Title: PSI 2018/2019 - Cosmology Review - Lecture 8

Date: Jan 16, 2019 09:00 AM

URL: http://pirsa.org/19010009

Abstract:

entropy of universe

how, when is CMB generated

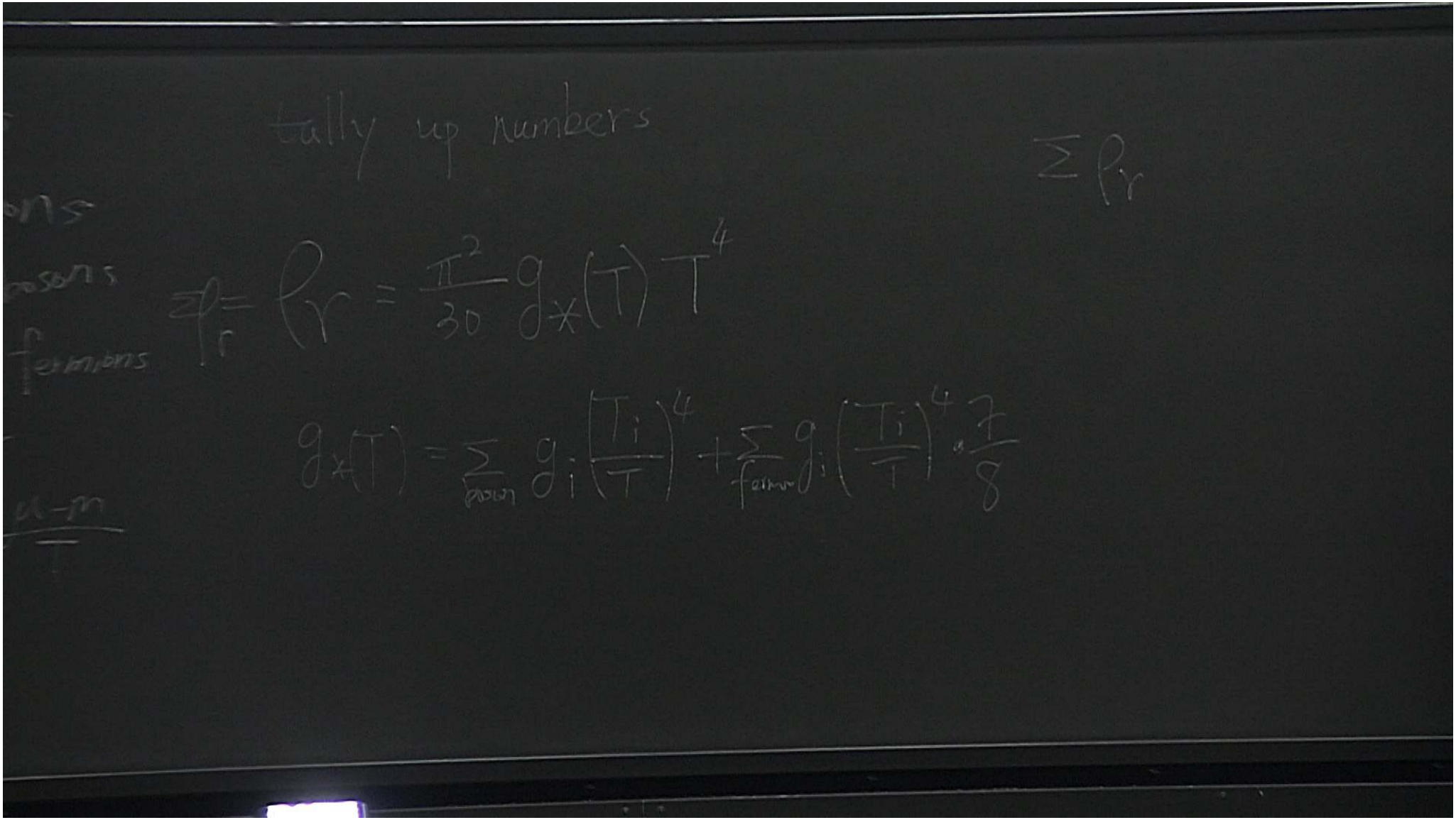
relativistic species

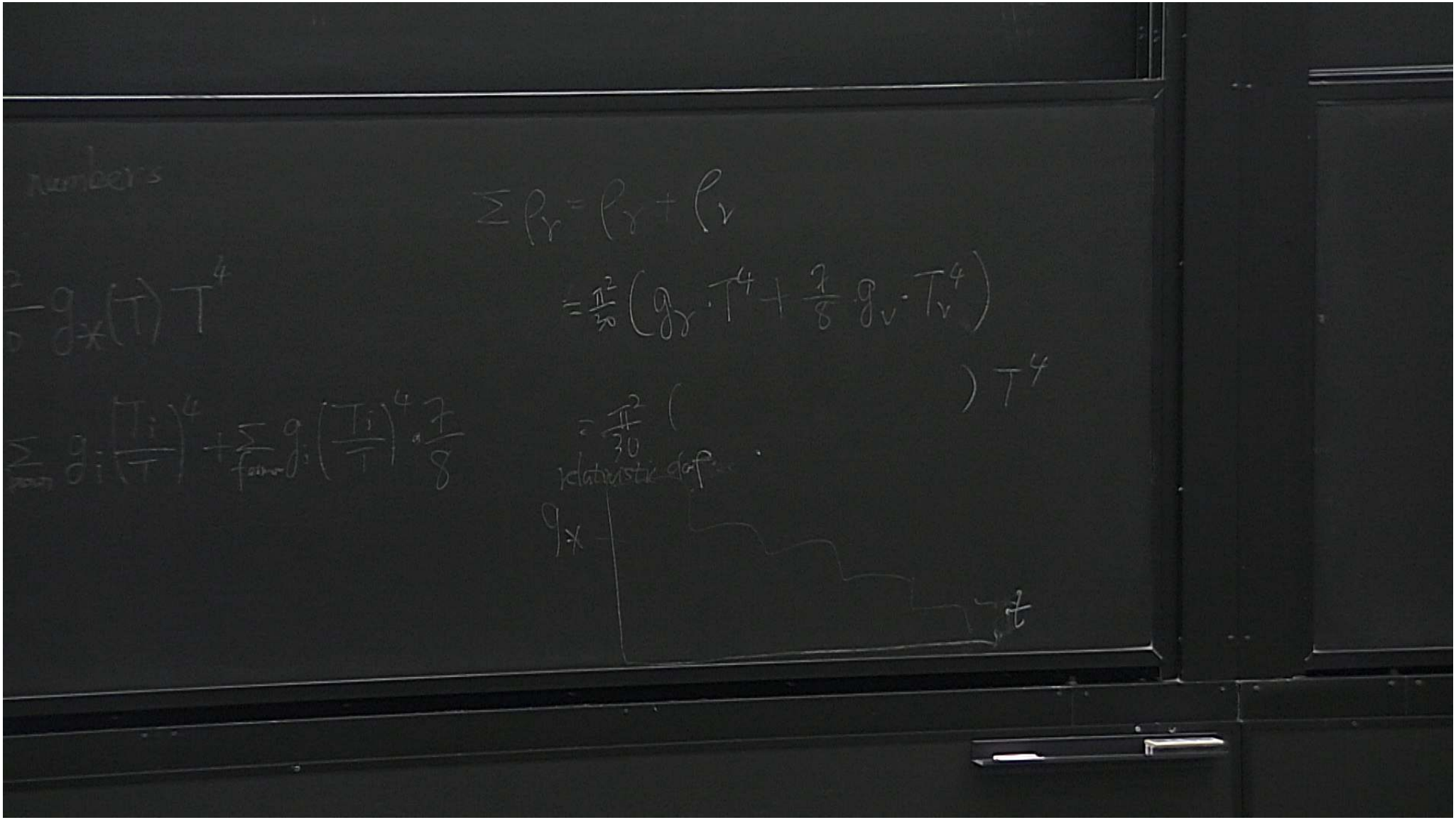
$$\rho \approx \frac{\pi^2}{30} g T^4 \begin{cases} 1 & \text{bosons} \\ \frac{7}{8} & \text{fermions} \end{cases}$$

$$n \approx \frac{\zeta(3)}{\pi^2} g T^3 \begin{cases} 1 & \text{bosons} \\ \frac{3}{4} & \text{fermions} \end{cases}$$

non relativistic particles

$$n = g \left(\frac{mT}{2\pi} \right)^{\frac{3}{2}} e^{\frac{\mu - m}{T}}$$


tally up numbers
$\sum \rho_r$
$\sum_r \rho_r = \rho_r = \frac{\pi^2}{30} g_*(T) T^4$
$g_*(T) = \sum_{\text{bosons}} g_i \left(\frac{T_i}{T}\right)^4 + \sum_{\text{fermions}} g_i \left(\frac{T_i}{T}\right)^4 \cdot \frac{7}{8}$


Numbers
$\Sigma \rho_r = \rho_\gamma + \rho_\nu$
$= \frac{\pi^2}{30}\left(g_\gamma \cdot T^4 + \frac{7}{8} \cdot g_\nu \cdot T_\nu^4\right)$
$= \frac{\pi^2}{30}($
$) T^4$
relativistic dof
g_*
t

Entropy of universe.

$$dQ = TdS = dU + PdV = 0$$

S: entropy is conserved. $\frac{dS}{dt} = 0$

$$\frac{dP}{dT} = \frac{P+\rho}{T}$$

$$\rho = \int_0^\infty E \cdot f(p) dp$$

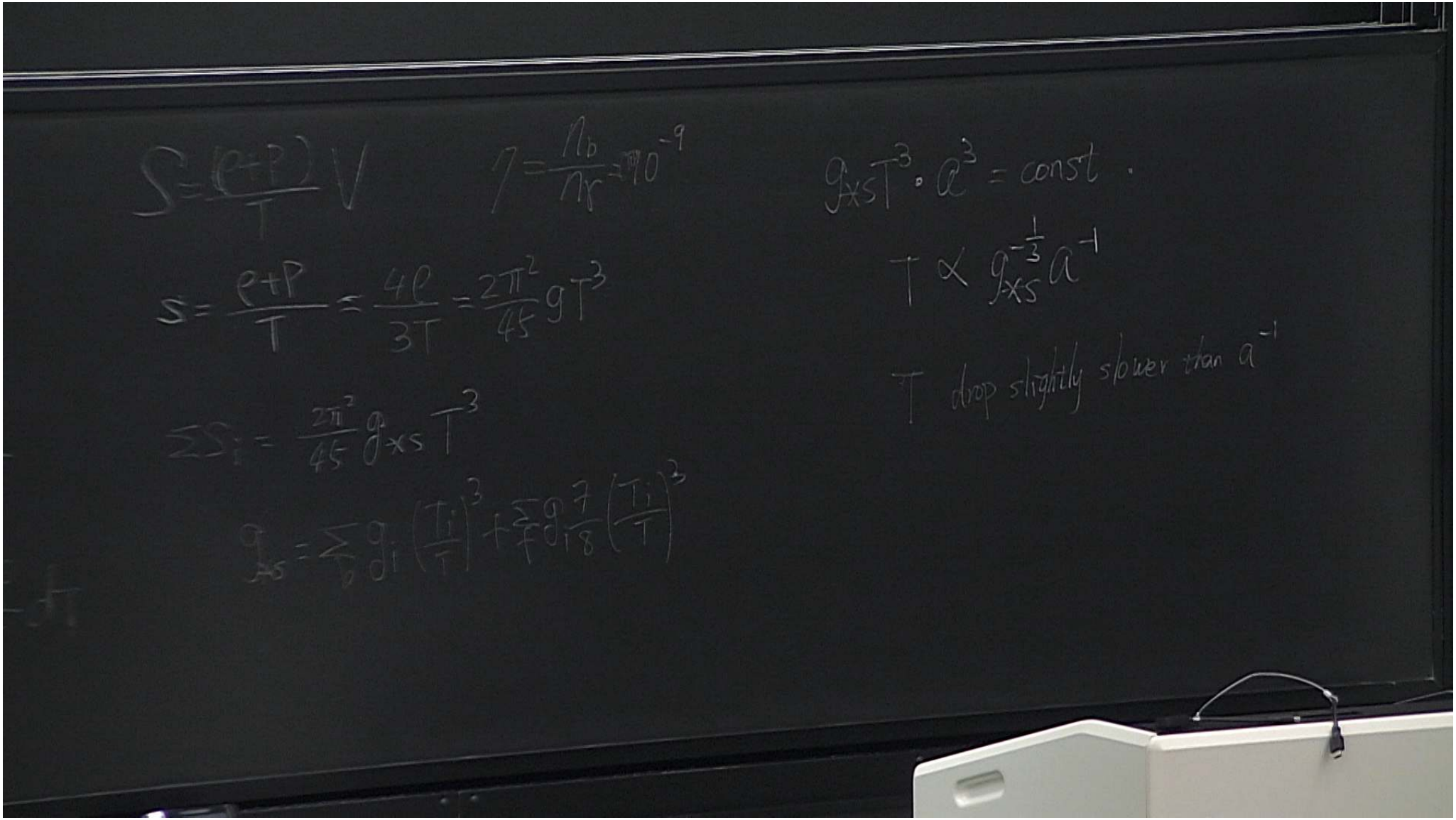
$$P = \int_0^\infty \frac{p^2}{E} \cdot f(p) dp$$

$$dS = \frac{dU + PdV + VdP - VdP}{T}$$

$$= \frac{d(\rho V) + d(PV) - VdP}{T}$$

$$= \frac{d((\rho+P)V)}{T} - \frac{V}{T}\frac{P+\rho}{T}dT$$

$$= d\left(\frac{(\rho+P)V}{T}\right)$$


$S = \frac{(\rho+P)}{T} V$
$\eta = \frac{n_b}{n_\gamma} \approx 10^{-9}$
$g_{*s} T^3 \cdot a^3 = \text{const.}$
$s = \frac{\rho+P}{T} = \frac{4\rho}{3T} = \frac{2\pi^2}{45} g T^3$
$T \propto g_{*s}^{-\frac{1}{3}} a^{-1}$
$\sum S_i = \frac{2\pi^2}{45} g_{*s} T^3$
T drop slightly slower than a^{-1}
$g_{*s} = \sum_b g_i \left(\frac{T_i}{T}\right)^3 + \frac{7}{8} \sum_f g_i \left(\frac{T_i}{T}\right)^3$

$$\sum \rho_r = \rho_\gamma + \rho_\nu$$

$$= \frac{\pi^2}{30}\left(g_\gamma \cdot T^4 + \frac{7}{8} g_\nu \cdot T_\nu^4\right)$$

$$= \frac{\pi^2}{30}\left(\quad \right) T^4$$

$$\left(\frac{T_i}{T}\right)^4 + \sum_{\text{fermions}} g_i \left(\frac{T_i}{T}\right)^4 \cdot \frac{7}{8}$$

$T_\nu < T_\gamma$

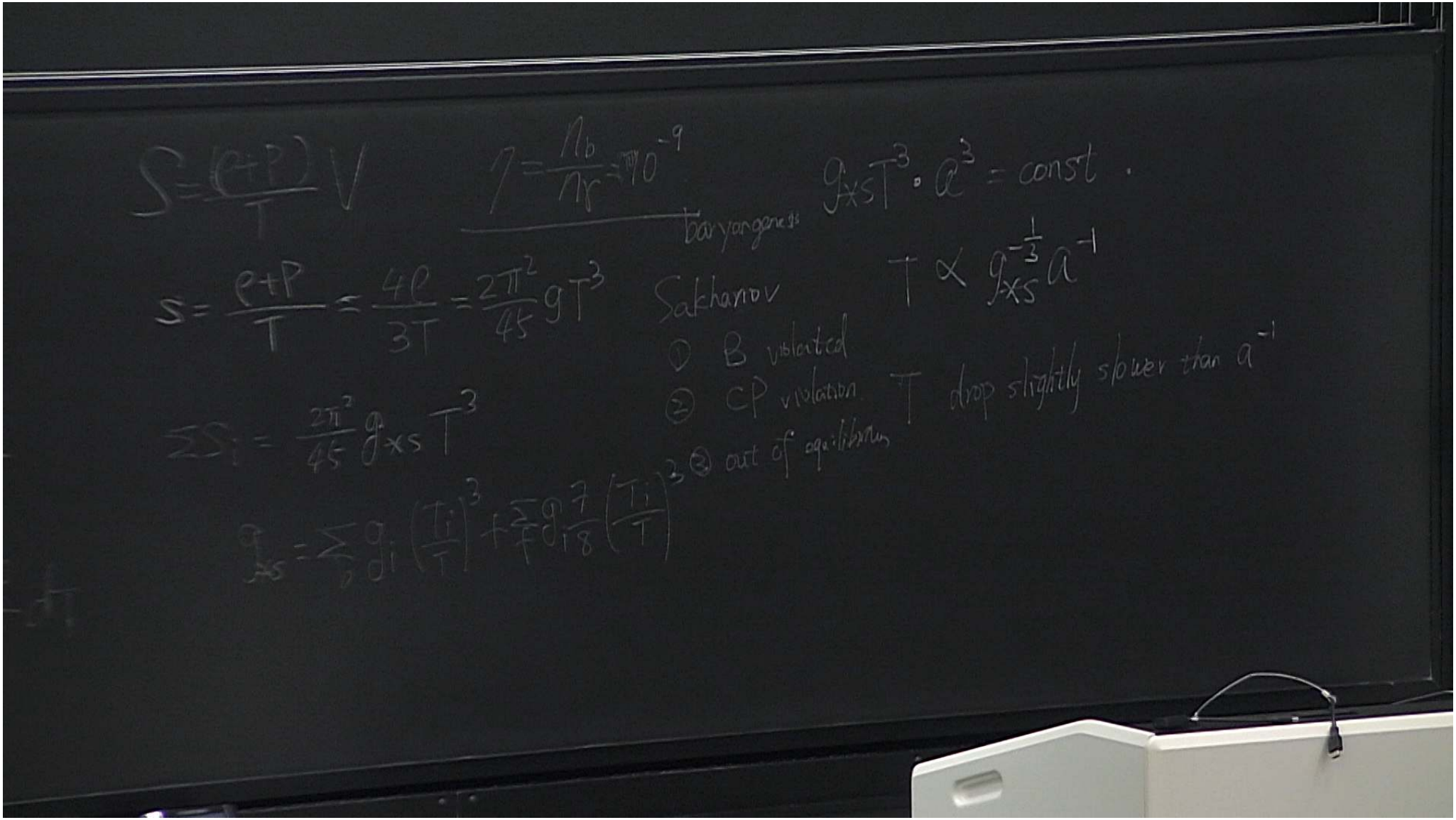
relativistic dof

g_*

g_{*s}

T_i g_* and g_{*s} decreases

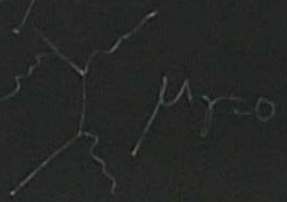
t

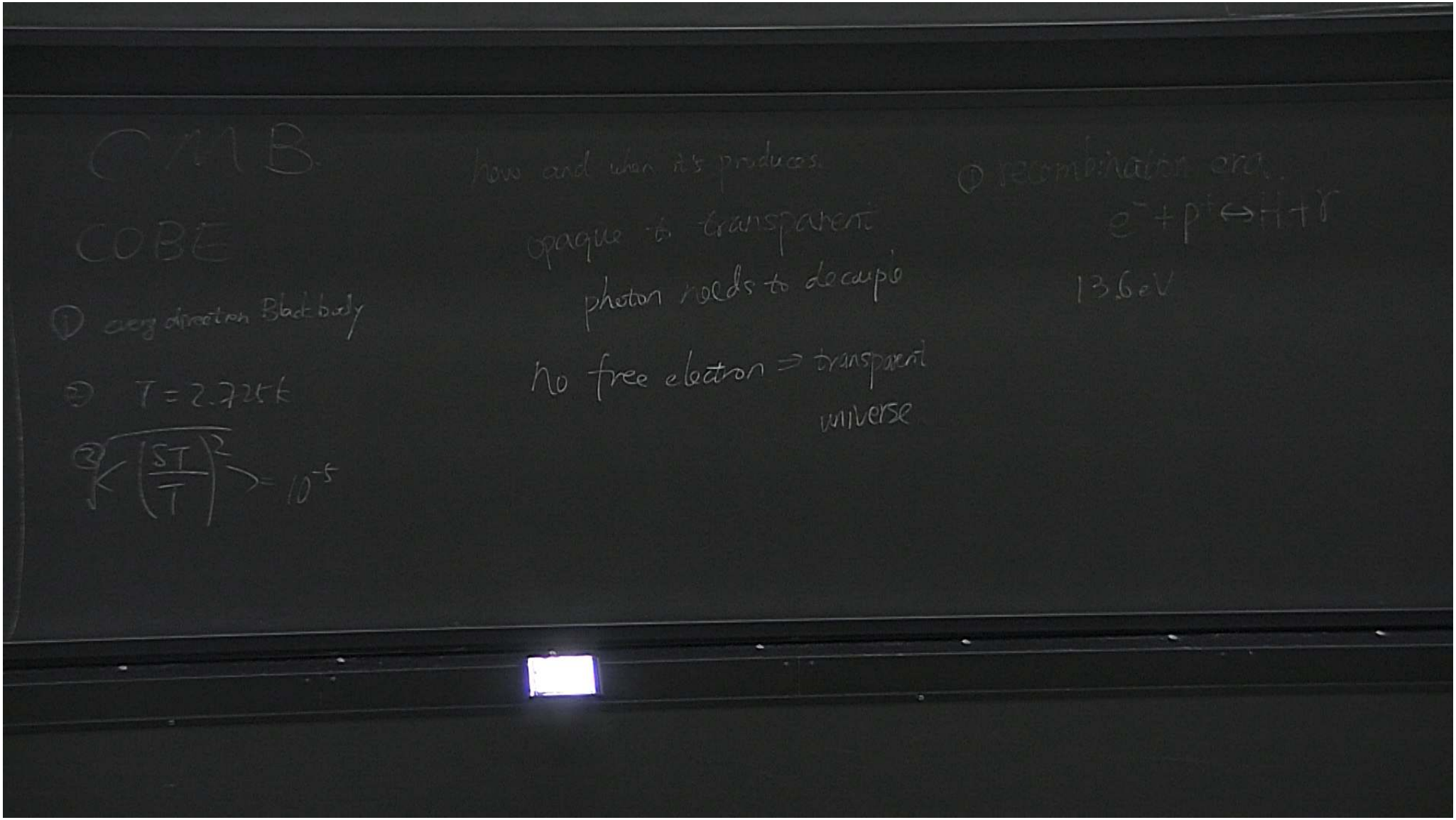

$S = \frac{(\rho+P)}{T} V$
$\eta = \frac{n_b}{n_\gamma} \approx 10^{-9}$
baryogenesis
$g_{*s} T^3 \cdot a^3 = \text{const.}$
$s = \frac{\rho+P}{T} = \frac{4\rho}{3T} = \frac{2\pi^2}{45} g T^3$
Sakharov
$T \propto g_{*s}^{-\frac{1}{3}} a^{-1}$
① B violated
② CP violation
③ out of equilibrium
T drop slightly slower than a^{-1}
$\sum S_i = \frac{2\pi^2}{45} g_{*s} T^3$
$g_{*s} = \sum_b g_i \left(\frac{T_i}{T}\right)^3 + \frac{7}{8} \sum_f g_i \left(\frac{T_i}{T}\right)^3$

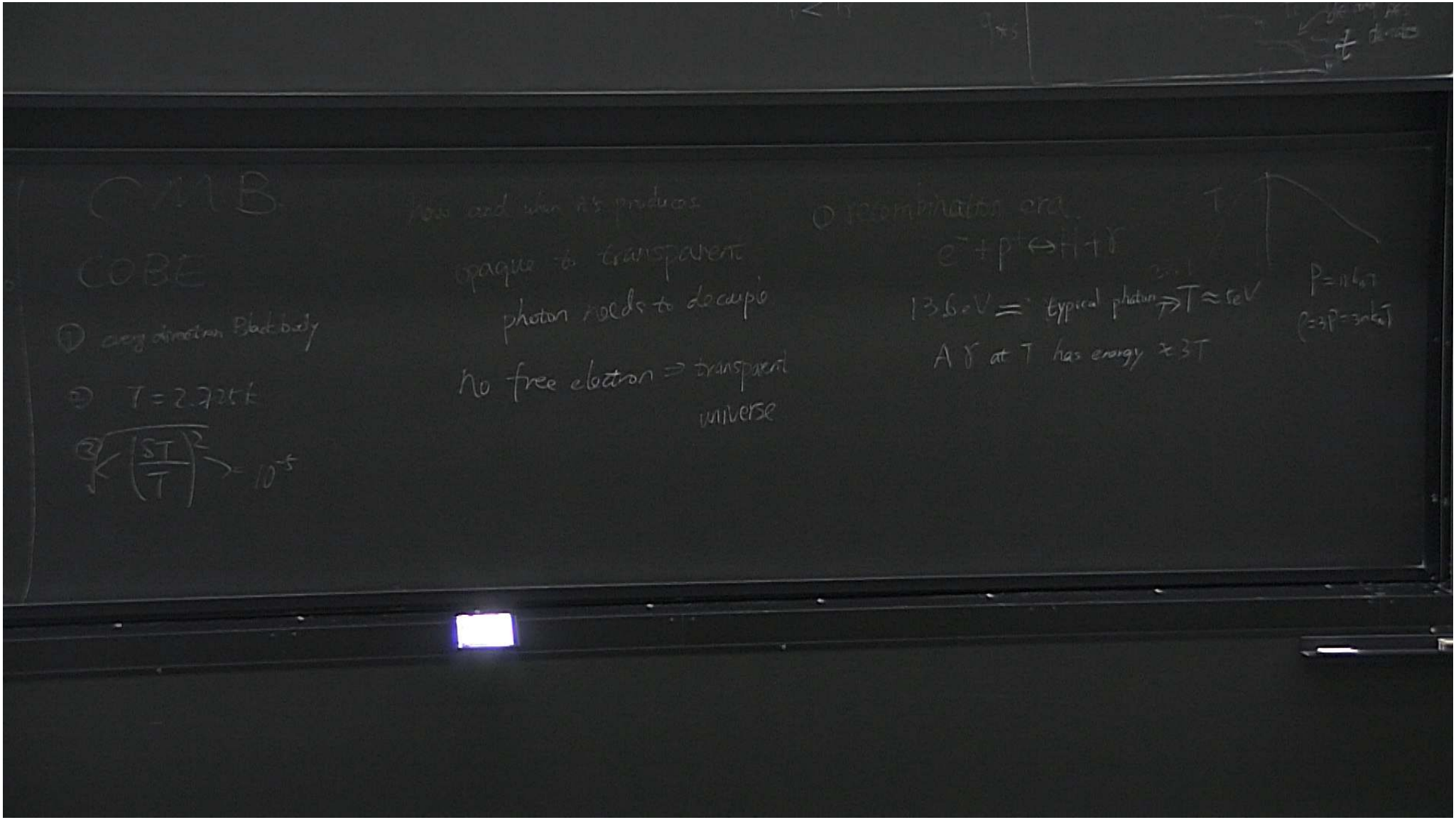
① $1+2 \leftrightarrow 3+4$ $\mu_1+\mu_2=\mu_3+\mu_4$

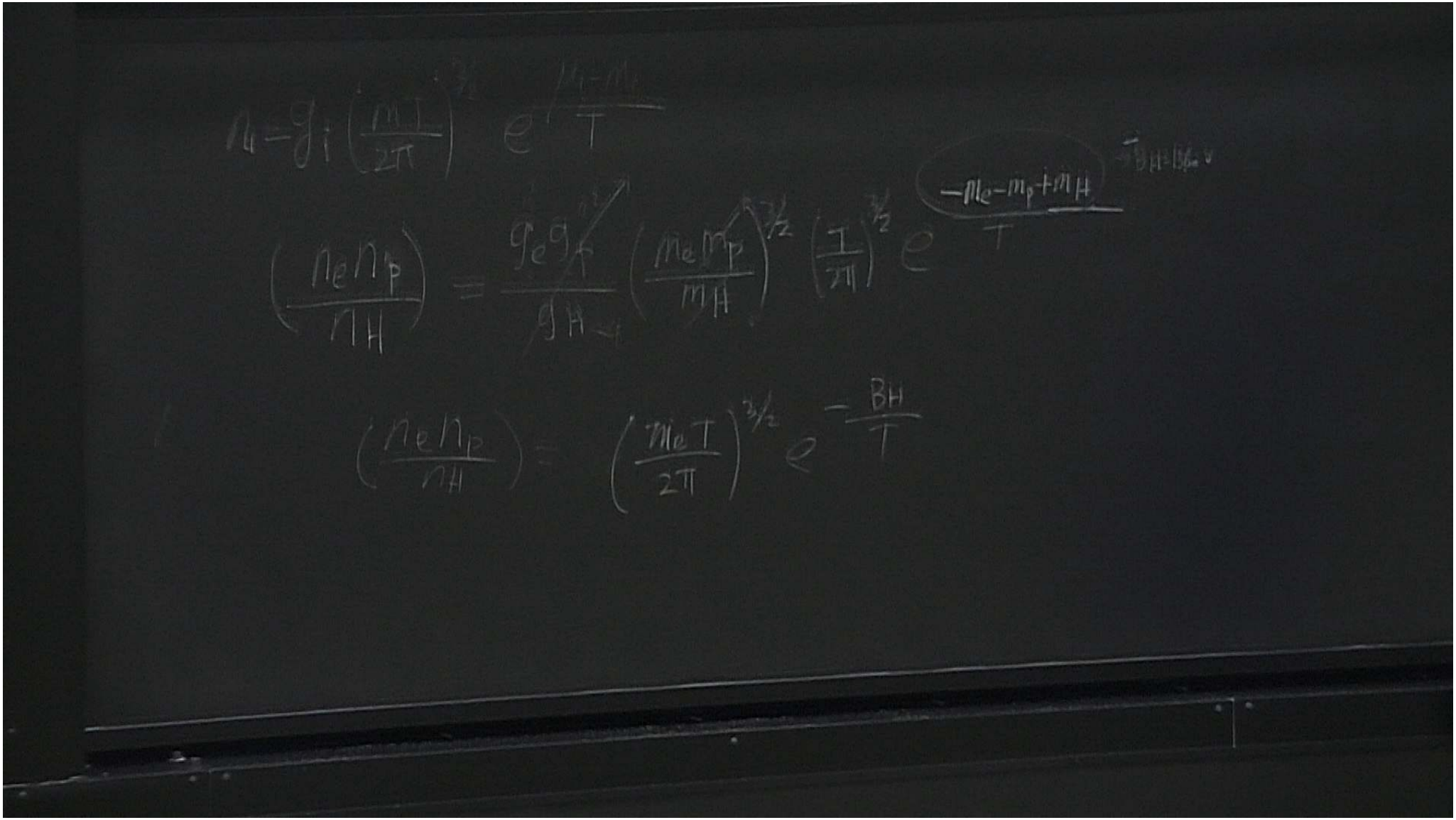
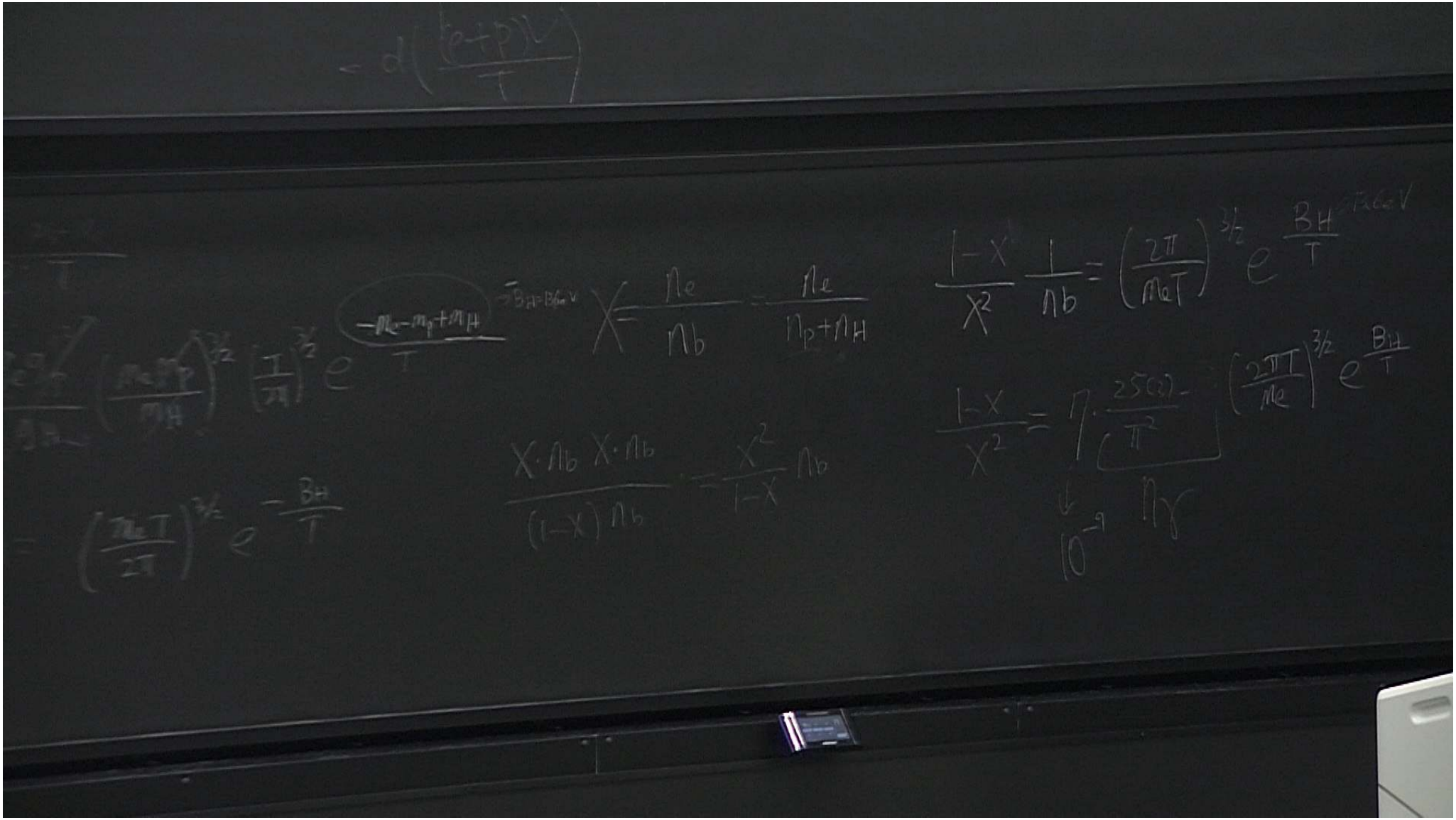
② $e+\gamma \rightarrow e^- + \gamma + \gamma$

③ take in the universe
$\mu_e = \mu_\gamma = 0$




CMB
COBE
① every direction Blackbody
② $T = 2.725\,k$
③ $\left\langle \left(\frac{\Delta T}{T}\right)^2 \right\rangle = 10^{-5}$
how and when it's produced
opaque → transparent
photon needs to decouple
no free electron ⇒ transparent universe
① recombination era
$e^- + p \leftrightarrow H + \gamma$
13.6 eV


CMB
COBE
① very similar Blackbody
② T = 2.725 K
$\left\langle \left(\frac{\Delta T}{T}\right)^2 \right\rangle \sim 10^{-5}$
how and when it's produced
opaque to transparent
photon needs to decouple
no free electron ⇒ transparent universe
① recombination era
$e^- + p \leftrightarrow H + \gamma$
13.6 eV ≈ typical photon ⇒ T ≈ eV
A γ at T has energy ≈ 3T
T
$P = nkT$
$(\epsilon = 3P = 3nkT)$


$n_i = g_i \left(\frac{m_i}{2\pi}\right)^{3/2} e^{\frac{\mu_i - m_i}{T}}$
$\left(\frac{n_e n_p}{n_H}\right) = \frac{g_e g_p}{g_H} \left(\frac{m_e m_p}{m_H}\right)^{3/2} \left(\frac{T}{2\pi}\right)^{3/2} e^{\frac{-m_e - m_p + m_H}{T}}$
$B_H = 13.6\,eV$
$\left(\frac{n_e n_p}{n_H}\right) = \left(\frac{m_e T}{2\pi}\right)^{3/2} e^{-\frac{B_H}{T}}$