Title: PSI 2018/2019 - Foundations of Quantum Mechanics - Lecture 1

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Abstract:

Quantum Foundations
Lecture 1

PSI Review Class: 7th January 2019

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Emlyn Hughes on Quantum Theory

- In 2013, Prof. Emlyn Hughes at Columbia University introduced his GenEd class on quantum theory in the following way.

- Warning: This video contains offensive imagery

  https://vimeo.com/59932634

- How does this video make you feel about quantum mechanics?

Emlyn Hughes on Quantum Theory

"In order to learn quantum mechanics, you have to strip to your raw, erase all the garbage from your brain, and start over again. Nothing you have learned in your life up till now is in any way helpful to prepare you for this, because everything you do in your everyday life is totally opposite to what you are going to learn in quantum mechanics. And so, I've been tasked with the impossible challenge of having to teach you quantum mechanics in one hour. What, basically the most brilliant minds, Einstein and so on, couldn't figure out working on it their whole life."

The not-so subtle subtext

"You are going to be very confused by quantum mechanics. As much as if your physics professor did a weird performance art piece for no apparent reason. The smartest people in the world do not understand it. Therefore, I, an extremely smart professor, cannot possibly be expected to teach you, with your small undergraduate brains, this subject in a way that you can understand it. Nevertheless, suck it up because you have to pass an exam on it at the end"

My interpretation of what Emlyn Hughes meant.

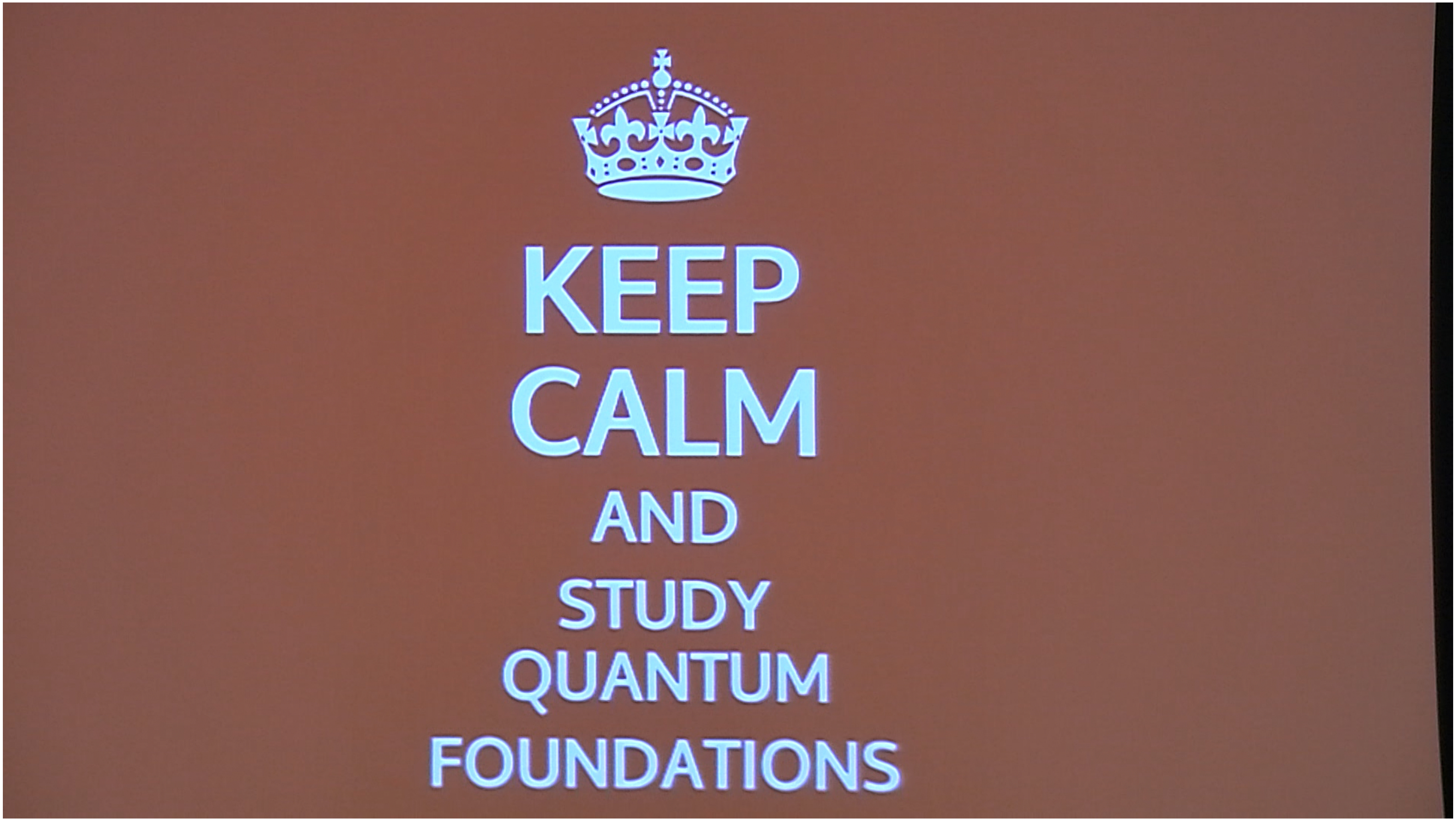
An Obligatory Feynman Quote

"I think I can safely say that nobody understands quantum mechanics" – "The Character of Physical Law", chapter 6, p. 129

- This quote appears in almost every popular science book about quantum theory, and many textbooks too.
- I think it is an excuse for teaching quantum theory badly, i.e. "I am confused about quantum theory. The smartest physicist was confused too. Therefore, you will be confused and it is not my fault."

A lesser known Feynman quote

"We always have had … a great deal of difficulty in understanding the world view that quantum mechanics represents. At least I do, because I'm an old enough man that I haven't got to the point that this stuff is obvious to me. Okay, I still get nervous with it. And therefore, some of the younger students … you know how it always is, every new idea, it takes a generation or two until it becomes obvious that there's no real problem. It has not yet become obvious to me that there's no real problem. **I cannot define the real problem, therefore I suspect there's no real problem, but I'm not sure there's no real problem."** – "Simulating Physics with Computers", *International Journal of Theoretical Physics*, volume 21, 1982, p. 467-488


KEEP
CALM
AND
STUDY
QUANTUM
FOUNDATIONS

A More Rational Approach

- Our task is to approach this as a scientific question. Specifically:
  - Define the theory in as clear and general a way as possible.
  - Define the real problem.
  - Show how we can use math, physics, philosophy and experiment to address the problem.
  - Explore the proposed solutions.

Some Wrong Answers

- Quantum theory describes a world far from our everyday experience, so there is no reason it should be comprehensible. We have to get used to abstraction.
    - The same is true of relativity.
- Interpretation of quantum theory is irrelevant for practical applications, let's leave it to philosophers.
    - This is a selection effect.
    - Modern applications like quantum information/computation show that thinking about foundations is useful.
    - It leads to novel experiments, e.g. Bell's theorem
    - It may suggest how to adapt the theory beyond its current scope, e.g. in quantum gravity.

Course Outline

- Week 1:
  - 1) A (Biased) History of Quantum Theory
  - 2) Postulates of Quantum Theory
  - 3) Quantum Phenomenology
  - 4) The Generalized Formalism
- Week 2:
  - 5) Realism vs. Antirealism
  - 6) Epistricted Classical Theories
  - 7) Ontological Models
  - 8) No-Go Theorems
- Week 3:
  - 9) The Classical Limit of Quantum Theory
  - 10) Interpretations of Quantum Theory

1) A (biased) History of Quantum Theory

Old Quantum theory

$E = hf$

$p = h/\lambda$

etc.

Quantum Mechanics

Heisenberg matrix mechanics (1925)

Schrödinger wave mechanics (1926)

Hilbert space formalism (1930-32)

Taking Quantum Theory Seriously As A Fundamental Theory

Bohm (1952), Everett (1957), Bell (1964-)

Wavefunction of the universe

Quantum Computers

1900 1910 1920 1930 1940 1950 1960 1970 1980 1990 2000

First Quantum Revolution

Second Quantum Revolution

Third Quantum Revolution

First Quantum Revolution

- Old quantum theory was not a full physical theory. Just a series of ad hoc rules that contradicted existing physics.
- It was necessary to judiciously choose which part of the system to apply quantum rules to, leaving the rest classical.
- This survived into Copenhagen-style quantum mechanics.
  - Physicists were not particularly bothered. They were used to doing it.
  - Copenhagen made this into a virtue rather than a vice.

Second Quantum Revolution

- Heisenberg's matrix mechanics was originally based on the idea that systems were always in (what we now call) stationary states, and from time to time would jump between them indeterministically.
  - This was inspired by the Bohr atom.
- Heisenberg found that he needed physical quantities (observables) to be matrices to get this to work.
- Non-stationary states were a later addition (with Born and Jordan), and quite alien to Heisenberg's initial thinking.

Second Quantum Revolution

- Schrödinger initially thought of his wavefunction as a physical field. There was no probability rule. Particles were supposed to emerge somehow from the dynamics.
- This was given up when it was found that, with realistic Hamiltonians, wavefunctions always spread in time.
- Entanglement also makes the physical field interpretation difficult.
- Max Born introduced the probability wave interpretation in 1926. Schrödinger was later forced to accept it.

Second Quantum Revolution

- The Heisenberg and Schrödinger theories were unified (by Schrödinger, Dirac, and von Neumann) resulting in the modern Hilbert space formalism.
  - Note that there were initially two perfectly coherent ideas of what quantum theory is about. The unification is true to neither of them.
  - Heisenberg had to “borrow” non-stationary states. Schrödinger had to “borrow” probabilities.
  - As a result, it became completely unclear what the theory was fundamentally about.

The Two Churches of Quantum Theory

- Schrödinger and Heisenberg are *mathematically* equivalent, but a conceptual divide still exists.
- The Church of the Larger Hilbert Space:
  - Quantum theory is a dynamical theory, much like a classical field theory, but with a weirder object called a wavefunction replacing the classical field. All is to be derived from a wavefunction evolving unitarily in time.
- The Church of the Smaller Hilbert Space
  - Something weird happened to the algebra of observables, they became non-commutative. Quantum theory is the only consistent probability theory for such observables.

Third Quantum Revolution

- Starting in the 1950's, People like Bohm and Everett were dissatisfied with the Copenhagen idea that there was a necessary split between the classical and quantum worlds.
  - If quantum theory is fundamental, we should be able to describe the whole universe as a quantum system, with no external classical world.
- This led to a reanalysis of foundations, leading to things like Bell's theorem and quantum information.
- This has been a very slow-burn revolution.

2) Postulates of Quantum Theory

1) Overview
2) State Space
3) Observables
4) Dynamics
5) Composite Systems

2.1) Overview

1. **State Space**: A physical system A is associated with a (complex) Hilbert space $\mathcal{H}_A$.
2. **States**: A (pure) state of a physical system is a unit vector

$$|\psi\rangle_A \in \mathcal{H}_A \text{ (up to a global phase).}$$

3. **Observables**: Measurable physical quantities correspond to self-adjoint operators $M^\dagger = M$, which can be written in spectral form

$$M = \sum_j \lambda_j P_j.$$

The possible outcomes of a measurement of M are the eigenvalues λ_j.

4. **The Born rule**: When M is measured on a system assigned the state $|\psi\rangle$, the outcome λ_j occurs with probability $\mathrm{Prob}(\lambda_j|\psi) = \langle\psi|P_j|\psi\rangle$.

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2.1) Overview

5. **Dynamics**: An isolated (not interacting with the environment or being measured) system evolves according to the Schrödinger equation

$$i\frac{\partial|\psi\rangle}{\partial t} = H|\psi\rangle.$$

Equivalently, dynamics is unitary $|\psi(t)\rangle = U(t)|\psi(0)\rangle$,

$$U^\dagger(t)U(t) = I$$

6. **Composite systems**: A system AB composed of two subsystems, A with Hilbert space $\mathcal{H}_A$ and B with Hilbert space $\mathcal{H}_B$ has a Hilbert space

$$\mathcal{H}_{AB} = \mathcal{H}_A \otimes \mathcal{H}_B$$

where $\otimes$ denotes the tensor product.

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2.2) State Space

1. **State Space**: A physical system A is associated with a (complex) Hilbert space $\mathcal{H}_A$.
2. **States**: A (pure) state of a physical system is a unit vector

$$|\psi\rangle_A \in \mathcal{H}_A \text{ (up to a global phase).}$$

- The fact that the state space is a *linear (vector)* space is usually attributed to the *superposition principle*:
  - If $|\psi\rangle_A$ and $|\phi\rangle_A$ are physical states then $\alpha|\psi\rangle_A + \beta|\phi\rangle_A$ is also a physical state for any $\alpha, \beta \in \mathbb{C}$.
- This does not explain why the field should be $\mathbb{C}$ (as opposed to $\mathbb{R}$ or something else) or why it needs to be an inner product space.

The Simplest State Space: A Qubit

- A system with a 2d Hilbert space is called a qubit. The basis vectors are labelled

$$|0\rangle = |z+\rangle = \begin{pmatrix}1\\0\end{pmatrix}$$

$$|1\rangle = |z-\rangle = \begin{pmatrix}0\\1\end{pmatrix}$$

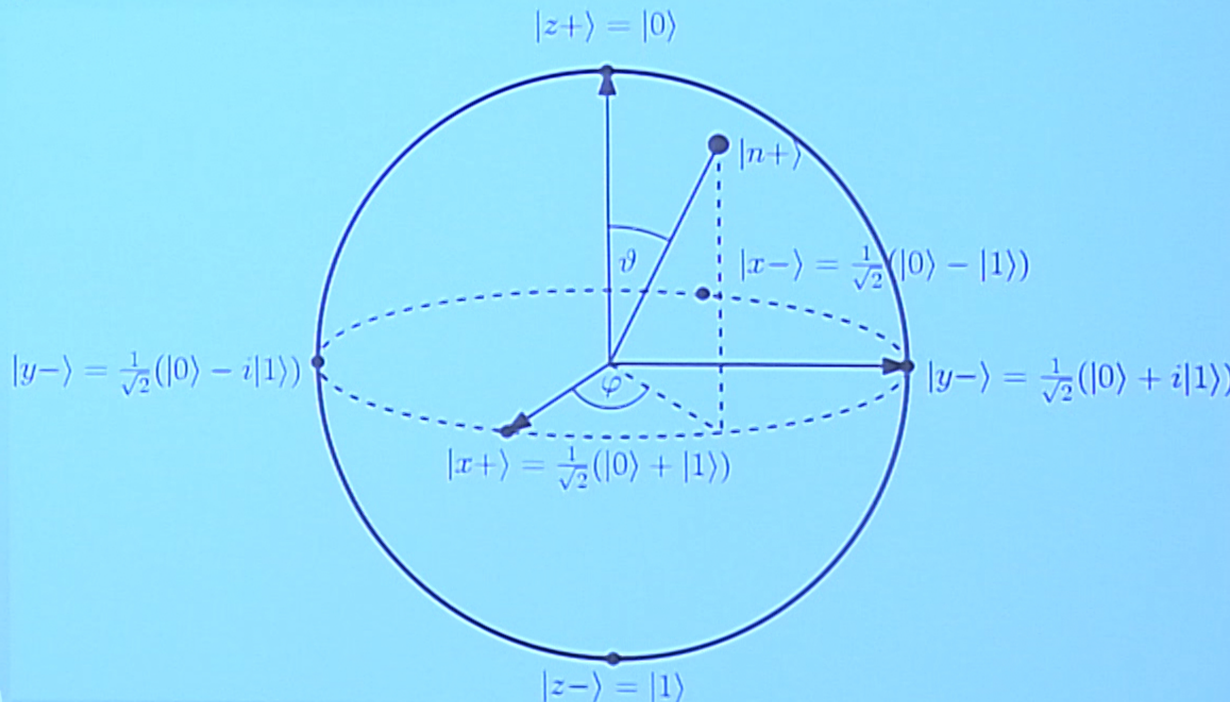
- A general state can be written as

$$|n+\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$$

corresponding to a unit vector

$\vec{n} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$

in the *Bloch Sphere.*



2.3) Observables

3. **Observables**: Measurable physical quantities correspond to self-adjoint operators $M^\dagger = M$, which can be written in spectral form

$$M = \sum_j \lambda_j P_j.$$

The possible outcomes of a measurement of M are the eigenvalues λ_j.

4. **The Born rule**: When M is measured on a system assigned the state $|\psi\rangle$, the outcome λ_j occurs with probability $\text{Prob}(\lambda_j|\psi) = \langle\psi|P_j|\psi\rangle$.

- As far as the probabilities are concerned, the eigenvalues λ_j are just labels for the outcomes of the measurements. All that matters is the set of projectors $\{P_j\}$, so we will often work with them directly.
- A set $\{P_j\}$ of orthogonal $P_j P_k = \delta_{jk}$ projectors that satisfy $\sum_j P_j = I$ is called a *Projector Valued Measure (PVM).*

2.3) Observables

- If all the projectors in a PVM are 1-dimensional $P_j = |\phi_j\rangle\langle\phi_j|$ then $\{|\phi_j\rangle\}$ is an orthonormal basis and we call the PVM a *measurement in the basis* $\{|\phi_j\rangle\}$. In this case, the Born rule is:

$$\mathrm{Prob}(j|\psi) = \langle\psi|P_j|\psi\rangle = \langle\psi|\phi_j\rangle\langle\phi_j|\psi\rangle = |\langle\phi_j|\psi\rangle|^2$$

Observable	$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$
Eigenspaces	$\lambda = +1{:}\ \|x+\rangle$ $\lambda = -1{:}\ \|x-\rangle$	$\lambda = +1{:}\ \mathrm{span}\{\|0\rangle, \|1\rangle\}$ $\lambda = -1{:}\ \|2\rangle$
PVM	$\{\|x+\rangle\langle x+\|, \|x-\rangle\langle x-\|\}$	$\{\|0\rangle\langle 0\| + \|1\rangle\langle 1\|, \|2\rangle\langle 2\|\}$
Basis	$\{\|x+\rangle, \|x-\rangle\}$	Not a basis measurement

$$|x\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$$

2.3) Observables

- After a measurement, the state of the system changes. The rule

$$|\psi\rangle \rightarrow \frac{P_j|\psi\rangle}{\sqrt{\langle\psi|P_j|\psi\rangle}}$$

is (correctly) called the *Lüders' rule* and is often stated as one of the postulates of quantum theory.

- I do not consider this a postulate because it does not always hold. A measurement that obeys the Lüders' rule is often called an *ideal* or measurement.

2.4) Dynamics

5. **Dynamics**: An isolated (not interacting with the environment or being measured) system evolves according to the Schrödinger equation

$$i\frac{\partial|\psi\rangle}{\partial t} = H|\psi\rangle.$$

Equivalently, dynamics is unitary $|\psi(t)\rangle = U(t)|\psi(0)\rangle$,

$$U^{\dagger}(t)U(t) = I$$

- This is not really an additional postulate. We can derive it from what we have so far.

2.4) Dynamics

- Quantum systems obey the *superposition principle*: If $|\psi_1(t)\rangle$ and $|\psi_2(t)\rangle$ are solutions to the equation of motion then so is
$$\alpha|\psi_1(t)\rangle + \beta|\psi_2(t)\rangle$$
- This implies that time evolution can be described by a linear operator $U(t)$, such that
$$|\psi(t)\rangle = U(t)|\psi(0)\rangle$$
- In order to conserve probabilities, dynamics should preserve normalization. To see this, consider an orthonormal basis $\{|j\rangle\}$. The Born rule probability of getting outcome $|j\rangle$ at time $t = 0$ is
$$\mathrm{Prob}(j,0) = |\langle j|\psi\rangle|^2 = \langle\psi|j\rangle\langle j|\psi\rangle$$
- so the probability of getting any outcome is
$$\sum_j \mathrm{Prob}(j,0) = \sum_j \langle\psi|j\rangle\langle j|\psi\rangle = \langle\psi|\psi\rangle = 1$$

2.4) Dynamics

- If we want this to also be true at time t then
$$\mathrm{Prob}(j,t) = |\langle j|\psi(t)\rangle|^2 = |\langle j|U(t)|\psi\rangle|^2 = \langle\psi|U^\dagger(t)|j\rangle\langle j|U(t)|\psi\rangle$$
and $\sum_j \mathrm{Prob}(j,t) = \langle\psi|U^\dagger(t)U(t)|\psi\rangle$.
- If we want this to equal 1 then we must have
$$\langle\psi|U^\dagger(t)U(t)|\psi\rangle = \langle\psi|\psi\rangle = \langle\psi|I|\psi\rangle$$
- This implies that $U^\dagger(t)U(t) = I$, which is the definition of a *unitary* matrix.
- We also want $U(0) = I$, and we want evolution to be continuous, so suppose, for small Δt,
$$U(t) = I + \Delta t A$$
to first order for some matrix A.

2.4) Dynamics

- Now, if $U(t)$ is unitary, then A must be *anti-Hermitian*, i.e. $A^\dagger = -A$.
- Proof: $U^\dagger(\Delta t)U(\Delta t) = I \quad \Rightarrow \quad (I + \Delta t A^\dagger)(I + \Delta t A) = I$

$$I + \Delta t\left(A^\dagger + A\right) + O\left(\Delta t^2\right) = I$$

Therefore, all the terms of order Δt and higher must be zero, so

$$A^\dagger + A = 0 \quad \text{or} \quad A^\dagger = -A.$$

- Therefore, we have

$$\frac{\partial|\psi\rangle}{\partial t} = \lim_{\Delta t \to 0}\left(\frac{|\psi(\Delta t)\rangle - |\psi(0)\rangle}{\Delta t}\right) = A|\psi(0)\rangle$$

- If we now define $H = iA$ then H will be Hermitian $H^\dagger = H$ and

$$\boxed{i\frac{\partial|\psi\rangle}{\partial t} = H|\psi\rangle}$$

which is the Schrödinger equation.

2.5) Composite Systems

6. **Composite systems**: A system AB composed of two subsystems, A with Hilbert space $\mathcal{H}_A$ and B with Hilbert space $\mathcal{H}_B$ has a Hilbert space
$$\mathcal{H}_{AB} = \mathcal{H}_A \otimes \mathcal{H}_B$$
where $\otimes$ denotes the tensor product.

- $\mathcal{H}_A \otimes \mathcal{H}_B$ is defined as the Hilbert space spanned by vectors of the form $|\psi\rangle_A \otimes |\phi\rangle_B$, where $|\psi\rangle_A \in \mathcal{H}_A$ and $|\phi\rangle_B \in \mathcal{H}_B$.
- If $|0\rangle_A, |1\rangle_A, \cdots |d_A\rangle_A$ is a basis for $\mathcal{H}_A$ and $|0\rangle_B, |1\rangle_B, \cdots |d_B\rangle_B$ is a basis for $\mathcal{H}_B$ then
$$\begin{gathered} |0\rangle_A \otimes |0\rangle_B, |0\rangle_A \otimes |1\rangle_B, \cdots |0\rangle_A \otimes |d_B\rangle_B, \\ |1\rangle_A \otimes |0\rangle_B, |1\rangle_A \otimes |1\rangle_B, \cdots |1\rangle_A \otimes |d_B\rangle_B, \\ \cdots \\ |d_A\rangle_A \otimes |0\rangle_B, |d_A\rangle_A \otimes |1\rangle_B, \cdots |d_A\rangle_A \otimes |d_B\rangle_B \end{gathered}$$
is a basis for $\mathcal{H}_A \otimes \mathcal{H}_B$.

2.5) Composite Systems

- In other words, $\mathcal{H}_A \otimes \mathcal{H}_B$ consists of all vectors of the form

$$|\psi\rangle_{AB} = \sum_{j=0}^{d_A} \sum_{k=0}^{d_B} \alpha_{jk} |j\rangle_A \otimes |k\rangle_B$$

  where $\alpha_{jk} \in \mathbb{C}$.

- Note: the dimension of $\mathcal{H}_A \otimes \mathcal{H}_B$ is $d_A \times d_B$.
- Physicists like to get sloppy with notation, by writing

$$|j\rangle_A \otimes |k\rangle_B = |j\rangle_A |k\rangle_B = |jk\rangle_{AB}$$

  or even

$$|j\rangle \otimes |k\rangle = |j\rangle |k\rangle = |jk\rangle$$

  if the system label is obvious from the ordering.

2.5) Composite Systems

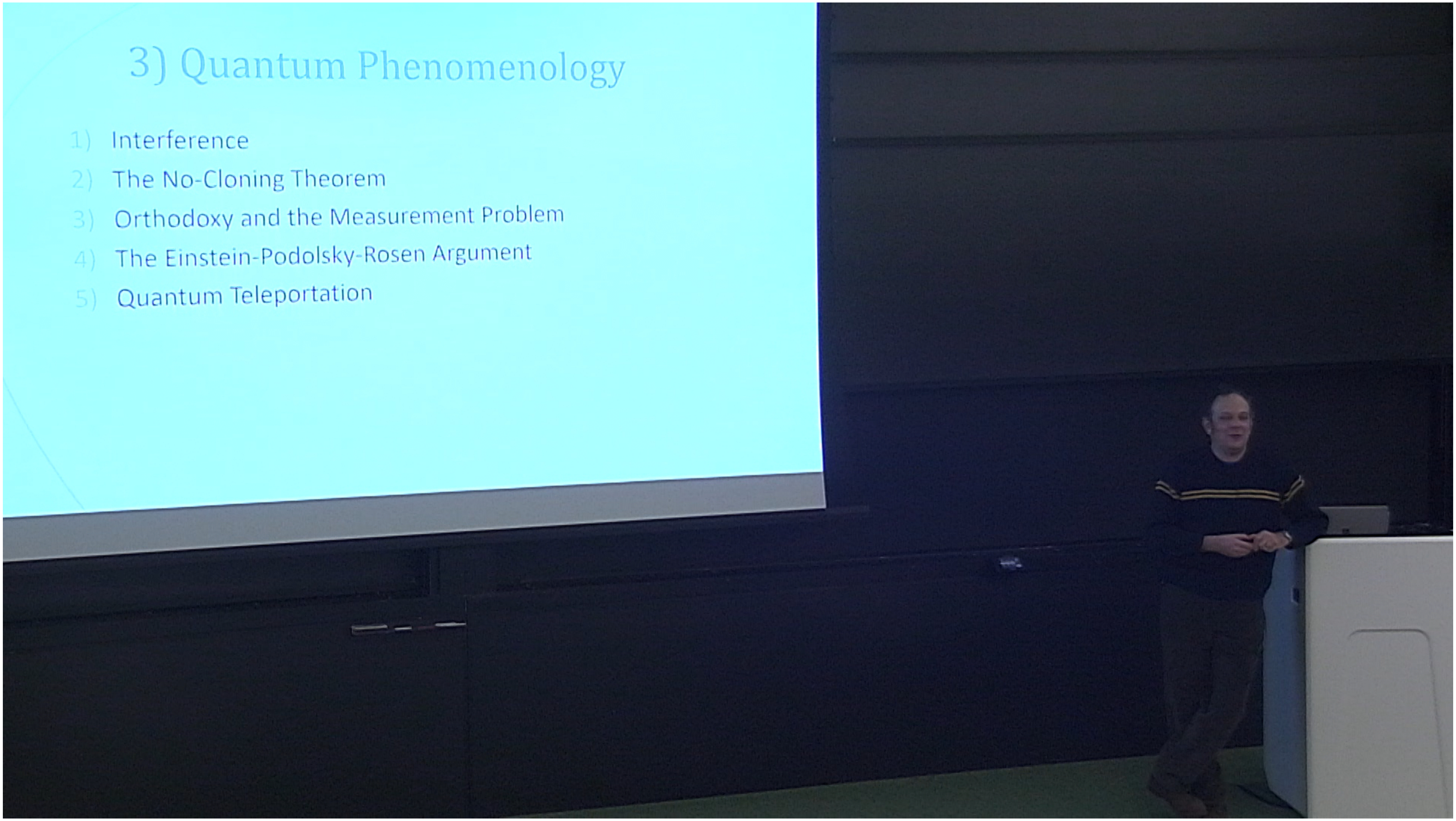
- This can obviously be generalized to more systems

$$|\psi\rangle_{ABC\cdots Z} = \sum_{j_A=0}^{d_A} \sum_{j_B=0}^{d_B} \sum_{j_C=0}^{d_C} \cdots \sum_{j_Z=0}^{d_Z} \alpha_{j_A j_B j_C \cdots j_Z} |j_A j_B j_C \cdots j_Z\rangle$$

- Note that the dimension increases exponentially with the number of systems. If a system has dimension d then the tensor product of n such systems has dimension d^n.

3) Quantum Phenomenology

1) Interference
2) The No-Cloning Theorem
3) Orthodoxy and the Measurement Problem
4) The Einstein-Podolsky-Rosen Argument
5) Quantum Teleportation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