Title: PSI 2018/2019 - Cosmology Review - Lecture 12

Date: Jan 22, 2019 09:00 AM

URL: http://pirsa.org/19010013

Abstract:

initical condition

Yesterday · puzzles

fluctuation natural

one solution : $\ddot{a} > 0$

toy model

$-1 < w < -\frac{1}{3}$

fairy tale

$N = \ln \frac{a_E}{a_I} \simeq 60$

$$ds^2 = -dt^2 + a^2 d\vec{x}^2$$

$$\mathcal{L}_\phi = \frac{1}{2}\dot{\phi}^2 - V \leftarrow \phi(t)$$

background evolution.

$$= -\frac{1}{2}(\partial\phi)^2 - V$$

$$S = \int d^4x \sqrt{-g}\left(\frac{M_{pl}^2}{2}R + \mathcal{L}_\phi\right)$$

$$T_{\mu\nu} = \partial_\mu\phi\,\partial_\nu\phi + g_{\mu\nu}\mathcal{L}_\phi$$

$$T_{00} = \dot{\phi}^2 - \left(\frac{1}{2}\dot{\phi}^2 - V\right)$$

$$= \frac{1}{2}\dot{\phi}^2 + V = \rho$$

$$T_{ii} = g_{ii}\left(\frac{1}{2}\dot{\phi}^2 - V\right)$$

$$P = \frac{1}{2}\dot{\phi}^2 - V$$

$$T_{\mu\nu} = \partial_\mu\phi\,\partial_\nu\phi + g_{\mu\nu}\mathcal{L}_\phi \qquad T^\mu{}_\nu = \begin{pmatrix} -\rho & & & \\ & p & & \\ & & p & \\ & & & p \end{pmatrix}$$

$$T_{00} = \dot\phi^2 - \left(\tfrac{1}{2}\dot\phi^2 - V\right)$$

$$= \tfrac{1}{2}\dot\phi^2 + V = \rho$$

$$w = \frac{p}{\rho} = \frac{\frac{1}{2}\dot\phi^2 - V}{\frac{1}{2}\dot\phi^2 + V}$$

$$T_{ii} = g_{ii}\left(\tfrac{1}{2}\dot\phi^2 - V\right)$$

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$$T_{00} = \dot\phi^2 - \left(\tfrac{1}{2}\dot\phi^2 - V\right)$$

$$= \tfrac{1}{2}\dot\phi^2 + V = \rho$$

$$w = \frac{p}{\rho} = \frac{\frac{1}{2}\dot\phi^2 - V}{\frac{1}{2}\dot\phi^2 + V}$$

$$\dot\phi^2 \to 0$$

$$w \to -1$$

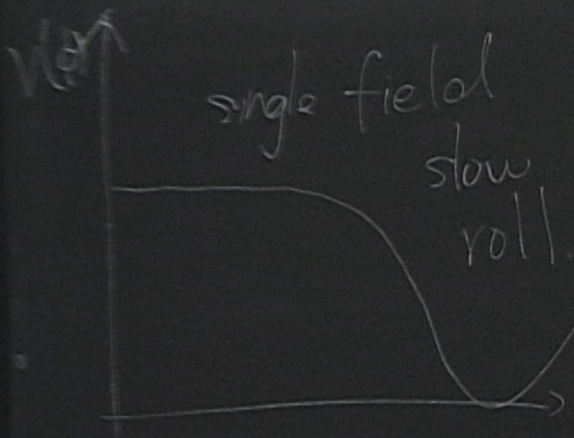
$$\dot\phi^2 = V$$

$$\frac{\frac{1}{2}-1}{\frac{1}{2}+1} = -\frac{1}{3}$$

$$T_{ii} = g_{ii}\left(\tfrac{1}{2}\dot\phi^2 - V\right)$$

$$\dot\phi^2 \ll V$$

$$p = \tfrac{1}{2}\dot\phi^2 - V$$



single field slow roll.

EOM

$$\boxed{H^2 = \frac{\rho}{3M_{pl}^2} = \frac{\frac{1}{2}\dot\phi^2 + V}{3M_{pl}^2}}$$

$$\boxed{\ddot\phi + 3H\dot\phi + V'(\phi) = 0}$$

slow roll approximation

$$\epsilon = -\frac{\dot H}{H^2} \ll 1 \sim \frac{\dot\phi^2}{H^2} \ll 1$$

$$\eta = \frac{\dot\epsilon}{\epsilon H} \ll 1 \sim \frac{\ddot\phi}{H\dot\phi} \ll 1$$

$\nabla_\mu T^{\mu\nu} = 0$

$$\dot\rho = -3H(\rho + P)$$

$$= -3H(\dot\phi^2)$$

$$\dot\rho = \dot\phi\ddot\phi + V'(\phi)\dot\phi$$

$a \ll 1$

$b \ll 1$

$ab \ll 1$

$a+b \ll 1$

$a-b$

om

$$= \frac{\rho}{3M_{pl}^2} = \frac{\frac{1}{2}\dot\phi^2 + V}{3M_{pl}^2}$$

$$= -3H(\rho + P)$$

$$= -3H(\dot\phi^2)$$

$$\dot\phi\ddot\phi + V'(\phi)\dot\phi$$

$$\boxed{\ddot\phi + 3H\dot\phi + V'(\phi) = 0}$$

$$H^2 \simeq \frac{V}{3M_{pl}^2} \qquad \dot\phi \sim \frac{V'}{3H}$$

$$\underline{3H\dot\phi + V'(\phi) \simeq 0}$$

slow roll approximation

$$\epsilon = -\frac{\dot H}{H^2} \ll 1 \sim \frac{\dot\phi^2}{H^2} \ll 1 \sim \epsilon_V = \frac{M_{pl}^2}{2}\left(\frac{V'}{V}\right)^2$$

$$\eta = \frac{\dot\epsilon}{\epsilon H} \ll 1 \sim \frac{\ddot\phi}{H\dot\phi} \ll 1 \sim \eta_V = M_{pl}^2\left(\frac{V''}{V}\right)$$

$$H^2 \simeq \frac{V}{3M_{pl}^2}$$

$$\dot{\phi} \sim \frac{V'}{3H}$$

$$3H\dot{\phi} + V'(\phi) \simeq 0$$

$$\ll 1 \sim \epsilon_V = \frac{M_{pl}^2}{2}\left(\frac{V'}{V}\right)^2$$

$$\ll 1 \sim \eta_V = M_{pl}^2\left(\frac{V''}{V}\right)$$

$$N_{tot} = \int_{N_I}^{N_E} dN = \int_{a_I}^{a_E} d\ln a$$

$$= \int_{t_I}^{t_f} H\, dt \overset{trick}{\simeq} \int_{t_I}^{t_f} \frac{H\dot{\phi}\, dt}{\dot{\phi}} = \int_{\phi_I}^{\phi_f} \frac{H\, d\phi}{\dot{\phi}}$$

$$= \int_{\phi_I}^{\phi_f} \frac{H^2\, d\phi}{H\dot{\phi}} = \int_{\phi_I}^{\phi_E} \frac{\frac{V}{3}}{-\frac{V'}{3}}\, d\phi = -\int_{\phi_I}^{\phi_E} \frac{V}{V'}\, d\phi$$

$$\mathcal{N} = -\int_{\phi_I}^{\phi_E} \frac{V}{V'}\, d\phi = -\int_{\phi_I}^{\phi_E} \frac{\phi}{2}\, d\phi$$

$$= -\frac{\phi^2}{4}\Big|_{\phi_I}^{\phi_E}$$

$$= \frac{\phi_I^2}{4} - \frac{\phi_E^2}{4}$$

$$= 60$$

$$\Phi_I \approx 15 M_{pl}$$

$$V = \frac{1}{2} m^2 \phi^2 << M_{pl}^4$$

$$V' = m^2 \phi$$

$$\frac{V}{V'} = \frac{\phi}{2}$$

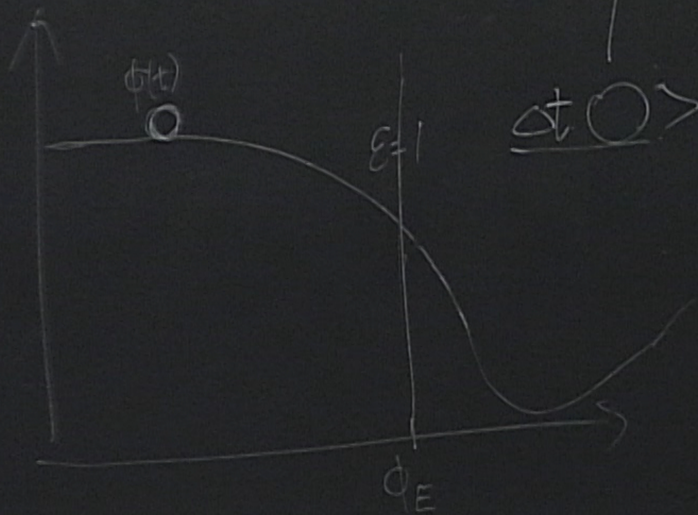
$$\epsilon_V = \frac{1}{2}\left(\frac{V'}{V}\right)^2$$

$$= \frac{1}{2}\left(\frac{2}{\phi}\right)^2$$

$$\phi_E = \sqrt{2} M_{pl} \quad \epsilon_V = 1$$

quantum Fluctuations. $\phi(t,x)$

$\delta t\, \bigcirc > \hbar$



toy model

$$S = \int d^3x\, dt\; a^3\left(\frac{1}{2}\dot{\phi}^2 + (\nabla\phi)^2\right).$$

$$\ddot{\phi} + 3H\dot{\phi} - \frac{\nabla^2\phi}{a^2} = 0$$

$$\frac{\ }{H^2}$$

$$\ll 1 \sim \frac{\ddot{\phi}}{H\dot{\phi}} \ll 1 \sim \eta_V = M_{pl}^2 \left(\frac{V''}{V}\right)$$

$$= \int_{\phi_I}^{\phi_f} \frac{H^2\, d\phi}{H\dot{\phi}} = \int_{\phi_I}^{\phi_f} \frac{\frac{V}{3}}{-\frac{V'}{3}}\, d\phi$$

ϕ_E

$$\phi(t,x) \rightarrow \phi_k(t)\, e^{i\vec{k}\cdot\vec{x}}$$

$$\ddot{\phi}_k + 3H\dot{\phi}_k + \frac{k^2}{a^2}\phi_k = 0$$

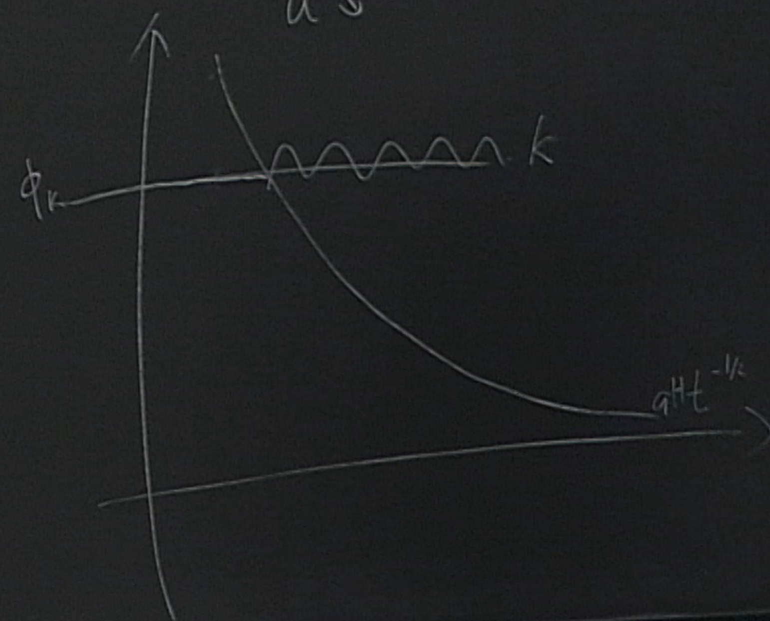
$H \sim \frac{|k|}{a}$

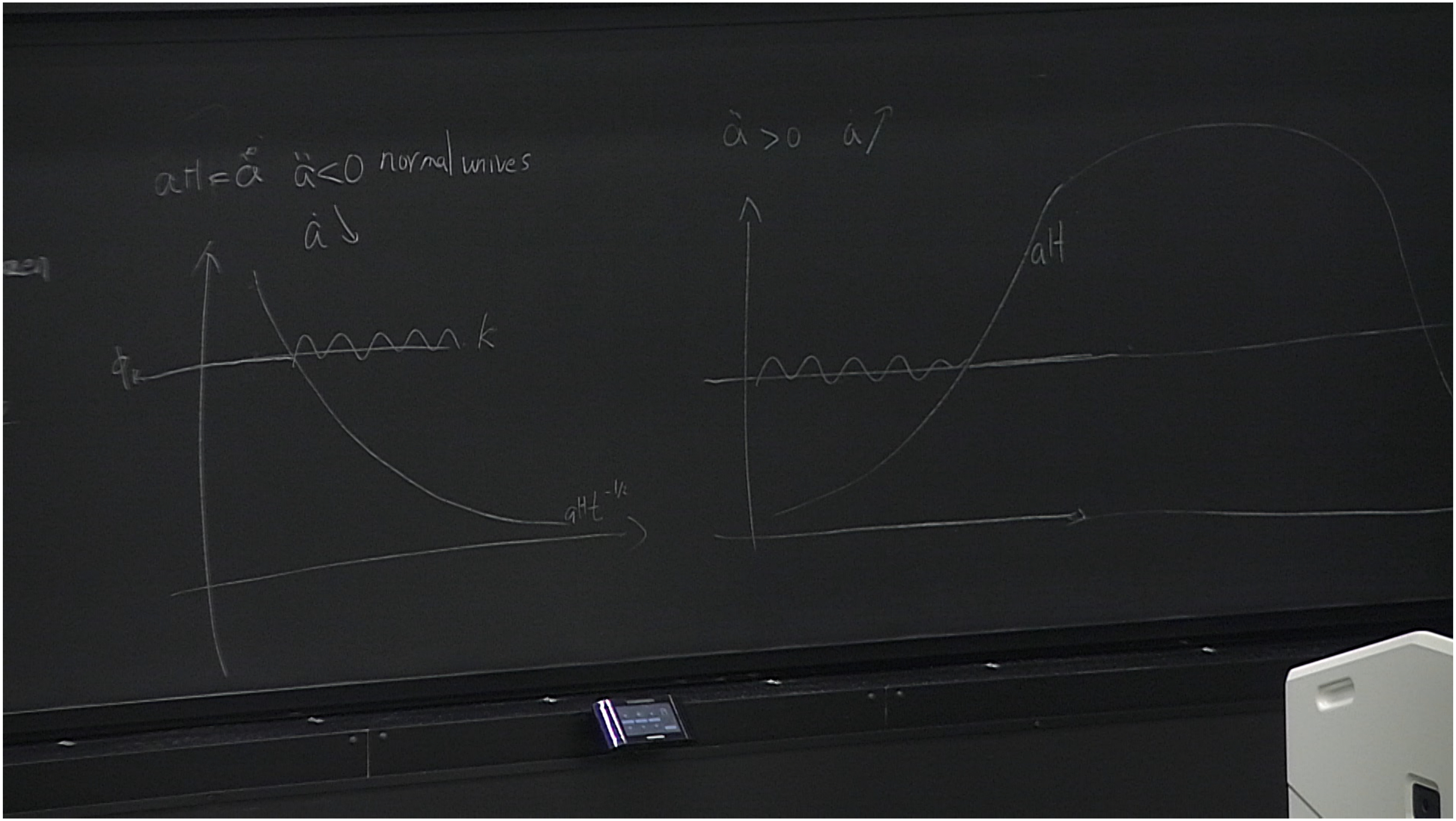
$|k| \ll aH \to$ outside the horizon   mode is frozen

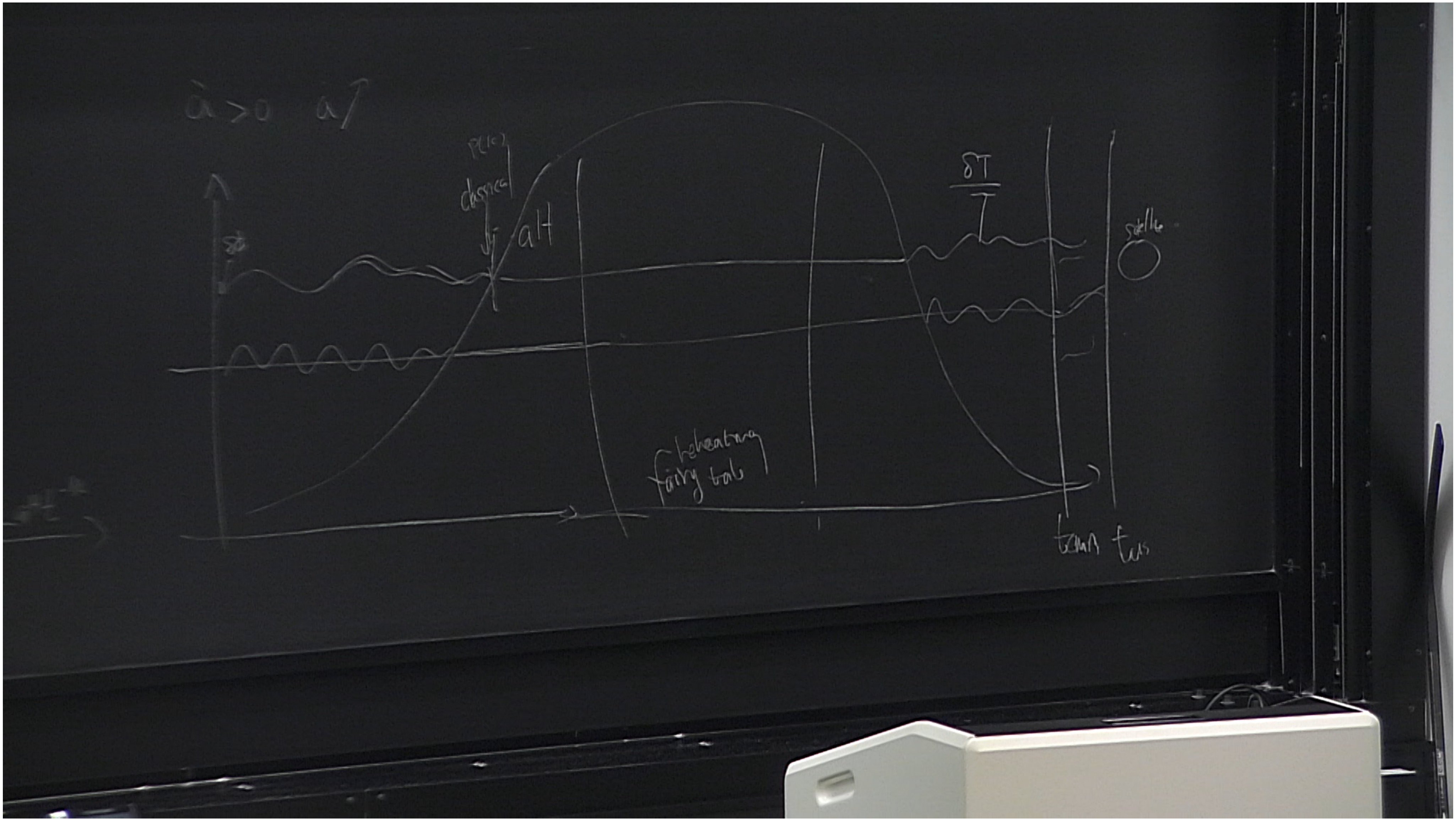
$|k| \gg aH \to$ inside the horizon   oscillate

$aH = \dot{a}$ $\ddot{a} < 0$ normal unives

$\dot{a} \downarrow$




aH = ȧ   ä<0 normal waves
ȧ ↘
k
aH
t^{-1/2}
ä > 0   ȧ ↗
aH


fairy tale
reheating

$H \sim \frac{|k|}{a}$

$|k| \ll aH \to$ outside the horizon   mode is frozen

$|k| \gg aH \to$ inside the horizon   oscillate

$aH = \dot{a}$ $\ddot{a} < 0$ normal universe

$\dot{a} \downarrow$