Title: PSI 2018/2019 - Foundations of Quantum Mechanics - Lecture 2

Date: Jan 08, 2019 10:15 AM

URL: http://pirsa.org/19010018

Abstract:

Quantum Foundations Lecture 2

PSI Review Class: 8th January 2019

Instructor: Matthew Leifer
leifer@chapman.edu
PI Office: 353

3) Quantum Phenomenology

1) Interference
2) Orthodoxy and the Measurement Problem
3) The Einstein-Podolsky-Rosen Argument
4) The No-Cloning Theorem
5) Quantum Teleportation

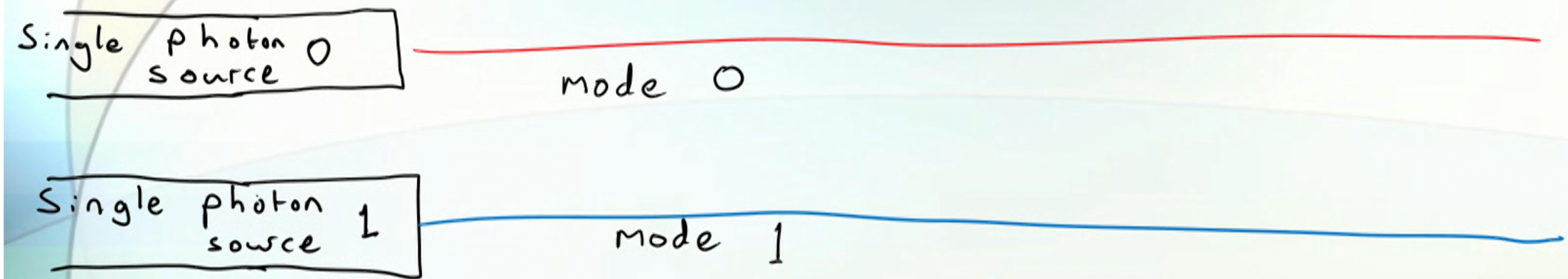
3.1) Interference

- Feynman on the double slit experiment:

"We choose to examine a phenomenon which is impossible, *absolutely* impossible, to explain in any classical way, and which has in it the heart of quantum mechanics. In reality, it contains the *only* mystery. We cannot make the mystery go away by "explaining" how it works. We will just *tell* you how it works. In telling you how it works we will have told you about the basic peculiarities of all quantum mechanics." – Feynman Lectures on Physics Vol. III 1-1

- I completely disagree with this quote, but quantum interference is one of the things we shall have to explain. Let's simplify and look at a photon in an interferometer.

Single Photon Interferometry

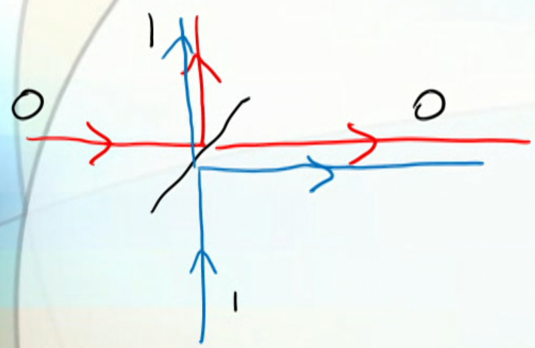


$|0\rangle = \begin{pmatrix}1\\0\end{pmatrix}$ photon in mode 0

$|1\rangle = \begin{pmatrix}0\\1\end{pmatrix}$ photon in mode 1

- We only consider single photon states, so we have a qubit.
- We always take the two path lengths through the inteferometer to be the same, so we only keep track of phase differences due to optical elements

Beam Splitters



- We always give the transmitted output mode the same label as the input mode, and opposite for the reflected mode
- Action of beamsplitter is given by a 2×2 unitary matrix

$$B = \begin{pmatrix} t & r \\ r & t \end{pmatrix}$$

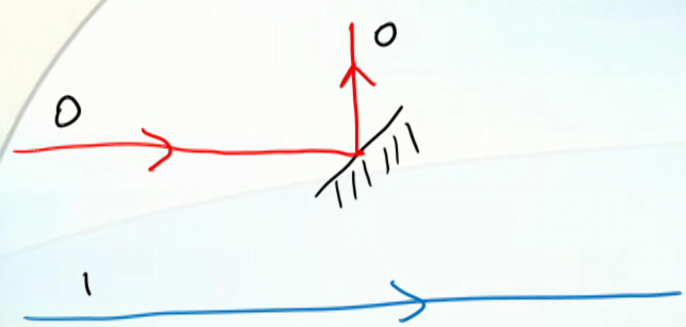
t = coefficient of transmission
r = coefficient of reflection

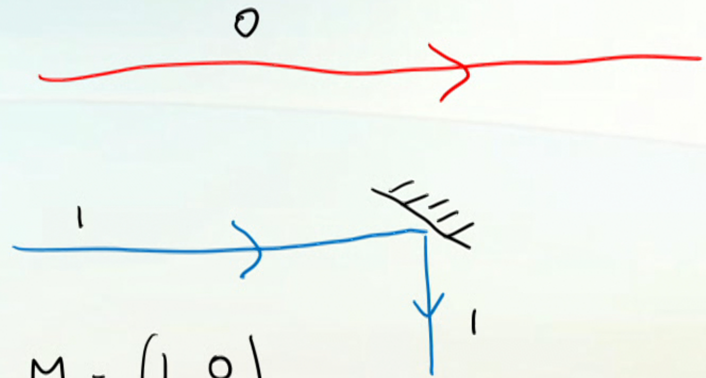
- $T = |t|^2$ is the transmittivity $R = |r|^2$ is the reflectivity

$$T + R = 1 \quad \text{by unitarity}$$

- A 50/50 beamsplitter has $B = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$

Mirrors

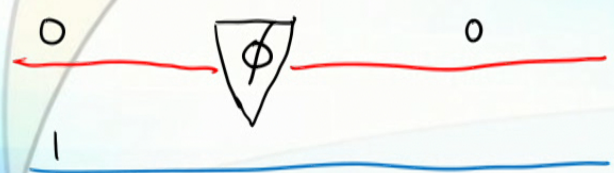




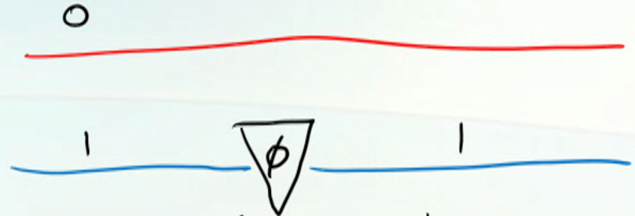
$$M_0 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \qquad M_1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

- Reflection at a mirror induces a π-phase change, so action is given by M_0 or M_1
- Remember, we will always make total path length of 0 and 1 the same, so we don't have to track more than this.

Phase Shifters



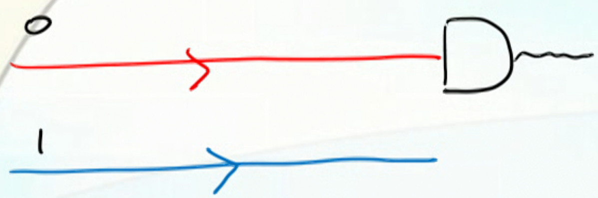
$$A_{\phi,0} = \begin{pmatrix} e^{i\phi} & 0 \\ 0 & 1 \end{pmatrix}$$



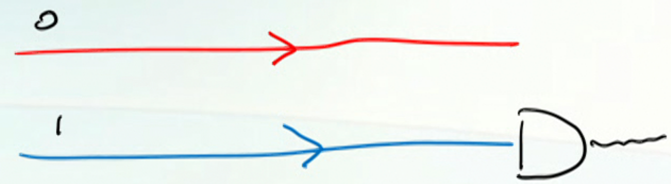
$$A_{\phi,1} = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix}$$

- We can induce a relative phase between modes 0 and 1 by inserting a block of refractive material in one of the paths, reducing the wave length.
- By altering the length/refractive index of the material, we can induce any relative phase difference ϕ we like.

Detectors



$$\text{Prob}(0) = |\langle 0|\psi\rangle|^2$$



$$\text{Prob}(1) = |\langle 1|\psi\rangle|^2$$

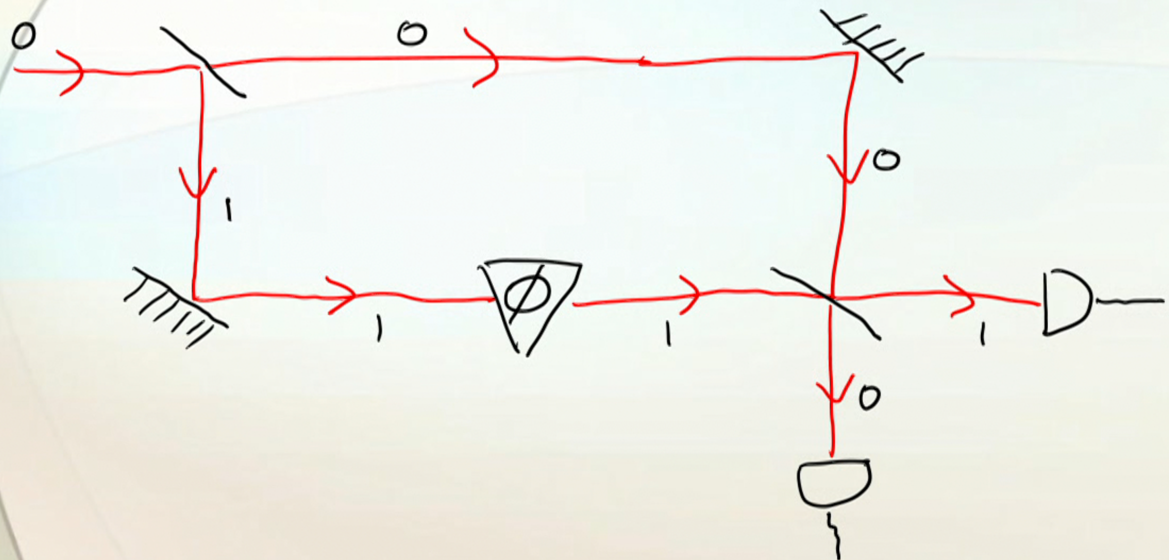
- If we place a detector in path 0/1, it will click with the probabilities indicated.
- If the measurement is ideal/nondemolition/nondestructive, then after detection, the state is updated to

  Found in path 0: $|\psi\rangle \to |0\rangle$

  Found in path 1: $|\psi\rangle \to |1\rangle$

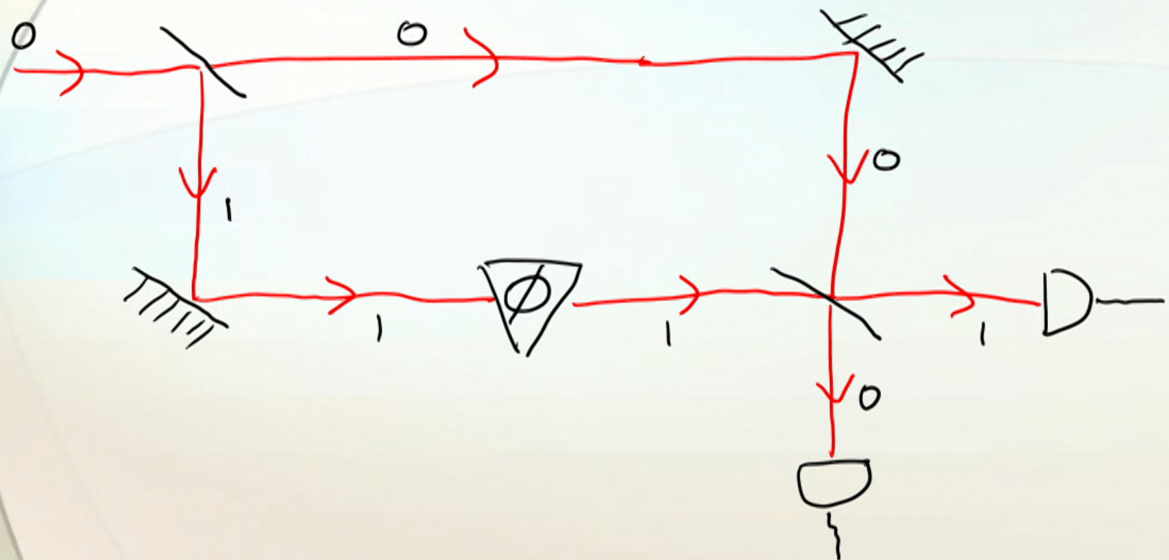
Hard to do in practice.

Mach-Zehnder Interferometer



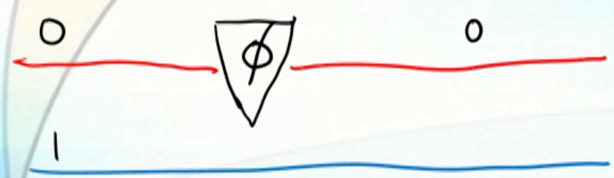
- A general Mach-Zehnder interferometer has 50/50 beamsplitters and a phase shift ϕ on path 1.
- The state just before detection is
  $|\psi\rangle = B\, A_{\phi,1} M_1 M_0 B\, |0\rangle$
- You will show that $\text{Prob}(0) = \sin^2\left(\frac{\phi}{2}\right)$ $\text{Prob}(1) = \cos^2\left(\frac{\phi}{2}\right)$
- With $\phi = 0$, the photon is always detected on path 1 due to
  constructive interference on path 1
  destructive " " " 0.

Mach-Zehnder Interferometer

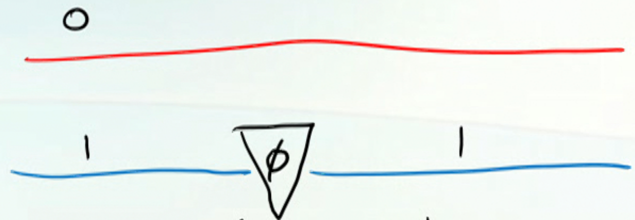


- A general Mach-Zehnder interferometer has 50/50 beamsplitters and a phase shift ϕ on path 1.
- The state just before detection is
  $|\psi\rangle = B\, A_{\phi,1} M_1 M_0 B\, |0\rangle$
- You will show that $\text{Prob}(0) = \sin^2\left(\frac{\phi}{2}\right)$ $\text{Prob}(1) = \cos^2\left(\frac{\phi}{2}\right)$
- With $\phi = 0$, the photon is always detected on path 1 due to
  constructive interference on path 1
  destructive " " " 0.

Phase Shifters



$$A_{\phi,0} = \begin{pmatrix} e^{i\phi} & 0 \\ 0 & 1 \end{pmatrix}$$



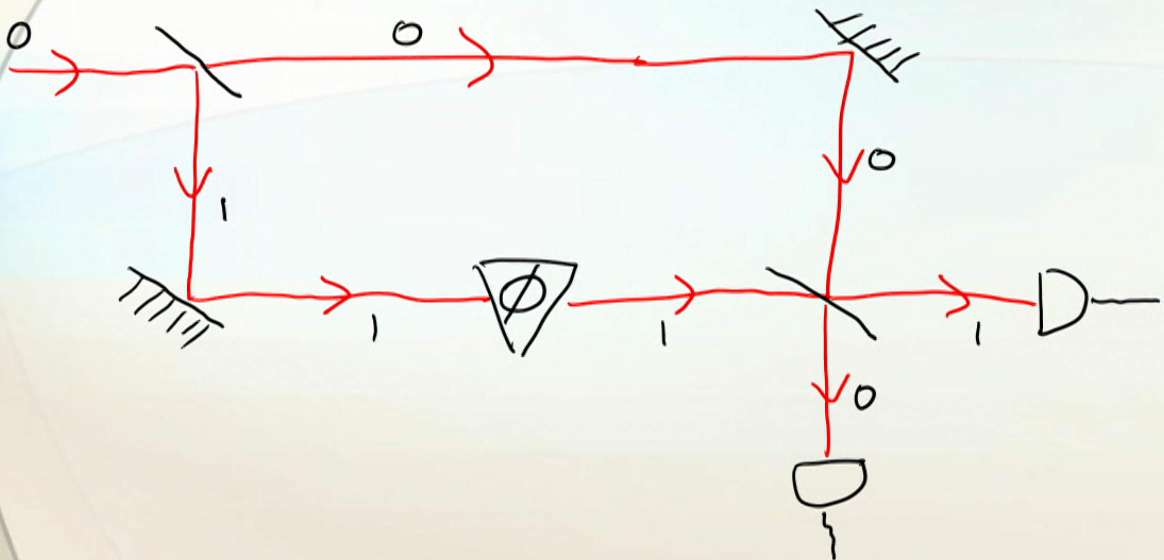
$$A_{\phi,1} = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix}$$

- We can induce a relative phase between modes 0 and 1 by inserting a block of refractive material in one of the paths, reducing the wave length.
- By altering the length/refractive index of the material, we can induce any relative phase difference ϕ we like.

Feynman's Interference "Paradox"

- Classically, particles and waves are mutually exclusive.
- The interference at the second beamsplitter indicates that the photon is behaving like a wave.
- But the fact that only one detector clicks at a time, indicates that it is behaving like a particle.
- If it is a particle, it ought to travel along either path 0 or path 1.
- So what happens if we try to check which one is the case?

Mach-Zehnder Interferometer



- A general Mach-Zehnder interferometer has 50/50 beamsplitters and a phase shift ϕ on path 1.
- The state just before detection is

$$|\psi\rangle = B\, A_{\phi,1} M_1 M_0 B\, |0\rangle$$

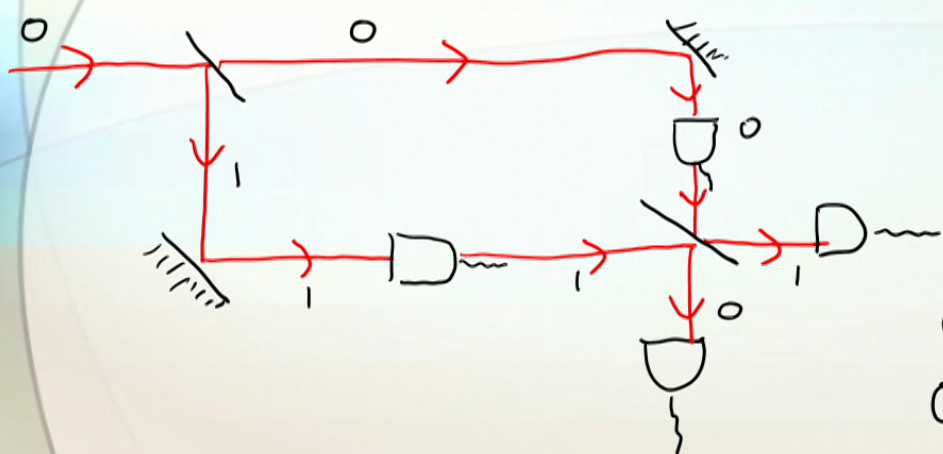
- You will show that $\text{Prob}(0) = \sin^2\left(\frac{\phi}{2}\right)$ $\quad \text{Prob}(1) = \cos^2\left(\frac{\phi}{2}\right)$
- With $\phi = 0$, the photon is always detected on path 1 due to
  constructive interference on path 1
  destructive " " " 0.

Feynman's Interference "Paradox"

- Classically, particles and waves are mutually exclusive.
- The interference at the second beamsplitter indicates that the photon is behaving like a wave.
- But the fact that only one detector clicks at a time, indicates that it is behaving like a particle.
- If it is a particle, it ought to travel along either path 0 or path 1.
- So what happens if we try to check which one is the case?

Feynman's Interference "Paradox"

- Suppose we put non-destructive detectors on both of the paths



- Before path detectors, state is

$$|\psi\rangle = M_1 M_0 B |0\rangle$$
$$= -\frac{1}{\sqrt{2}}\left(|0\rangle + i|1\rangle\right)$$

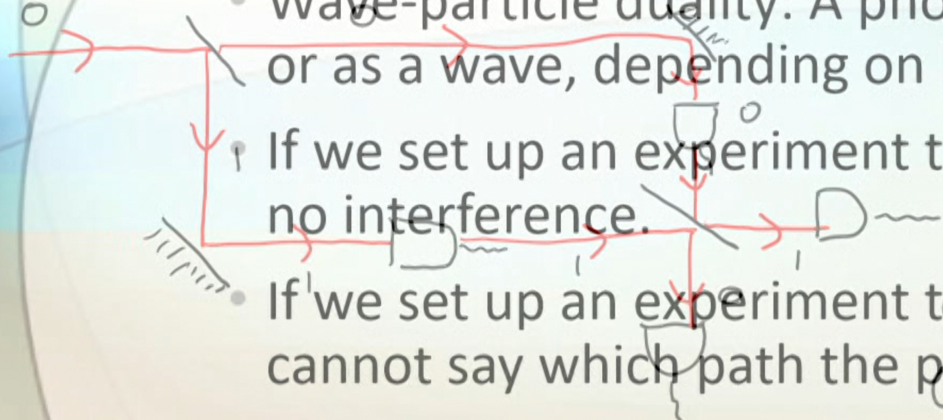
- ∴ Each detector clicks with 50/50 probability
- Afterwards, the state will be $|0\rangle$ or $|1\rangle$ with 50/50 probability
- Before final detectors, state will be either

$$B|0\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + i|1\rangle\right) \quad \text{or} \quad B|1\rangle = \frac{1}{\sqrt{2}}\left(i|0\rangle + |1\rangle\right)$$

- ∴ Final detectors will each click with 50/50 probability
- Attempting to check which path the photon travels along destroys any interference.

Traditional Resolution

Suppose we put non-destructive detectors on both of the paths



- Wave-particle duality: A photon will behave either as a particle or as a wave, depending on how the experiment is set up.
- If we set up an experiment to detect particle paths, there will be no interference.
- If we set up an experiment to detect interference, we necessarily cannot say which path the photon travels along.
- The question of which path a photon travels along during an interference experiment has no meaning.
  - People sometimes say "it travels along both paths" or "neither path", but "has no meaning" is closer to Copenhagen and textbook accounts.

Before path detectors, state is $|\psi\rangle = M_1 M_2 B|0\rangle = -\frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$

∴ Each detector clicks with 50/50 probability

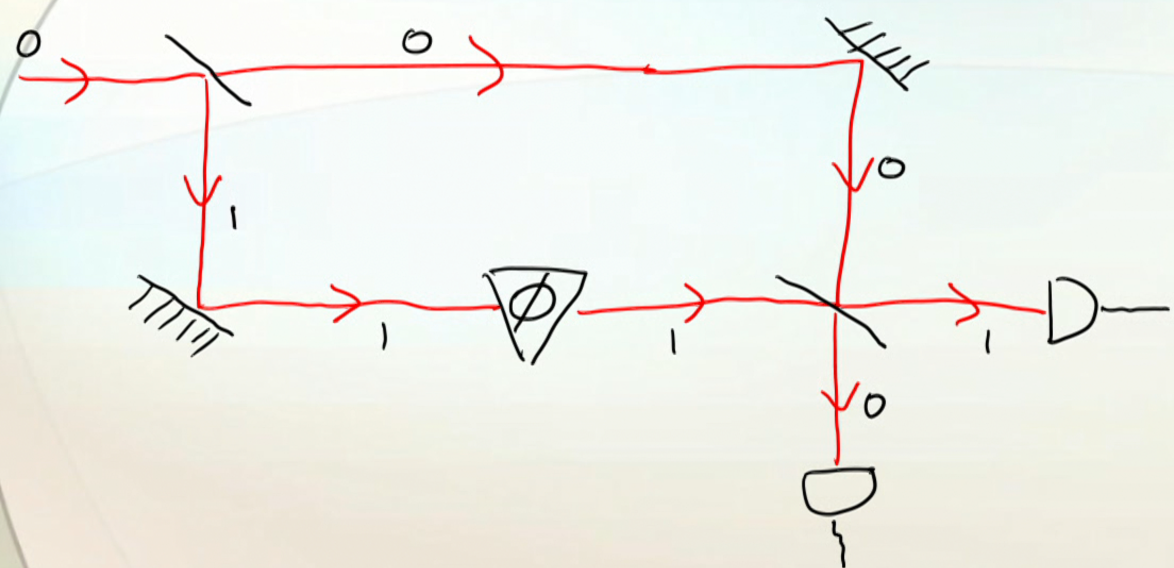
Afterwards, the state will be $|0\rangle$ or $|1\rangle$ with 50/50 probability

Before final detector state will be either $B|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$ or $B|1\rangle = \frac{1}{\sqrt{2}}(i|0\rangle + |1\rangle)$

∴ Final detectors click with 50/50 probability

Attempting to check which path the photon travels along destroys any interference.

Mach-Zehnder Interferometer



- A general Mach-Zehnder interferometer has 50/50 beamsplitters and a phase shift ϕ on path 1.
- The state just before detection is

  $$|\psi\rangle = B\, A_{\phi,1} M_1 M_0 B\, |0\rangle$$

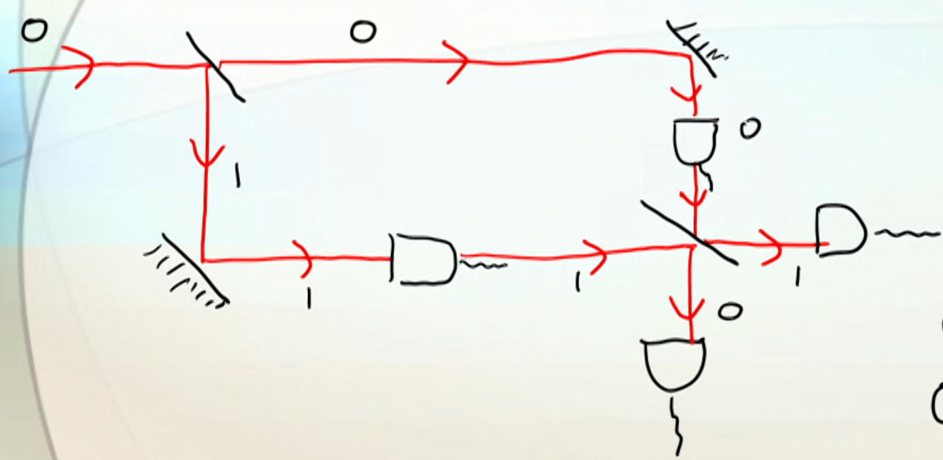
- You will show that $\text{Prob}(0) = \sin^2\left(\frac{\phi}{2}\right)$ $\quad \text{Prob}(1) = \cos^2\left(\frac{\phi}{2}\right)$
- With $\phi = 0$, the photon is always detected on path 1 due to
  constructive interference on path 1
  destructive " " " 0.

Criticism of Traditional Resolution

- It assumes that photons must behave either like waves or particles.
  - Why not both at the same time (c.f. de Broglie-Bohm theory)?
  - Why not something else entirely?
- It assumes that having a property is synonymous with being able to measure that property without disturbing anything else.
  - Why can’t the photon have a trajectory that is either unmeasurable or not measurable without disturbing some other property that is responsible for interference.
- Interference is rather weak evidence for “quantum weirdness”.
  - We shall have to explain it, but there are much more difficult problems.

Feynman's Interference "Paradox"

- Suppose we put non-destructive detectors on both of the paths



- Before path detectors, state is

$$|\psi\rangle = M_1 M_0 B |0\rangle$$
$$= -\frac{1}{\sqrt{2}}\left(|0\rangle + i|1\rangle \right)$$

- ∴ Each detector clicks with 50/50 probability
- Afterwards, the state will be $|0\rangle$ or $|1\rangle$ with 50/50 probability
- Before final detectors, state will be either

$$B|0\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + i|1\rangle \right) \quad \text{or} \quad B|1\rangle = \frac{1}{\sqrt{2}}\left(i|0\rangle + |1\rangle \right)$$

- ∴ Final detectors will each click with 50/50 probability
- Attempting to check which path the photon travels along destroys any interference.

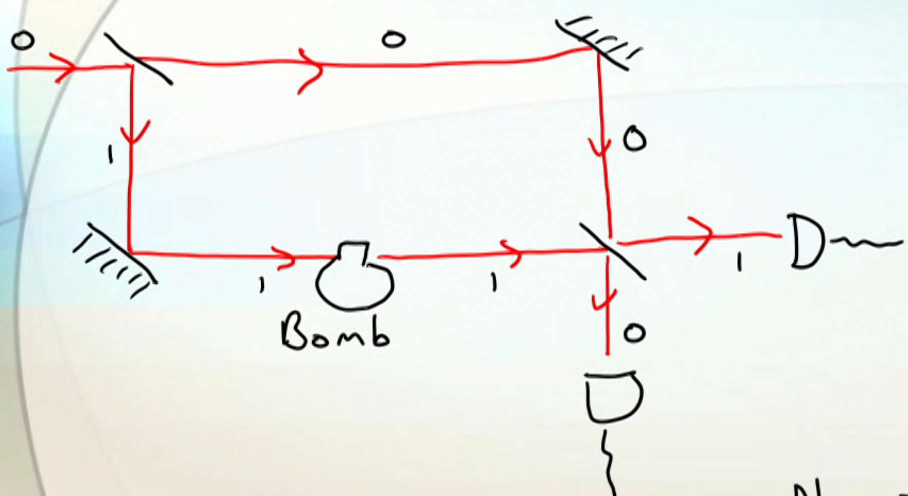
Criticism of Traditional Resolution

- It assumes that photons must behave either like waves or particles.
  - Why not both at the same time (c.f. de Broglie-Bohm theory)?
  - Why not something else entirely?
- It assumes that having a property is synonymous with being able to measure that property without disturbing anything else.
  - Why can't the photon have a trajectory that is either unmeasurable or not measurable without disturbing some other property that is responsible for interference.
- Interference is rather weak evidence for "quantum weirdness".
  - We shall have to explain it, but there are much more difficult problems.

Elitzur-Vaidman Bomb Test

- This is a wrinkle on the Feynman paradox that makes it much more dramatic.
- Consider a very sensitive bomb that explodes as soon as even the tiniest amount of electromagnetic radiation is incident on it.
- Can we detect whether a bomb is good or a dud without blowing ourselves up?
  - Classically: No. The only way to tell is to shine light on it and see if it blows up.
  - Quantumly: Yes. We can use Mach-Zehnder.

Elitzur-Vaidman Bomb Test



- If bomb is a dud, it does nothing, so we have an ordinary M-Z interferometer
  - Photon will always be detected on output path 1.
- If bomb is good it will act as a detector

KABOOM ⇒ Photon is on path 1

No KABOOM ⇒ Photon is on path 0

- But if we learn that photon is on path 0, it will be detected on output path 0 or 1 with 50/50 probability.
- Conclusion: If detector 0 clicks, we know the bomb is good <u>and</u> that the photon did not touch the bomb. This happens with prob $\frac{1}{2}$.

Elitzur-Vaidman Bomb Test

- Note: We can make the probability of detection without kaboom $1 - \epsilon$ for any $\epsilon > 0$ by using a more sophisticated interferometer.
- It is sometimes claimed that EV bomb test is evidence for nonlocality. We know for sure that the photon was nowhere near the bomb, but the presence of the bomb still influences what happens to it.
  - This assumes that if the photon goes along path 0 then there is literally nothing that goes along path 1 that could mediate the influence.
  - QFT should make us skeptical of this, as the quantum vacuum has substructure (see epistricted theories later in course).

Elitzur-Vaidman Bomb Test

- Note: We can make the probability of detection without kaboom $1 - \epsilon$ for any $\epsilon > 0$ by using a more sophisticated interferometer.
- It is sometimes claimed that EV bomb test is evidence for nonlocality. We know for sure that the photon was nowhere near the bomb, but the presence of the bomb still influences what happens to it.
    - This assumes that if the photon goes along path 0 then there is literally nothing that goes along path 1 that could mediate the influence.
    - QFT should make us skeptical of this, as the quantum vacuum has substructure (see epistricted theories later in course).

3.2) Orthodoxy and the Measurement Problem

- Macroscopic superpositions and the measurement problem are often thought to be the most pressing problems in the foundations of quantum theory.
- But they have been solved multiple times. They are not problems with quantum theory per se, but rather with the interpretation of quantum theory usually given in textbooks.
  - This is why I prefer to define the problem differently (see next week's lectures).
- This is known as the Orthodox, Textbook, or Dirac-von Neumann interpretation.
- It is often mislabeled as the Copenhagen interpretation, but it differs drastically from the views of Bohr, Heisenberg, etc. that it is not even in the same category.

The Orthodox Interpretation

1. Physical systems have objective properties:
   - The possible properties of a system are its observables. The possible values of those properties are the corresponding eigenvalues.

2. The eigenvalue-eigenstate link:
   - When the system is in an eigenstate of an observable M with eigenvalue m then M is a property of the system and it takes value m.
   - The system has no objective physical properties other than these.

The Orthodox Interpretation

1. Physical systems have objective properties:
   - The possible properties of a system are its observables. The possible values of those properties are the corresponding eigenvalues.

2. The eigenvalue-eigenstate link:
   - When the system is in an eigenstate of an observable M with eigenvalue m then M is a property of the system and it takes value m.
   - The system has no objective physical properties other than these.

The Orthodox Interpretation

- The eigenvalue-eigenstate link is equivalent to saying that the quantum state $|\psi\rangle$ is an objective property of an individual quantum system and that it is the only objective property of the system.
- Why?
  - By e-e link $|\psi\rangle\langle\psi|$ is a property of the system with value 1.
  - This uniquely determines $|\psi\rangle$ (up to global phase), so $|\psi\rangle$ is a property.
  - All other objective physical properties are uniquely determined by $|\psi\rangle$.

Some terminology

- ψ-ontic:
  - A theory in which the quantum state $|\psi\rangle$ is an objective physical property of an individual quantum system.
- ψ-complete:
  - A theory that is ψ-ontic and in which $|\psi\rangle$ is the only objective physical property, e.g. orthodox interpretation.
- ψ-epistemic:
  - A theory in which the quantum state has a similar status to a probability distribution, which you might call an epistemic, ensemble, or statistical state depending on how you think about probabilities.

Some terminology

- ψ-ontic:
  - A theory in which the quantum state $|\psi\rangle$ is an objective physical property of an individual quantum system.
- ψ-complete:
  - A theory that is ψ-ontic and in which $|\psi\rangle$ is the only objective physical property, e.g. orthodox interpretation.
- ψ-epistemic:
  - A theory in which the quantum state has a similar status to a probability distribution, which you might call an epistemic, ensemble, or statistical state depending on how you think about probabilities.

Schrödinger's Cat

"One can even set up quite ridiculous cases. A cat is penned up in a steel chamber, along with the following device (which must be secured against direct interference by the cat): in a Geiger counter, there is a tiny bit of radioactive substance, so small, that perhaps in the course of the hour one of the atoms decays, but also, with equal probability, perhaps none; if it happens, the counter tube discharges and through a relay releases a hammer that shatters a small flask of hydrocyanic acid. If one has left this entire system to itself for an hour, one would say that the cat still lives if meanwhile no atom has decayed. The first atomic decay would have poisoned it. The psi-function of the entire system would express this by having in it the living and dead cat (pardon the expression) mixed or smeared out in equal parts.

It is typical of these cases that an indeterminacy originally restricted to the atomic domain becomes transformed into macroscopic indeterminacy, which can then be resolved by direct observation. That prevents us from so naively accepting as valid a "blurred model" for representing reality. In itself, it would not embody anything unclear or contradictory. There is a difference between a shaky or out-of-focus photograph and a snapshot of clouds and fog banks." – J. Trimmer, "The Present Situation in Quantum Mechanics: A Translation of Schrödinger's 'Cat Paradox' Paper" Proc. Am. Phil. Soc. vol. 124 pp. 323-338 (1980).

Schrödinger's Cat

- If we interact a macroscopic system with a microscopic system in a superposition, then we can generate superpositions of macroscopically distinct states, e.g.

$$\frac{1}{\sqrt{2}}(|\text{Cat is alive}\rangle + |\text{Cat is dead}\rangle)$$

- In orthodox interpretation this is physically distinct from

$$|\text{Cat is alive}\rangle \quad \text{or} \quad |\text{Cat is dead}\rangle$$

- The macroscopic superposition does not correspond to anything in our direct experience.

The Measurement Problem

- A related problem is that there are two ways of handling measurements in quantum theory.
  1. The measurement postulates.
  2. A measurement device is a physical system, made of atoms, so we ought to be able to describe it as a quantum system, which interacts unitarily with the system being measured.
- As an example, consider a qubit in state

$$\alpha|0\rangle + \beta|1\rangle$$

upon which we perform an ideal measurement in the basis $\{|0\rangle, |1\rangle\}$.

The Measurement Problem

- According to the measurement postulates, the system will either collapse to

  $|0\rangle$ with probability $|\alpha|^2$

  or $|1\rangle$ with probability $|\beta|^2$.

- Now consider the measurement device as a physical system. Let $|R\rangle$ be the state in which it is ready to perform the measurement, i.e. initial state is

$$(\alpha|0\rangle + \beta|1\rangle) \otimes |R\rangle$$

The Measurement Problem

- The measurement is an interaction between the system and the measuring device, described by a unitary operator U.
- Let $|M_0\rangle$ be the state in which the measuring device registers 0.
- Let $|M_1\rangle$ be the state in which the measuring device registers 1.
- Then,

$$U|0\rangle \otimes |R\rangle = |0\rangle \otimes |M_0\rangle$$
$$U|1\rangle \otimes |R\rangle = |1\rangle \otimes |M_1\rangle$$

The Measurement Problem

- By the superposition principle, we should then have:

  $$U[(\alpha|0\rangle + \beta|1\rangle) \otimes |R\rangle] = \alpha|0\rangle \otimes |M_0\rangle + \beta|1\rangle \otimes |M_1\rangle.$$

- On the orthodox interpretation, this is physically distinct from

  $|0\rangle$ with probability $|\alpha|^2$

  or $|1\rangle$ with probability $|\beta|^2$.

- So this is a flat out contradiction. The orthodox interpretation is straightforwardly wrong.

Comments on the Measurement Problem

- The measurement problem is a problem for ψ-complete theories.
  - For a ψ-ontic, but not ψ-complete, theory, additional variables may determine which branch of the superposition describes reality. The measurement postulates could be a mathematical shortcut to avoid tracking the true, but mostly irrelevant, joint state of the system and measuring device.
  - For a ψ-epistemic theory, the measurement postulates may be viewed as no different from updating a probability distribution on the acquisition of new information.

The Measurement Problem

- By the superposition principle, we should then have:

  $U[(\alpha|0\rangle + \beta|1\rangle) \otimes |R\rangle] = \alpha|0\rangle \otimes |M_0\rangle + \beta|1\rangle \otimes |M_1\rangle.$
- On the orthodox interpretation, this is physically distinct from

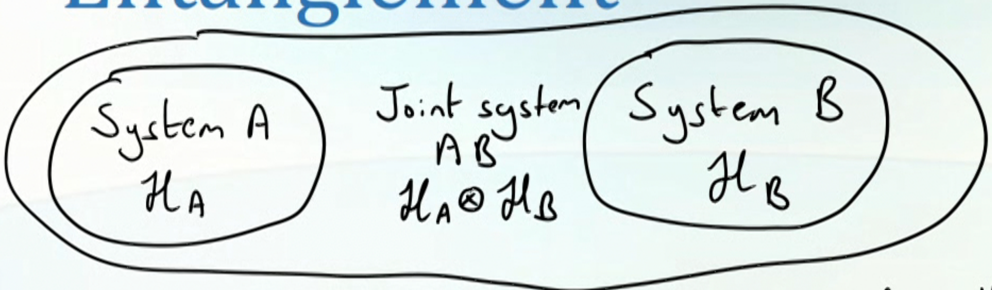
  $|0\rangle$ with probability $|\alpha|^2$

  or $|1\rangle$ with probability $|\beta|^2$.
- So this is a flat out contradiction. The orthodox interpretation is straightforwardly wrong.

3.3) The Einstein-Podolsky-Rosen (EPR) Argument

- In 1935, Einstein, Podolsky and Rosen pointed out a conflict between orthodox quantum mechanics and locality. – A. Einstein, B. Podolsky, N. Rosen, "Can Quantum-Mechanical Description of Physical Reality be Considered Complete?," *Phys. Rev.*, vol. 47 pp. 777–780 (1935).
- "When two systems, of which we know the states by their respective representatives, enter into temporary physical interaction due to known forces between them, and when after a time of mutual influence the systems separate again, then they can no longer be described in the same way as before, viz. by endowing each of them with a representative of its own. I would not call that *one* but rather *the* characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought. By the interaction the two representatives [the quantum states] have become entangled." – E. Schrödinger "Discussion of Probability Relations Between Separated Systems," *Proc. Cambridge Phil. Soc.*, 31, pp. 555–563 (1935).

Entanglement



- The Hilbert space $\mathcal{H}_{AB} = \mathcal{H}_A \otimes \mathcal{H}_B$ consists of all states of the form

$$|\psi\rangle_{AB} = \sum_{jk} \alpha_{jk} |j\rangle_A \otimes |k\rangle_B$$

- A state $|\psi\rangle_{AB}$ is a product state if it can be written as

$$|\psi\rangle_{AB} = |\phi\rangle_A \otimes |\chi\rangle_B \quad \text{for some } |\phi\rangle_A \in \mathcal{H}_A, \ |\chi\rangle_B \in \mathcal{H}_B$$

- Otherwise it is an entangled state
- According to the orthodox interpretation A and B have no individual propoties when AB is entangled.

Entanglement

- For 2-qubits it is straightforward to prove that

$$|\psi\rangle_{AB} = \alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle$$

is a product state iff

$$\alpha_{00}\alpha_{11} = \alpha_{01}\alpha_{10}$$

- So, in particular,

$$|\Phi^+\rangle_{AB} = \frac{1}{\sqrt{2}}\left(|00\rangle_{AB} + |11\rangle_{AB}\right)$$

is an entangled state.

- Note: If $\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ and $\vec{n}.\vec{\sigma} = n_1\sigma_1 + n_3\sigma_3$ with $\vec{n} = \begin{pmatrix} n_1 \\ n_3 \end{pmatrix}$ a unit vector in the x-z plane then

$$|\Phi^+\rangle_{AB} = \frac{1}{\sqrt{2}}\left(|\vec{n}_+\rangle_A|\vec{n}_+\rangle_B + |\vec{n}_-\rangle_A|\vec{n}_-\rangle_B\right)$$

with $\vec{n}.\vec{\sigma}\,|\vec{n}_\pm\rangle = \pm|\vec{n}_\pm\rangle$

Partial Measurement

- If we measure one of the subsystems of a joint system in a complete orthonormal basis, then after the measurement the state gets updated to a product state.

⊙ Joint system starts in state

$$|\psi\rangle_{AB} = \sum_{jk} \alpha_{jk} |j\rangle_A \otimes |k\rangle_B$$

⊙ A is measured in basis $\{|\phi_m\rangle\}$, outcome $|\phi_n\rangle$ is obtained.

⊙ B gets updated to

$$\frac{{}_A\langle\phi_n|\psi\rangle_{AB}}{\| {}_A\langle\phi_n|\psi\rangle_{AB}\|^2} = \frac{\sum_{jk} \alpha_{jk} \langle\phi_n|j\rangle |k\rangle_B}{\sum_{k} |\alpha_{jk}\langle\phi_n|j\rangle|^2}$$

Partial Measurement

- In particular, if $|\Phi^+\rangle_{AB}$ is measured in the basis $\{|0\rangle_A, |1\rangle_A\}$ then system B ends up in the state

  $|0\rangle_B$ if $|0\rangle_A$ is found

  or $|1\rangle_B$ if $|1\rangle_A$ is found

- More generally, if $|\Phi^+\rangle_{AB}$ is measured in the basis $\{|\vec{n}+\rangle_A, |\vec{n}-\rangle_A\}$ then B ends up in

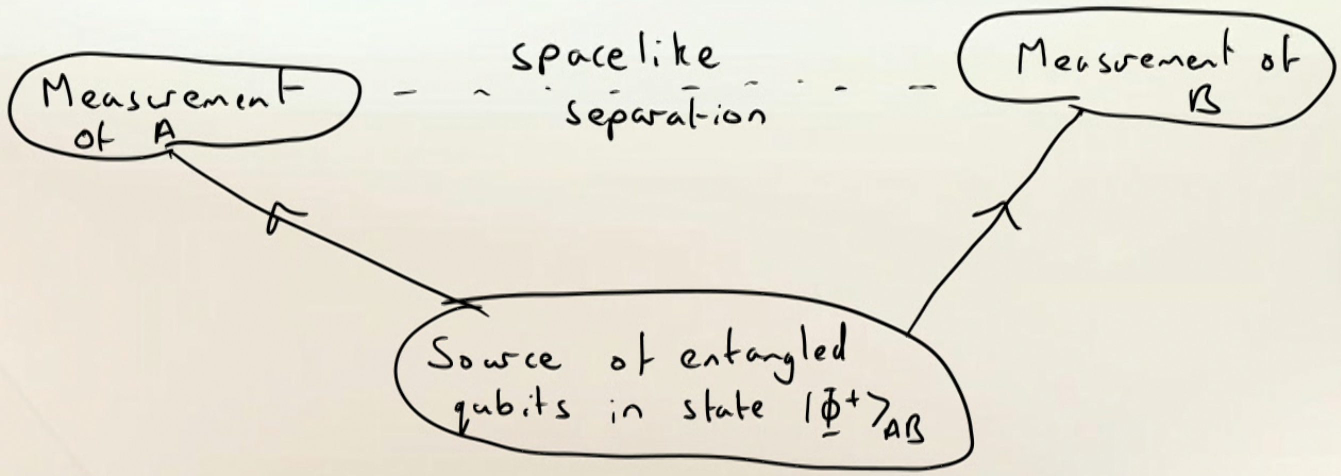
  $|\vec{n}+\rangle_B$ if $|\vec{n}+\rangle_A$ is found

  $|\vec{n}-\rangle_B$ if $|\vec{n}-\rangle_A$ is found

- We will be able to predict the result of a $\vec{n}\cdot\vec{\sigma}$ measurement on system B with certainty.

The EPR Criterion of Reality

- "If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of reality corresponding to that quantity." – A. Einstein, B. Podolsky, N. Rosen, "Can Quantum-Mechanical Description of Physical Reality be Considered Complete?," *Phys. Rev.*, vol. 47 pp. 777–780 (1935).
- We can ensure that a measurement of A "does not disturb" B by locality.



The EPR Argument

- By the EPR criterion and locality, system B must have an element of reality that determines the outcome of a $\{|0\rangle_B, |1\rangle_B\}$ measurement before A is measured.
- The orthodox interpretation is nonlocal, because this "pops into existence" from nothing when A is measured.
- But note: Any interpretation in which measurement of $\{|0\rangle_B, |1\rangle_B\}$ is undetermined before A is measured would also be nonlocal by the EPR criterion.
- Note that, because of the perfect correlations in all $\{|\vec{n}+\rangle, |\vec{n}-\rangle\}$ measurements, the same is true for all possible measurement directions. Having all of these elements of reality would violate the uncertainty principle for B.
  - This is irrelevant to the main argument, which holds for just one measurement.
  - However, one can use this to show that a local theory must also be ψ-epistemic – N. Harrigan, R. Spekkens, Found. Phys. 40, 125 (2010).
- Bell's Theorem will show that no completion of quantum theory can restore locality.