Title: PSI 2018/2019 - Foundations of Quantum Mechanics - Lecture 8

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Abstract:

Quantum Foundations
Lecture 8

PSI Review Class: 16th January 2019

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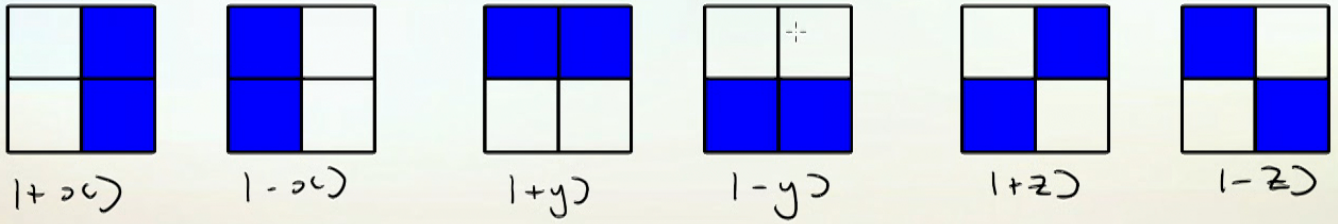
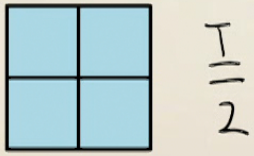
7.1.b) Epistemic States

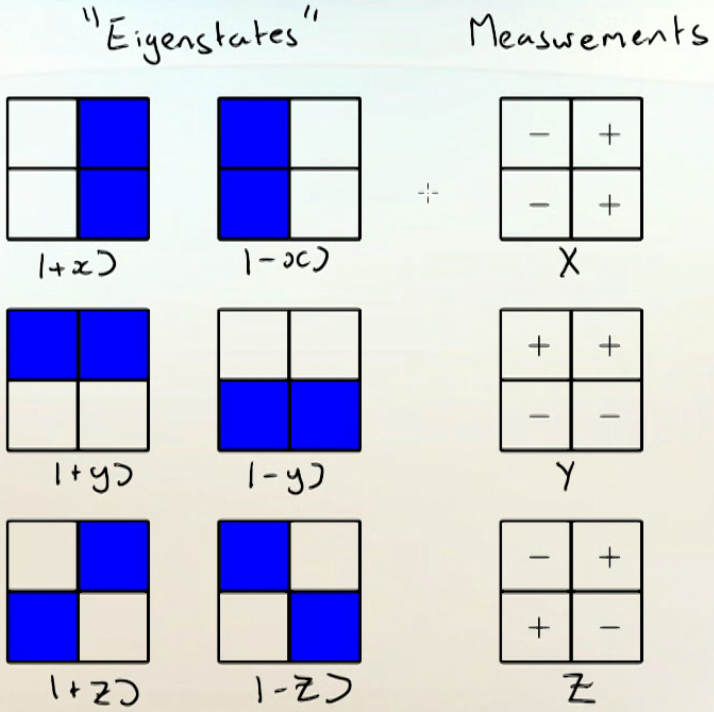
- There are six epistemic states (probability distributions) compatible with the knowledge-balance principle.



- We can also have a state of non-maximal knowledge:

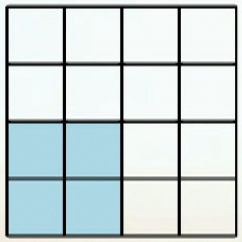


Valid Measurements on a toy bit and their "eigenstates"

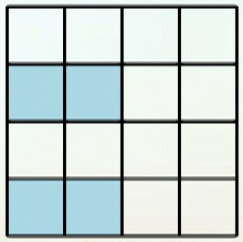


Product and Correlated States

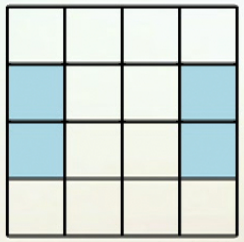
- Of the valid epistemic states, some of them are products of independent distributions of the two toy bits.



$|+x\rangle \otimes |+x\rangle$

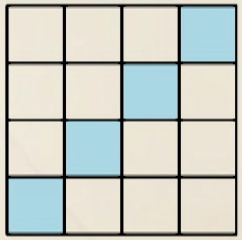


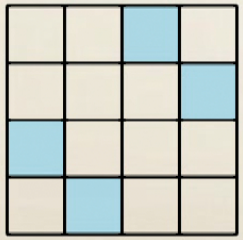
$|+x\rangle \otimes |+y\rangle$

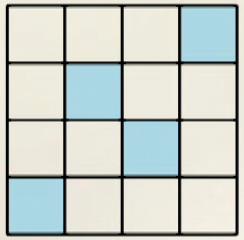


$|+z\rangle \otimes |-z\rangle$

- And some of them are correlated ("entangled")



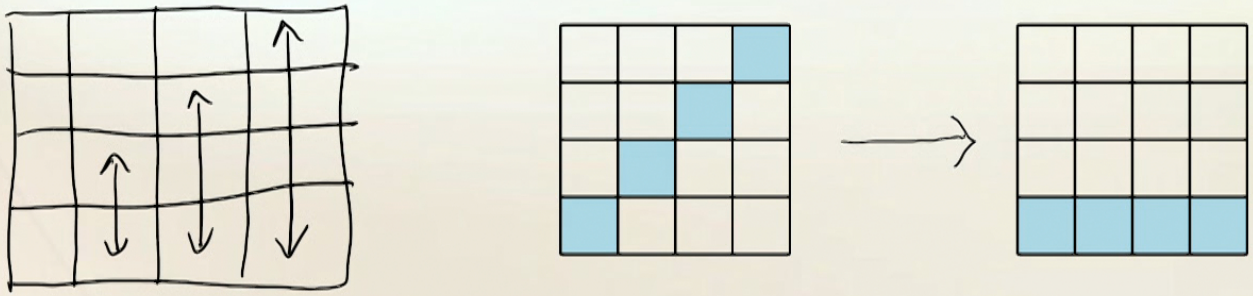




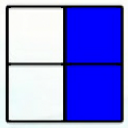
These have uniform marginal distributions

Reversible Dynamics on Composites

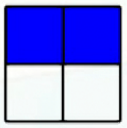
- Because we need to preserve the knowledge-balance principle for subsystems, not all permutations represent valid dynamics for a composite system.
- Example:



7.2.c) Indistinguishability of Pure States



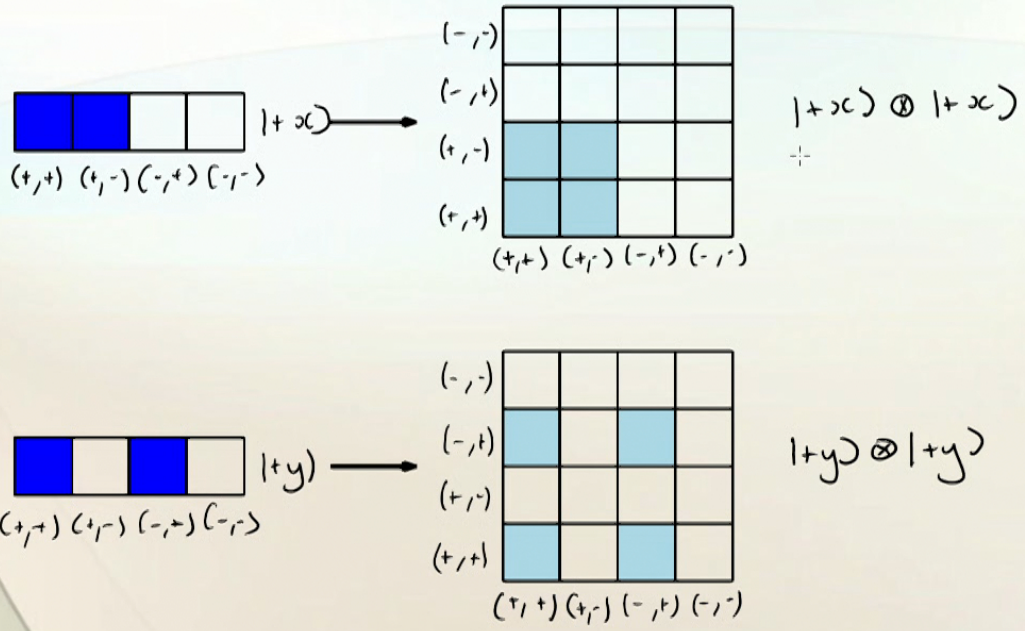
$|+x\rangle$



$|+y\rangle$

- It is impossible to perform a measurement that distinguishes $|+x\rangle$ from $|+y\rangle$ with certainty because, in both cases, the ontic state is $(+,+)$ with probability $\frac{1}{2}$

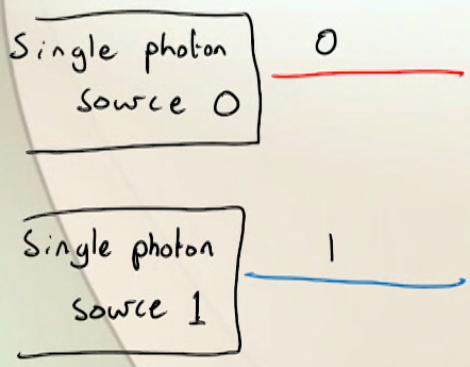
7.2.d) No-Cloning Theorem



This transition is impossible because initial states overlap with probability $\frac{1}{2}$ but final states overlap with probability $\frac{1}{4}$

7.2.e) Interference

- We can get an exact analogy with the Feynman interference paradox and the Elitzur-Vaidman Bomb.
- In the quantum mechanical version, we first switch to a Fock Space (optical mode) description.



We previously used $|0\rangle$ = photon in mode 0
$|1\rangle$ = photon in mode 1

Now use: $|0\rangle_0$ = vacuum of mode 0
$|1\rangle_0$ = 1 photon in mode 0
$|0\rangle_1$ = vacuum of mode 1
$|1\rangle_1$ = 1 photon in mode 1

$|0\rangle \rightarrow |1\rangle_0|0\rangle_1$
$|1\rangle \rightarrow |0\rangle_0|1\rangle_1$
} We now have 2 qubits if we stick to states with at most one photon in each mode

Beamsplitters and Dectectors

- Action of a 50/50 beamsplitter is

OLD

$$|0\rangle \longrightarrow \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$$
$$|1\rangle \longrightarrow \frac{1}{\sqrt{2}}(i|0\rangle + |1\rangle)$$

NEW

$$|1\rangle_0|0\rangle_1 \longrightarrow \frac{1}{\sqrt{2}}(|1\rangle_0|0\rangle_1 + i|0\rangle_0|1\rangle_1)$$
$$|0\rangle_0|1\rangle_1 \longrightarrow \frac{1}{\sqrt{2}}(i|1\rangle_0|0\rangle_1 + |0\rangle_0|1\rangle_1)$$

In new description, the beamsplitter entangles the two mode qubits.

- Detectors: These now act as local measurements on a tensor product

Detector on mode 0: $\{|0\rangle_0\langle 0|, |1\rangle_0\langle 1|\}$ } Click if $|1\rangle\langle 1|$ outcome

Detector on mode 1: $\{|0\rangle_1\langle 0|, |1\rangle_0\langle 1|\}$ } No click if $|0\rangle\langle 0|$ outcome

- We won't worry about the two mirrors in a Mach-Zehnder because they just give a global phase factor.

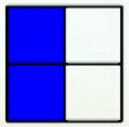
Toy Theory Model

- Since we have 2 qubits, we will use a model with 2 toy bits. One toy bit travels along mode 0 and the other along mode 1.
- We make the following correspondence:

Vacuum states:

$|0\rangle_0 = |-x\rangle_0$

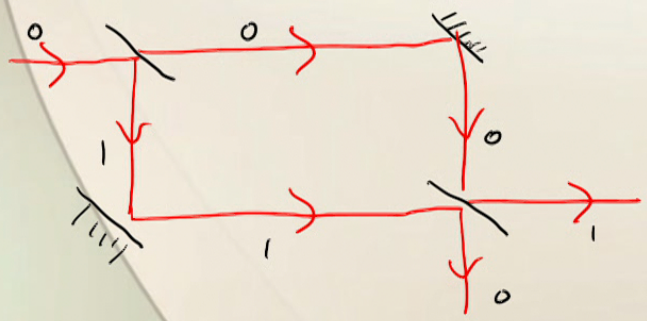
$|0\rangle_1 = |-x\rangle_1$



One-photon states:

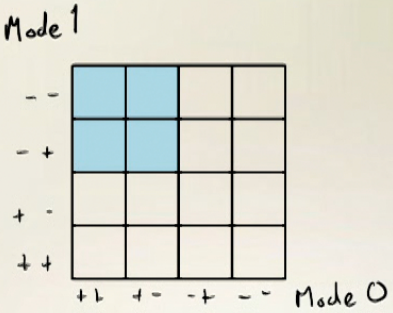
$|1\rangle_0 = |+x\rangle_0$

$|1\rangle_1 = |+x\rangle_1$



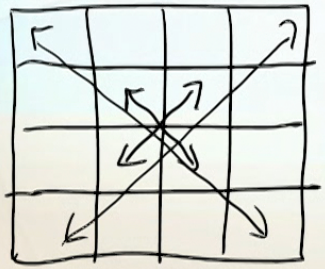
Initial state of a Mach-Zehnder

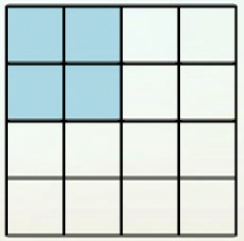
$|1\rangle_0|0\rangle_1 \rightarrow |+x\rangle_0 |-x\rangle_1$



Toy Theory Beamsplitter

- To model the entangling 50/50 beamsplitter in the toy theory, we use the following permutation

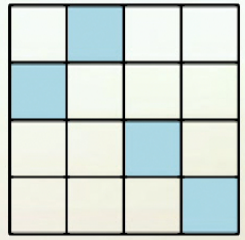




$|+x\rangle_0 |-x\rangle_1$

analogue of

$|1\rangle_0 |0\rangle_1$



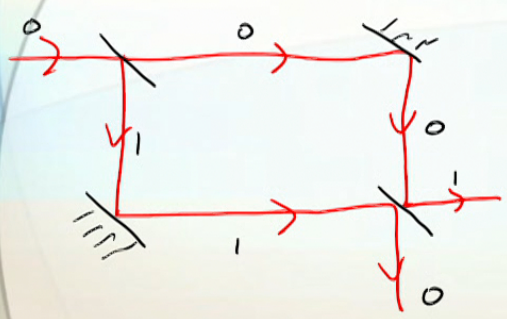
analogue of

$\frac{1}{\sqrt{2}} (|1\rangle_0|0\rangle_1 + i|0\rangle_0|1\rangle_1)$

- You can check that this permutation satisfies the knowledge-balance principle

Toy Theory Mach-Zehnder

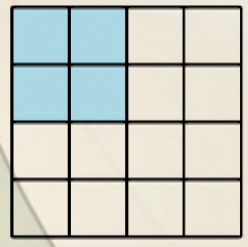
○ Mach-Zehnder without detectors on the paths



An X measurement of mode 0 always yields +1

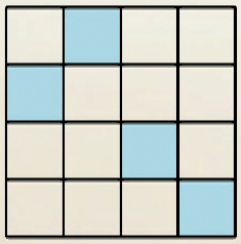
An X measurement of mode 1 always yields -1

⇒ Photon is always found in mode 0

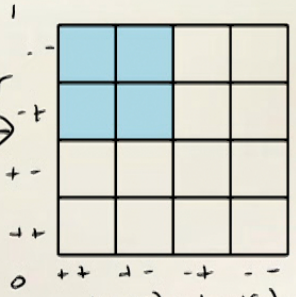


$|+x\rangle_0 |-x\rangle_1$

First Beamsplitter →



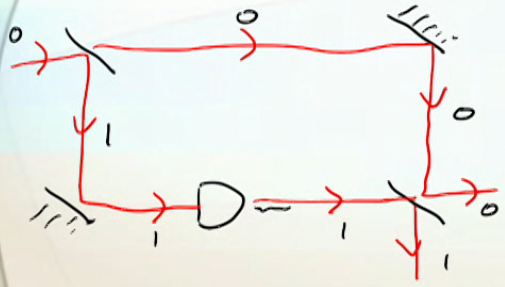
Second beamsplitter →



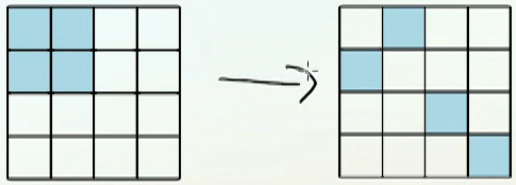
$|+x\rangle_0 |-x\rangle_1$

Toy Theory Mach-Zehnder

- Suppose we now put a detector in mode 1, which could be an Elitzur-Vaidman bomb

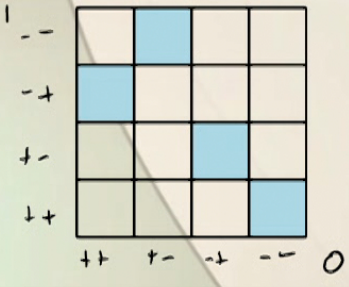


First beamsplitter works as before

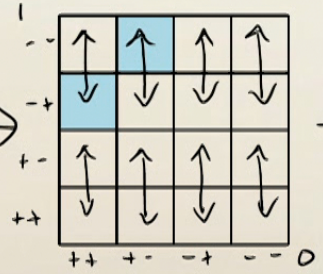


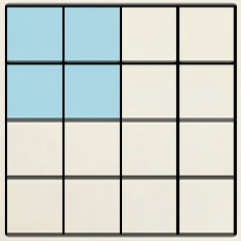
Probabilities of each detector firing are now 50/50

Now we measure in mode 1: suppose the detector does not click

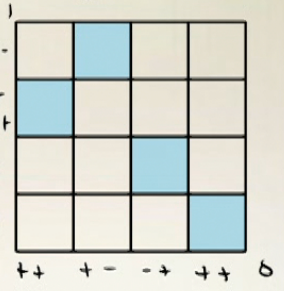


mode 1 is -+ or --





Second beamsplitter



Summary

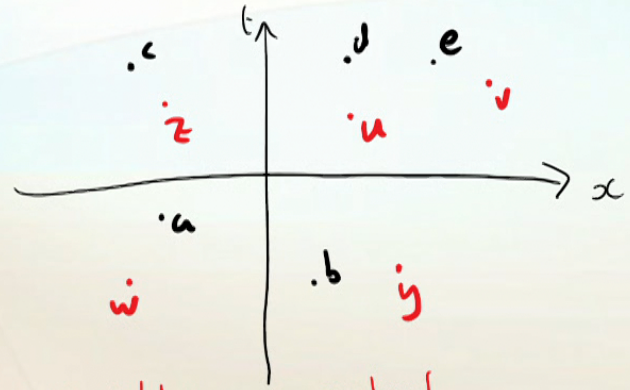
- A large number of apparently strange phenomena are accounted for (at least qualitatively) in the toy theory.
- They occur because it is ψ-epistemic:
  - Pure states are states of incomplete knowledge. Nonorthogonal pure states correspond to overlapping probability distributions.
  - This accounts for non-uniqueness of mixed state decompositions, indistinguishability, and no-cloning.
  - For interference, we should recognize that the vacuum is a quantum state, so also a state of incomplete knowledge. The degrees of freedom in the vacuum can mediate information about whether or not a detector is present, even when there is no photon in the mode.

8) Ontological Models

1) Definitions
2) Examples
3) Excess Baggage
4) Contextuality
5) Ψ-ontology
6) Bell's Theorem
7) The Colbeck-Renner Theorem

8.1) Definitions

- An operational theory looks like this:



• = variables we control

• = variables we observe

Theory predicts $\text{Prob}(a,b,c,d,e|u,v,w,y,z)$

- Ontic states:

In addition to the variables we control and observe, there may be additional physical properties denoted λ that take values in a set Λ.

λ is called an ontic state.

Λ is called the ontic state space

λ can be anything you like: particle trajectories, field configurations, quantum states, green aliens, none of the above.

Single World Realism

- (Single world) Realism :

  On each run of the experiment, the operational variables and λ each take a definite value.

- Independence :

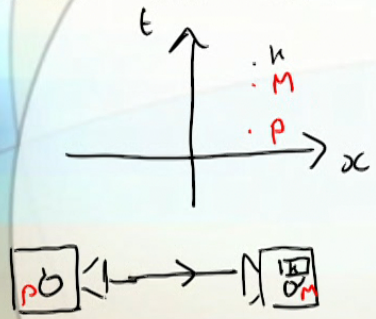
  Each run of the experiment is independent and identically distributed

$\Leftrightarrow \exists$ a joint probability distribution

$$P_r(a, b, c, d, e, \lambda \mid u, v, w, y, z)$$

Prepare and Measure Experiments

- For the next few lectures we will be considering a very simple kind of experiment



  - P is a choice of preparation
  - M is a choice of measurement
  - k is the outcome of the measurement
  - An operational theory predicts $Prob(k|P,M)$

- If there are ontic states with single world realism and independence, we should have $P_r(k,\lambda|P,M)$
- We reproduce the operational predictions if

$$Prob(k|P,M) = \int_\Lambda d\lambda \, P_r(k,\lambda|P,M) .$$

Ontological Models

- In general, we can write

$$\text{Prob}(k|P,M) = \int_\Lambda d\lambda \, \text{Pr}(k,\lambda|P,M) = \int_\Lambda d\lambda \, \text{Pr}(k|\lambda,P,M)\,\text{Pr}(\lambda|P,M)$$

- We now want to think of λ as representing the physical properties of the system we are experimenting on between preparation and measurement. We introduce two assumptions.

1) Measurement independence: $\text{Pr}(\lambda|P,M) = \text{Pr}(\lambda|P)$

   Often motivated as free choice / free will / no retrocausality

2) λ-mediation: $\text{Pr}(k|\lambda,P,M) = \text{Pr}(k|\lambda,M)$

   If λ represents all the physical properties of the system, it ought to be soley responsible for mediating any observed correlation between preparation and measurement.

Ontological Models

- With measurement independence and λ-mediation we have

$$\text{Prob}(k|P,M) = \int_\Lambda \Pr(k|M,\lambda)\,\Pr(\lambda|P)\,d\lambda$$

- An ontological model is a model with ontic states satisfying: single world realism, independence, measurement independence, and λ-mediation.

- In other words:
  - There is an ontic state space Λ
  - To each preparation we assign a probability distribution $\Pr(\lambda|P)$ over Λ
  - To each measurement we assign a probability distribution $\Pr(k|M,\lambda)$ over measurement outcomes that depends on λ
  - These should reproduce the operational predictions

$$\text{Prob}(k|P,M) = \int_\Lambda \Pr(k|M,\lambda)\,\Pr(\lambda|P)\,d\lambda$$

Quantum Models

- We are usually interested in experiments that are well-described by quantum theory. In which case:
  - We assign a Hilbert space $\mathcal{H}$ to the system.
  - To each preparation P we assign a density operator $\rho_P \in \mathcal{L}(\mathcal{H})$
  - To each measurement M we assign a POVM E_k^M
  - The operational predictions are $Prob(k|P,M) = Tr(E_k^M \rho_P)$
- Then, we'll have

$$Tr(E_k^M \rho_P) = \int_\Lambda d\lambda \, Pr(k|M,\lambda) Pr(\lambda|P)$$

Naughtiness

- In general there are many different ways of preparing the same quantum state

  many P's $\longrightarrow$ same ρ

- And many ways of implementing the same quantum measurement

  many M's $\longrightarrow$ same POVM $\{E_k\}$

- Nothing I have said so far implies that

$$\rho_P = \rho_Q \Rightarrow \Pr(\lambda|P) = \Pr(\lambda|Q)$$

$$\text{or } E^M_k = E^N_k \Rightarrow \Pr(k|M,\lambda) = \Pr(k|N,\lambda)$$

$\therefore$ It is extremely naughty to write $\Pr(E\rho) = \int_\Lambda d\lambda \Pr(k|E,\lambda)\Pr(\lambda|\rho)$

- Nonetheless, I will sometimes do this if I am implicitly only considering one way of preparing a state / implementing a measurement or if what I need to say is independent of implementation.

9.2) Examples
Spekkens' Toy Theory

$\Lambda = \{++, +-, -+, --\}$

Preparation P	Epistemic State $Pr(\lambda|P)$
$+x$	
$-x$	
$+y$	
$-y$	
$+z$	
$-z$	

Measurement M	Outcome k	Response Function $Pr(k|M,\lambda)$
X	$+1$	
X	-1	
Y	$+1$	
Y	-1	
Z	$+1$	
Z	-1	

The Beltrametti-Bugajski Model

- This is just an encoding of the orthodox interpretation in the ontological models framework.
- $\Lambda = \{$ set of projectors onto 1-d subspaces of $\mathcal{H}\}$
- When a pure state $|\psi\rangle$ is prepared

$$Pr(\lambda|\psi) = \delta(|\lambda\rangle\langle\lambda| - |\psi\rangle\langle\psi|)$$

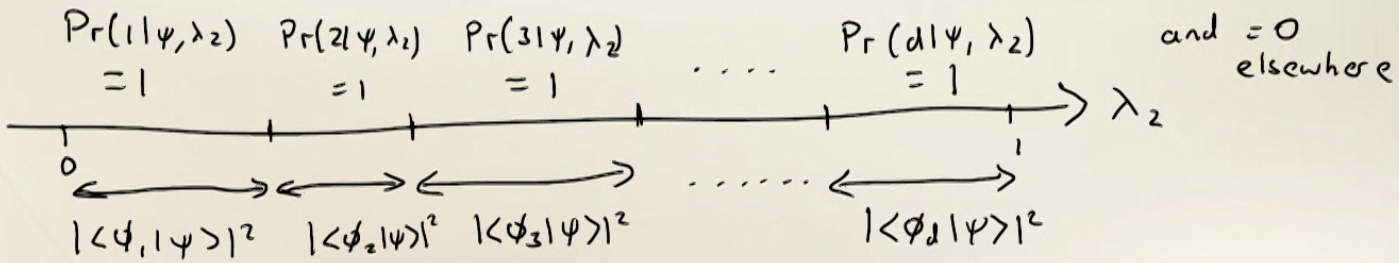
- When a POVM $M = \{E_k\}$ is measured

$$Pr(k|M,\lambda) = Tr(E_k|\lambda\rangle\langle\lambda|)$$

- By construction $Tr(E_k|\psi\rangle\langle\psi|) = \int_\Lambda d\lambda\, Pr(k|M,\lambda) Pr(\lambda|\psi)$
- Note that different preparations of mixed states correspond to different epistemic states.

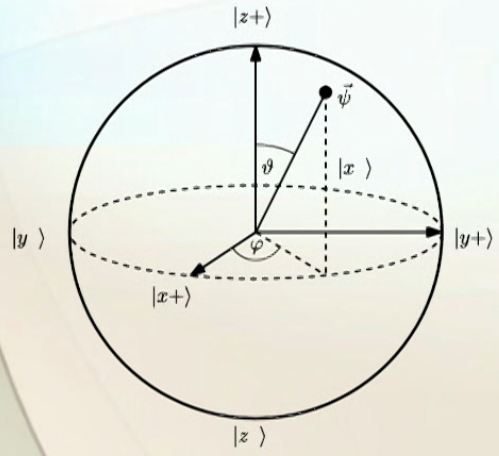
The Bell Model

- This was intended as an example to show that a deterministic hidden variable theory is always possible in principle
- $\Lambda = \{$ set of projectors onto 1-d subspaces of $\mathcal{H}\} \times [0,1]$
  λ_1 λ_2
- When pure state $|\psi\rangle$ is prepared
  $Pr(\lambda_1, \lambda_2 | \psi) = \delta(|\lambda_1\rangle\langle\lambda_1| - |\psi\rangle\langle\psi|)$ ← uniform distribution over λ_2
- When an orthonormal basis $M = \{|\phi_k\rangle\langle\phi_k|\}_{k=1}^{d}$ is measured



The Kochen-Specker Model

- The Kochen-Specker model works for a qubit. We can use the Bloch sphere representation of state vectors.



Λ = the unit sphere

$$P_r(\vec{\lambda}\,|\,\vec{\psi}) = \frac{1}{\pi} H(\vec{\psi}\cdot\vec{\lambda})\,\vec{\psi}\cdot\vec{\lambda}$$

$$H(x) = \begin{cases} 1, & x > 0 \\ 0, & x \le 0 \end{cases}$$

For an orthonormal basis measurement $M = \{|\phi\rangle\langle\phi|, |\phi^\perp\rangle\langle\phi^\perp|\}$

$$P_r(\vec{\phi}\,|\,\vec{\lambda}, M) = H(\vec{\phi}\cdot\vec{\lambda})$$

$$P_r(\vec{\phi}^\perp\,|\,\lambda, M) = 1 - H(\vec{\phi}\cdot\vec{\lambda})$$

Proving $|\langle\phi|\psi\rangle|^2 = \int_\Lambda d\lambda\, P_r(\vec{\phi}\,|\,\vec{\lambda}, M)\, P_r(\vec{\lambda}\,|\,\vec{\psi})$ is a hard ~~4-qubit problem~~ exercise.

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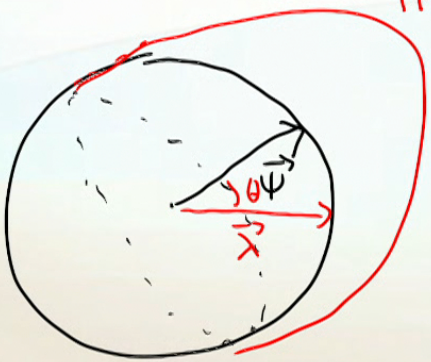
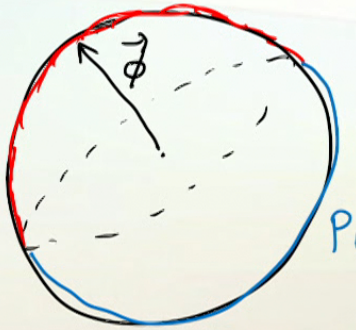


$Pr(\vec{\lambda}|\vec{\psi}) \propto \cos\theta$ in hemisphere of $\vec{\psi}$

$Pr(\vec{\phi}|\lambda, M) = +1$



$Pr(\vec{\phi}|\lambda, M) = 0$

When computing integrals in KS model, don't forget that

$d\vec{\lambda} = \sin\theta \, d\theta \, d\phi$ in spherical polar co-ordinates.