Title: PSI 2018/2019 - Foundations of Quantum Mechanics - Lecture 11

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Abstract:

Quantum Foundations
Lecture 11

PSI Review Class: 21th January 2019

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8) Ontological Models

1) Definitions
2) Examples
3) Excess Baggage
4) Contextuality
5) Ψ-ontology
6) Bell's Theorem

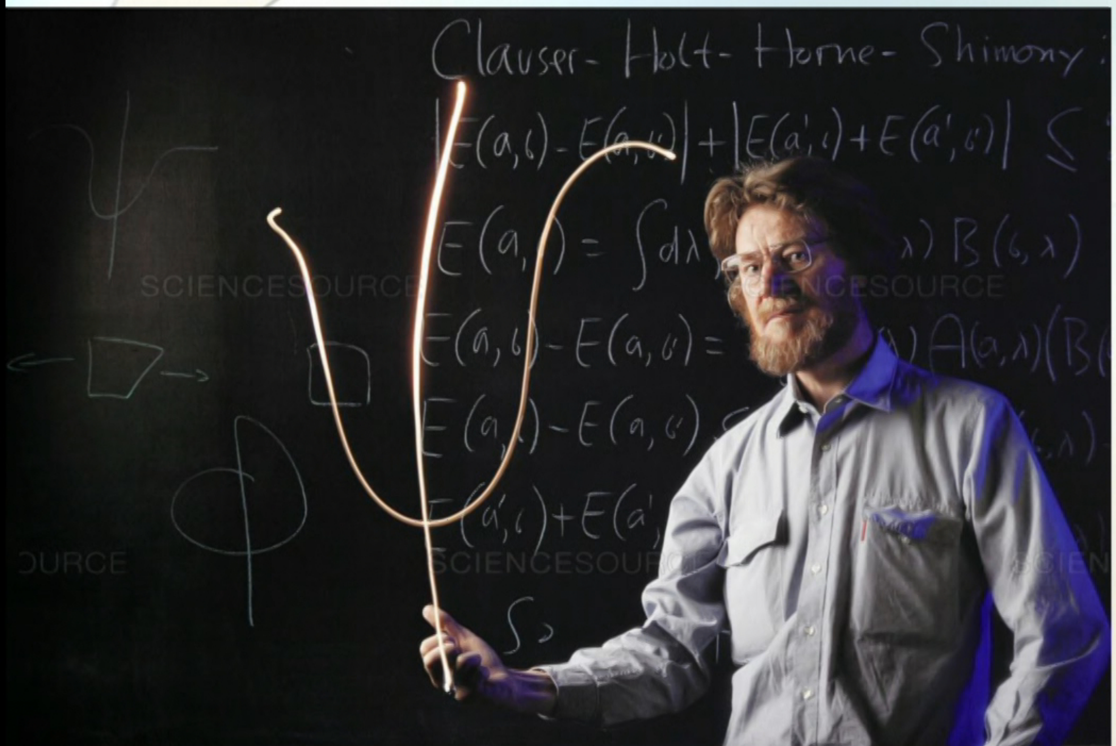
8.6) Bell's Theorem

- The entangled state

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |0\rangle_B + |1\rangle_A \otimes |1\rangle_B)$$

exhibits perfect correlations when both Alice and Bob measure in the $\{|0\rangle, |1\rangle\}$ basis (or indeed the $\{|\vec{n}+\rangle, |\vec{n}-\rangle\}$ basis with $\vec{n}$ in the $x - z$ plane.

- According to the orthodox interpretation, Bob's outcomes "pop into existence" nonlocally when Alice makes her measurement and the quantum state collapses.
- EPR argued that the measurement outcomes must pre-exist in order to avoid nonlocality.
- This is exactly how it works in the Spekkens toy theory.



- In 1964, John Stewart Bell proved that the correlations of entangled quantum systems cannot be explained in this way.
- We will explain a version due to Clauser, Horne, Shimony and Holt.

The CHSH Game

- Get into groups of four.
  - In each group, choose one person to be:

Alice

Bob

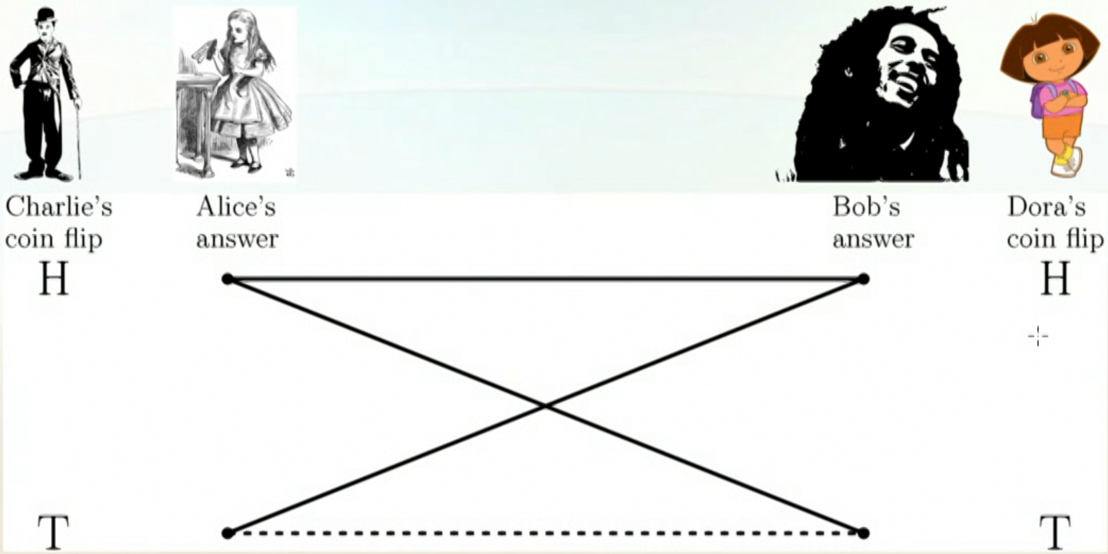
Charlie

Dora

The CHSH Game

1. Alice and Bob get together for a few minutes to decide their strategy.
2. Bob leaves the room with Dora. Alice and Charlie stay.
3. Charlie and Dora each flip a coin. Write down the outcome.
4. Alice and Bob have to write either +1 or -1 in response.
5. Alice and Bob win the game if:
   - Whenever the coin flips are HH, HT, or TH, they give the same answer.
   - Whenever the coin flips are TT, they give a different answer.
6. Repeat steps 3-5.
7. Bob and Dora come back in the room. They count how many times Alice and Bob won as well as the total number of times they played.
8. Report the results back to me. The goal is to win the game 85% of the time.

Why can't Alice and Bob always win?



- Assuming the coin flips are uniformly random, Alice and Bob will win at most 75% of the time in the long run.

List of All Deterministic Strategies

Alice	**H**	+	+	+	+	+	+	+	+	-	-	-	-	-	-	-	-
	T	+	+	+	+	-	-	-	-	+	+	+	+	-	-	-	-
Bob	**H**	+	+	-	-	+	+	-	-	+	+	-	-	+	+	-	-
	T	+	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-
Winning Probability		75%	75%	25%	25%	75%	25%	75%	25%	25%	75%	25%	75%	25%	25%	75%	75%

Do Nondterministic Strategies Matter?

- Suppose Alice and Bob do not choose a fixed strategy, but use classical randomness (coin flips, dice throws, etc.) to choose it each time, i.e. they decide to use strategy j, k, l, m with probability $p_{j,k,l,m}$ $(j, k, l, m = \pm 1)$.
- On each round of the game they will still end up using a deterministic strategy with winning probability $\leq 75\%$.
- The average of the winning probability cannot be higher than the winning probability for the best deterministic strategy.
- Alice and Bob might as well just pick the best deterministic strategy.

What About Delaying the Decision?

- Alice and Bob each have four local deterministic strategies (ignoring what the other person is doing)

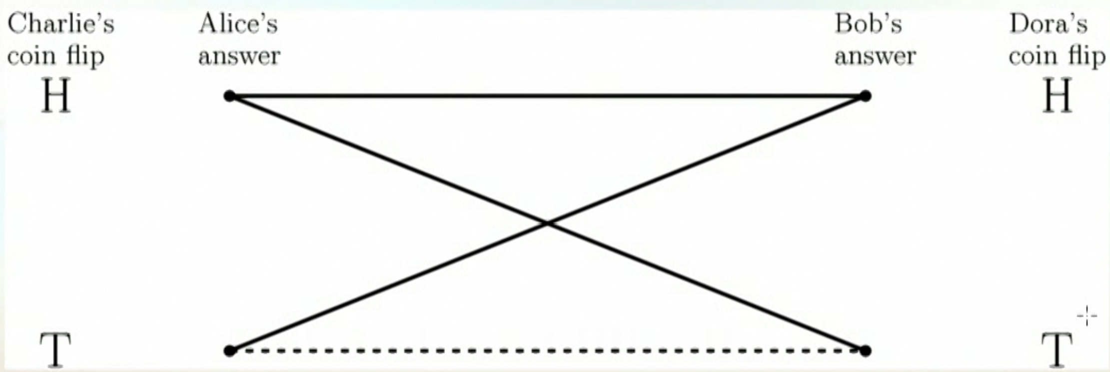
H	+	+	-	-
T	+	-	+	-

- Alice and Bob could decide as follows:
  - Alice waits until she sees the outcome of her coin flip.
  - If it is H, she picks $+/-$ with probability $p_{\pm}^{\mathrm{H}}$ (e.g. by flipping a biased coin)
  - If it is T, she picks $+/-$ with probability $p_{\pm}^{\mathrm{T}}$
  - Bob does similarly with distributions $q_{\pm}^{\mathrm{H}}$ and $q_{\pm}^{\mathrm{T}}$
- But this just amounts to picking strategy j, k, l, m with probability

$$p_{j,k,l,m} = p_j^{\mathrm{H}} p_k^{\mathrm{T}} q_l^{\mathrm{H}} q_m^{\mathrm{H}}$$

- In other words, Alice and Bob could just have flipped all their coins at the beginning, so we are back to the previous case.

The CHSH Inequality



- Whatever strategy Alice and Bob use (deterministic, nodeterministic, delayed), their outcome probabilities satisfy

$$P(a = b|H,H) + P(a = b|H,T) + P(a = b|T,H) + P(a \neq b|TT) \leq 3$$

- This is (a version of) the CHSH inequality.

Quantum Violation

- Now suppose that we allow Alice and Bob to use quantum systems to play the game.
- They initially prepare two qubits in a state $|\psi\rangle_{AB}$. Alice takes system A with her and Bob takes system B.
- If Alice's coin flip is heads, she measures her system in the basis $\{|\vec{n}_{\mathrm{H}}+\rangle, |\vec{n}_{\mathrm{H}}-\rangle\}$. If she gets the $|\vec{n}_{\mathrm{H}}\pm\rangle$ outcome she answers $a = \pm 1$.
- If Alice's coin flip is tails, she measures her system in the basis $\{|\vec{n}_{\mathrm{T}}+\rangle, |\vec{n}_{\mathrm{T}}-\rangle\}$. If she gets the $|\vec{n}_{\mathrm{T}}\pm\rangle$ outcome she answers $a = \pm 1$.
- Bob does the same thing on his system with the bases $\{|\vec{m}_{\mathrm{H}}+\rangle, |\vec{m}_{\mathrm{H}}-\rangle\}$ and $\{|\vec{m}_{\mathrm{T}}+\rangle, |\vec{m}_{\mathrm{T}}-\rangle\}$.

Quantum Violation

- Suppose Alice and Bob prepare the state

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle_A \otimes |0\rangle_B + |1\rangle_A \otimes |1\rangle_B\right)$$

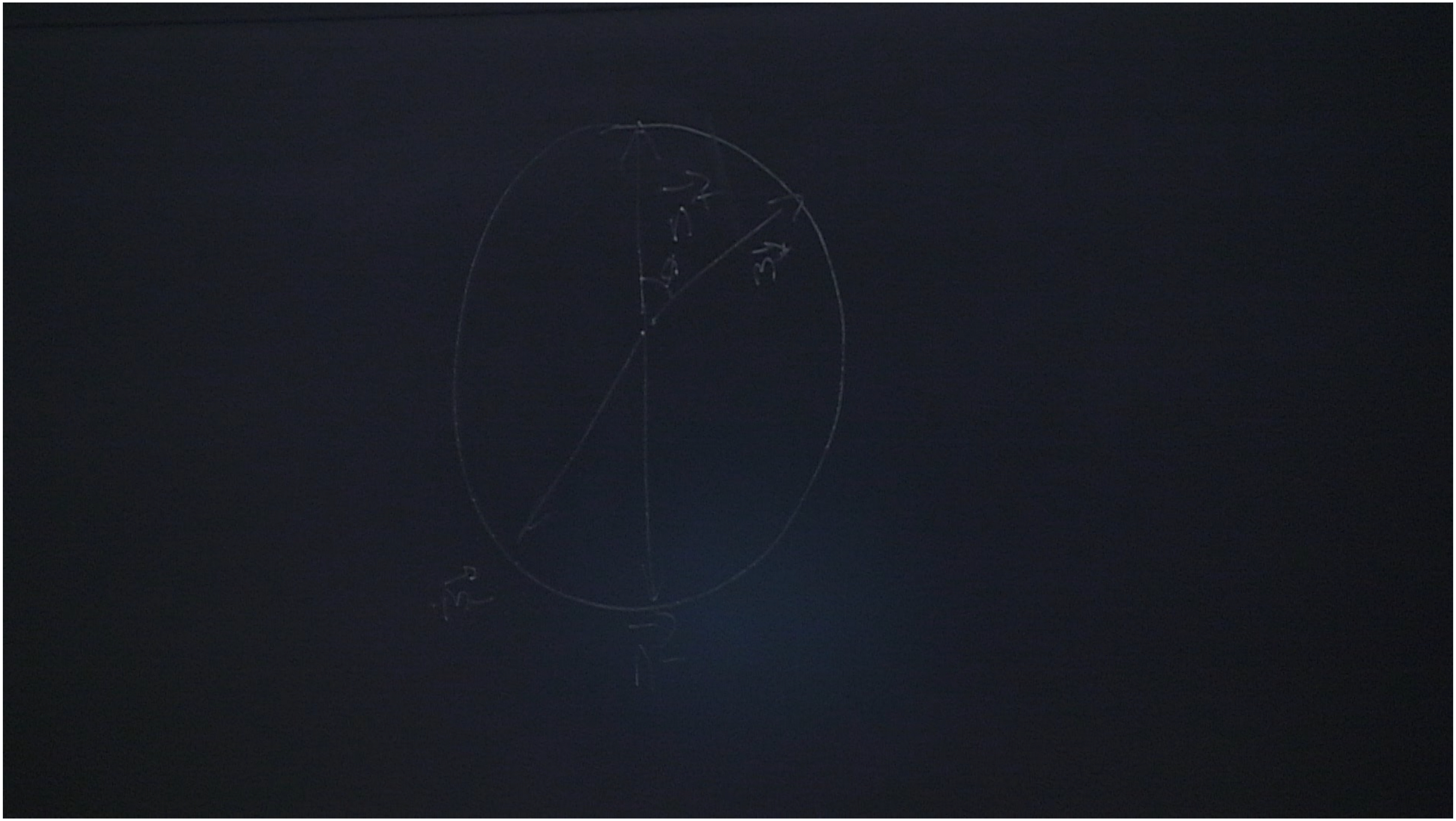
- The outcome probabilities are

$$P(\vec{n}+, \vec{m}+) = \tfrac{1}{2}\cos^2\tfrac{\phi}{2}, \qquad P(\vec{n}+, \vec{m}-) = \tfrac{1}{2}\sin^2\tfrac{\phi}{2}$$

$$P(\vec{n}-, \vec{m}-) = \tfrac{1}{2}\cos^2\tfrac{\phi}{2}, \qquad P(\vec{n}-, \vec{m}+) = \tfrac{1}{2}\sin^2\tfrac{\phi}{2}$$

where ϕ is the angle between $\vec{n}$ and $\vec{m}$ on the x-z plane of the Bloch sphere.

- So we just have to choose the measurement angles and see what we get.



Quantum Violation

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- The outcome probabilities are

$$P(\vec{n}+, \vec{m}\,+) = \tfrac{1}{2}\cos^2\tfrac{\phi}{2}, \qquad P(\vec{n}+, \vec{m}\,-) = \tfrac{1}{2}\sin^2\tfrac{\phi}{2}$$

$$P(\vec{n}-, \vec{m}\,-) = \tfrac{1}{2}\cos^2\tfrac{\phi}{2}, \qquad P(\vec{n}-, \vec{m}\,+) = \tfrac{1}{2}\sin^2\tfrac{\phi}{2}$$

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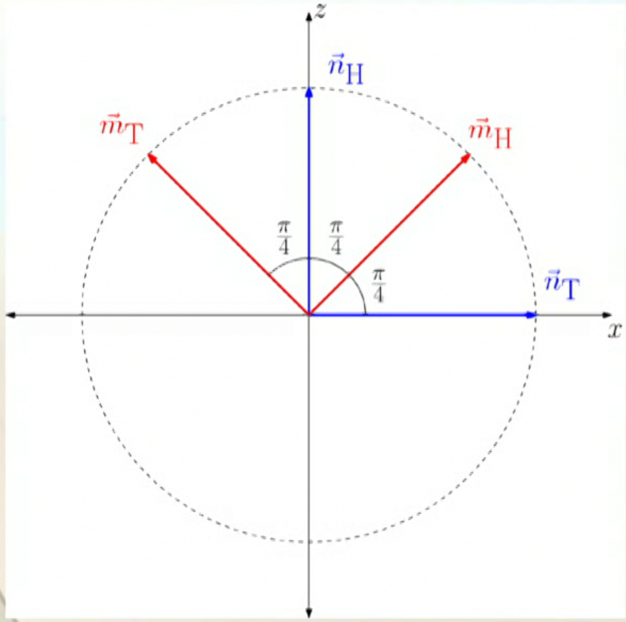
Quantum Violation



$$P(a = b|\mathrm{H},\mathrm{H})$$

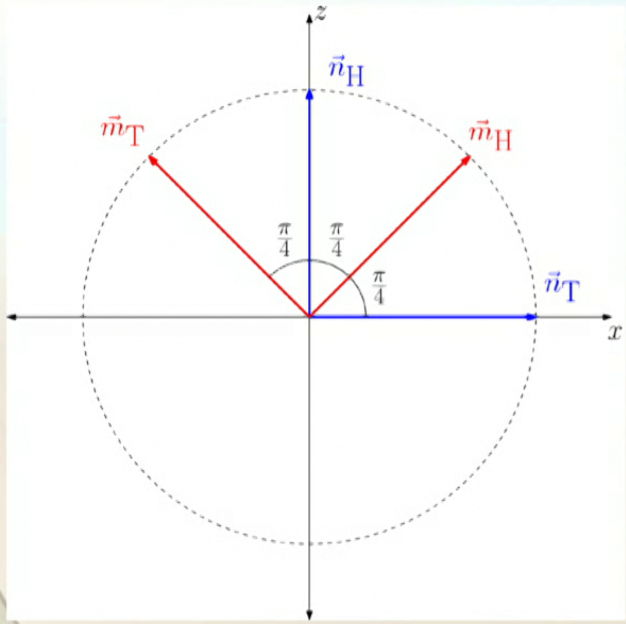
$$= P(\vec{n}_\mathrm{H}+, \vec{m}_\mathrm{H}+) + P(\vec{n}_\mathrm{H}-, \vec{m}_\mathrm{H}-)$$

$$= \cos^2\left(\frac{\pi}{8}\right)$$

$$= \frac{1+\cos\left(\frac{\pi}{4}\right)}{2}$$

$$= \frac{1}{2}\left(1 + \frac{1}{\sqrt{2}}\right)$$

Quantum Violation



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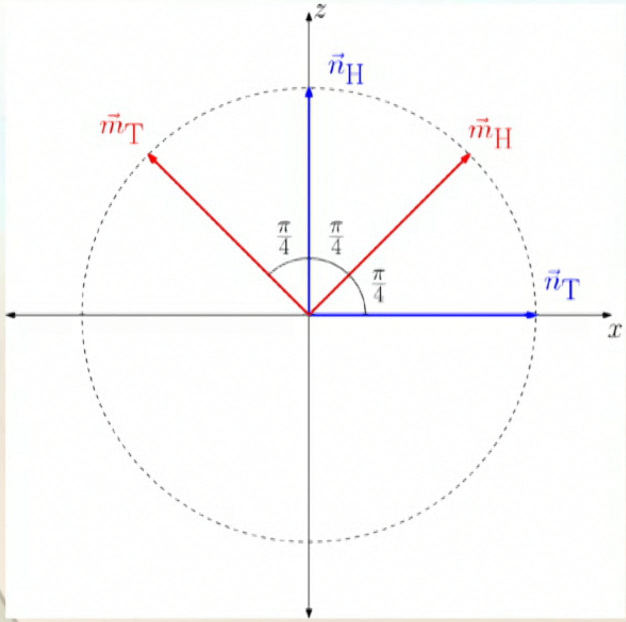
Quantum Violation



$$P(a \neq b|\mathrm{T}, \mathrm{T})$$

$$= P(\vec{n}_{\mathrm{T}}+, \vec{m}_{\mathrm{T}}-) + P(\vec{n}_{\mathrm{T}}-, \vec{m}_{\mathrm{T}}+)$$

$$= \sin^2\left(\frac{3\pi}{8}\right) = \cos^2\left(\frac{\pi}{8}\right)$$

$$= \frac{1 + \cos\left(\frac{\pi}{4}\right)}{2}$$

$$= \frac{1}{2}\left(1 + \frac{1}{\sqrt{2}}\right)$$

Quantum Violation

- Therefore, in the quantum case, we can get

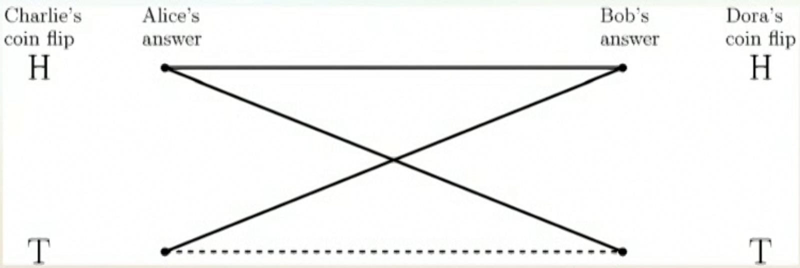
$$P(a = b|H,H) + P(a = b|H,T) + P(a = b|T,H) + P(a \neq b|TT)$$

$$4\cos^2\left(\frac{\pi}{8}\right) = 2\left(1 + \frac{1}{\sqrt{2}}\right) \approx 3.141 > 3$$

- Therefore, with quantum mechanics you can win the game with probability
$$\cos^2\left(\frac{\pi}{8}\right) = \frac{1}{2}\left(1 + \frac{1}{\sqrt{2}}\right) \approx 85.4\% > 75\%$$
- This is actually the maximum possible success probability in quantum mechanics, known as the *Tsirelson bound*.

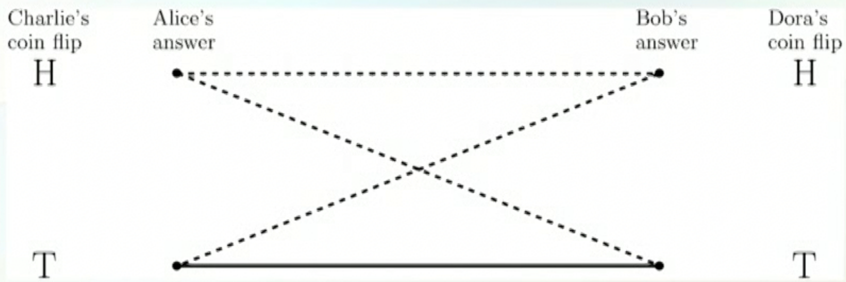
The Usual Form of the CHSH Inequality

- The CHSH inequality is usually expressed in terms of expectation values of observables rather than probabilities.
- To do this, note that we actually have four inequalities



$$1 \leq P(a = b|H,H) + P(a = b|H,T) + P(a = b|T,H) + P(a \neq b|TT) \leq 3$$

The Usual Form of the CHSH Inequality



$$1 \leq P(a \neq b|H,H) + P(a \neq b|H,T) + P(a \neq b|T,H) + P(a = b|TT) \leq 3$$

or

$$-3 \leq -P(a \neq b|H,H) - P(a \neq b|H,T) - P(a \neq b|T,H) - P(a = b|TT) \leq -1$$

The Usual Form of the CHSH Inequality

- Because Alice and Bob's answers a, b take values ± 1

$$\langle ab \rangle = P(a = b) - P(a \neq b)$$

$$1 \leq P(a = b|H,H) + P(a = b|H,T) + P(a = b|T,H) + P(a \neq b|TT) \leq 3$$
$$-3 \leq -P(a \neq b|H,H) - P(a \neq b|H,T) - P(a \neq b|T,H) - P(a = b|TT) \leq -1$$

- Summing these gives:

$$-2 \leq \langle ab \rangle_{\mathrm{HH}} + \langle ab \rangle_{\mathrm{HT}} + \langle ab \rangle_{\mathrm{TH}} - \langle ab \rangle_{\mathrm{TT}} \leq 2$$

which is the usual CHSH inequality.

- And our quantum strategy gives

$$4\cos^2\left(\frac{\pi}{8}\right) - 4\left(1 - \cos^2\left(\frac{\pi}{8}\right)\right) = 8\cos^2\left(\frac{\pi}{8}\right) - 4 = \frac{8}{2}\left(1 + \frac{1}{\sqrt{2}}\right) - 4 = 2\sqrt{2} \approx 2.828$$

which is what is usually called the Tsirelson bound.

Conditional Independence

- Two random variables, A and B are *independent,* denoted $A \perp B$ if
$$P(A,B) = P(A)P(B)$$
- The *conditional probability* of B given A is
$$P(B|A) = \frac{P(A,B)}{P(A)}$$
- Independence can equivalently be written as
$$P(B|A) = P(B) \qquad \text{or} \qquad P(A|B) = P(A)$$
- Two random variables, A and B are *conditionally independent* given C, denoted $A \perp B|C$ if any of the following three equivalent conditions holds
  1. $P(A|B,C) = P(A|C)$
  2. $P(B|A,C) = P(B|C)$
  3. $P(A,B|C) = P(A|C)P(B|C)$

Reichenbach's Principle

- Scientific realists usually think that correlations need to have causes.
- *Reichenbach's principle* encapsulates how this is supposed to work.

  - If A and B are correlated $P(A,B) \neq P(A)P(B)$ then either:

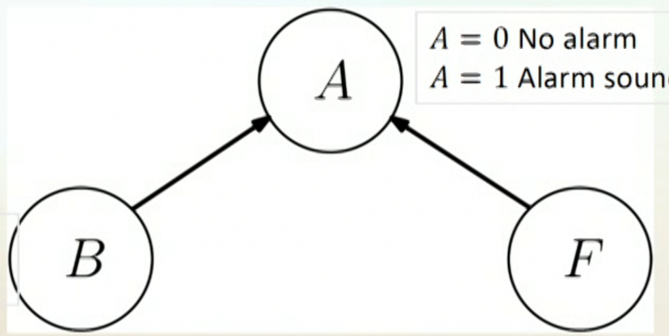
1. A is the cause of B
2. B is the cause of A
3. There is a common cause C for both A and B, and $A \perp B|C$

$$P(A,B|C) = P(A|C)P(B|C)$$

The Markov Condition

- Reichenbach's principle can be formulated in the language of *Causal (Bayesian) Networks*.

$A = 0$ No alarm
$A = 1$ Alarm sounding



$B = 0$ No burglar in house
$B = 1$ Burglar in house

$F = 0$ No fire in house
$F = 1$ Fire in house

$$P(A,B,F) = P(A|B,F)P(B)P(F)$$

The Markov Condition

- We draw a *directed acyclic graph:*
  - The vertices are the random variables.
  - We draw an edge from A to B if A is a direct cause of B.
  - The probabilities factor according to the *Markov Condition*

$$P(X_1, X_2, \cdots, X_n) = P(X_n|\text{pa}(X_n)) \cdots P(X_2|\text{pa}(X_2))P(X_1|\text{pa}(X_1))$$

where $\text{pa}(X)$ denotes the parents of X in the graph.

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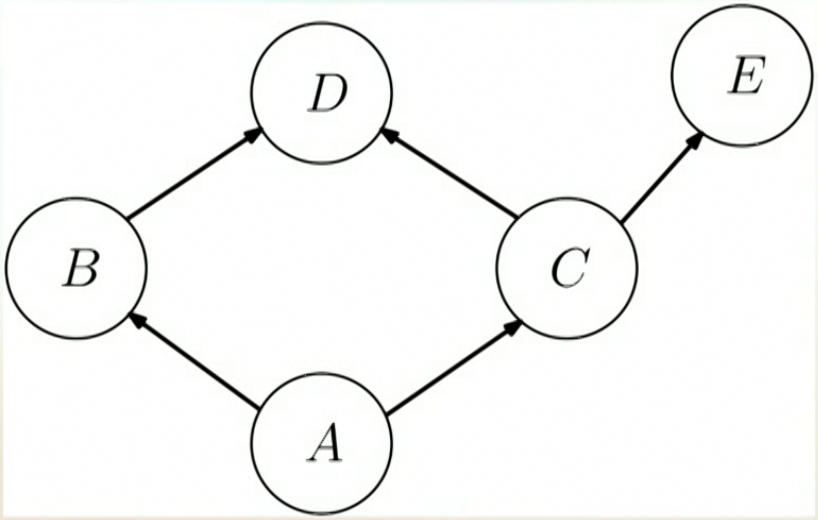
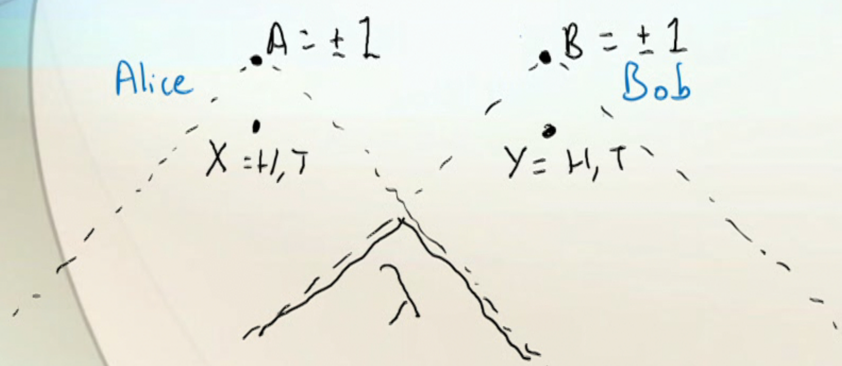
Another Example



$$P(A,B,C,D,E) = P(E|C)P(D|B,C)P(C|A)P(B|A)P(A)$$

Application to Bell Experiments

- Suppose Alice's coin flip and answer happen at spacelike separation to Bob's coin flip and answer.



- Since Alice and Bob's wings of the experiment are spacelike separated, according to special relativity (X, A) cannot be direct causes of (Y, B) and vice versa.

- Let λ be a complete description of the state of affairs in a region that screens off (X, A) from (Y, B)
  - Any lightlike path from (X, A) to (Y, B) via the past must intersect the region.

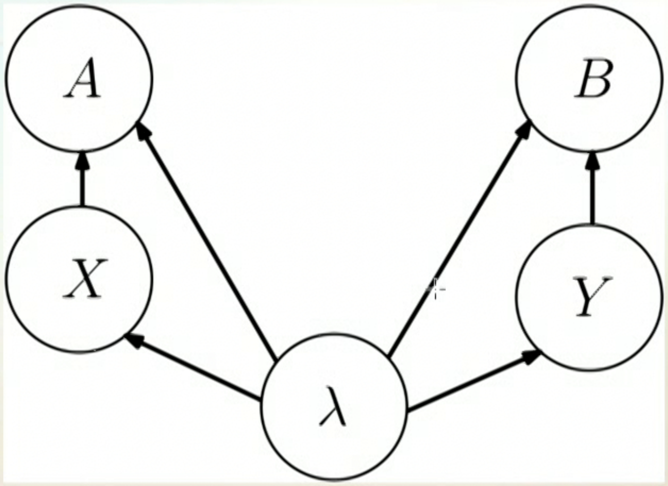
- $\Rightarrow$ Any common cause of (X, A) and (Y, B) must be contained in λ.

Application to Bell Experiments

- According to special Relativity, the possible causal relationships are:

$$P(A, B, X, Y, \lambda)$$

$$= P(B|Y,\lambda)P(A|X,\lambda)P(Y|\lambda)P(X|\lambda)P(\lambda)$$



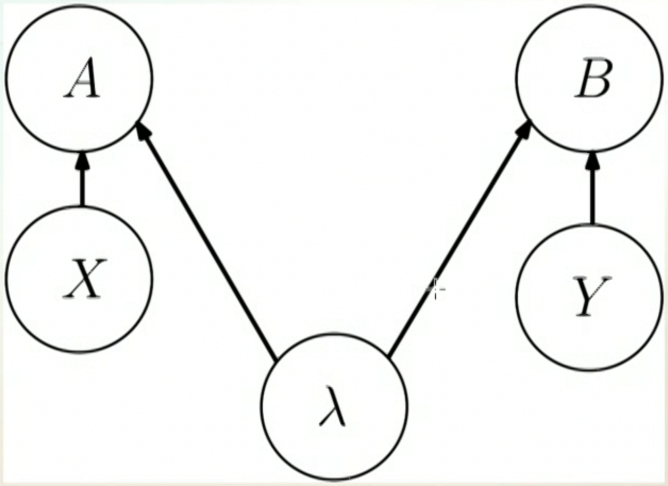
Application to Bell Experiments

- However, we normally assume that the coin flips X and Y are freely chosen, independently from the system being measured.
- This leads to the *measurement independence* assumption

$$X, Y \perp \lambda$$

$$P(X, Y|\lambda) = P(X, Y)$$

- With this, we have



$$P(A, B, X, Y, \lambda)$$

$$= P(B|Y, \lambda)P(A|X, \lambda)P(Y)P(X)P(\lambda)$$

Application to Bell Experiments

$$P(A,B,X,Y,\lambda) = P(B|Y,\lambda)P(A|X,\lambda)P(Y)P(X)P(\lambda)$$

- If we conditionalize on X, Y and λ, we get
$$P(A,B|X,Y,\lambda) = P(B|Y,\lambda)P(A|X,\lambda)$$
- This condition is known as *local causality*
- To reiterate, it follows from:
  - The Markov condition (Reichenbach's principle)
  - The causal structure given by special relativity (spacelike separation)
  - The assumption that X and Y are chosen independently of the system being investigated.

Application to Bell Experiments

- If we now compute the observed conditional probabilities, we will get
$$P(A,B|X,Y) = \sum_{\lambda} P(B|Y,\lambda)P(A|X,\lambda)P(\lambda)$$
- Let's think about what this says in terms of the CHSH game.
  - Alice and Bob get together to determine a joint strategy – call it λ.
  - Based on λ and X, Alice flips a biased coin to determine A with probability $P(A|X,\lambda)$.
  - Based on λ and Y, Bob flips a biased coin to determine B with probability $P(B|Y,\lambda)$.
- But this is exactly the sort of strategy we showed must satisfy the CHSH inequality.
- The quantum violation therefore rules out a locally causal model.

Application to Bell Experiments

$$P(A,B,X,Y,\lambda) = P(B|Y,\lambda)P(A|X,\lambda)P(Y)P(X)P(\lambda)$$

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$$P(A,B|X,Y,\lambda) = P(B|Y,\lambda)P(A|X,\lambda)$$
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- To reiterate, it follows from:
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  - The causal structure given by special relativity (spacelike separation)
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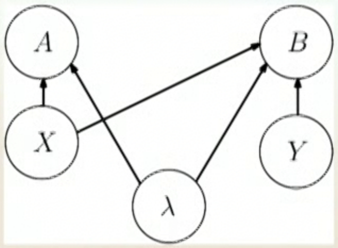
Implications

- If you accept the Markov condition and measurement independence, then there must be a superluminal causal influence (nonlocality). For example:



- Your model violates relativity at the ontological level.

- We could instead reject the Markov condition:
  - Correlations do not have to have causal explanations.
  - This is appealing to anti-realists.
- We could modify the Markov condition:
  - Causal explanations work differently in quantum theory.
- We could reject measurement independence:
  - There is no free choice.
  - Superdeterminism
  - Retrocausality

Summary of Ontological Models

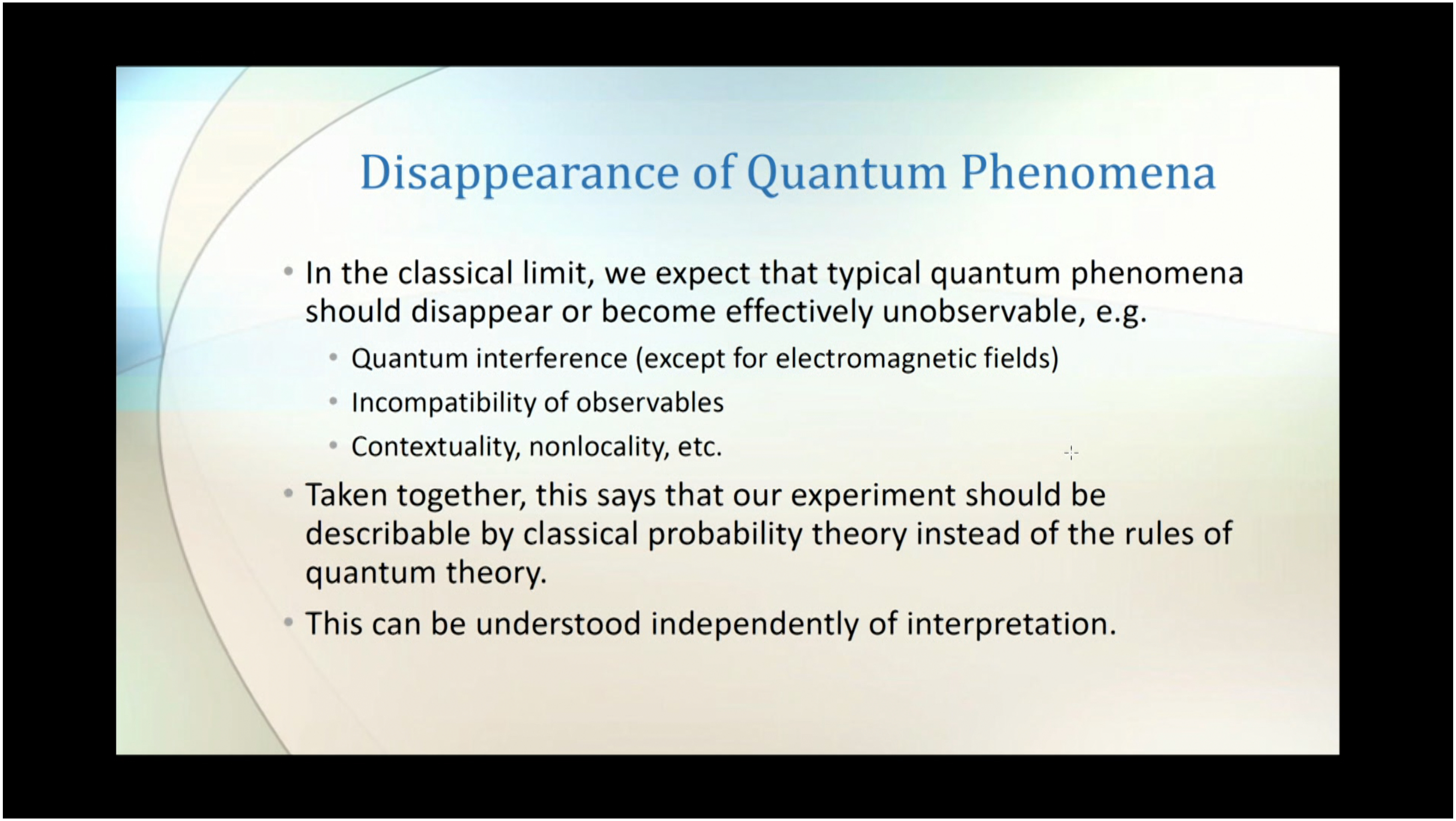
- If our interpretation of quantum mechanics fits into the ontological models framework then it has to have a number of unappealing features:
  - Excess baggage
  - Contextuality
  - ψ-ontology
  - Nonlocality
- Two options:
  - Bite the bullet and adopt an interpretation that has these features, viewing the no-go theorems as justification for why we have to have these features (de Broglie-Bohm, Spontaneous Collapse theories).
  - Go anti-realist or adopt a more exotic ontology that does not fit into the ontological models framework (Copenhagenish, many-worlds).

9) The Classical Limit of Quantum Theory

1) Requirements for the Classical Limit
2) "Classiscal" Limit of the Path Integral
3) Ehrenfest's Theorem
4) Wigner-Moyal Formalism
5) Spreading of Wavepackets
6) Coherent States
7) Environmentally Induced Decoherence

9.1) Requirements of the Classical Limit

- One of our requirements of an interpretation of quantum theory was that it should "save the phenomena". In particular, this means we should be able to understand why the macroscopic world looks "classical" to us.
- There are several aspects of the classical limit, some of which can be dealt with independently of interpretation, and some which cannot.


Disappearance of Quantum Phenomena
In the classical limit, we expect that typical quantum phenomena should disappear or become effectively unobservable, e.g.
Quantum interference (except for electromagnetic fields)
Incompatibility of observables
Contextuality, nonlocality, etc.
Taken together, this says that our experiment should be describable by classical probability theory instead of the rules of quantum theory.
This can be understood independently of interpretation.

Actualization of Measurement Outcomes

- If we solve the first question then we will have a description of our system in terms of classical probabilities.
- How do we know that those probabilities can be interpreted in terms of classical uncertainty?
  - e.g. even if we specify $\mathrm{Prob(cat\ alive)} = \mathrm{Prob(cat\ dead)} = \frac{1}{2}$ and it is not practical to perform an interference experiment, how do we know that this describes a situation in which EITHER the cat is dead OR it is alive and we simply do not know which.
  - It could still be that the cat is in some kind of indefinite state until we actually observe it.
- This cannot be solved without positing an ontology for the theory, so it is definitely an interpretation dependent question.

Recovery of the Equations of Classical Mechanics

- We should be able to identify quantities that obey the equations of classical mechanics, but note that there are two options here:
  - The Liouville Limit: A probability density on phase space obeys the Liouville equation

$$\frac{\partial \rho(\boldsymbol{q}, \boldsymbol{p})}{\partial t} = -\{\rho, H\} \quad \text{where} \quad \{\rho, H\} = \sum_{j=1}^{n} \left[\frac{\partial H}{\partial p_j} \frac{\partial \rho}{\partial q_j} - \frac{\partial H}{\partial q_j} \frac{\partial \rho}{\partial p_j} \right]$$

  - The Trajectory Limit: There exist canonically conjugate variables that obey Hamilton's equations

$$\frac{\mathrm{d}p_j}{\mathrm{d}t} = -\frac{\partial H}{\partial q_j}, \qquad \frac{\mathrm{d}q_j}{\mathrm{d}t} = \frac{\partial H}{\partial p_j}$$

Recovery of the Equations of Classical Mechanics

- In a ψ-complete theory, we should be able to derive the trajectory limit:
  - We observe classical systems travelling along definite trajectories. If the quantum state is all there is, we somehow need to derive this from the unitary evolution of a quantum state.
- In a ψ-epistemic theory, we only expect the Liouville limit:
  - The quantum state has the same status as a probability density, so we should expect it to behave as such in the classical limit. Whatever travels along trajectories is not described by standard quantum theory.
- In a ψ-ontic, but not ψ-complete, interpretation, it is completely non-obvious what to expect.
- So this is a subtly interpretation dependent question: what might be perceived as a failure of classicality on one view need not be from other points of view.