian Quantum Theory

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Everettian Schrödinger's cat

- The primitive ontology of Everett does not directly correspond to our everyday experience, so we must regard the latter as emergent.
- In a Schrödinger cat state like:

$$\alpha|\uparrow\rangle \otimes |\text{alive cat}\rangle + \beta|\downarrow\rangle \otimes |\text{dead cat}\rangle$$

we know that the world we experience corresponds to one of the two branches, not both. The ontology is just the entire quantum state, so there is nothing in it that picks out one branch over the other.

- Our world of experience exists + ontology is just the quantum state $\Rightarrow$ There are many worlds.
- Everett's idea was to define relative states:
  - Relative to $|\uparrow\rangle$, the cat is alive.
  - Relative to $|\downarrow\rangle$, the cat is dead.
- But both are equally real.

Everettian Observers

- If I model an observer (you) as a quantum system then how do we know that you see a definite outcome in a measurement?

$$\alpha|\uparrow\rangle \otimes |\text{you saw } \uparrow\rangle + \beta|\downarrow\rangle \otimes |\text{you saw } \downarrow\rangle$$

- Deutsch's answer: I should just ask you.
  - I'll pass you a blank piece of paper on which you write "yes" if you saw a definite outcome and "no" otherwise.
  - This should be described as a unitary interaction.
  - If you are honest then we should have:

$$|\uparrow\rangle \otimes |\text{you saw } \uparrow\rangle \otimes |\text{blank}\rangle \rightarrow |\uparrow\rangle \otimes |\text{you saw } \uparrow\rangle \otimes |\text{yes}\rangle$$
$$|\downarrow\rangle \otimes |\text{you saw } \downarrow\rangle \otimes |\text{blank}\rangle \rightarrow |\downarrow\rangle \otimes |\text{you saw } \downarrow\rangle \otimes |\text{yes}\rangle$$

  - Then, by linearity:

$$(\alpha|\uparrow\rangle \otimes |\text{you saw } \uparrow\rangle + \beta|\downarrow\rangle \otimes |\text{you saw } \downarrow\rangle) \otimes |\text{blank}\rangle$$
$$\rightarrow (\alpha|\uparrow\rangle \otimes |\text{you saw } \uparrow\rangle + \beta|\downarrow\rangle \otimes |\text{you saw } \downarrow\rangle) \otimes |\text{yes}\rangle$$

  - So you will always report a definite outcome, even in superposition.

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The Basis Problem

- There are two big problems for Everett/many-worlds:
  - The Basis Problem: How do we identify emergent worlds?
  - The Probability problem: Why should we assign Born rule probabilities to the worlds?
- Basis problem: I can write any quantum state in any orthonormal basis.

$$\alpha|\uparrow\rangle \otimes |\text{you saw } \uparrow\rangle + \beta|\downarrow\rangle \otimes |\text{you saw } \downarrow\rangle$$
$$= \tfrac{1}{2}([|\uparrow\rangle + |\downarrow\rangle] \otimes [\alpha|\text{you saw } \uparrow\rangle + \beta|\text{you saw } \downarrow\rangle])$$
$$+ \tfrac{1}{2}([|\uparrow\rangle - |\downarrow\rangle] \otimes [\alpha|\text{you saw } \uparrow\rangle - \beta|\text{you saw } \downarrow\rangle])$$

- Why shouldn't I interpret the second decomposition as representing two worlds in which you are in a horribly nonclassical state?
- Answer: We also have the dynamics. Identify worlds as structures in the quantum state that persist in time ⇒ decoherence theory.

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Decoherence

- It is important to realize that, at this stage, we cannot rely on any structures in quantum theory that derive their meaning from the quantum probability rule.
  - This includes reduced density operators.
- In Zurek's approach to decoherence, we assume a preferred decomposition of the Hilbert space of the universe into subsystems.
  - This is an additional structure.
- Oxford Everettians prefer to use the decoherent histories formalism, which does not need this additional structure.
  - In my opinion, the decoherent histories formalism presupposes the probability rule, so I prefer Zurek's approach.
- See M. Schlosshauer, Rev. Mod. Phys. 76:1267-1305 (2004) for a review of decoherence and its role in interpretations.

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The Probability Problem

- Now that we have defined worlds, we need to explain why, in a typical measurement interaction:

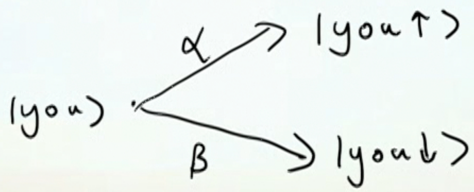
$$(\alpha|\uparrow\rangle + \beta|\downarrow\rangle) \otimes |\text{you}\rangle \otimes |E\rangle \rightarrow \alpha|\uparrow\rangle \otimes |\text{you}_\uparrow\rangle \otimes |E_\uparrow\rangle + \beta|\downarrow\rangle \otimes |\text{you}_\downarrow\rangle \otimes |E_\downarrow\rangle$$

you should, before the measurement, assign probabilities $|\alpha|^2$ and $|\beta|^2$ to the two worlds that will be created.

- Even the meaning of the probabilities is nontrivial:
  - It is not as if you will become either $\text{you}_\uparrow$ or $\text{you}_\downarrow$ and you don't know which. Both have equal claim to be your successor.
  - It is as if you are going to be cloned twice, the original you killed, and you have to assign probabilities to your two successors.

World Counting

- An intuitively appealing rule is "world counting": If there are N worlds after a branching event, then the probability of each world should be $\frac{1}{N}$.



$\text{Prob}(\uparrow) = \text{Prob}(\downarrow) = \frac{1}{2}$ which is wrong.

- This is so intuitively appealing to some people that they take it to obviously rule out many-worlds.
- However, it is not at all obvious why you should do world counting when all worlds exist on an equal footing.
- The precise number of worlds is not even well-defined in general.

Interpretations of Probability

- There are broadly three ways of interpreting probabilities:
  1. Frequentist: probability is the long run relative frequency of an outcome in multiple repetitions of an experiment.
  2. Bayesian: Probabilities represent the degrees of belief of a rational agent – they constrain rational decision making.
  3. Objective chance: Probabilities represent objective facts about the way a single experiment is performed – perhaps a disposition to produce a certain outcome, or facts about what the relative frequency would be if repeated.
- All interpretations of probability are controversial. When deriving probability in many worlds we should distinguish:
  - Problems that are common to classical probability, which we can't blame on Everett.
  - Problems that are specific to the many-worlds interpretation.

Attempt at Frequentist Derivation

- Everett attempted a frequentist derivation. Later refinements by Graham, Hartle, and others.
- Imagine multiple systems prepared in the same quantum state

$$|\psi\rangle_{S_1} \otimes |\psi\rangle_{S_2} \otimes \cdots \otimes |\psi\rangle_{S_N}, \quad \text{where } |\psi\rangle_{S_j} = \alpha|\uparrow\rangle_{S_j} + \beta|\downarrow\rangle_{S_j}.$$

- After measurement and branching we can write the state as

$$|\Phi\rangle = |\Psi\rangle_{S_1E_1} \otimes |\Psi\rangle_{S_2E_2} \otimes \cdots \otimes |\Psi\rangle_{S_NE_N}, \text{ where } |\Psi\rangle_{S_j} = \alpha|\uparrow\rangle_{S_j} \otimes |E_\uparrow\rangle_{E_j} + \beta|\downarrow\rangle_{S_j} \otimes |E_\downarrow\rangle_{E_j}.$$

- To simplify notation, let

$$|0\rangle_{S_jE_j} = |\uparrow\rangle_{S_j} \otimes |E_\uparrow\rangle_{E_j} \quad \text{and} \quad |1\rangle_{S_jE_j} = |\downarrow\rangle_{S_j} \otimes |E_\downarrow\rangle_{E_j}$$

- We can now define relative frequency projectors, e.g. for $N = 3$:

$$\Pi_0 = |000\rangle\langle 000|, \qquad \Pi_{1/3} = |100\rangle\langle 100| + |010\rangle\langle 010| + |001\rangle\langle 001|$$

$$\Pi_{2/3} = |110\rangle\langle 110| + |101\rangle\langle 101| + |011\rangle\langle 011|, \qquad \Pi_1 = |111\rangle\langle 111|$$

Attempt at Frequentist Derivation

- In the limit $N \to \infty$ you can show: $\Pi_{|\beta|^2}|\Phi\rangle = |\Phi\rangle$.
- The state is an eigenstate of the relative frequency $|\beta|^2$ projector with eigenvalue 1.
- The idea is to interpret this as saying that the relative frequency of $|1\rangle$'s is $|\beta|^2$ with certainty.
- Problems:
  - This requires the eigenvalue-eigenstate link, but for infinite spaces probability 1 is not the same as certainty.
  - For any finite N, you get $\langle\Phi|\Pi_{|\beta|^2}|\Phi\rangle \geq 1 - \epsilon$, where ϵ decreases exponentially with N, but you can't interpret this as "the relative frequency is likely to be $|\beta|^2$ without interpreting $\langle\Phi|\Pi_{|\beta|^2}|\Phi\rangle$ as a probability, which is what we are trying to derive in the first place.
- Refinements define approximate relative frequency projectors. They still have the same problems.

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Decision Theoretic Approach

- Deutsch and Wallace have developed an approach based on subjective Bayesian probability and objective chance. – D. Deutsch, Proc. Roy. Soc. A 455:3129-3137 (1999). D. Wallace, The Emergent Multiverse (OUP, 2012).

- The full argument requires some sophisticated decision theory. Will present a more heuristic version here.

Classical Subjective Bayesianism

- Suppose we are uncertain about which of a finite set X of possibilities might occur, e.g. $X = \{1,2,3,4,5,6\}$ for the outcome of a dice roll.
- A subset of X is called an event. It represents a proposition we can state about the outcome, e.g. $\text{outcome is odd} = \{1,3,5\}$.
- The axioms of classical probability are:
  - $0 < P(E) < 1$
  - $P(\emptyset) = 0, P(X) = 1$
  - If E and F are disjoint then $P(E \cup F) = P(E) + P(F)$
- In subjective Bayesianism, probabilities represent the subjective degrees of belief of a decision making agent. Why should they obey these axioms?

The Dutch Book argument

- We define a way of measuring degrees of belief:
  - Your probability for E is the price $\$P(E)$ at which you would be willing to buy or sell any number of the following lottery tickets

Price $P(E)	Pays $1 if E occurs Pays $0 otherwise

- Rationality criterion: You should not enter into a system of bets that causes you to make a loss for every possible outcome.
  - From this, we can *derive* the axioms of probability.

Example Dutch Book Derivations

- Suppose you set $P(E) > 1$.

- Then you would be willing to buy a ticket that pays \$1 if E occurs and nothing otherwise for a price $\$P(E)$.

  - If E occurs then you have lost: $\$P(E) - \$1 > \$0$.

  - If E does not occur then you have lost: $\$P(E) > 0$.

- Sure loss in both cases, so rationality implies $\$P(E) \leq 1$.

Example Dutch Book Derivations

- Suppose you set $P(E) + P(F) > P(E \cup F)$ for disjoint E and F.
- Then you'd be willing to buy these two tickets:

Price	Pays \$1 if E occurs
\$P(E)	\$0 otherwise

Price	Pays \$1 if F occurs
\$P(F)	Pays \$0 otherwise

- And sell this one:

Price	Pays \$1 if E∪F occurs
\$P(E∪F)	Pays \$0 otherwise

Possible outcome	E	F	neither E nor F
Loss	$P(E)+P(F)-P(E\cup F) -1+1 > 0$	$P(E)+P(F)-P(E\cup F) -1+1 > 0$	$P(E)+P(F)-P(E\cup F) > 0$

Application to Many-Worlds

- Keep definition of probabilities in terms of willingness to make bets on measurement outcomes.
- Modified Rationality criterion: You should not enter into a system of bets that causes all of your successors to make a loss.

$$|\psi\rangle \otimes |you\rangle \longrightarrow \alpha_1|\psi_1\rangle \otimes |you_1 ☹\rangle + \alpha_2|\psi_2\rangle \otimes |you_2 ☹\rangle \quad ✗$$

- This gives a meaning to the probabilities of future worlds, and the Dutch Book implies you should assign probabilities $P(j)$ to the worlds that satisfy the usual axioms:

$$P(j) \geq 0, \quad \sum_j P(j) = 1.$$

- It still remains to argue that $P(j) = |\alpha_j|^2$

Deutsch-Wallace Postulates

1. State supervenience: The probabilities you should assign depend only on the quantum state (here's where objective chance comes in).

2. Microstate indifference: You only care about the $ you win on a branch. The rest of it can be changed without affecting your betting preferences.

3. Branching indifference: You don't care if worlds branch into even more worlds later on, provided you have the same $ on the new branches.

4. Continuity: Probabilities should be a continuous function of the quantum state.

Equal Amplitude Case

- You measure a system prepared in the state $\frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)$, get $\$a$ for $\uparrow$ and $\$b$ for $\downarrow$.

$$\rightarrow \frac{1}{\sqrt{2}}(|\uparrow\rangle|\$a\rangle + |\downarrow\rangle|\$b\rangle)$$

- Suppose you did the same thing with opposite prizes $\$a$ for $\downarrow$ and $\$b$ for $\uparrow$.

$$\rightarrow \frac{1}{\sqrt{2}}(|\downarrow\rangle|\$a\rangle + |\uparrow\rangle|\$b\rangle)$$

- By microstate indifference, you can flip the spin afterwards without changing your betting preferences.

$$\rightarrow \frac{1}{\sqrt{2}}(|\uparrow\rangle|\$a\rangle + |\downarrow\rangle|\$b\rangle)$$

- But now both cases give the same physical state. Since this applies for all possible choices of bets and prizes $P(\uparrow) = P(\downarrow) = \frac{1}{2}$.
- Clearly this generalizes to an equal superposition of N branches, which would give $P(j) = \frac{1}{N}$.

Rational and General Amplitude Case

- For rational amplitudes, we can use branching indifference to branch into an equal superposition, e.g.

$$\sqrt{\tfrac{2}{3}}|\psi_1\rangle|\$a\rangle + \sqrt{\tfrac{1}{3}}|\psi_2\rangle|\$b\rangle \quad \rightarrow \quad \sqrt{\tfrac{1}{3}}\left(|\psi_1^1\rangle|\$a\rangle + |\psi_1^2\rangle|\$a\rangle + |\psi_2\rangle|\$b\rangle\right)$$

- We know the probabilities are $\frac{1}{3}$ in the equal superposition, so we'll get:

$$P(\psi_1) = P(\psi_1^1) + P(\psi_1^2) = \tfrac{2}{3}$$

- From this, we'll get $P(j) = |\alpha_j|^2$ for rational amplitudes.

- We then get the general case by continuity.

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Some Objections

- Bayesian probability is supposed to be about reasoning in the face of uncertainty. There is no uncertainty here – all successors exist. You have skewed the meaning of decision theory:
  - Reinterpret probability as degree with which you should care about your successors.
  - Apply argument after you have become entangled with the measuring device, but before you are aware of the outcome – self-locating uncertainty about which branch you are on.
- I don't believe in state supervenience. Suppose I create two clones of you, put one in a room with α painted on the wall and the other in a room with β on the wall, and then kill the original. Why should I assign probabilities $|\alpha|^2$ and $|\beta|^2$ to the two clones?
  - The quantum state is the only ontology available to determine objective chances. If not that then what?
- See S. Saunders et. al. (eds.), Many Worlds?, (OUP, 2010) for many papers pro and contra.

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Summary so Far

- Everett starts from the premise that a quantum state evolving unitarily is the entire ontology of quantum theory – Church of the Larger Hilbert Space writ large.
- From this we *derive* that there must be many-worlds.
- The basis problem is solved to most people's satisfaction by decoherence.
- The probability problem is much more controversial:
  - To the extent that probabilities make sense at all in many-worlds, I believe it is solved by the Deutsch-Wallace or related approaches.
- Even if everything works out, we can still deny the premise of the interpretation, which is heavily ψ-ontic.
- ψ-epistemicists should look elsewhere, but many worlds is a decent choice for a ψ-ontologist.

The Ironic Many-Worlds Interpretation

- A ψ-epistemicist like myself does not like the starting premise of many-worlds, i.e. the quantum state is real.
- But we don't actually need all of the quantum state to determine the pointer basis in decoherence – many different starting states will decohere into the same pointer states, e.g.

$$(\alpha|\uparrow\rangle + \beta|\downarrow\rangle)|E\rangle \quad \rightarrow \quad \alpha|\uparrow\rangle|E_{\uparrow}\rangle + \beta|\uparrow\rangle|E_{\downarrow}\rangle\,,$$

Decoheres the same way regardless of the values of α and β.

- So we can believe that the structure that determines pointer states is real, but not the amplitudes in that decomposition.
- But now how are we going to derive the Born rule?