Title: PSI 2018/2019 - Foundations of Quantum Mechanics - Lecture 6

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Abstract:

Quantum Foundations Lecture 6

PSI Review Class: 14th January 2019

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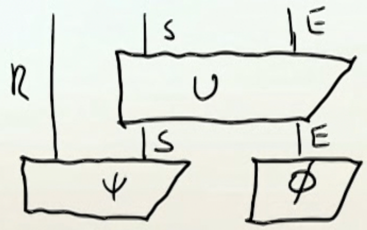
5) The Generalized Formalism

1) The Two Churches of Quantum Theory
2) The Hilbert Space of Hermitian Matrices
3) Density Operators
4) Positive Operator Valued Measures (POVMs)
5) Completely Positive Trace Preserving (CPT) Maps
6) The Lindblad Equation

Summary

- Church of the Larger Hilbert Space view on dynamics:
  - The dynamics of a system interacting with its environment is to be derived from unitary interaction and then tracing out the environment.



  - This gives $\rho_S \to \sum_j M^{(j)} \rho_S M^{(j)\dagger}$, where $M^{(j)} = \langle j|_E U_{SE} |\phi\rangle_E$
  - The operators $M^{(j)}$ can be *any* set of operators such that $\sum_j M^{(j)\dagger} M^{(j)} = I_S$

Summary

- Church of the Smaller Hilbert Space view on dynamics
  - Dynamics should be a map $\mathcal{E}_{B|A}$ from states on $\mathcal{H}_A$ to states on $\mathcal{H}_B$.
  - The map should preserve convex mixtures

$$\mathcal{E}_{B|A}(p\rho_A + (1-p)\sigma_A) = p\mathcal{E}_{B|A}(\rho_A) + (1-p)\mathcal{E}_{B|A}(\sigma_A)$$

  - This implies that the map can be extended to be a linear map from $\mathcal{L}(\mathcal{H}_A)$ to $\mathcal{L}(\mathcal{H}_B)$.

$$\mathcal{E}_{B|A} \in \mathcal{L}(\mathcal{L}(\mathcal{H}_A) \to \mathcal{L}(\mathcal{H}_B))$$

The View from the Smaller Space

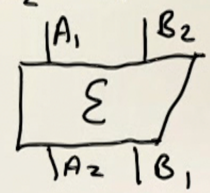
- Now comes the fun part:

$$\mathcal{L}(\mathcal{H}_A) = \mathcal{H}_{A_2} \otimes \mathcal{H}_{A_1}^{\dagger} \qquad \mathcal{L}(\mathcal{H}_B) = \mathcal{H}_{B_2} \otimes \mathcal{H}_{B_1}^{\dagger}$$

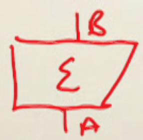
These are both $\mathcal{H}_A$ but it helps to keep track of which is the input and which is the output

$$\therefore\ \mathcal{L}(\mathcal{L}(\mathcal{H}_A) \to \mathcal{L}(\mathcal{H}_B)) = \mathcal{L}(\mathcal{H}_{A_2} \otimes \mathcal{H}_{A_1}^{\dagger} \to \mathcal{H}_{B_2} \otimes \mathcal{H}_{B_1}^{\dagger}) = \mathcal{H}_{B_2} \otimes \mathcal{H}_{B_1}^{\dagger} \otimes \mathcal{H}_{A_2}^{\dagger} \otimes \mathcal{H}_{A_1}$$

$$\therefore\ \mathcal{E}_{B|A} = \sum_{jklm} \mathcal{E}^{jm}_{kl}\, |j\rangle_{A_1} \otimes {}_{A_2}\langle k| \otimes {}_{B_1}\langle l| \otimes |m\rangle_{B_2} \qquad \mathcal{E}^{j_{A_1} m_{B_2}}_{k_{A_2} l_{B_1}}$$

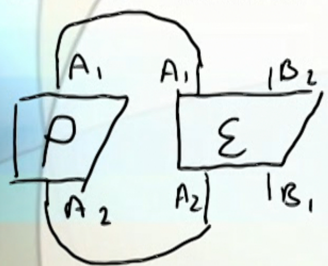


$$\mathcal{E}_{BA} = \sum_{(jk)(lm)} \mathcal{E}^{(jk)}_{(lm)}\, |jk\rangle_A \otimes {}_B\langle lm|$$



The View from the Smaller Church

- The action of $\mathcal{E}_{B|A}$ on a density operator ρ_A is going to be



$$\mathcal{E}_{B|A}(\rho_A) = \sum_{jklm} \mathcal{E}^{jm}_{kl}\, {}_{A_2}\langle k|\rho_A|j\rangle_{A_1} \otimes |m\rangle_{B_2}\, {}_{B_1}\langle l|$$

$$= \mathcal{E}^{j_{A_1} m_{B_2}}_{k_{A_2} l_{B_1}}\, \rho^{k_{A_2}}_{j_{A_1}}$$

- The space $\mathcal{L}(\mathcal{L}(\mathcal{H}_A) \to \mathcal{L}(\mathcal{H}_B)) = \mathcal{H}_{A_1} \otimes \mathcal{H}^{\dagger}_{A_2} \otimes \mathcal{H}^{\dagger}_{B_1} \otimes \mathcal{H}_{B_2}$ can be decomposed in a different way, which will end up giving us the operator-sum decomposition.

$$\left(\mathcal{H}_{A_1} \otimes \mathcal{H}^{\dagger}_{B_1}\right) \otimes \left(\mathcal{H}_{B_2} \otimes \mathcal{H}^{\dagger}_{A_2}\right) = \mathcal{L}(\mathcal{H}_{B_1} \to \mathcal{H}_{A_1}) \otimes \mathcal{L}(\mathcal{H}_{A_2} \to \mathcal{H}_{B_2})$$

$$= \mathcal{L}(\mathcal{H}_{A_1} \to \mathcal{H}_{B_1})^{\dagger} \otimes \mathcal{L}(\mathcal{H}_{A_2} \to \mathcal{H}_{B_2})$$

$$= \mathcal{L}(\mathcal{L}(\mathcal{H}_A \to \mathcal{H}_B))$$

The View from the Smaller Church

- On this decomposition, we would write the action of $\mathcal{E}_{B|A}$ as

$$\mathcal{E}_{B|A}(\rho_A) = \sum_{jklm} \mathcal{E}^{jm}_{kl} \, |m\rangle_{B_2} \, {}_{A_2}\langle k| \rho_A |j\rangle_{A_1} \, {}_{B_1}\langle l|$$

- If we view $\mathcal{L}(\mathcal{H}_A \to \mathcal{H}_B)$ as a space of kets $|jk\rangle_{AB} = |j\rangle_B \, {}_A\langle k|$
  and $\mathcal{L}(\mathcal{H}_B \to \mathcal{H}_A)$ as a space of bras $\langle jk|_{AB} = |k\rangle_A \, {}_B\langle j|$

  Then $\mathcal{E}_{B|A}$ has the form

$$\mathcal{E}_{B|A} = \sum_{jklm} \mathcal{E}^{jm}_{kl} \, |mk\rangle_{AB} \langle lj|$$

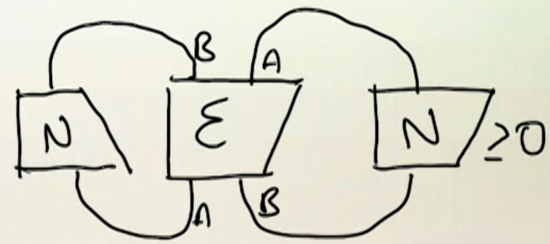
The View from the Smaller Church

⊙ If I can show that $\mathcal{E}_{B|A}$ is a positive operator on this space

i.e. $\forall \; N \in \mathcal{L}(\mathcal{H}_A \to \mathcal{H}_B)$

$$\langle N|\mathcal{E}_{B|A}|N\rangle = \sum_{jklm} \mathcal{E}^{jm}_{kl} \langle N|mk\rangle\langle lj|N\rangle \geq 0$$

which means $\quad N^{\dagger\, k_A}_{m_B} \, \mathcal{E}^{j_A m_B}_{k_A l_B} \, N^{l_B}_{j_A} \geq 0$



Then we will have an eigenoperator decomposition with positive eigenvalues

$$\mathcal{E}_{B|A} = \sum_j \lambda_j |R^{(j)}\rangle\langle R^{(j)}| = \sum_j |M^{(j)}\rangle\langle M^{(j)}| \quad \text{where} \quad M^{(j)} = \sqrt{\lambda_j}\, R^{(j)}$$

The View from the Smaller Church

- Now if $\mathcal{E}_{B|A} = \sum_j |M^{(j)}\rangle\langle M^{(j)}|$

  then $\mathcal{E}_{B|A}(\rho_A) = \sum_j M^{(j)} \rho_A M^{(j)\dagger}$ so we'll have an operator sum decomposition

- We'll prove that the required positivity holds soon.
- However, if the operator sum decomposition holds, we can prove that

$$\sum_j M^{(j)\dagger} M^{(j)} = I$$

- If $\mathcal{E}_{B|A}(\rho_A)$ is a density operator then $\mathrm{Tr}(\mathcal{E}_{B|A}(\rho_A)) = \mathrm{Tr}(\rho_A) = 1$

  for any input density operator ρ_A, i.e. $\mathcal{E}_{B|A}$ is trace preserving

Trace Preservation

- If $\sum_j M^{(j)\dagger} M^{(j)} = I$ then

$$\mathrm{Tr}\left(\mathcal{E}_{B|A}(\rho_A)\right) = \mathrm{Tr}\left[\sum_j M^{(j)} \rho_A M^{(j)\dagger}\right] = \mathrm{Tr}\left[\left(\sum_j M^{(j)\dagger} M^{(j)}\right)\rho_A\right]$$

$$= \mathrm{Tr}\left[I \rho_A\right] = \mathrm{Tr}(\rho_A) = 1$$

- Conversely, suppose $\mathcal{E}_{B|A}$ is trace preserving and let $\rho_A = |\psi\rangle_A\langle\psi|$

Then $\mathrm{Tr}\left[\sum_j M^{(j)} |\psi\rangle_A \langle\psi| M^{(j)\dagger}\right] = 1$

$${}_A\langle\psi| \left(\sum_j M^{(j)\dagger} M^{(j)}\right) |\psi\rangle_A = 1$$

$\Rightarrow$ ${}_A\langle\psi|N|\psi\rangle_A = 1$ with $N = \sum_j M^{(j)\dagger} M^{(j)}$

Trace Preservation

① Now, if $\langle\psi|N|\psi\rangle = 1$ for all unit vectors $|\psi\rangle$ then $N = I$

Proof: Let $|\psi\rangle = |k\rangle$ (a basis vector), then $\langle k|N|k\rangle = 1$, so diagonal entries are equal to 1

Let $|\psi\rangle = \frac{1}{\sqrt{2}}(|k\rangle + |m\rangle)$

$\Rightarrow \frac{1}{2}(\langle k|N|k\rangle + \langle k|N|m\rangle + \langle m|N|k\rangle + \langle m|N|m\rangle) = 1$

$\Rightarrow \langle k|N|m\rangle + \langle m|N|k\rangle = 0$ ①

Let $|\psi\rangle = \frac{1}{\sqrt{2}}(|k\rangle + i|m\rangle)$

$\Rightarrow i\langle k|N|m\rangle - i\langle m|N|k\rangle = 0$

$\Rightarrow \langle k|N|m\rangle - \langle m|N|k\rangle = 0$ ②

$\frac{① + ②}{2} \Rightarrow \langle k|N|m\rangle = 0$, so off-diagonal entries are 0.

Positivity vs. Complete Positivity

- It would be nice if we could show that $\langle N | \mathcal{E}_{B|A} | N \rangle \geq 0$ follows from the requirement that $\mathcal{E}_{B|A}(\rho_A)$ is a positive operator.
- Such a superoperator is called a positive superoperator.
- What happens instead is something of an embarassment for the Church of the Smaller Hilbert space.
- If system A is correlated with its environment E, then acting with $\mathcal{E}_{B|A}$ on A alone should keep the state of AE positive

  $\mathcal{E}_{B|A}(\rho_{AE})$ should be a positive operator
- A superoperator with this property is called completely positive.

Positivity vs. Complete Positivity

- An example of a superoperator that is positive but not completely positive is the transpose map

$$\mathcal{E}_{B|A}(|j\rangle_A\langle k|) = |k\rangle_B\langle j| \quad \text{(here } \mathcal{H}_A \text{ and } \mathcal{H}_B \text{ have same dimension)}$$

- This maps $\rho_{j_A}^{k_A}$ to $\rho_{k_B}^{j_B}$, which preserves its eigenvalues and hence positivity.
- However, let A, B, E be qubits and consider the initial state

$$\rho_{AE} = |\Phi^+\rangle_{AE}\langle\Phi^+| \quad \text{with} \quad |\Phi^+\rangle_{AE} = \frac{1}{\sqrt{2}}(|00\rangle_{AE} + |11\rangle_{AE})$$

$$= \frac{1}{2}(|00\rangle\langle 00| + |00\rangle\langle 11| + |11\rangle\langle 00| + |11\rangle\langle 11|)$$

$$\mathcal{E}_{B|A}(\rho_{AE}) = \frac{1}{2}(|00\rangle\langle 00| + |10\rangle\langle 01| + |01\rangle\langle 10| + |11\rangle\langle 11|)$$

$\frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$ is an eigenvector of this with eigenvalue $-\frac{1}{2}$

$\Rightarrow$ not a positive operator.

Complete Positivity

- For complete positivity, we require

$$\mathcal{E}_{B|A}(\rho_{AE}) \text{ is a positive operator for any positive } \rho_{AE}$$

$$\Leftrightarrow \; {}_{BE}\langle\psi|\mathcal{E}_{B|A}(\rho_{AE})|\psi\rangle_{BE} \geq 0 \text{ for any } |\psi\rangle_{AE} \in \mathcal{H}_A \otimes \mathcal{H}_E \text{ and any positive } \rho_{AE}$$

- In particular

$${}_{A'B}\langle\psi|\mathcal{E}_{B|A}(|\delta\rangle_{AA'}\langle\delta|)|\psi\rangle_{A'B} \geq 0 \quad \text{for} \quad |\delta\rangle_{AA'} := \sum_j |j\rangle_A \otimes |j\rangle_{A'}$$

but this turns out to be equivalent to

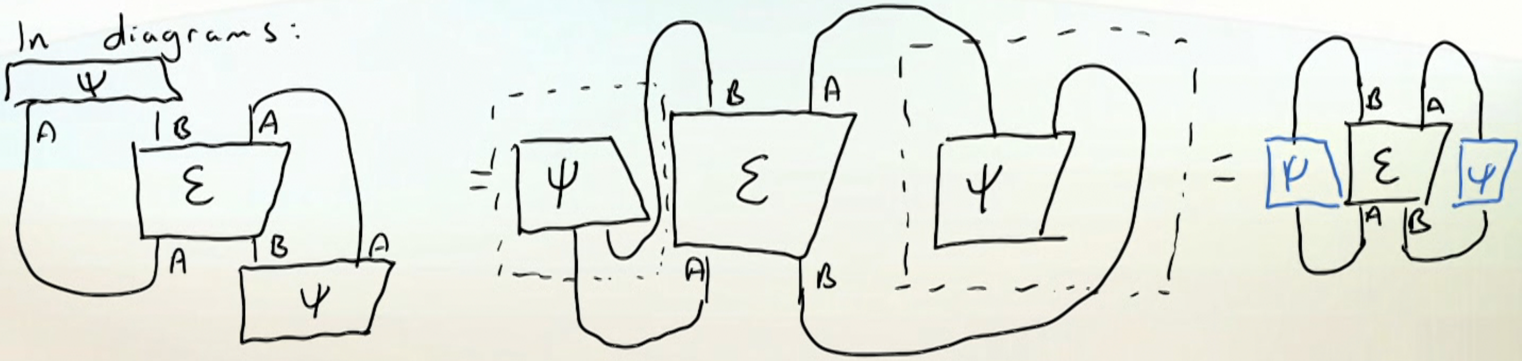
$$\langle\psi|\, \mathcal{E}_{B|A}|\psi\rangle \geq 0 \quad \text{for any} \quad \psi \in \mathcal{L}(\mathcal{H}_A \to \mathcal{H}_B)$$

which is the positivity condition needed for the operator sum decomposition

Complete Positivity

$${}_{A'B}\langle\psi|\, \mathcal{E}_{B|A}\left(|\delta\rangle_{AA'}\langle\delta|\right)|\psi\rangle_{A'B} = \langle\psi|\mathcal{E}_{B|A}|\psi\rangle$$

Proof: In diagrams:



In index notation:

$$\psi^{\dagger}_{r_A m_B}\,\mathcal{E}^{j_A m_B}_{k_A l_B}\,\delta_{j_A n_A}\,\delta^{k_A r_A}\,\psi^{n_A l_B} = \psi^{\dagger k_A}_{m_B}\,\mathcal{E}^{j_A m_B}_{k_A l_B}\,\psi^{l_B}_{j_A} = \langle\psi|\mathcal{E}_{B|A}|\psi\rangle$$

Complete Positivity

- We have now proved that a completely positive, trace-preserving map must have the form

$$\mathcal{E}_{B|A}(\rho_A) = \sum_j M^{(j)} \rho_A M^{(j)\dagger} \quad \text{with} \quad \sum_j M^{(j)\dagger} M^{(j)} = I_A$$

- We still need to check that $\mathcal{E}_{B|A}(\rho_{AE})$ is positive for all ρ_{AE}

Since ρ_{AE} is positive, we can define $\rho_{AE}^{1/2}$ and then

$$M^{(j)} \rho_{AE} M^{(j)\dagger} = \left(M^{(j)} \rho_{AE}^{1/2}\right)\left(M^{(j)} \rho_{AE}^{1/2}\right)^\dagger \quad \text{is of the form } N^\dagger N$$

for $N = \left(M^{(j)} \rho_{AE}^{1/2}\right)^\dagger$, so is positive

A sum of positive operators is positive, so we are done.

Summary

⊙ A physical map from density operators in $\mathcal{L}(\mathcal{H}_A)$ to density operators in $\mathcal{L}(\mathcal{H}_B)$ must be of the form

$$\mathcal{E}_{B|A}(\rho_A) = \sum_j M^{(j)} \rho_A M^{(j)\dagger} \quad \text{where } M^{(j)} \in \mathcal{L}(\mathcal{H}_A \to \mathcal{H}_B)$$
$$\text{and } \sum_j M^{(j)\dagger} M^{(j)} = I_A$$

⊙ We can derive this from

1) The system acts unitarily with an environment it is initially uncorrelated with.

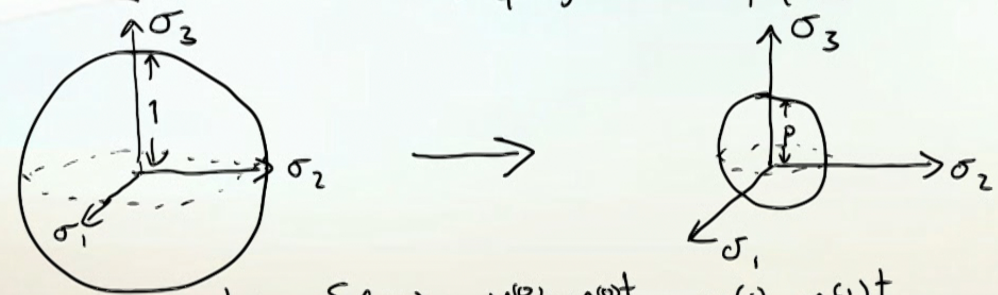
or 2) The map must be completely positive $\mathcal{E}_{B|A}(\rho_{AE})$ is positive for all positive ρ_{AE}

and trace preserving $\mathrm{Tr}(\mathcal{E}_{B|A}(\rho_A)) = \mathrm{Tr}(\rho_A)$

Examples of Qubit CPT Maps

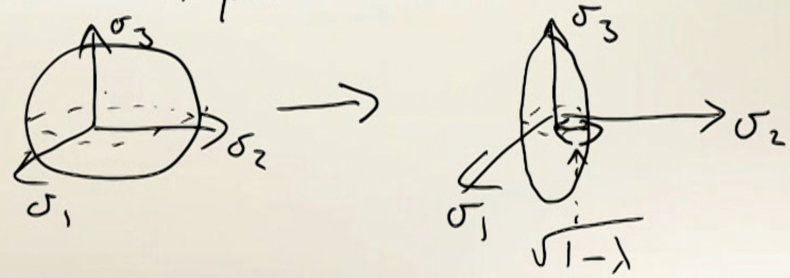
- Depolarizing channel:

$$\mathcal{E}(\rho) = p\frac{I}{2} + (1-p)\rho = \left(1-\frac{3p}{4}\right) I\rho I + \frac{p}{4}\left(\sigma_1\rho\sigma_1 + \sigma_2\rho\sigma_2 + \sigma_3\rho\sigma_3\right)$$



- Dephasing channel: $\mathcal{E}(\rho) = M^{(0)}\rho M^{(0)\dagger} + M^{(1)}\rho M^{(1)\dagger}$

$$M^{(0)} = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-\lambda} \end{pmatrix} \quad M^{(1)} = \begin{pmatrix} 0 & 0 \\ 0 & \sqrt{\lambda} \end{pmatrix}$$



5.6) The Lindblad Equation

- A density operator evolves under unitary dynamics according to

$$\rho \rightarrow U \rho U^{\dagger}$$

- If the unitary is generated by a fixed Hamiltonian $U(t) = e^{-iH(t-t_0)}$ then

$$\rho(t) = e^{-iH(t-t_0)} \rho(t_0) e^{iH(t-t_0)}$$

$$\rho(t+\Delta t) - \rho(t) = [I - iH\Delta t]\rho(t)[I + iH\Delta t] - \rho(t) \quad \text{to 1st order}$$

$$= -i\Delta t \left(H\rho(t) - \rho(t) H \right)$$

$$= -i\Delta t [H, \rho(t)]$$

$$\Rightarrow \boxed{\frac{d\rho}{dt} = -i[H, \rho]}$$

This is called the von-Neuman equation.

Continuous Time Dynamics

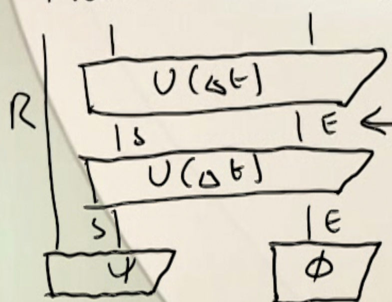
- But we know that finite time dynamics need not be unitary. We can have a completely positive, trace preserving map.

$$\rho \rightarrow \mathcal{E}(\rho) = \sum_j M^{(j)} \rho M^{(j)\dagger}$$

- What is the corresponding continuous-time dynamics?
- You might have thought that we can just parameterize $\mathcal{E}$ by t and assume that $\mathcal{E}_{t+\Delta t} = \mathcal{E}_{\Delta t} \circ \mathcal{E}_t$ i.e. $\rho(t_0 + t + \Delta t) = \mathcal{E}_{\Delta t}\left(\mathcal{E}_t(\rho(t_0))\right)$
- This would give $\mathcal{E}_t$ the structure of a continuous semi-group.
- But there is a problem with this from the point of view of the larger church.

The View from the Larger Church

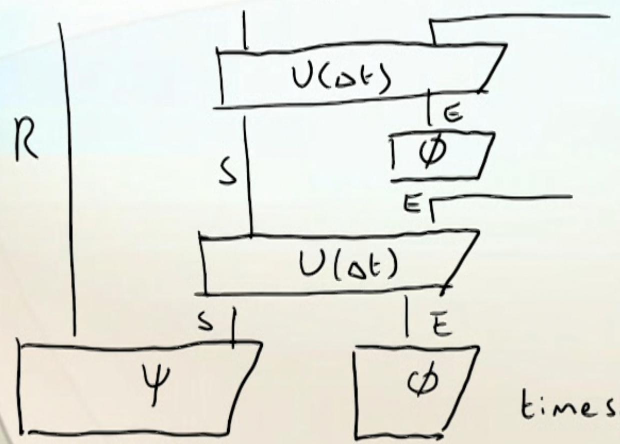
- Recall that, in order to derive CPT maps, we assumed that the system was initially uncorrelated from its environment.
- Thus, if we want $\mathcal{E}_{2\Delta t} = \mathcal{E}_{\Delta t} \circ \mathcal{E}_{\Delta t}$ with $\mathcal{E}_{\Delta t}$ CPT, we need the system to be uncorrelated with its environment after every Δt timestep.
- If the system is interacting with the environment under a fixed Hamiltonian H_{SE} then this won't be true in general



Expect a correlated state at this stage already.

The View from the Larger Church

- So we will have to assume that the interaction with the environment is approximately like this

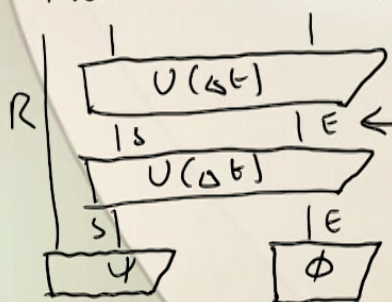


- The system behaves as if it is interacting with a new uncorrelated environment at every time step.
- This is called the weak coupling limit.
- E.g. suppose the environment is a thermal bath

timescale for rethermalization of the bath << timescale on which system gets significantly correlated with environment.

The View from the Larger Church

- Recall that, in order to derive CPT maps, we assumed that the system was initially uncorrelated from its environment.
- Thus, if we want $\mathcal{E}_{2\Delta t} = \mathcal{E}_{\Delta t} \circ \mathcal{E}_{\Delta t}$ with $\mathcal{E}_{\Delta t}$ CPT, we need the system to be uncorrelated with its environment after every Δt timestep.
- If the system is interacting with the environment under a fixed Hamiltonian H_{SE} then this won't be true in general



← Expect a correlated state at this stage already.

Deriving the Lindblad Equation

- $\mathcal{E}_{\Delta t}$ will have the usual operator sum form

$$\rho(t+\Delta t) = \mathcal{E}_{\Delta t}(\rho(t)) = \sum_{j=0}^{N} M^{(j)}\rho(t)M^{(j)\dagger} \simeq \rho(t) + O(\Delta t)$$

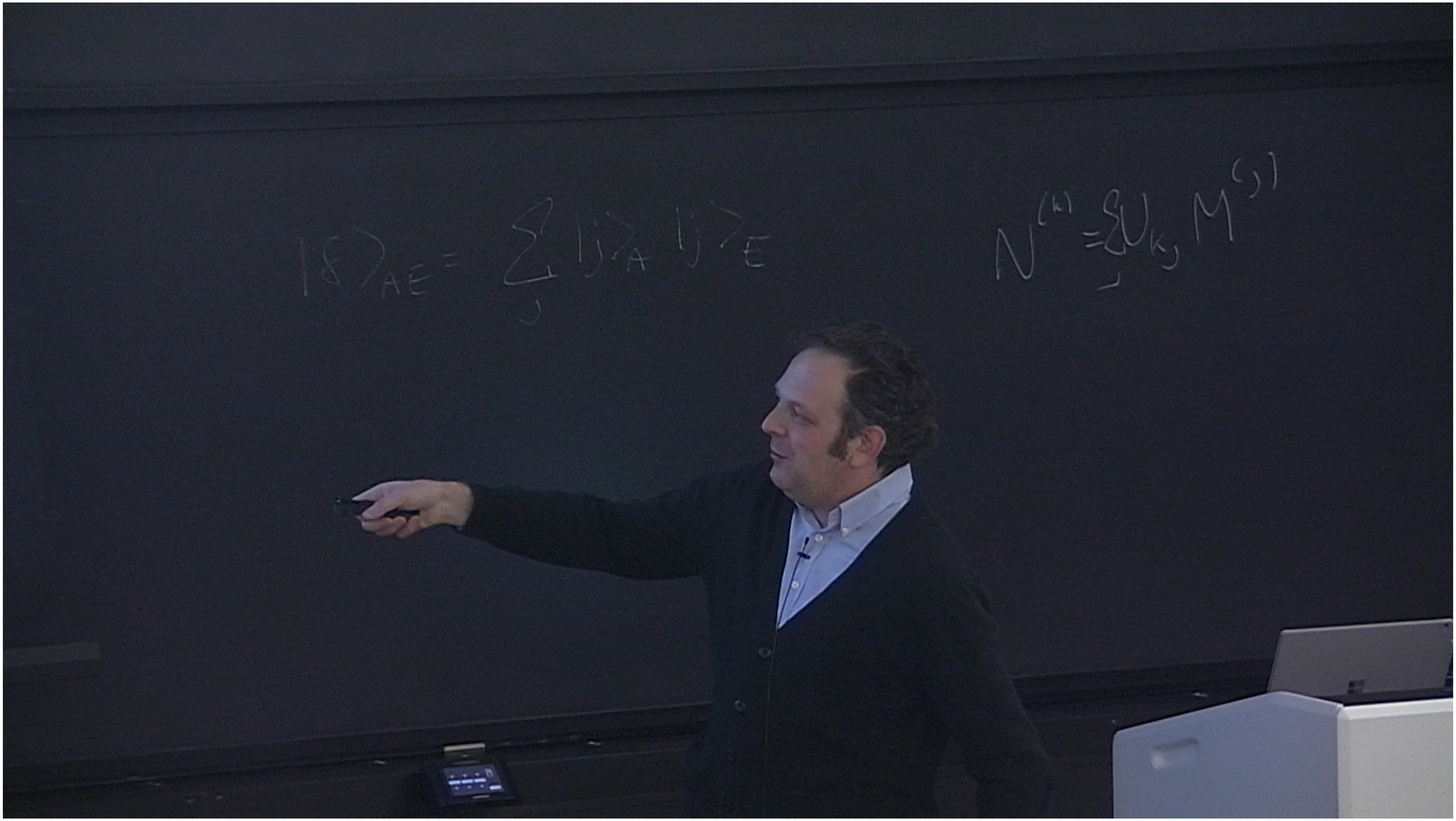
- We want to expand each term up to order Δt.
- We can, without loss of generality, put all of the $O(1)$ term in a single Kraus operator

$$M^{(0)} = I + (\underbrace{L^{(0)} - iH}_{})\Delta t + O(\Delta t^2)$$

general decomposition of an operator into two Hermitian operators

- In order for $M^{(j)}\rho M^{(j)\dagger}$ to contribute for $j=1,2,\dots,N$ we need

$$M^{(j)} = L^{(j)}\sqrt{\Delta t} + O(\Delta t)$$


|ψ⟩_AE = Σ_j |j⟩_A |j⟩_E
N^(k) = Σ_j U_kj M^(j)

Deriving the Lindblad Equation

- $\mathcal{E}_{\Delta t}$ will have the usual operator sum form

$$\rho(t+\Delta t) = \mathcal{E}_{\Delta t}(\rho(t)) = \sum_{j=0}^{N} M^{(j)} \rho(t) M^{(j)\dagger} \simeq \rho(t) + O(\Delta t)$$

- We want to expand each term up to order Δt.
- We can, without loss of generality, put all of the $O(1)$ term in a single Kraus operator

$$M^{(0)} = I + \big(\underbrace{L^{(0)} - iH}_{\text{general decomposition of an operator into two Hermitian operators}}\big)\Delta t + O(\Delta t^2)$$

- In order for $M^{(j)} \rho M^{(j)\dagger}$ to contribute for $j = 1, 2, \ldots, N$ we need

$$M^{(j)} = L^{(j)} \sqrt{\Delta t} + O(\Delta t)$$

Deriving the Lindblad Equation

- Plugging these terms into $\rho(t+\Delta t) = \mathcal{E}_{\Delta t}(\rho(t))$ gives

$$\rho(t+\Delta t) - \rho(t) = \left[(L^{(0)} - iH)\rho(t) + \rho(t)(L^{(0)} + iH) + \sum_{j=1}^{N} L^{(j)} \rho(t) L^{(j)\dagger} \right] \Delta t$$

$$= \left[\underbrace{-i[H,\rho]}_{\text{unitary part}} + \underbrace{\{L^{(0)}, \rho(t)\}}_{\substack{\text{anticommutator} \\ L^{(0)}\rho(t) + \rho(t) L^{(0)}}} + \sum_{j=1}^{N} L^{(j)} \rho(t) L^{(j)\dagger} \right] \Delta t$$

$$\therefore \quad \frac{d\rho}{dt} = -i[H,\rho] + \{L^{(0)}, \rho(t)\} + \sum_{j=1}^{N} L^{(j)} \rho(t) L^{(j)\dagger}$$

Deriving the Lindblad Equation

- We still have to impose the trace preserving condition

$$\sum_i M^{(i)\dagger} M^{(i)} = I$$

$$M^{(0)\dagger} M^{(0)} = \left[I + (L^{(0)} + iH)\Delta t\right]\left[I + (L^{(0)} - iH)\Delta t\right]$$

$$= I + (L^{(0)} + \cancel{iH} + L^{(0)} - \cancel{iH})\Delta t + O(\Delta t^2)$$

$$= I + 2L^{(0)}\Delta t + O(\Delta t^2)$$

$$\sum_{i=1}^{N} M^{(i)\dagger} M^{(i)} = \left(\sum_{j=1}^{N} L^{(j)\dagger} L^{(j)}\right)\Delta t + O(\Delta t^2)$$

$$\Rightarrow L^{(0)} = -\frac{1}{2}\sum_{j=1}^{N} L^{(j)\dagger} L^{(j)}$$

$$\boxed{\frac{d\rho}{dt} = -i[H,\rho] + \sum_{j=1}^{N}\left(L^{(j)}\rho L^{(j)\dagger} - \frac{1}{2}\left\{L^{(j)\dagger} L^{(j)}, \rho\right\}\right)}$$

Deriving the Lindblad Equation

- Plugging these terms into $\rho(t+\Delta t) = \mathcal{E}_{\Delta t}(\rho(t))$ gives

$$\rho(t+\Delta t) - \rho(t) = \left[(L^{(0)} - iH)\rho(t) + \rho(t)(L^{(0)} + iH) + \sum_{j=1}^{N} L^{(j)}\rho(t)L^{(j)\dagger}\right]\Delta t$$

$$= \Big[\underbrace{-i\,[H,\rho]}_{\text{unitary part}} + \underbrace{\{L^{(0)},\rho(t)\}}_{\substack{\text{anticommutator} \\ L^{(0)}\rho(t)+\rho(t)L^{(0)}}} + \sum_{j=1}^{N} L^{(j)}\rho(t)L^{(j)\dagger}\Big]\Delta t$$

$$\therefore \quad \frac{d\rho}{dt} = -i\,[H,\rho] + \{L^{(0)},\rho(t)\} + \sum_{j=1}^{N} L^{(j)}\rho(t)L^{(j)\dagger}$$

Deriving the Lindblad Equation

- We still have to impose the trace preserving condition

$$\sum_i M^{(j)\dagger} M^{(j)} = I$$

$$\left.\begin{aligned} M^{(0)\dagger} M^{(0)} &= \left[I + (L^{(0)} + iH)\Delta t\right]\left[I + (L^{(0)} - iH)\Delta t\right] \\ &= I + (L^{(0)} + \cancel{iH} + L^{(0)} - \cancel{iH})\Delta t + O(\Delta t^2) \\ &= I + 2L^{(0)}\Delta t + O(\Delta t^2) \\ \sum_{i=1}^{N} M^{(j)\dagger} M^{(j)} &= \left(\sum_{j=1}^{N} L^{(j)\dagger} L^{(j)}\right)\Delta t + O(\Delta t^2) \end{aligned}\right\} \Rightarrow L^{(0)} = -\frac{1}{2}\sum_{j=1}^{N} L^{(j)\dagger} L^{(j)}$$

$$\boxed{\frac{d\rho}{dt} = -i[H,\rho] + \sum_{j=1}^{N}\left(L^{(j)}\rho L^{(j)\dagger} - \frac{1}{2}\left\{L^{(j)\dagger}L^{(j)}, \rho\right\}\right)}$$

Example: Decoherence

- Consider a qubit with Hamiltonian $H=0$ and a single Lindblad operator
  $L = \gamma \sigma_3$
- Then we get $\frac{d\rho}{dt} = \gamma^2 (\sigma_3 \rho \sigma_3 - \rho)$
- In terms of components $\begin{pmatrix} \dot\rho_{00} & \dot\rho_{01} \\ \dot\rho_{10} & \dot\rho_{11} \end{pmatrix} = \begin{pmatrix} 0 & -2\gamma^2 \rho_{01} \\ -2\gamma^2 \rho_{10} & 0 \end{pmatrix}$

so we get the solution:

$$\rho(t) = \begin{pmatrix} \rho_{00}(0) & \rho_{01}(0) e^{-2\gamma^2 t} \\ \rho_{10}(0) e^{-2\gamma^2 t} & \rho_{11}(0) \end{pmatrix}$$

The off-diagonal elements decay exponentially

System decoheres in the $|0\rangle, |1\rangle$ basis.

6) Realism vs. Anti-Realsim

1) Realism
2) Anti-Realism
3) Realism vs. Anti-Realism
4) A Synthesis?
5) Making the Problem Precise

6) Realism vs. Anti-Realism

- So far, we have taken an approach to quantum theory that is *operational*, i.e. it treats measurements as a primitive and refers only to the outcomes of experimental procedures, and not properties of systems that exist independently of that.
- Our goal in this section is to make the central question in the foundations of quantum mechanics precise. To do so, we shall have to delve a little more deeply into the philosophy of science.
- See J. Ladyman, Understanding Philosophy of Science (Routledge, 2003) for more details.

6.1) Realism

- Scientific Realism is the idea that:
  - There exists an objectively real physical world, independent of observers.
  - The job of a physical theory is to attempt to describe it.
  - Successful physical theories are approximately correct descriptions of the objectively real physical world.
- It is more accurate to think of theoretical entities, e.g. electrons, quarks, as referring to things that actually exist than to do otherwise.

What realism is not

- Realism does not necessarily mean:
  - All quantum observables must have definite values at all times.
  - The quantum state is a description of reality.
  - Nature must be deterministic.
  - Our theories are literally true.
- Warning: “Realism” is often used in these ways in the literature on quantum foundations. It is much harder to deny “real” realism.

6.2) Anti-Realism

- Varieties: idealism, logical positivism, empiricism, instrumentalism, operationalism.
- The only things we have direct access to are our own perceptions and/or the records of results from our experimental apparatuses.
- Theories are simply systems for organizing/predicting regularities in those perceptions/results.
- Theoretical entities, e.g. electrons, are a convenient fiction used in our calculations.
- Operationalism: Every statement of a theory should boil down to a list of instructions for what to do in the lab and what will be seen as a result.

What realism is not

- Realism does not necessarily mean:
  - All quantum observables must have definite values at all times.
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  - Nature must be deterministic.
  - Our theories are literally true.
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6.3) Realism vs. Anti-Realism
Putnam's "No Miracles" Argument

When they argue for their position, realists typically argue against some version of idealism - in our time, this would be positivism or operationalism. (...) And the typical realist argument against idealism is that it makes the success of science a miracle

(...)The modern positivist has to leave it without explanation (the realist charges) that 'electron calculi' and 'space-time calculi' and 'DNA calculi' correctly predict observable phenomena if, in reality, there are no electrons, no curved space-time, and no DNA molecules. If there are such things, then a natural explanation of the success of theories is that they are partially true accounts of how they behave. And a natural account of the way scientific theories succeed each other (...) is that a partially correct/incorrect account of a theoretical object (...) is replaced by a better account of the same object or objects. But if those objects don't really exist at all, then it is a miracle that a theory which speaks of gravitational action at a distance successfully predicts phenomena; it is a miracle that a theory which speaks of curved space-time successfully predicts phenomena; and the fact that the laws of the former theory are derivable 'in the limit' from the laws of the latter theory has no methodological significance

H. Putnam, Meaning and the Moral Sciences, Routledge (1978)

6.3) Realism vs. Anti-Realism
Eddington's Fishy Story

Let us suppose that an ichthyologist is exploring the life of the ocean. He casts a net into the water and brings up a fishy assortment. Surveying his catch, he proceeds in the usual manner of a scientist to systematise what it reveals. He arrives at two generalisations: (1) No sea-creature is less than two inches long. (2) All sea-creatures have gills. These are both true of his catch, and he assumes tentatively that they will remain true however often he repeats it.

In applying this analogy, the catch stands for the body of knowledge which constitutes physical science, and the net for the sensory and intellectual equipment which we use in obtaining it. The casting of the net corresponds to observation; for knowledge which has not been or could not be obtained by observation is not admitted into physical science.

An onlooker may object that the first generalisation is wrong. "There are plenty of sea-creatures under two inches long, only your net is not adapted to catch them." The icthyologist dismisses this objection contemptuously. "Anything uncatchable by my net is *ipso facto* outside the scope of icthyological knowledge. In short, "what my net can't catch isn't fish." Or — to translate the analogy — "If you are not simply guessing, you are claiming a knowledge of the physical universe discovered in some other way than by the methods of physical science, and admittedly unverifiable by such methods. You are a metaphysician. Bah!"

A. Eddington, The Philosophy of Physical Science (1938)