Title: PSI 2018/2019 - Cosmology Review - Lecture 13

Date: Jan 23, 2019 09:00 AM

URL: http://pirsa.org/19010014

Abstract:

$$v = \int a u_k + a^{\dagger} u_k^*$$

$$v'' - \left(\nabla^2 + \frac{a''}{a}\right) v = 0$$

$$u_k(x,t) = \frac{1}{(2\pi)^{3/2}} f_k(\tau) e^{i\vec{k}\cdot\vec{x}}$$

$$f_k'' + \left(k^2 - \frac{a''}{a}\right) f_k = 0$$

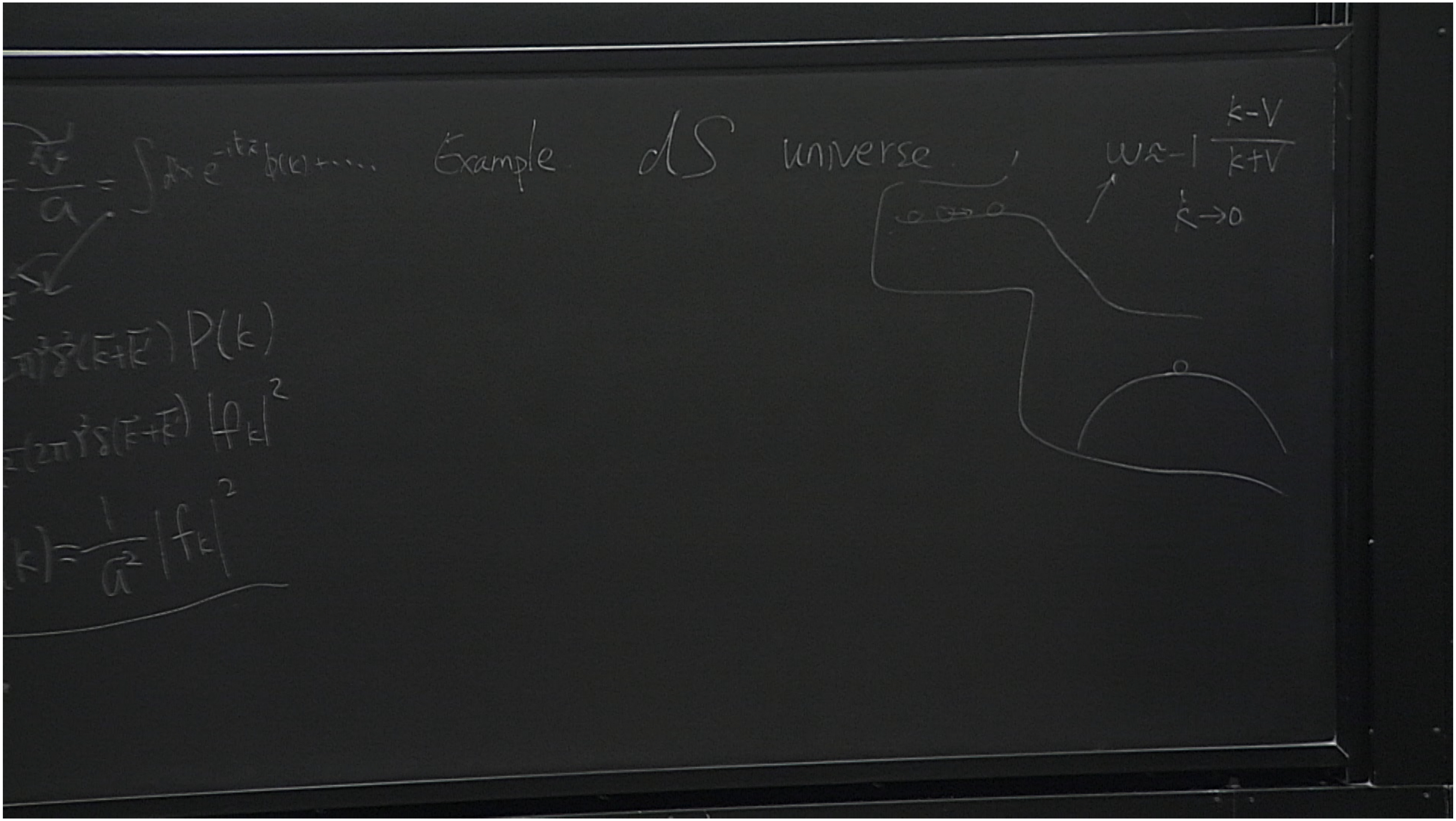
w_- w_+

$$\phi = \frac{v}{a} = \int d^3x \, e^{-i\vec{k}\cdot\vec{x}} \phi(k) + \ldots$$

$$\langle \phi_{\vec{k}} \phi_{\vec{k}'} \rangle \equiv (2\pi)^3 \delta^3(\vec{k}+\vec{k}') P(k)$$

$$= \frac{1}{a^2} (2\pi)^3 \delta(\vec{k}+\vec{k}') |f_k|^2$$

$$P(k) = \frac{1}{a^2} |f_k|^2$$


Example dS universe
k-V
k+V
k→0

Last toy model.

$$ds^2 = a^2(\tau)(-d\tau^2 + dx^i dx_i)$$

$$= -dt^2 + a^2(t) d\vec{x}^2$$

$$S = -\frac{1}{2}\int d^4x \sqrt{-g}\,(\partial\phi)^2$$

$$= \int d\tau\, d^3x\, \frac{1}{2} a^2 \left((\phi')^2 - (\nabla\phi)^2\right)$$

$$v = a\phi$$

$$= \int d\tau\, d^3x\, \frac{1}{2}\left((v')^2 - (\nabla v)^2 + \frac{a''}{a} v^2\right)$$

$$v = \int a_k u_k + a_k^\dagger u_k^*$$

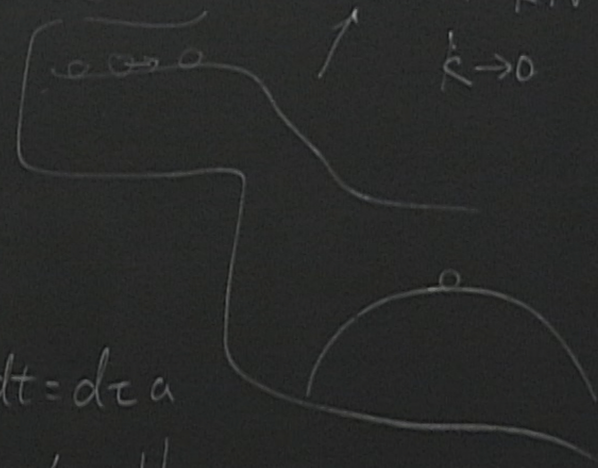
$$v'' - \left(\nabla^2 + \frac{a''}{a}\right) v = 0$$

$$u_k(x,\tau) = \frac{1}{(2\pi)^{3/2}} f_k(\tau)$$

$$f_k'' + \left(k^2 - \frac{a''}{a}\right) f_k =$$

Example dS universe , $w \approx -1 \frac{K-V}{K+V}$

$\dot{K} \to 0$



$a(t) \propto e^{Ht}$

$a(\tau)$

$$\boxed{\frac{\dot a}{a} = H} \leftarrow \text{const}$$

$dt = d\tau\, a$

$$\dot a = \frac{da}{dt} = \frac{da}{d\tau}\frac{d\tau}{dt} = \frac{1}{a} a' = aH$$

$$a' = a^2 H \qquad \frac{da}{a^2} = H d\tau \Rightarrow -\frac{1}{a} = H\tau$$

$$a = -\frac{1}{H\tau}$$

$\tau \to -\infty$

$\frac{a''}{a} \to 0$ initial condition

$f_k = \frac{1}{\sqrt{2|k|}} e^{-i|k|\tau}$

$\frac{a''}{a} = \frac{2}{\tau^2}$ $\quad \omega^2(\tau) = k^2 - \frac{2}{\tau^2}$

solve

$f_k = \frac{1}{\sqrt{2k}} e^{-ik\tau}\left(1 - \frac{i}{k\tau}\right)$

$P(k) = H^2\tau^2 \frac{1}{2k}\left(1 + \frac{1}{k^2\tau^2}\right)$

$\tau \to 0$

$P(k) = \frac{H^2}{2k^3} \propto \frac{1}{k^3}$

$\vec{x} \to \lambda \vec{x}$ $\quad \delta(\phi(x)\phi(y)) = 0$

Real thing.

$$S = M_{pl}^2 \int d^4x \sqrt{-g}\left(\frac{1}{2}R - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right)$$

$\downarrow$ action 2nd orde

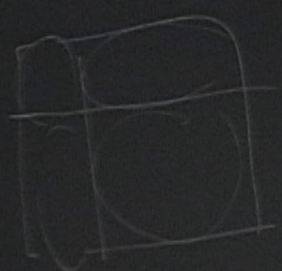
gauge invariant

$$g_{\mu\nu} \to g_{\mu\nu} + h_{\mu\nu}$$

$$\phi \to \phi + \varphi$$

$\phi(t,\vec{x}) = \phi(t)$ comoving

$\boxed{\text{Miracle Box}}$



$$S_\zeta = \int d^4x\, a^3 \epsilon \left(\dot{\zeta}^2 + \frac{1}{a^2}\zeta\nabla^2\zeta\right)$$

$$S_{\gamma_{ij}} = \frac{1}{8}\int d^4x\, a^3\left(\dot{\gamma}^2 + \frac{1}{a^2}\gamma_{ij}\nabla^2\gamma^{ij}\right)$$

Real thing.

$$S = M_{pl}^2 \int d^4x \sqrt{-g}\left(\frac{1}{2}R - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right)$$

$\downarrow$ action 2nd order

$g_{\mu\nu} \to g_{\mu\nu} + h_{\mu\nu}$

gauge invariant

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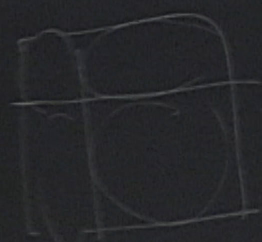
$\phi(t,\vec{x}) = \phi(t)$ comoving

$$\boxed{\text{Miracle Box}}$$



scalar

$$S_\zeta = \int d^4x\, a^3 \epsilon\left(\dot\zeta^2 + \frac{1}{a^2}\zeta\nabla^2\zeta\right)$$

$$S_{\gamma_{ij}} = \frac{1}{8}\int d^4x\, a^3\left(\dot\gamma^2 + \frac{1}{a^2}\gamma_{ij}\nabla^2\gamma_{ij}\right)$$

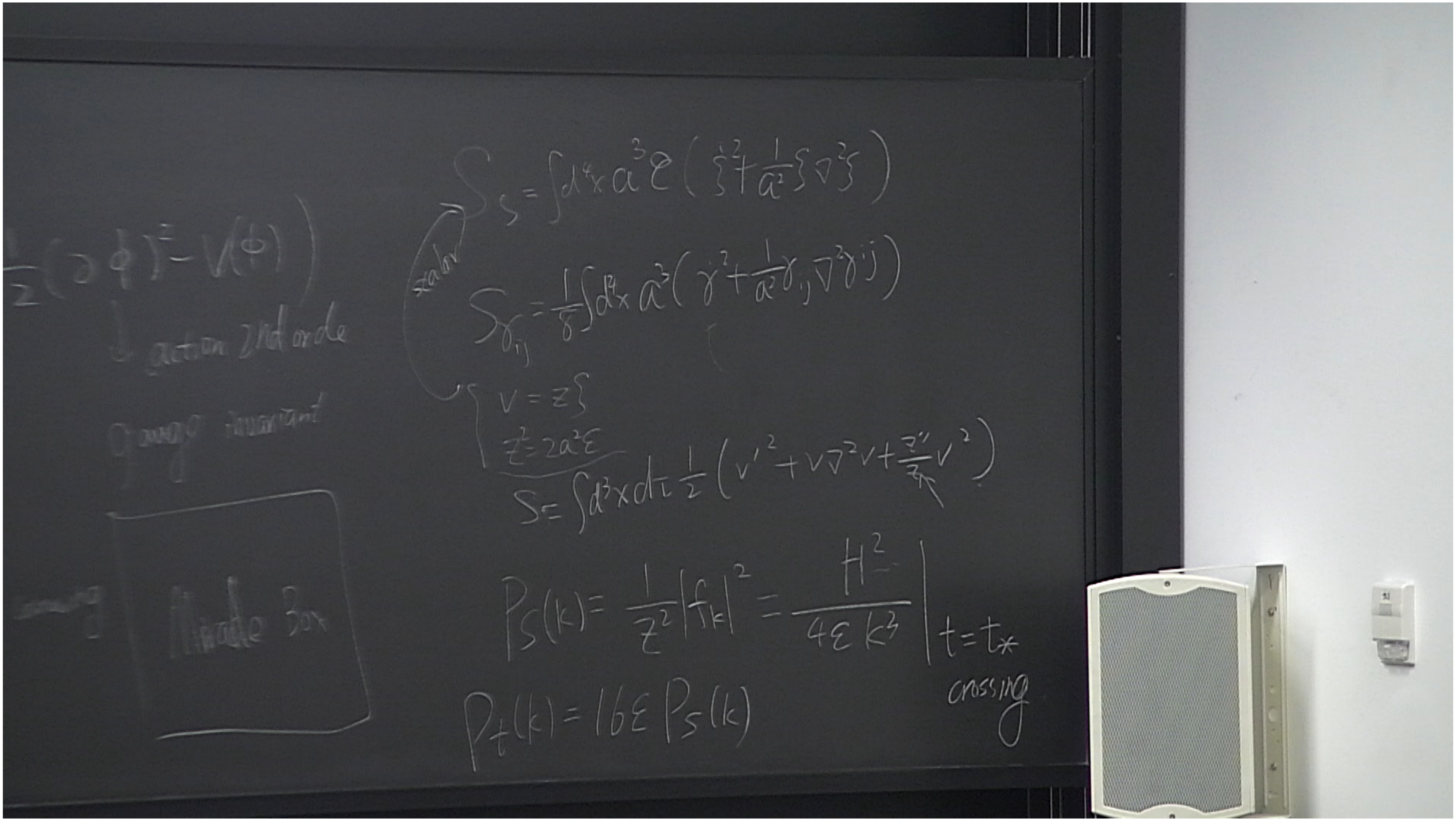
$v = z\zeta$

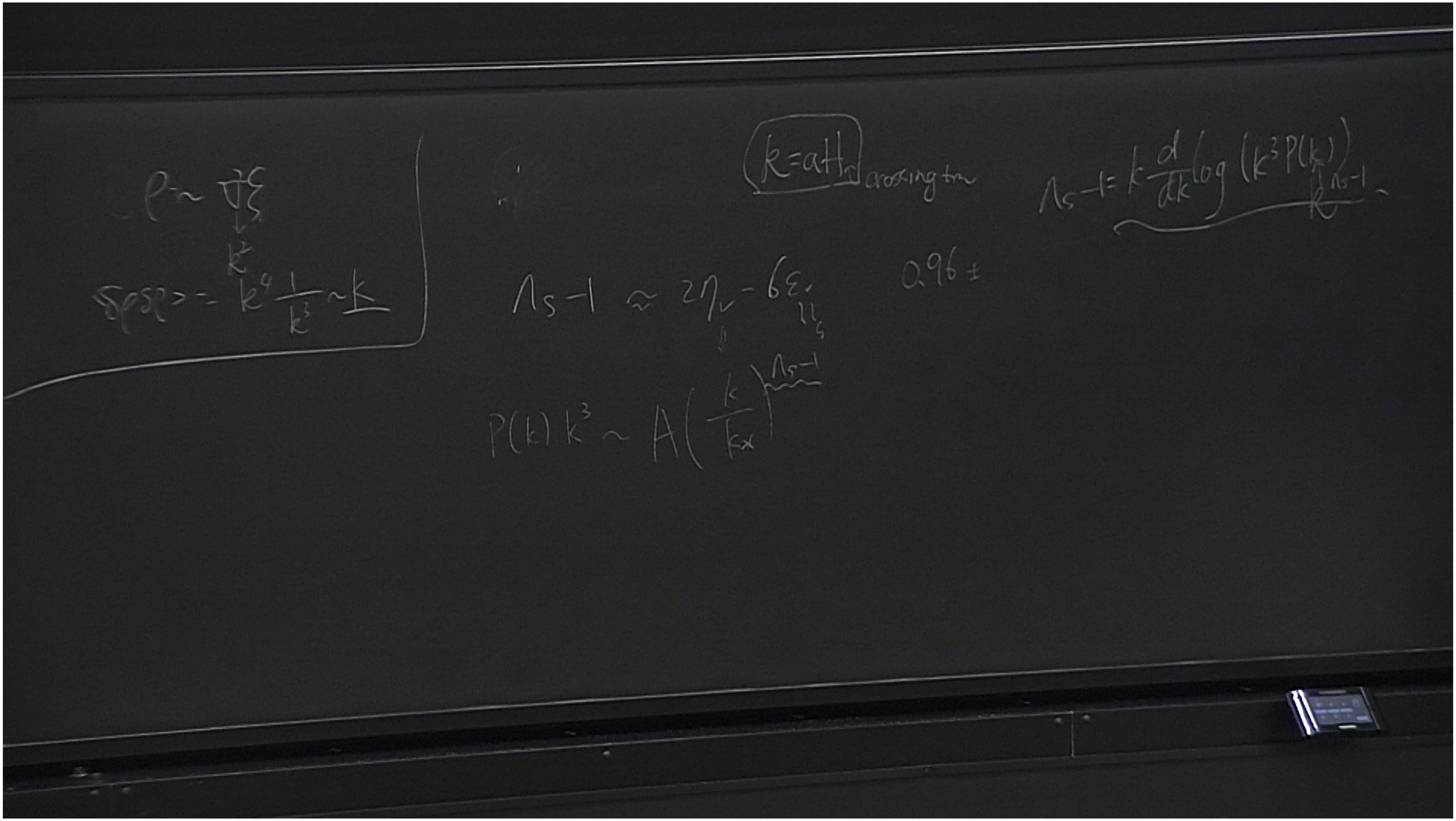
$z^2 = 2a^2\epsilon$

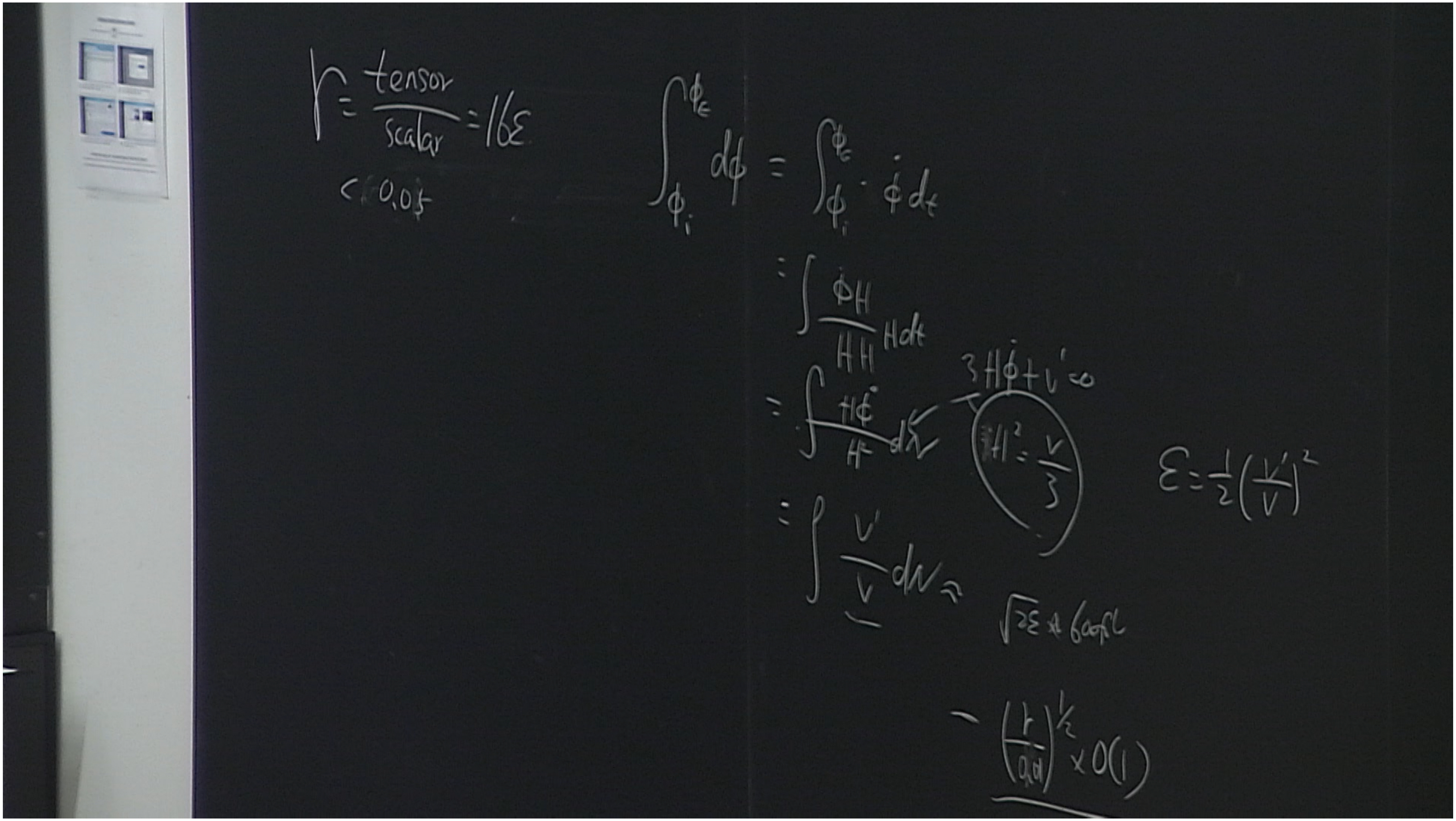
$$S = \int d^3x\, d\tau\, \frac{1}{2}\left(v'^2 + v\nabla^2 v + \frac{z''}{z}v^2\right)$$

$$P_\zeta(k) = \frac{1}{z^2}|f_k|^2 = \frac{H^2}{4\epsilon k^3}\Big|_{t=t_*}$$

crossing


$\frac{1}{2}(\partial\phi)^2 - V(\phi)$
action 2nd order
gauge invariant
Miracle Box
scalar
$S_\zeta = \int d^4x\, a^3 \epsilon \left(\dot\zeta^2 + \frac{1}{a^2}\zeta\nabla^2\zeta\right)$
$S_{\gamma_{ij}} = \frac{1}{8}\int d^4x\, a^3\left(\dot\gamma^2 + \frac{1}{a^2}\gamma_{ij}\nabla^2\gamma_{ij}\right)$
$v = z\zeta$
$z^2 = 2a^2\epsilon$
$S = \int d^3x\, d\tau\, \frac{1}{2}\left(v'^2 + v\nabla^2 v + \frac{z''}{z}v^2\right)$
$P_\zeta(k) = \frac{1}{z^2}|f_k|^2 = \frac{H^2}{4\epsilon k^3}\Big|_{t=t_*}$
crossing
$P_t(k) = 16\epsilon P_\zeta(k)$


k=aH crossing time
n_s − 1 = k d/dk log (k^3 P(k))
n_s − 1 ≈ 2η_V − 6ε_V
0.96 ±
P(k) k^3 ~ A (k/k_*)^(n_s−1)


$r = \frac{\text{tensor}}{\text{scalar}} = 16\epsilon$
< 0.05
$\int_{\phi_i}^{\phi_e} d\phi = \int_{\phi_i}^{\phi_e} \dot{\phi}\, dt$
$= \int \frac{\dot{\phi} H}{H H} H dt$
$= \int \frac{H\dot{\phi}}{H^2} dN$
$3H\dot{\phi} + V' = 0$
$H^2 = \frac{V}{3}$
$= \int \frac{V'}{V} dN \approx$
$\sqrt{2\epsilon} \times 60\,\text{efl}$
$\sim \left(\frac{r}{0.01}\right)^{1/2} \times O(1)$
$\epsilon = \frac{1}{2}\left(\frac{V'}{V}\right)^2$

$$V = \frac{1}{2}\phi^2 + \frac{1}{4}\phi^4 + \cancel{\left(\frac{\phi}{\Lambda}\right)^6} \cdots$$

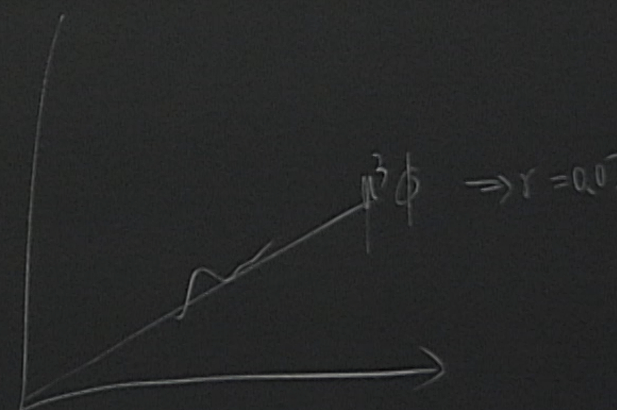
axion monodromy model:

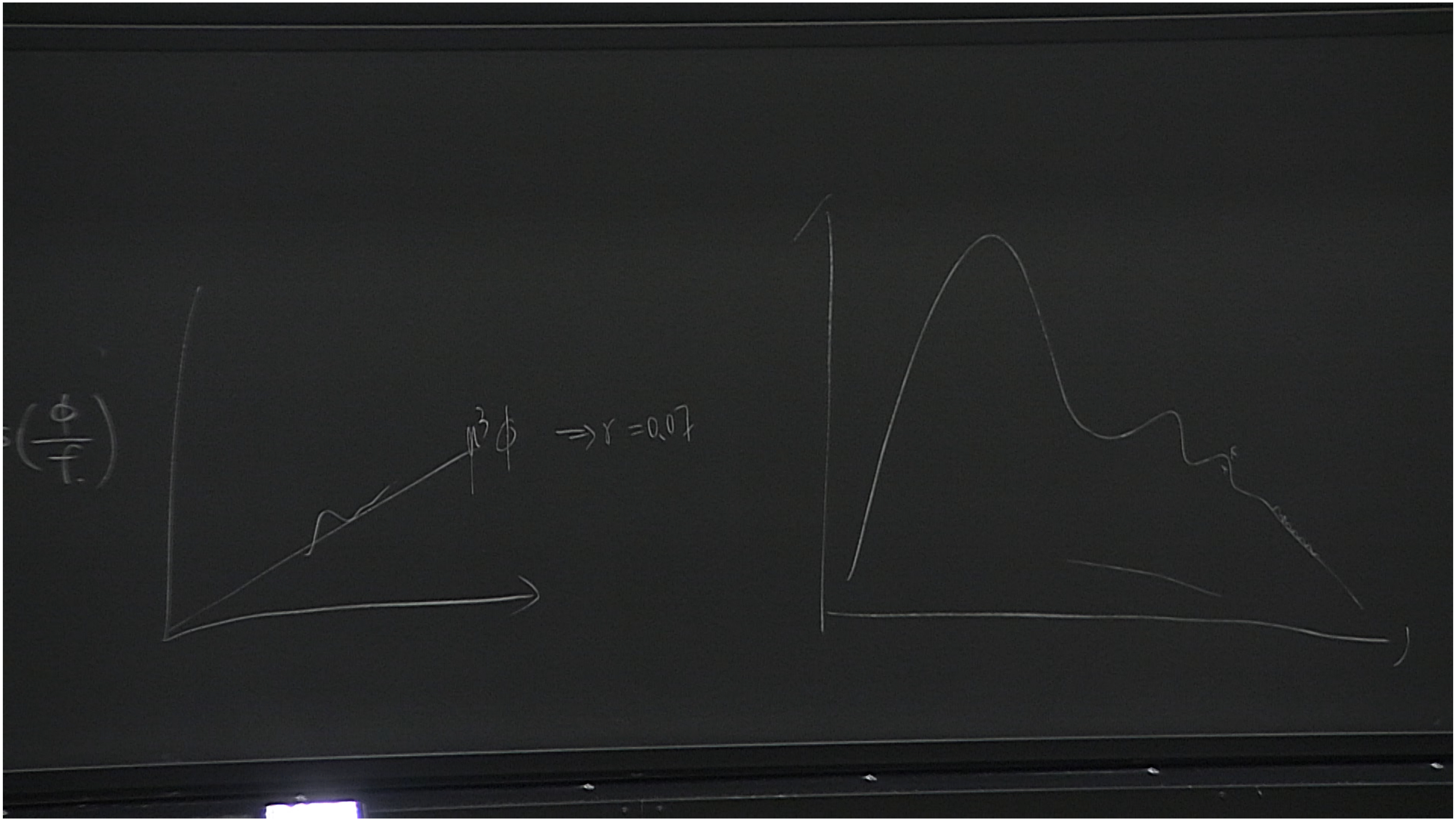
$$a \to a + 2\pi f$$

$\uparrow$ fixed number

$$\cos\left(\frac{a}{f}\right)$$

$$\phi + \cos\left(\frac{\phi}{f}\right)$$




$\mu^3\phi \Rightarrow r = 0.07$