

Title: Renormalization and Effective Field Theory - Lecture 13

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URL: <http://pirsa.org/18120026>

Abstract:

If we have some field theory on  $M$

Fields  $\mathcal{E}$

$\mathcal{O}_x(\mathcal{E})$  = local functionals  
=  $\int$  of Log. densities

Differential

$$S_j - \{ = \mathcal{O}_{j+1} + \{ \mathcal{L}_j \} \cdot \mathcal{O}_j(\mathcal{E}) \rightarrow \mathcal{O}_j(\mathcal{E})$$

$H^1(\mathcal{O}_x)$  = positive deformations in a neighborhood

$H^1(\mathcal{O}_x(\mathcal{E}))$  = positive deformations

$H^1(\mathcal{O}_x(\mathcal{E}))$  = symplectic

How to compute?

If  $x \in M$  let  $\mathcal{A}_x$  = cochain complex of local operators at  $x$

= functions of fields + derivatives, differential is  $\{S_j - \}$

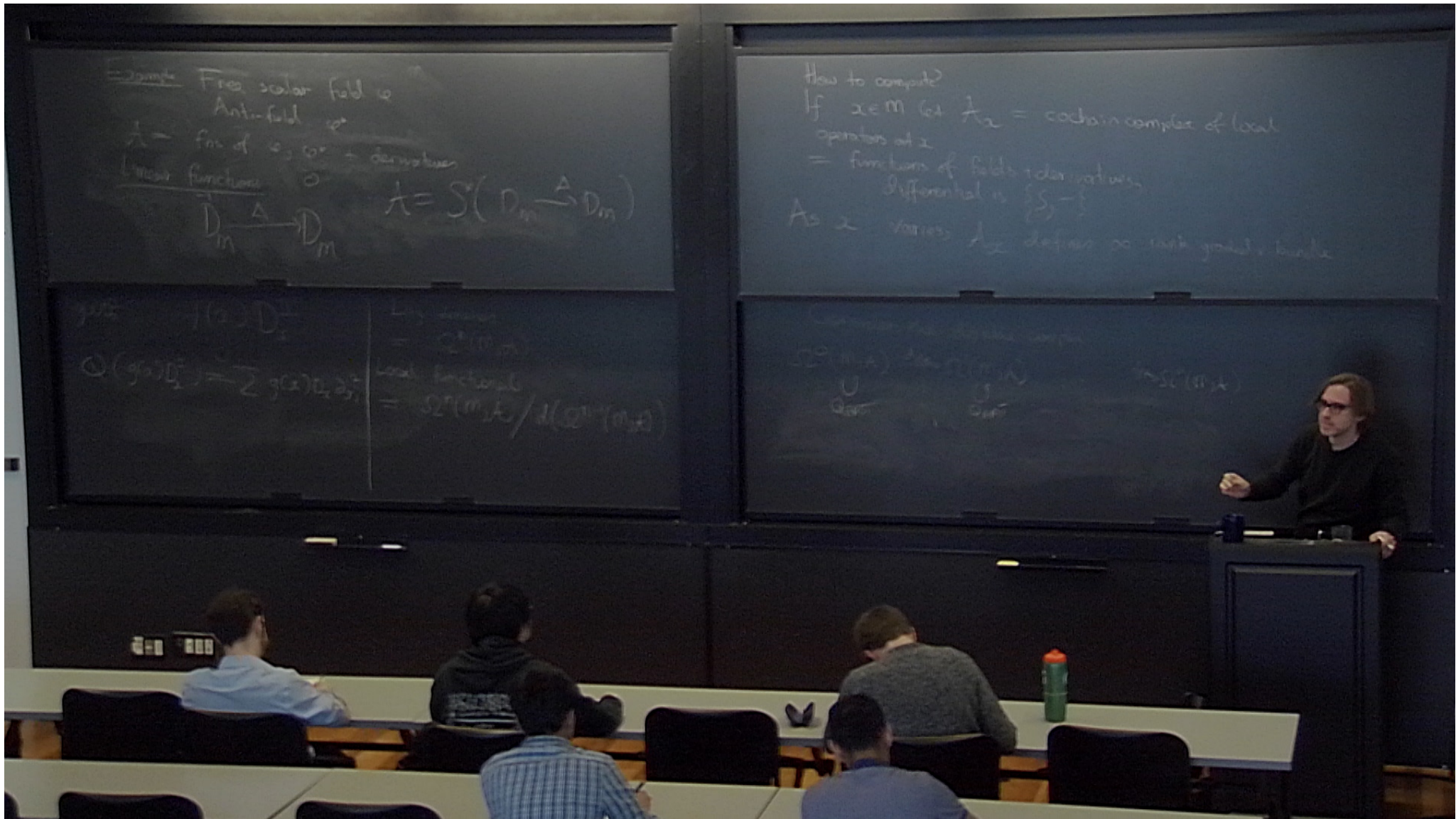
As  $x$  varies,  $\mathcal{A}_x$  defines an exact graded bundle

$\mathcal{A}_x$  = cochain of differential operators  
local at  $\mathcal{O}_x$  in a local manner  
=  $\mathcal{O}_x \in \mathbb{R}^n$ ,  $\partial_i \mathcal{O}_i$  makes sense

$$\partial_i \sum f_j(x) \mathcal{O}_j = \sum \frac{\partial f_j}{\partial x_i} \mathcal{O}_j + f_j \partial_i \mathcal{O}_j$$

Math terms  $\mathcal{A}$  is a  $D_M$ -commutative dg algebra



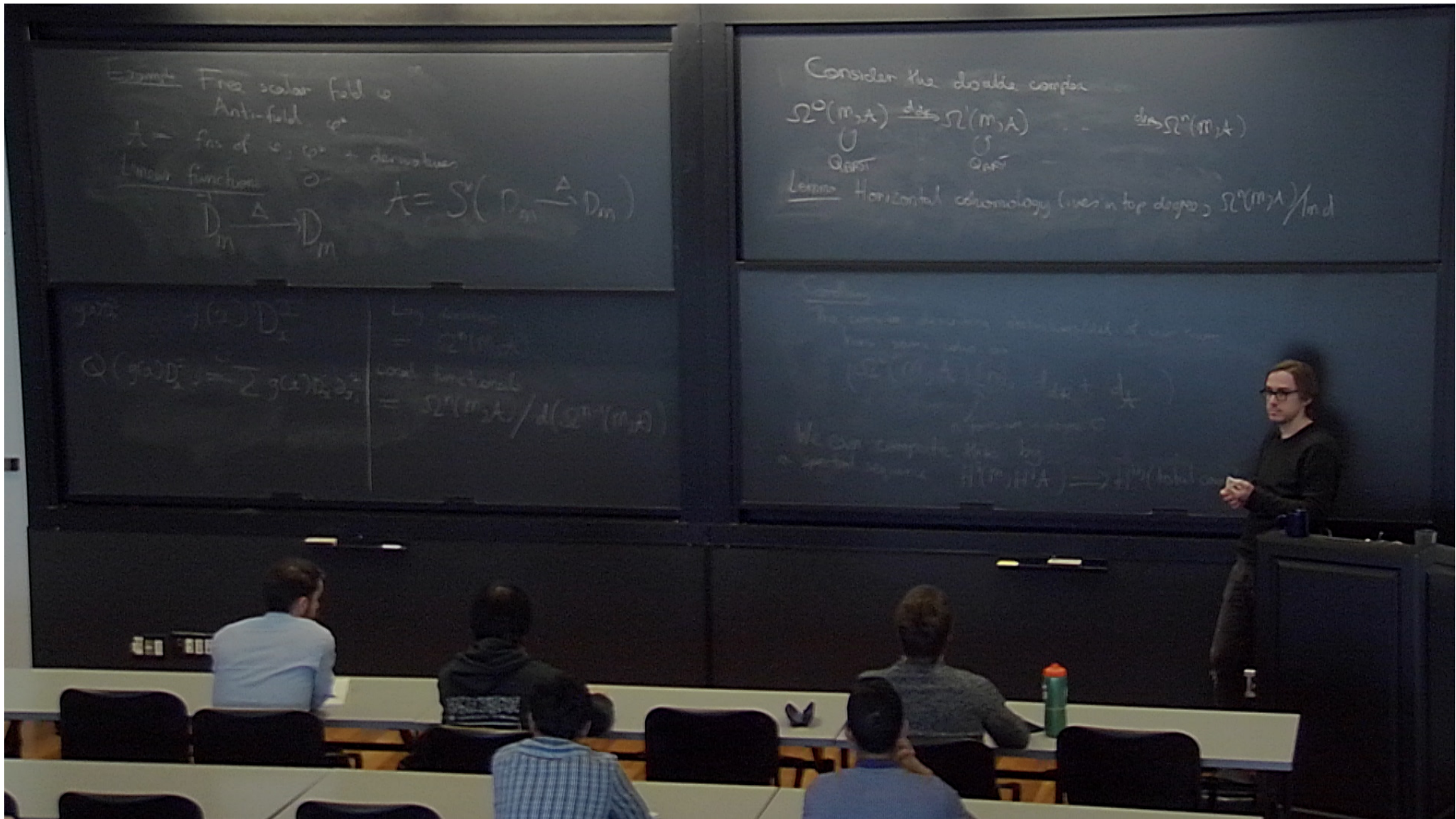


Example Free scalar field  $\phi$   
 Anti-field  $\psi$   
 $\Lambda = \text{fn of } \phi, \psi + \text{derivatives}$   
 Linear functional  $\int \Lambda$   
 $D_m \xrightarrow{\Lambda} D_m$   
 $\Lambda = S(D_m \xrightarrow{\Lambda} D_m)$

How to compute?  
 If  $x \in M$  let  $\mathcal{A}_x = \text{cochain complex of local operators at } x$   
 $= \text{function of fields + derivatives, differential is } \delta$   
 As  $x$  varies,  $\mathcal{A}_x$  defines a rank graded bundle

$g(x) D_x^+$   
 $\mathcal{Q}(g(x) D_x^+) = \int g(x) D_x^+$   
 Local functional  
 $= \int_{\mathcal{Q}(M)} \mathcal{Q}(g(x) D_x^+) / d(\mathcal{Q}(M))$

Computation of the complex  
 $\mathcal{Q}(M) \xrightarrow{\delta} \mathcal{Q}(M) \xrightarrow{\delta} \mathcal{Q}(M) \xrightarrow{\delta} \dots$   
 $\cup$   
 $\mathcal{Q}(M) \xrightarrow{\delta} \mathcal{Q}(M) \xrightarrow{\delta} \mathcal{Q}(M) \xrightarrow{\delta} \dots$   
 $\sim \mathcal{Q}(M)$

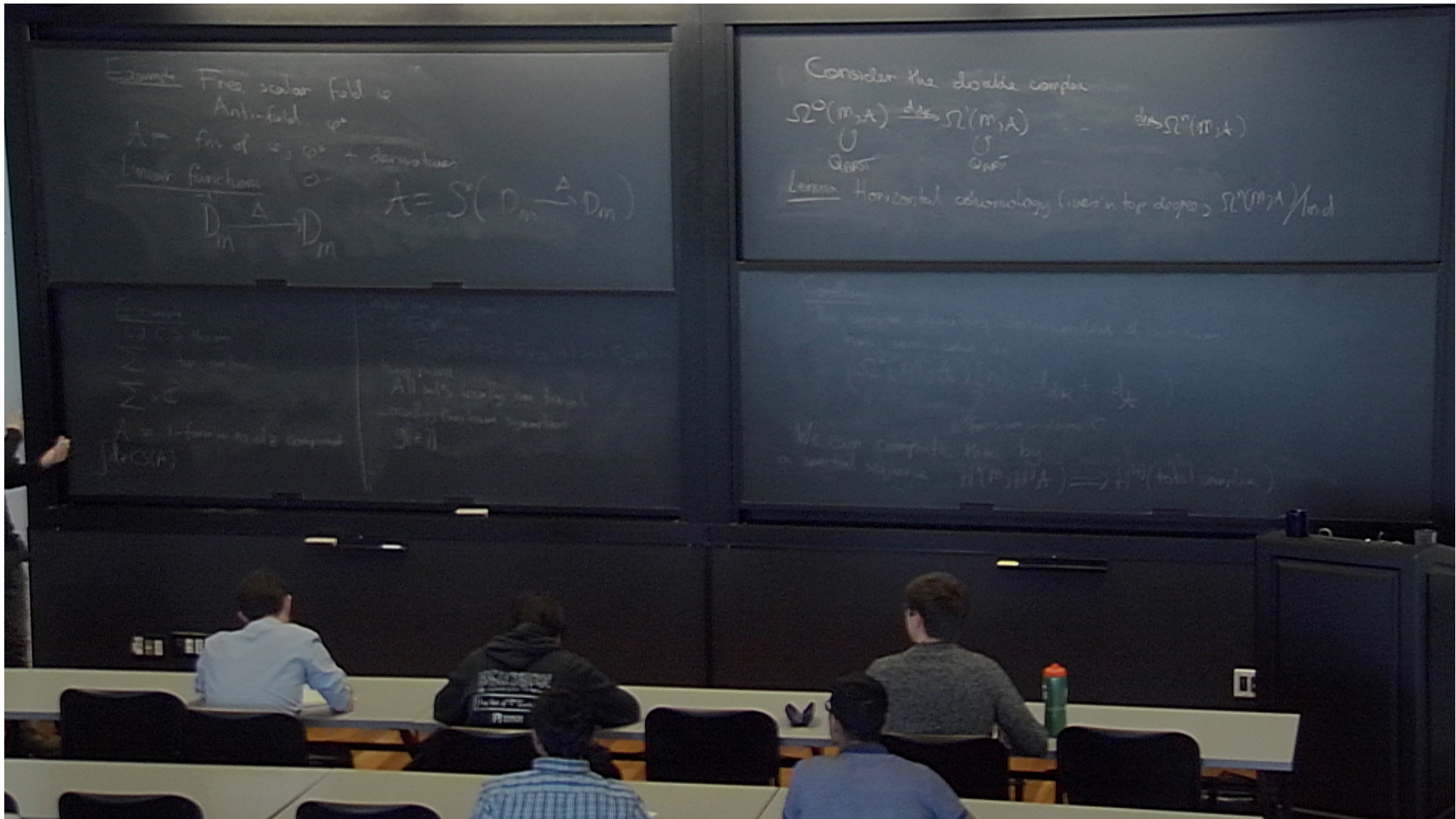


Example Free scalar field  $\varphi$   
 Anti-field  $\varphi^*$   
 $A =$  fun. of  $\varphi, \varphi^* +$  derivatives  
 Linear functions  $\mathcal{O}$   
 $D_m \xrightarrow{A} D_m$   $A = S(D_m \xrightarrow{A} D_m)$

Consider the double complex  
 $\Omega^0(m, A) \xrightarrow{d_{hor}} \Omega^1(m, A) \xrightarrow{d_{hor}} \Omega^2(m, A)$   
 $\cup \quad \cup$   
 $\mathcal{O}^{hor} \quad \mathcal{O}^{hor}$   
 Lemma Horizontal cohomology lives in top degree,  $\Omega^0(m, A) / \text{Im } d$

$g(x) = \int \mathcal{L}(x, \partial_x) dx$   
 $\mathcal{O}(g(x)) = \sum g(x) \partial_x^k$   
 Local functional  $= \Omega^0(m, A) / d(\Omega^0(m, A))$

We can compute this by  
 $H^0(\mathcal{O}^{hor}) \rightarrow H^0(\text{total})$



Example Free scalar field  $\phi$   
 Anti-field  $\phi^*$   
 $A =$  fns of  $\phi, \phi^* +$  derivatives  
 Linear functions  $\mathcal{O}$   
 $D_m \xrightarrow{\Delta} D_m$   $A = S^*(D_m \xrightarrow{\Delta} D_m)$

Consider the double complex  
 $\Omega^0(m, A) \xrightarrow{d_{\text{ext}}} \Omega^1(m, A) \xrightarrow{d_{\text{ext}}} \Omega^2(m, A)$   
 $\downarrow \text{d}_{\text{ext}} \quad \downarrow \text{d}_{\text{ext}}$   
 Lemma Horizontal cohomology lives in top degree  $\Omega^0(m, A) / \text{im } d$

Example  
 $\Sigma \rightarrow \Sigma$   
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 $A =$  1-forms on  $\Sigma$   
 $[A, \mathcal{O}(A)]$

Example  
 The spectral sequence associated to a double complex  
 $E_1 = H^0(\text{horizontal})$   
 $E_2 = H^0(\text{vertical})$   
 $E_3 = H^0(\text{total})$   
 We can compute this by  
 a spectral sequence  $H^0(m, H^1(A)) \Rightarrow H^1(\text{total complex})$

Example

4-d CS theory  
 $\Sigma$  4d surface  
 $\Sigma \times \mathbb{C}$   
 $A = 1$ -form w no  $d_2$  component  
 $\int \text{tr}(A)$

What are local operators?  
 = EOM say  
 $F_{3,2}(A) = 0$   $F_{2,2}(A) = 0$   $F_{1,2}(A) = 0$   
 Key point  
 All sol's locally are trivial  
 Locally flat, non-trivial globally  
 $g \in \mathbb{Z}$   
 Operators built from  $c_1, d_2^c, d_2^s$

Local operators have  
 $C(g) \cong \mathbb{Z}$   
 Calculating computed by  
 Equal to intersection of elements  
 Only following case  
 $C(g)$  gives us no information

$$H^1(\Sigma) = H^1_{\text{de}}(\mathbb{C})$$

$$H^2(\Sigma) = H^2(\mathbb{C})$$

In our context/did work

$$\frac{1}{2\pi} \int_{\Sigma} \text{tr}(A \wedge A) = \int_{\Sigma} \text{tr}(A \wedge A)$$

Corollary

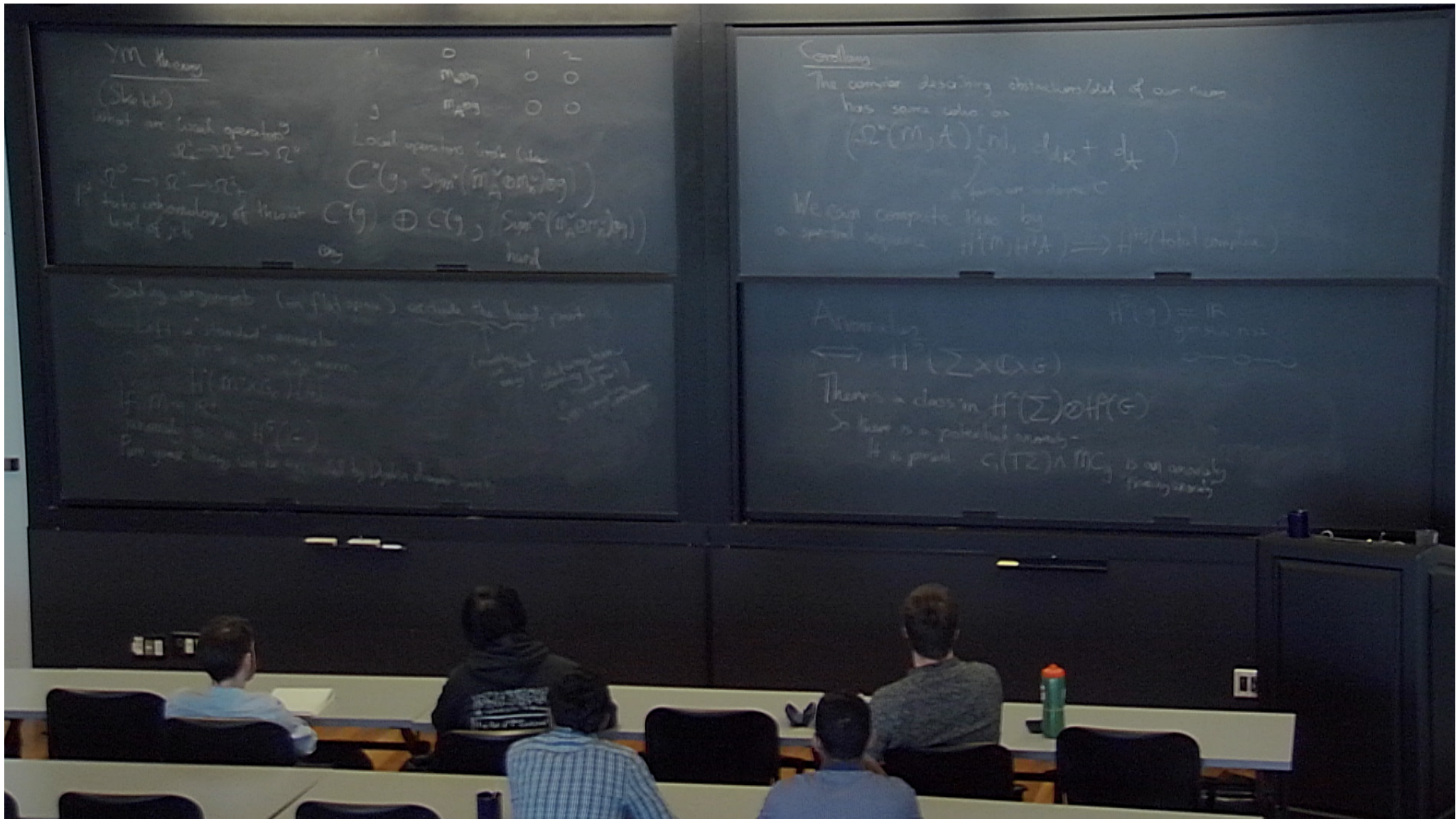
The complex describing obstructions/did of our theory  
 has some values on  
 $(\Omega^*(M, A)) \otimes (n_1, d_{\text{dR}} + d_A)$

We can compute this by  
 a spectral sequence  $H^*(M, H^*(A)) \Rightarrow H^*(\text{total complex})$

Assumption

$$\Leftrightarrow H^1(\Sigma \times \mathbb{C}) = 0$$

There is a class in  $H^1(\Sigma) \otimes H^1(\mathbb{C})$   
 So there is a potential anomaly -  
 It is fixed  $c_1(T\Sigma) \wedge \text{tr}(A)$  is an anomaly



YM theory

(Sketch) what are local operators  
 $\Omega^0 \rightarrow \Omega^1 \rightarrow \Omega^2$   
 takes cohomology of this at level of  $\mathfrak{g}$

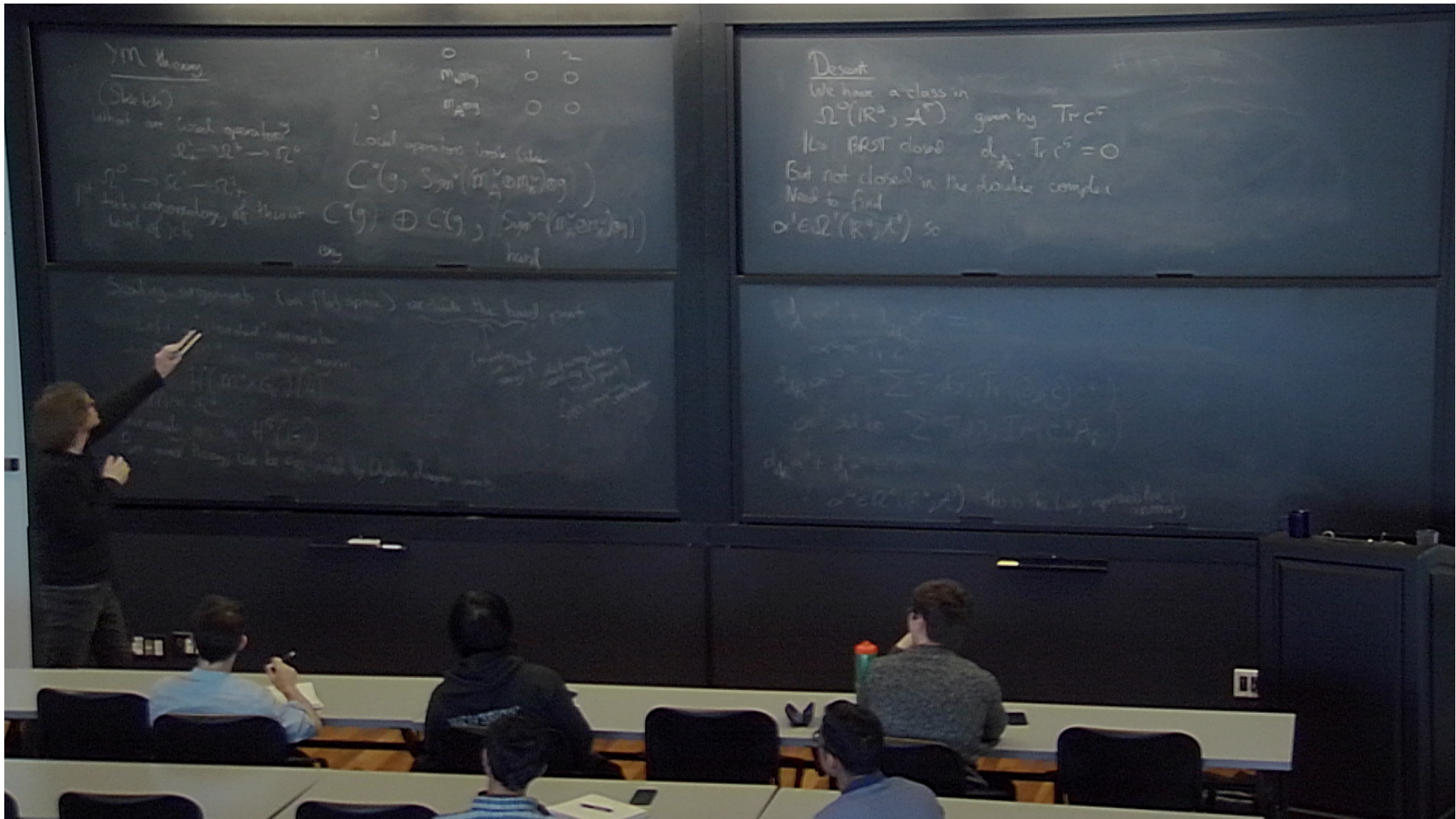
1	0	1	2
	$\mathfrak{m}_{\text{reg}}$	0	0
	$\mathfrak{m}_{\text{reg}}$	0	0

Local operators (in  $\mathfrak{g}$ )  
 $C^*(\mathfrak{g}, \text{Sym}(\mathfrak{m}_{\text{reg}}^{\otimes k} \otimes \mathfrak{g}))$   
 $C^*(\mathfrak{g}) \oplus C^*(\mathfrak{g}, \text{Sym}^{\otimes 2}(\mathfrak{m}_{\text{reg}}^{\otimes 2} \otimes \mathfrak{g}))$   
 on hard

Scaling arguments (in flat space) describe the hard part  
 $H^1(\Sigma \times \mathbb{R}^2)$   
 $H^1(\Sigma) \oplus H^1(\mathbb{R}^2)$   
 $H^1(\Sigma) \oplus H^1(\mathbb{R}^2)$   
 For some things can be computed by explicit diagrams

Corollary  
 The complex describing obstructions/det of our maps has same value as  
 $(\Omega^*(M, A)(n), d_{\mathbb{R}} + d_A)$   
 We can compute this by a spectral sequence  $H^i(M)H^j(A) \Rightarrow H^{i+j}(\text{total complex})$

Asymptotic  $H^1(\Sigma) = \mathbb{R} \oplus \mathbb{R} \oplus \mathbb{R}$   
 $\Leftrightarrow H^1(\Sigma \times \mathbb{R}^2)$   
 There's a class in  $H^1(\Sigma) \otimes H^1(\mathbb{R}^2)$   
 So there is a potential energy -  
 If a point  $C(\mathbb{R}^2) \wedge \mathfrak{m}_{\text{reg}}$  is an anomaly free theory



YM theory

(Sketch)  
 what are local operators?  
 $\Omega^0 \rightarrow \Omega^1 \rightarrow \Omega^2$   
 $\Omega^0 \rightarrow \Omega^1 \rightarrow \Omega^2$   
 takes cohomology of this at level of  $\mathfrak{g}$

-1	0	1	2
	$\mathfrak{m}_{\text{sym}}$	0	0
	$\mathfrak{m}_{\text{asym}}$	0	0

Local operators involve (class)  
 $C^*(\mathfrak{g}, \text{Sym}^*(\mathfrak{m}_{\text{sym}} \oplus \mathfrak{m}_{\text{asym}}))$   
 $C^*(\mathfrak{g}) \oplus C^*(\mathfrak{g}, \text{Sym}^*(\mathfrak{m}_{\text{sym}} \oplus \mathfrak{m}_{\text{asym}}))$   
 on hard

Descent

We have a class in  $\Omega^0(\mathbb{R}^2, \mathcal{A}^*)$  given by  $\text{Tr } c^5$   
 $\mathbb{K} \rightarrow \text{PRST closed } d_{\mathcal{A}} \text{Tr } c^5 = 0$   
 But not closed in the double complex  
 Need to find  $\alpha^1 \in \Omega^1(\mathbb{R}^2, \mathcal{A}^*)$  so

Scaling arguments (on flat space) include the hard part

local operators

$H^0(S^1) = \mathbb{R}$   
 $H^1(S^1) = \mathbb{R}$   
 $H^2(S^1) = 0$

local operators can be represented by De Rham cohomology

$d_{\mathcal{A}}^2 = 0$   
 $d_{\mathcal{A}}^2 = \sum \langle d_{\mathcal{A}}^2, \text{Tr } c^5 \rangle$   
 $d_{\mathcal{A}}^2 = \sum \langle d_{\mathcal{A}}^2, \text{Tr } c^5 \rangle$   
 $d_{\mathcal{A}}^2 = \sum \langle d_{\mathcal{A}}^2, \text{Tr } c^5 \rangle$   
 $d_{\mathcal{A}}^2 = \sum \langle d_{\mathcal{A}}^2, \text{Tr } c^5 \rangle$   
 this is the Lie algebra cohomology