Title: PSI 2018/2019 - Foundations of Quantum Mechanics - Lecture 13

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Abstract:

Quantum Foundations Lecture 24

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HSC112

A Map Of The Madness

	Realist		**Copenhagenish**	
	Ontological Model	**Exotic Ontology**	**Objective**	**Perspectival**
ψ-epsitemic		Ironic Many Worlds	Copenhagen	QBism
		Quantum Logical Realism	Healy's Quantum Pragmatism	Rovelli's Relational Quantum Mechanics
			Bub's "Information" Interpretation	
ψ-ontic	de Broglie-Bohm	Everett/Many Worlds		
	Spontaneous Collapse			
	Modal Interpretations			

10.ii) de Broglie-Bohm Theory

- A brief history:
  - The 1st order form of dBB theory was discovered and then abandoned by de Broglie in the 1920's.
  - dBB was rediscovered, in 2nd order form, by Bohm in 1952.
  - The forgotten 1st order form was promoted by Bell in the 1970's and 80's.
  - Proponents still fight over which form is better. I will follow Bell's approach here.
- See T. Norsen, "Foundations of Quantum Mechanics" (Springer,2017) for an overview of this theory.

Ontology of dBB Theory

- The goal of any interpretation is to:
  - Provide an ontology: a statement of what exists and how it behaves.
  - Save the phenomena: Explain the quantum predictions and our everyday experience in terms of the ontology.
- Bohmians typically divide the ontology into two pieces:
  - Primitive ontology: The things that determine what we experience. Usually assumed to be localized in spacetime – local beables. In dBB this is particle trajectories.
  - The rest: Needed to determine how the primitive ontology behaves. In dBB this is the quantum state.

Single Particle Theory in 1-Dimension

- For particles with no internal degrees of freedom (spin), we use the wavefunction

$$\psi(x,t) = \langle x|\psi(t)\rangle$$

- The quantum state obeys the Schrödinger equation: $i\frac{\partial|\psi\rangle}{\partial t} = H|\psi\rangle$
- dBB also has an actual particle with position X.

- This obeys the guidance equation:

$$\frac{dX}{dt} = \frac{1}{m}\frac{\mathrm{Im}\left(\psi^*(x,t)\frac{\partial\psi(x,t)}{\partial x}\right)}{\psi^*(x,t)\psi(x,t)}\Bigg|_{x=X}$$

Single Particle Theory in 3-Dimensions

- In 3-dimensions, we introduce the basis $|\vec{q}\rangle = |x\rangle \otimes |y\rangle \otimes |z\rangle$
- For particles with no internal degrees of freedom (spin), we use the wavefunction

$$\psi(\vec{q},t) = \langle\vec{q}|\psi(t)\rangle = \langle x|\langle y|\langle z|\psi(t)\rangle$$

- The quantum state obeys the Schrödinger equation: $i\frac{\partial|\psi\rangle}{\partial t} = H|\psi\rangle$
- dBB also has an actual particle with position vector $\vec{Q}$
- This obeys the guidance equation:

$$\frac{d\vec{Q}}{dt} = \frac{1}{m}\frac{\mathrm{Im}\left(\psi^*(\vec{q},t)\vec{\nabla}\psi(\vec{q},t)\right)}{\psi^*(q,t)\psi(q,t)}\Bigg|_{\vec{q}=\vec{Q}}$$

General Case

- To describe N particles, we need to specify a position vector for each of them

$$\boldsymbol{q} = (\vec{q}_1, \vec{q}_2, \cdots, \vec{q}_3)$$

- Notation: $\vec{q}$ denotes a vector in $\mathbb{R}^3$. $\boldsymbol{q}$ denotes a vector in $\mathbb{R}^{3N}$, called a *configuration vector*.
- $\mathbb{R}^{3N}$ is called *configuration space*.
- We can write a quantum state as a wavefunction on configuration space:

$$\psi(\boldsymbol{q}, t) = \psi(\vec{q}_1, \vec{q}_2, \dots, \vec{q}_N, t) = \langle \boldsymbol{q} | \psi(t) \rangle = \langle \vec{q}_1, \vec{q}_2, \dots, \vec{q}_N | \psi(t) \rangle$$

- The wavefunction obeys the Schrödinger equation: $i \frac{\partial |\psi\rangle}{\partial t} = H |\psi\rangle$
- dBB also has an actual point in configuration space:

$$\boldsymbol{Q} = (\vec{Q}_1, \vec{Q}_2, \dots, \vec{Q}_N)$$

- This obeys the guidance equation:

$$\frac{d\vec{Q}_k}{dt} = \frac{\hbar}{m_k} \frac{\mathrm{Im}\left(\psi^*(\boldsymbol{q}, t) \vec{\nabla}_k \psi(\boldsymbol{q}, t)\right)}{\psi^*(\boldsymbol{q}, t) \psi(\boldsymbol{q}, t)} \Bigg|_{\boldsymbol{q}=\boldsymbol{Q}}$$

Equilibrium Hypothesis and Equivariance

- One more postulate is required to obtain the same predictions as standard quantum theory - Quantum Equilibrium Hypothesis:
  - At time $t = t_0$, the probability density of the system occupying configuration point $\boldsymbol{Q}$ is:

$$\rho(\boldsymbol{Q}) = |\psi(\boldsymbol{Q})|^2$$

- Under the dBB evolution we will show that if this holds at $t = t_0$ then it holds at all times. This is known as equivariance.
- There is controversy about what $\rho(\boldsymbol{Q})$ means as dBB is applied to the *entire universe*, which only has a single configuration space point.
  - Roughly speaking, if we prepare many systems in the state $|\psi\rangle \otimes |\psi\rangle \otimes \cdots \otimes |\psi\rangle$, the probability density of configurations is $\rho(\boldsymbol{Q})$.
- Note that the quantum state is playing two *independent* roles:
  - It governs dynamics via the guidance equation.
  - It is used to set the probability density.

Continuity Equation For Probability

- Substituting these into

$$\frac{\partial \rho}{\partial t} = \psi^*(x,t)\frac{\partial \psi(x,t)}{\partial t} + \frac{\partial \psi^*(x,t)}{\partial t}\psi(x,t)$$

gives

$$\frac{\partial \rho}{\partial t} = \frac{i}{2m}\left[\psi^*(x,t)\frac{\partial^2 \psi(x,t)}{\partial^2 x} - \frac{\partial^2 \psi^*(x,t)}{\partial^2 x}\psi(x,t)\right]$$

$-iV(x)[\psi^*(x,t)\psi(x,t) - \psi(x,t)\psi^*(x,t)]$ (this term cancels)

Multiple particles

- For multiple particles in 3D, this generalizes to

$$\frac{\partial \rho(\boldsymbol{q}, t)}{\partial t} + \boldsymbol{\nabla} \cdot \boldsymbol{J}(\boldsymbol{q}, t) = 0$$

with probability current $\boldsymbol{J} = (\vec{J}_1, \vec{J}_2, \cdots, \vec{J}_n)$

$$\vec{J}_k(\boldsymbol{q}, t) = \frac{1}{m_k} \mathrm{Im}\left(\psi^*(\boldsymbol{q}, t) \vec{\nabla}_k \psi(\boldsymbol{q}, t)\right)$$

Bell's derivation of the guidance equation and equivariance

- Solutions of the Schrödinger equation satisfy the continuity equation:

$$\frac{\partial |\psi(\boldsymbol{q},t)|^2}{\partial t} + \boldsymbol{\nabla}\cdot \boldsymbol{J}(\boldsymbol{q},t) = 0$$

where $\boldsymbol{J}(\boldsymbol{q},t)$ is the probability current:

$$\boldsymbol{J} = (\vec{J}_1, \vec{J}_2, \dots, \vec{J}_N) \qquad \vec{J}_k(\boldsymbol{q}) = \frac{\hbar}{m_k}\mathrm{Im}(\psi^* \vec{\nabla}_k \psi)(\boldsymbol{q})$$

- If we consider a preparation of $|\psi\rangle \otimes |\psi\rangle \otimes \cdots$ we want to consider $\boldsymbol{J}$ as a flow of particle density rather than probability.

- If we assume this is generated by a velocity field $\boldsymbol{v}(\boldsymbol{q})$, e.g. as in hydrodynamics, then $\boldsymbol{J} = \rho\boldsymbol{v}$, so the equation for the velocity field should be:

$$\boldsymbol{v}(\boldsymbol{q}) = \frac{\boldsymbol{J}(\boldsymbol{q})}{\rho(\boldsymbol{q})} \qquad \vec{v}_k(\boldsymbol{q}) = \frac{\hbar}{m_k}\frac{\mathrm{Im}(\psi^* \vec{\nabla}_k \psi)}{\rho}(\boldsymbol{q})$$

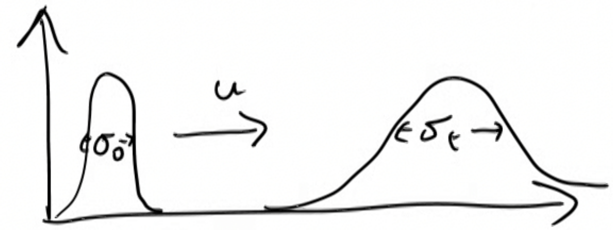
which gives the dBB velocities if we set $\rho(\boldsymbol{Q}) = |\psi(\boldsymbol{Q})|^2$.

Trajectories for a 1D Gaussian Wavepacket

- Consider an initial Gaussian wavepacket moving towards the right

$$\psi(x,0) = \frac{1}{(2\pi\sigma_0^2)^{1/4}} \exp\left[-\frac{x^2}{4\sigma_0^2} + ikx\right]$$

- Under free-particle evolution this moves with group velocity $u = \frac{\hbar k}{m}$ and spreads $\sigma_t = \sigma_0\sqrt{1 + \frac{\hbar^2 t^2}{4m^2\sigma_0^4}}$



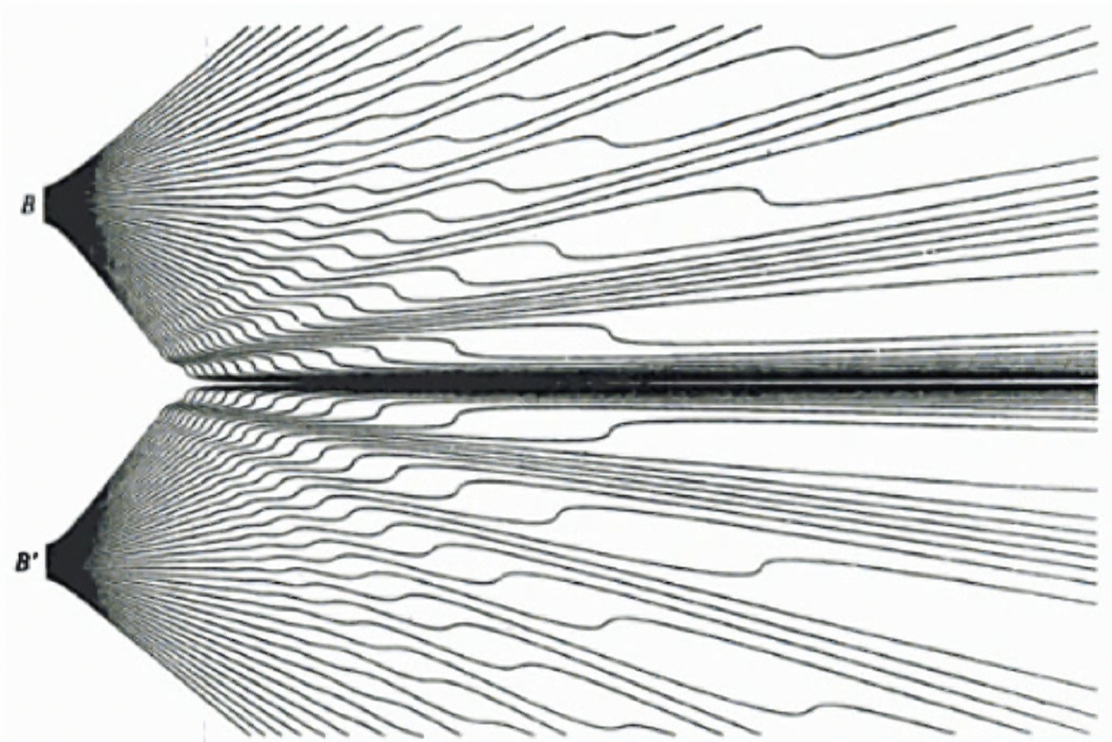
- If we consider a timescale s.t. spreading is negligible $t^2 \ll \frac{2m\sigma_0^2}{\hbar}$

  then the dBB velocity $\frac{dX}{dt} \simeq u$

Particle is dragged along with wavepacket at group velocity

See e.g. A. Pan, Pramana J. Phys. 74:867 (2010)

Double-Slit Trajectories



C.Philippidis et. al. Il Nuovo Cimento, vol.52B, No.1 (1979)

Model with Gaussian slits.

$\psi(\vec{q},t) = \psi_B(\vec{q},t) + \psi_{B'}(\vec{q},t)$

- When ψ_B, ψ_B have approximately no overlap (close to slits)

$$\vec{J} \simeq \vec{J}_B + \vec{J}_{B'}$$

$$\vec{J}_B = \frac{1}{m} \, \mathrm{Im} \, \psi_B^* \vec{\nabla} \psi_B$$

$$\vec{J}_{B'} = \frac{1}{m} \, \mathrm{Im} \, \psi_{B'}^* \vec{\nabla} \psi_B$$

The trajectories are as in geometric optics, i.e. perpendicular to wavefronts

- When they overlap there are cross-terms (interference) in the current, causing deflections which give the charateristic double-slit pattern.

Measurements in de Broglie-Bohm Theory

- Dividing the universe into system S and environment E allows us to define a pure state for the system called the conditional quantum state.

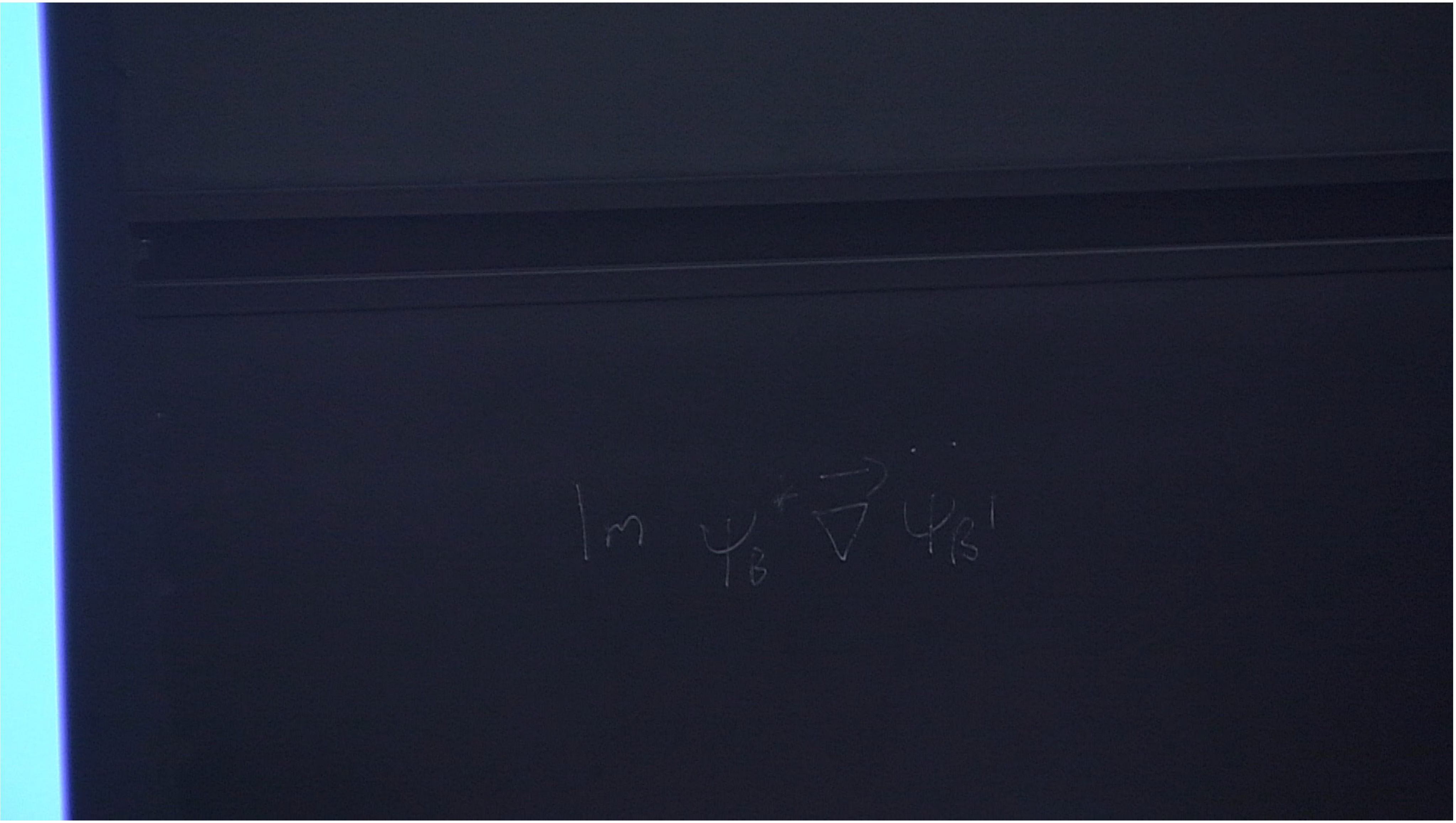
$$|\psi_{\boldsymbol{Q}_E}\rangle_S = \;_E\langle \boldsymbol{Q}_E|\psi\rangle_{SE}$$

where $\boldsymbol{Q}_E$ is the actual configuration point of the environment.

- Generally, these do not evolve according to the Schrödinger equation, but they do if there is decoherence into localized environment states.
  - For example, if $\boldsymbol{Q}_E$ is the pointer variable after a von Neumann measurement interaction.
- Model the measurement device as a large number of particles, with outcomes represented by macroscopically distinct states with very small overlap:

- In a measurement interaction:

$$[\alpha\psi_0(\boldsymbol{q}_S) + \beta\psi_1(\boldsymbol{q}_S)]\Phi_R(\boldsymbol{q}_E) \;\rightarrow\; \alpha\psi_0(\boldsymbol{q}_S)\Phi_0(\boldsymbol{q}_E) + \beta\psi_1(\boldsymbol{q}_S)\Phi_1(\boldsymbol{q}_E)$$


$\mathrm{Im}\, \psi_B^* \vec{\nabla} \psi_{B'}$

$$|\psi_1\rangle_S |\Phi_1\rangle_E + |\psi_2\rangle_S |\Phi_2\rangle_E$$

$$\langle \Phi_1 | \Phi_2 \rangle \simeq 0$$

$$\Phi_1(x)\, \Phi_2(x) \simeq 0$$

Measurements in de Broglie-Bohm Theory

$$\alpha\psi_0(\boldsymbol{q}_S)\Phi_0(\boldsymbol{q}_E) + \beta\psi_1(\boldsymbol{q}_S)\Phi_1(\boldsymbol{q}_E)$$

- If the lack of position overlap between $\Phi_0(\boldsymbol{q}_E)$ and $\Phi_1(\boldsymbol{q}_E)$ persists in time then:
  - The actual configuration of the environment $\boldsymbol{Q}_E$ is either in the support of $\Phi_0(\boldsymbol{q}_E)$ or the support of $\Phi_1(\boldsymbol{q}_E)$.
  - By equivariance, it will be in the support of $\Phi_0(\boldsymbol{q}_E)$ with probability $|\alpha|^2$ and in the support of $\Phi_1(\boldsymbol{q}_E)$ with probability $|\beta|^2$.
  - The conditional state of the system will either be $\propto \psi_0(\boldsymbol{q}_S)$ or $\propto \psi_1(\boldsymbol{q}_S)$.
  - $\psi_0(\boldsymbol{q}_S)$ and $\psi_1(\boldsymbol{q}_S)$ each evolve according to the Schrödinger equation.
  - The current breaks into two terms $\boldsymbol{J} = \boldsymbol{J}_0 + \boldsymbol{J}_1$, with $\boldsymbol{J}_0 = 0$ in the support of $\Phi_1(\boldsymbol{q}_E)$ and vice versa, i.e. no cross terms in the guidance equation.
- We get an effective collapse into either $\psi_0(\boldsymbol{q}_S)\Phi_0(\boldsymbol{q}_E)$ or $\psi_1(\boldsymbol{q}_S)\Phi_1(\boldsymbol{q}_E)$ and we can use the corresponding current $\boldsymbol{J}_0$ or $\boldsymbol{J}_1$ in the guidance equation to compute subsequent evolution.

Measurements in de Broglie-Bohm Theory

- If the measurement is an (approximate) position measurement then also $\psi_0(\boldsymbol{q}_S)\psi_1(\boldsymbol{q}_S) \approx 0$.
- The initial configuration $\boldsymbol{Q}_S$ of the system is either in the support of $\psi_0(\boldsymbol{q}_S)$ with probability $|\alpha|^2$ or in the support of $\psi_1(\boldsymbol{q}_S)$ with probability $|\beta|^2$.
- The measurement outcome is a deterministic function of $\boldsymbol{Q}_S$: position measurements simply reveal the pre-existing position.
- However, for other observables, e.g. momentum, $\psi_0(\boldsymbol{q}_S)\psi_1(\boldsymbol{q}_S) \neq 0$, i.e. the initial configuration does not necessarily "belong" to one of the two eigenstates.
- Which measurement outcome occurs is a function of *both* $\boldsymbol{Q}_S$ and $\boldsymbol{Q}_E$.
- Momentum measurement does not measure the dBB momentum $m_k \frac{d\vec{Q}_k}{dt}$.
- The theory is deterministic: outcome uniquely determined by ontic states of system and measuring device.
- But not outcome deterministic: outcome uniquely determined by ontic state of system on its own.

Treatment of Spin

- In the minimalist Bell approach to dBB, no observables apart from position are part of the primitive ontology.
- Spin only appears in the wavefunction.
- We can write a wavefunction including spin as a spinor, e.g. for a single particle:

$$\psi_0(\vec{q}) \otimes |\uparrow\rangle + \psi_1(\vec{q}) \otimes |\downarrow\rangle \quad \rightarrow \quad \bar{\psi}(\vec{q}) = \begin{pmatrix} \psi_0(\vec{q}) \\ \psi_1(\vec{q}) \end{pmatrix}$$

- For N spin-$1/2$ particles, we would have a 2^N dimensional spinor vector.
- The guidance equation is now:

$$\frac{d\vec{Q}_k}{dt} = \frac{\hbar}{m_k} \frac{\mathrm{Im}(\bar{\psi}^* \cdot \vec{\nabla}_k \bar{\psi})}{\bar{\psi}^* \cdot \bar{\psi}} (\boldsymbol{Q}),$$

  where $\cdot$ is spinor inner product.
- It is possible instead to have primitive ontic states for any complete orthonormal basis, but discrete bases require a stochastic guidance equation.

Counterintuitive Features of dBB Trajectories

- dBB trajectories display several features that violate classical intuitions about particle trajectories.
- It is important to note that, if decoherence occurs in an environmental basis that is localized in position, dBB trajectories of the system will approximately follow classical trajectories.
- dBB doesn't *owe* us anything more than that. So long as:
  - It reproduces the predictions of quantum theory in measurements.
  - Macroscopic systems typically have approximately classical trajectories.

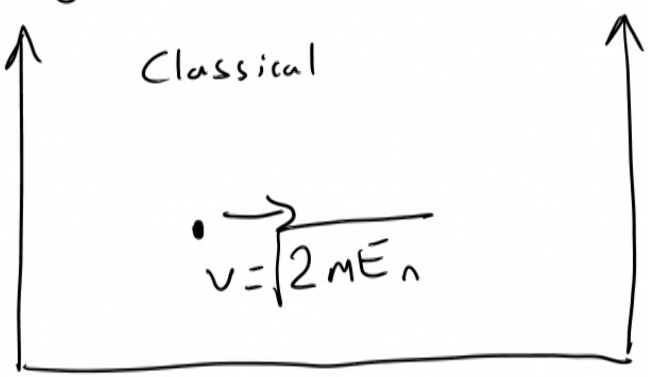
  then the theory saves the phenomena.
- Since quantum and classical predictions are different, dBB trajectories *must* differ from classical ones in some situations.
- The question is only if they are weirder than absolutely necessary to reproduce quantum theory, and whether that is a bad thing.

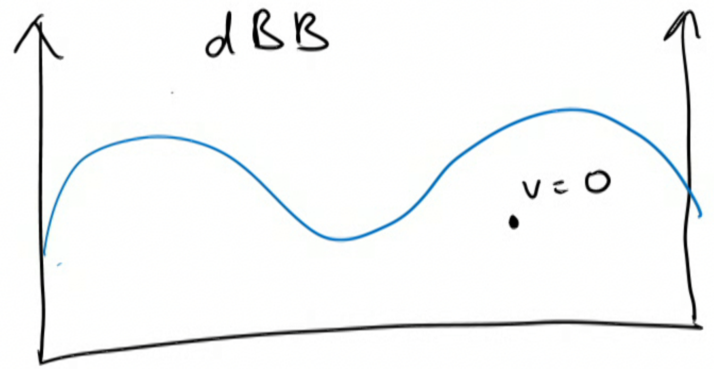
Real Stationary States

- Consider a stationary state: $\psi(\boldsymbol{q},t) = \psi_n(\boldsymbol{q})e^{-iE_n t/\hbar}$
- The current is: $\vec{J}_k(\boldsymbol{q}) = \frac{\hbar}{m_k}\mathrm{Im}(\psi_n^*\vec{\nabla}_k\psi_n)(\boldsymbol{q})$, i.e. is independent of t.
- However, if $\psi_n(\boldsymbol{q})$ is also a real valued function then:

$$\vec{J}_k(\boldsymbol{q}) = \frac{\hbar}{2im_k}(\psi_n^*\vec{\nabla}_k\psi_n - \psi_n\vec{\nabla}_k\psi_n^*)(\boldsymbol{q}) = 0$$

- The particles are also stationary, e.g. particle in an infinite well, hydrogen atom eigenstates.





The No-Crossing Rule

- In classical mechanics, phase space trajectories do not cross (except at singularities) because equations are 2nd order and so $(\boldsymbol{q}, \boldsymbol{p})$ contains enough data to specify a unique trajectory.
- In dBB the guidance equations is 1st order and there is no back action on the quantum state from the configuration space point:
- $[\psi(\boldsymbol{q}, t_0), \boldsymbol{Q}(t_0)]$ and $[\psi(\boldsymbol{q}, t_0), \boldsymbol{Q}'(t_0)]$ specify unique trajectories.
- Trajectories associated with the same wavefunction evolution cannot cross in *configuration space*.
- This is responsible for almost all the weird features of dBB trajectories.
- Note: with decoherence into localized environment states:

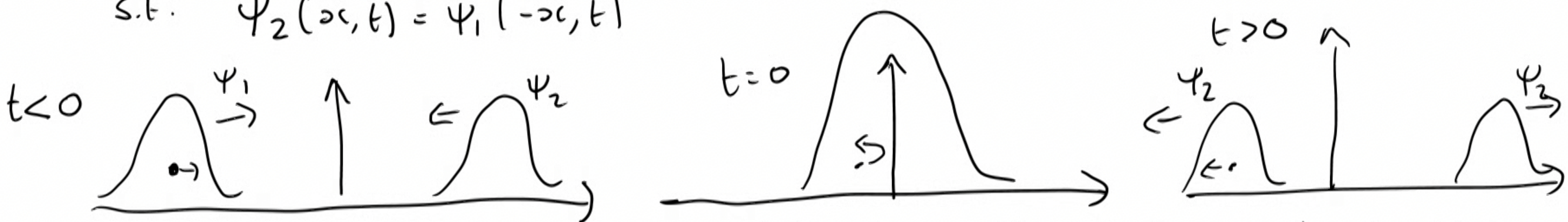
$$\alpha\psi_0(\boldsymbol{q}_S)\Phi_0(\boldsymbol{q}_E) + \beta\psi_1(\boldsymbol{q}_S)\Phi_1(\boldsymbol{q}_E)$$

trajectories can cross in the *system* configuration space because $\boldsymbol{Q}_E$ is necessarily different in the two branches. This is needed to recover classical trajectories.

Empty Waves Steal the Particle

- Consider a superposition of 2 wavepackets $\psi(x,t) = \frac{1}{\sqrt{2}}\left(\psi_1(x,t) + \psi_2(x,t)\right)$

  s.t. $\psi_2(x,t) = \psi_1(-x,t)$

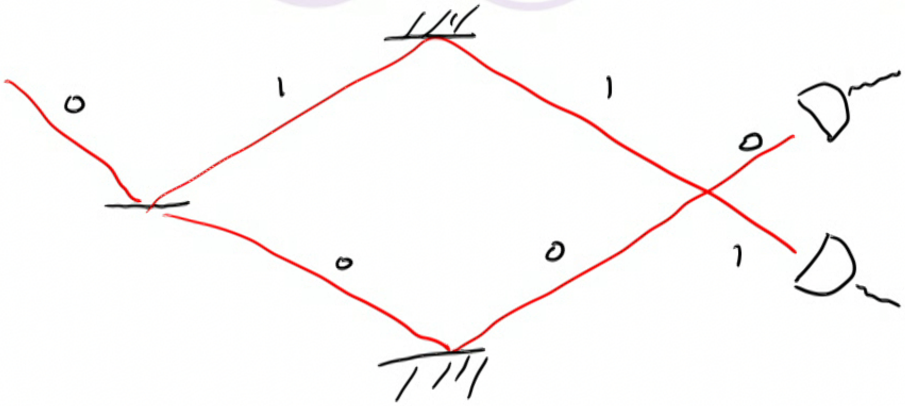


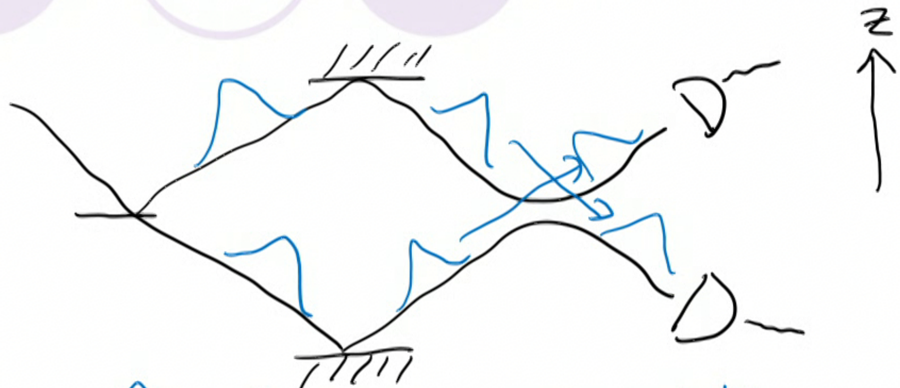
- The dBB particle will switch wavepackets during the interference due to the no-crossing rule: empty wave steals the particle.
- Here we can explicitly see that $J(0,t)=0$ for all times b.c. $J(x,t)$ is an odd function of x

$$J(x,t) = \left[J_{11}(x,t) + J_{22}(x,t) + J_{12}(x,t) + J_{21}(x,t)\right] = \left[J_{11}(x,t) - J_{11}(-x,t) + J_{12}(x,t) - J_{12}(-x,t)\right]$$

where $J_{km}(x,t) = \frac{\hbar}{m} \operatorname{Im}\left(\psi_k^* \frac{\partial \psi_m}{\partial x}\right)$

Consequences for Mach-Zehnder





Λ = Gaussian wavepackets
— = dBB trajectories

- If we remove the final beamsplitter from a Mach-Zehnder, many physicists would be inclined to say that detector 0 firing is evidence that the particle took path 0.
- The opposite happens in dBB. No crossing ⇒ the empty wave steals the particle

Detector 0 clicks ⇒ The particle travelled along path 1.

Surreal Trajectories

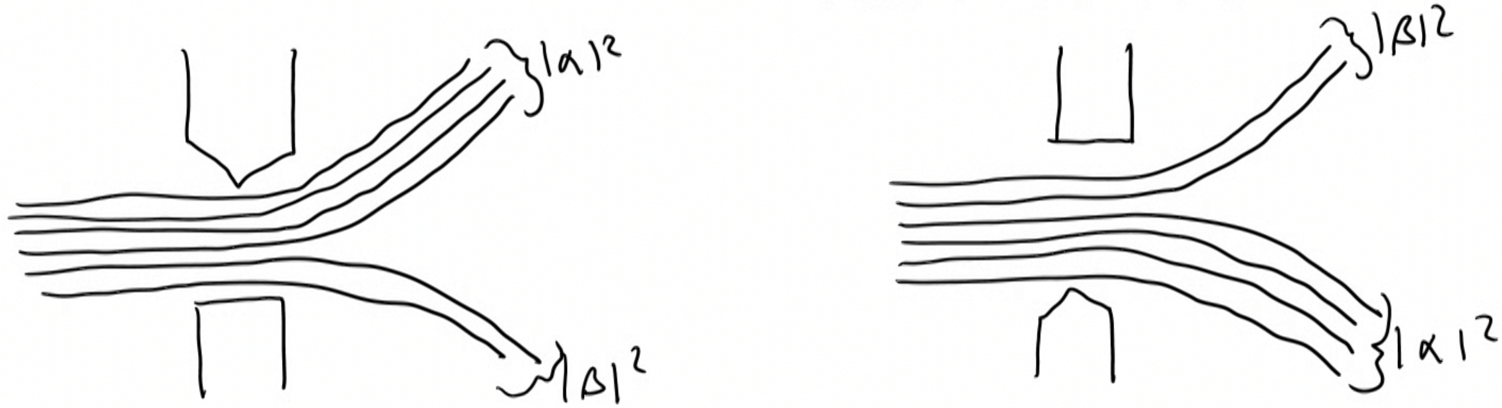
- To make things more dramatic, we can place a localized spin-$\frac{1}{2}$ system in path 0 initially placed in $|\uparrow\rangle$ and have the interaction

$$\psi_0(\vec{q}) \otimes \Phi_P(\vec{q}_P) \otimes |\uparrow\rangle \rightarrow \psi_0(\vec{q}) \otimes \Phi_P(\vec{q}_P) \otimes |\downarrow\rangle$$
$$\psi_1(\vec{q}) \otimes \Phi_P(\vec{q}_P) \otimes |\uparrow\rangle \rightarrow \psi_0(\vec{q}) \otimes \Phi_P(\vec{q}_P) \otimes |\uparrow\rangle$$

- Because $\vec{Q}_P$ is unaffected by this interaction the current will still be zero in the interference region.
- If we detect the particle at detector 0 and subsequently measure the spin, we will find it spin down.
- You might want to take this as evidence that the particle travelled along path 0, but the dBB trajectory is path 1.
- This can happen because the spin flip does not lead to decoherence that is localized in position.

KS Contextuality in de Broglie-Bohm

- KS Contextuality occurs in dBB because the outcome of an experiment depends on $\boldsymbol{Q}_S, \psi(\boldsymbol{q}_S), \boldsymbol{Q}_E, \Phi_R(\boldsymbol{q}_E)$, and the interaction Hamiltonian, and not on $\boldsymbol{Q}_S, \psi(\boldsymbol{q}_S)$ alone.
- Example: Stern-Gerlach measurement of $\psi(\boldsymbol{q}_S) \otimes (\alpha|\uparrow\rangle + \beta|\downarrow\rangle)$



- No-crossing rule $\Rightarrow$ some $\boldsymbol{q}_S$ switch between giving spin up and spin down outcomes when we rotate the magnets by 180°.
- This is *more contextual* than implied by KS, which can only be proved in $d \geq 3$.

Underdetermination

- The only property of the guidance equation needed to reproduce the quantum predictions is equivariance: $\rho(\boldsymbol{Q}, t_0) = |\psi(\boldsymbol{Q}, t_0)|^2 \rightarrow \rho(\boldsymbol{Q}, t) = |\psi(\boldsymbol{Q}, t)|^2$ for all other t.
- Any other equivariant dynamics would do just as well, e.g. (E. Deotto, G. Ghiradri, Found.Phys. 28:1-30 (1998))

$$\frac{d\vec{Q}_k}{dt} = \frac{\hbar}{m_k}\frac{\mathrm{Im}(\psi^*\vec{\nabla}_k\psi)}{\psi^*\psi}(\boldsymbol{Q}) + \frac{\vec{J}_0(\vec{Q}_k)}{\psi^*\psi(\boldsymbol{Q})} \quad \text{with} \quad \vec{\nabla}\cdot\vec{J}_0 = 0$$

- Further:
  - We could add more primitive variables, e.g. spin with stochastic dynamics.
  - We could use a different basis, e.g. momentum.
  - We could even use a POVM, e.g. coherent states.

The Equilibrium Hypothesis

- The quantum state plays two roles in dBB:
  - Dynamical: it appears in the guidance equation.
  - Probabilistic: We set $\rho(\boldsymbol{q}, t_0) = |\psi(\boldsymbol{q}, t_0)|^2$ as a postulate – quantum equilibrium hypothesis.
- These two roles are independent, we could set the probability density to anything else.
- There is evidence (analytic and numerical) that, under suitable coarse-graining, other densities relax to $|\psi(\boldsymbol{q}, t_0)|^2$ over time, akin equilibriation in statistical mechanics.
- Valentini posits that nonequilibrium states may have occurred in the early universe.
  - This would resolve some of the underdetermination, but leads to the bold hypothesis that superluminal signaling occurs in our universe.

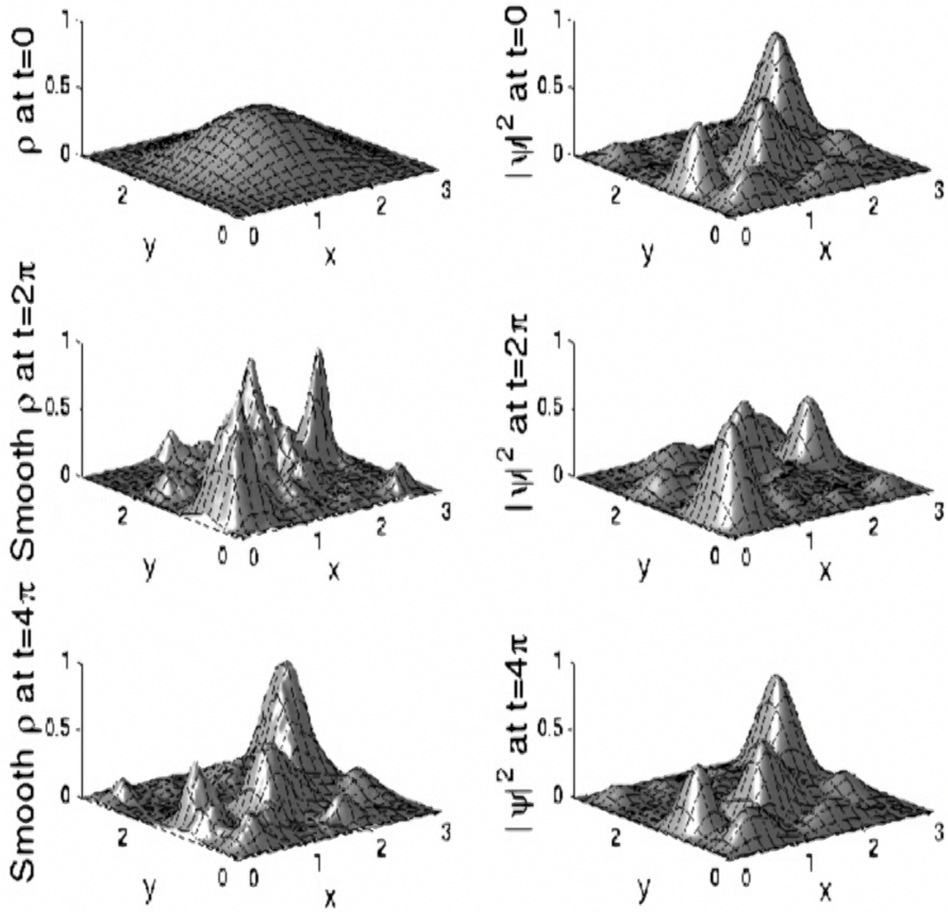


Figure 7: Smoothed $\tilde{\rho}$, compared with $|\psi|^2$, at times $t = 0$, 2π and 4π. While $|\psi|^2$ recurs to its initial value, the smoothed $\tilde{\rho}$ shows a remarkable evolution towards equilibrium.

A. Valentini, H. Westman, Proc. Roy. Soc. Lond. A 461:253-272 (2005)

Relativistic Generalizations of de Broglie-Bohm

- Generalizations of dBB to relativistic QFT have been developed. There are various versions:
  - Particle ontology vs. field ontology.
  - An ontology with particle occupation numbers requires stochastic dynamics.
  - A mixture of the two, e.g. particles for fermions and fields for bosons, only fermions and treat bosons like spin or vice versa.

- These theories cannot be *fundamentally* Lorentz invariant:
  - Under the equilibrium hypothesis, the operational predictions are Lorentz invariant.
  - But the theories violate parameter independence – there is superluminal signaling at the ontic level.
  - These effects would become observable in nonequilibrium states.

Summary

- dBB provides a coherent ontology with straightforward equations of motion, and saves the phenomena.
- Trajectories do not obey common intuitions, but arguably this must be so if they are to reproduce quantum phenomena.
- dBB arguably *more weird* than an interpretation has to be, i.e.
  - Contextual in ways that QM does not require.
  - Nonlocal in experiments that have local explanations.
  - ψ-ontic even for experiments that have good ψ-epistemic explanations.
- Taking the equilibrium hypothesis as a postulate is a fine tuning and leads to underdetermination of the theory.
- Viewing it as emergent removes the underdetermination, but leads to the bold hypothesis that we should expect to see explicit Lorentz violation, i.e. signaling, somewhere in nature.
- dBB is a good counterexample to many exaggerated claims about QM.