Title: PSI 2018/2019 - Cosmology Review - Lecture 4

Date: Jan 10, 2019 09:00 AM

URL: http://pirsa.org/19010005

Abstract:

n= Today. Our Universe

Ia Supernovae: dark energy exist

$w \approx -1$

$$H^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}$$

$$\rho_{crit,0} \equiv \frac{3H_0^2}{8\pi G} \sim \frac{10 \text{ hydrogen}}{m^3} \sim \frac{10^{11} M_\odot}{Mpc^3}$$

$$\Omega_I \equiv \frac{\rho_I}{\rho_{crit,0}} \qquad \rho \propto a^{-3(1+w)} \qquad P = w\rho$$

$$\Omega_k \equiv \frac{-k}{H_0^2}$$

$$\frac{H^2}{H_0^2} = \Omega_r a^{-4} + \Omega_m a^{-3} + \Omega_\Lambda + \Omega_k a^{-2}$$

Simple exercise: Single component universe.

$$\frac{H^2}{H_0^2} = a^{-3(1+w_I)}$$

$\Omega_I = 1$: single component

$$\left(\frac{\dot a}{a}\right) = H_0\, a^{-\frac{3}{2}(1+w_I)}$$

$$a \propto t^q \qquad \frac{\dot a}{a} = \frac{q t^{q-1}}{t^q} = \frac{q}{t} = q t^{-1}$$

$$H_0\, t^{-\frac{3}{2}q(1+w_I)}$$

$$a^{-3(1+w_I)}$$

$$-1 = -\frac{3}{2}q(1+w_I)$$

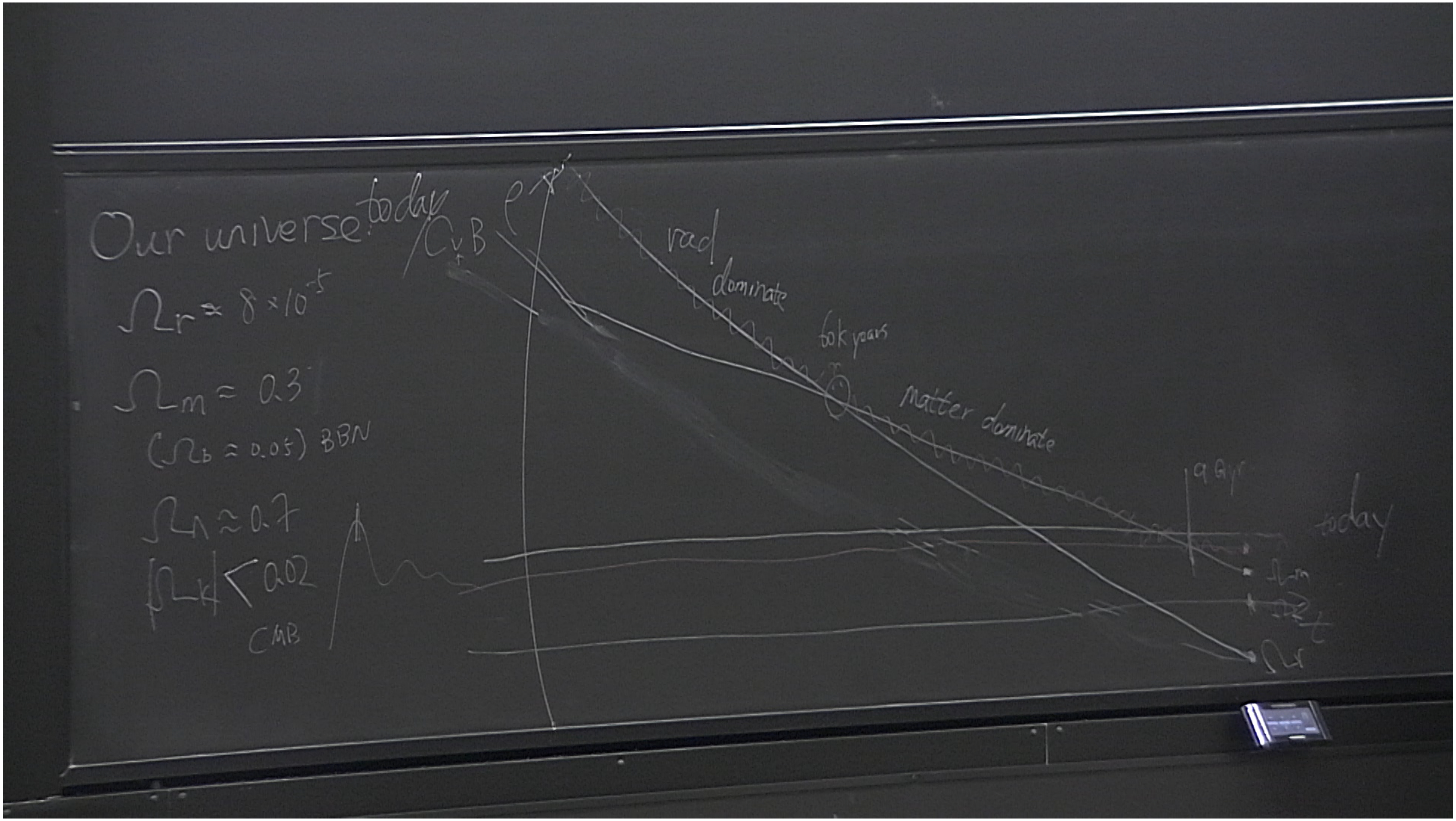
$$q = \frac{2}{3(1+w_I)}$$

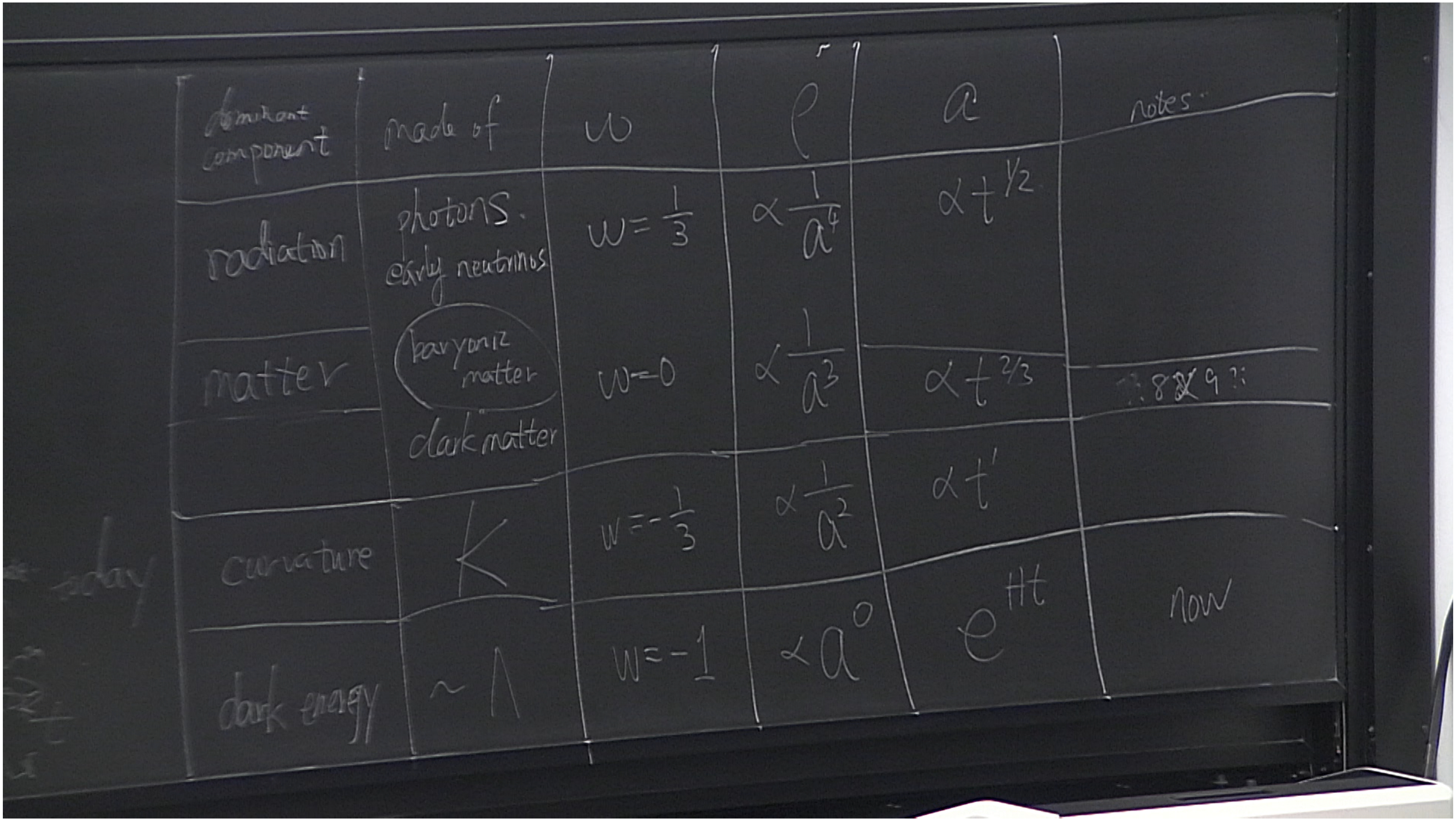
matter $w=0$ $\quad q=\frac{2}{3}$ $\quad a\propto t^{2/3}$

radiation $w=\frac{1}{3}$ $\quad q=\frac{1}{2}$ $\quad a\propto t^{1/2}$

curvature $w=-\frac{1}{3}$ $\quad q=1$ $\quad a\propto t$

Λ: $w=-1$ $\quad a\propto e^{Ht}$


Our universe
$\Omega_r \approx 8\times10^{-5}$
$\Omega_m \approx 0.3$
$(\Omega_b \approx 0.05)$ BBN
$\Omega_\Lambda \approx 0.7$
$|\Omega_k| < 0.02$
CMB
today
$C\nu B$
rad
dominate
60k years
matter dominate
9 Gyr
today
Ω_m
t
Ω_r



dominant component
made of
w
ρ
a
notes
radiation
photons. early neutrinos
w = 1/3
∝ 1/a^4
∝ t^{1/2}
matter
baryonic matter
dark matter
w = 0
∝ 1/a^3
∝ t^{2/3}
curvature
K
w = -1/3
∝ 1/a^2
∝ t^1
dark energy
~ Λ
w = -1
∝ a^0
e^{Ht}
now
today

supernovae dark energy exist $w=-1$

$$\Omega_I \equiv \frac{\rho_I}{\rho_{crit,0}} \qquad \rho \propto a^{-3(1+w)} \qquad P = w\rho$$

$$\Omega_k \equiv \frac{-k}{H_0^2}$$

$$\frac{H^2}{H_0^2} = \Omega_r a^{-4} + \Omega_m a^{-3} + \Omega_\Lambda + \Omega_k a^{-2} = \sum_I \Omega_I a^{-3(1+w_I)}$$

$$\left(\frac{\dot a}{a}\right) = H_0 a^{-\frac{3}{2}(1+w_I)}$$

$$a \propto t^q \qquad \frac{\dot a}{a} = \frac{q t^{q-1}}{t^q} = \frac{q}{t} \propto q t^{-1}$$

$$H_0 t^{-\frac{3}{2}q(1+w_I)}$$

matter $w=0$ $q=\frac{2}{3}$

radiation $w=\frac{1}{3}$ $q=\frac{1}{2}$

curvature $w=-\frac{1}{3}$ $q=1$

Λ: $w=-1$ $a \propto e^{Ht}$

Supernovae → dark energy

observed flux $\frac{\text{Power}}{\text{Area}}$

$$F_{lux} = \frac{\Delta E_{ob}}{\Delta t\ \text{Area}}$$

relate this with redshift

detour

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right) = a^2(t)\left(-d\tau^2 + d\chi^2 + S_k^2(\chi)\, d\Omega^2\right).$$

$$\frac{dr^2}{1-kr^2} = d\chi^2 \qquad r = S_k(\chi) = \begin{cases} \sin\chi & k=+1 \\ \chi & k=0 \\ \sinh\chi & k=-1 \end{cases}$$

$d\tau = d\chi$ light travels

$$dt^2 = a^2(t)\, d\tau^2$$

← conformal time

$$F_{flux} = \frac{\Delta E_{ob}}{\Delta t\, Area} \qquad E \propto \frac{1}{a} \qquad \Delta E_{ob} = \frac{\Delta E_{em}\, a}{a_0} = \frac{\overset{L=const}{L\Delta t_{em}}\, a}{a_0} = \frac{L\, a\, a\, \Delta\tau}{a_0}$$

$$z = \frac{\lambda_0 - \lambda_{em}}{\lambda_{em}}$$

$$1+z = \frac{a_0}{a}$$

$$= \frac{L a^2 \Delta\tau}{a_0 a\, a_0 \Delta\tau\, 4\pi S_k^2(\chi)}$$

function of a

$\Omega_m, \Omega_\Lambda, \ldots$

$$dz = a_0 \frac{-\dot{a}}{a^2}\, dt = -\frac{a_0}{a} H\, dt$$

$$\chi = \int d\chi = \int d\tau = \int \frac{dt}{a} = \int \frac{dz}{a_0 H}$$

$$F_{rax} = \frac{\Delta E_{ob}}{\Delta t\ \text{Area}} \qquad E \propto \frac{1}{a}$$

$$\Delta E_{ob} = \frac{\Delta E_{em}\, a}{a_0} = \frac{\overbrace{L \Delta t_{em}}^{L = const}\, a}{a_0} = \frac{L\, a\, a\, \Delta\tau}{a_0}$$

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function of a

Ω_m, Ω_Λ

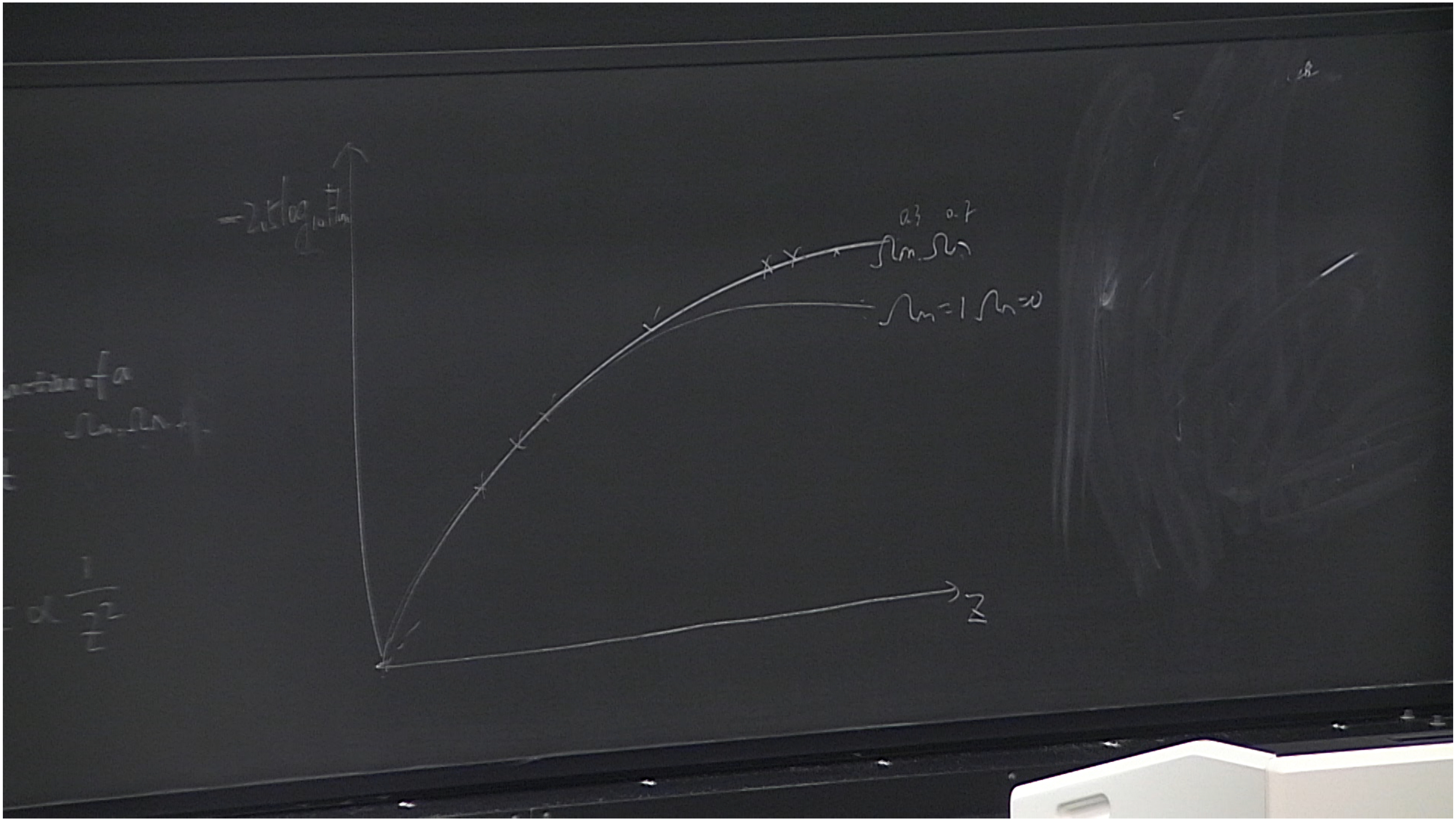
$$\chi = \int d\chi = \int d\tau = \int \frac{dt}{a} = \int \frac{dz}{a_0 H}$$

$$F \propto \frac{1}{z^2}$$

nearby

$$F \propto \frac{1}{z^2}$$

$-2.5 \log_{10} F$



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Supernovae
+
dark energy

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$d\tau = d\chi$ light travels

$$dt^2 = a^2(t)\, d\tau^2$$ ← conformal time