

Title: Conformal Horizon Fluctuations in de Sitter Space, Dynamical Dark Energy & the CMB

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Abstract: <p>Infrared sensitivity of the de Sitter decay rate due to particle creation requires that gravitational backreaction be taken into account on the horizon scale. At lowest order, backreaction can be studied by Linear Response of the geometry to quantum matter perturbations around the Bunch-Davies state. In Linear Response the scalar degree of freedom derived from the conformal anomaly gives rise to scalar gravitational waves that grow without bound on the de Sitter horizon scale,&nbsp; which implies substantial non-linear quantum backreaction effects in cosmology. Fluctuations in the conformal scalar are potentially responsible for the primordial density fluctuations observed in the CMB anisotropy without an inflaton, and can also lead to dynamical, spacetime dependent dark energy. Possible observational tests of the conformal origin of primordial density fluctuations is suggested by the prediction of equality of scalar and tensor weight spectral indices, and the bispectral shape function of non-Gaussian correlations in the CMB predicted by conformal invariance.</p>

# Instability of de Sitter Space (Expanding FRW)

- Particle Creation & Backreaction

$$\frac{\partial H}{\partial \tau} = -4\pi G (\rho + p)$$

Compare to ‘Cosmological Electric Field Problem’

‘Shorting’ the vacuum

$$\frac{\partial \vec{E}}{\partial t} = -\vec{j}$$

- Maximally Symmetric BD State is **Unstable** to Spontaneous Real Particle Creation in each case
- Vacuum must be defined in Real Time by Stueckelberg-Feynman-Schwinger-deWitt  **$m^2-i\epsilon$**  not Euclidean continuation
- Once created, particles must be allowed to **interact** & to induce a particle **cascade**
- Backreaction, interactions necessary, non-trivial dynamics

# What happens in Global De Sitter Spacetime ?

- $S^3$  Hyperboloid Coordinates

$$T = H^{-1} \sinh u$$

$$W = H^{-1} \cosh u \cos \chi$$

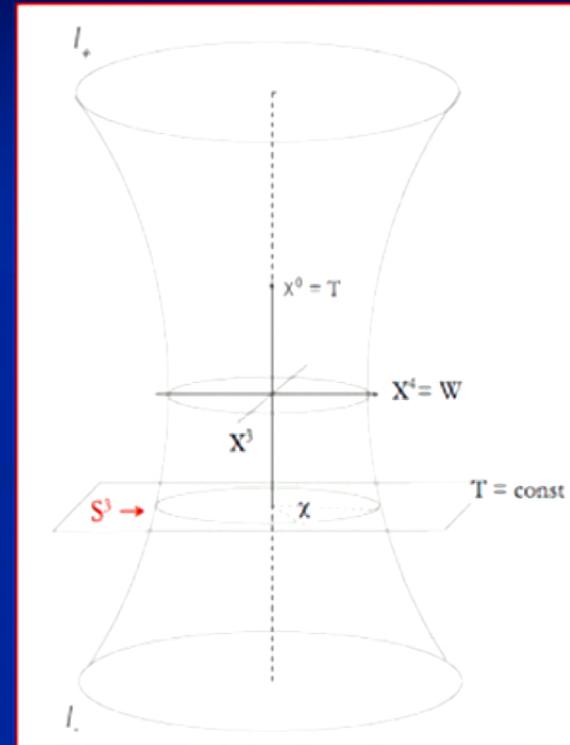
$$X^i = H^{-1} \cosh u \sin \chi \hat{n}^i$$

$$i = 1, 2, 3$$

$$ds^2 = H^{-2} [-du^2 + \cosh^2 u d\Omega_3^2]$$

$$d\Omega_3^2 \equiv d\chi^2 + \sin^2 \chi d\Omega^2$$

- Cover Complete Manifold
- Symmetry Group of  $S^3$  at const.  $u$  is  $O(4)$
- Maximally Symmetric State is CTBD State



# Potential Scattering on de Sitter Hyperboloid

- Solns. of Scalar Wave Eq. on Full Hyperboloid  $\Phi \sim y_k(u) Y_{klm}(\hat{N})$

$$\left[ \frac{d^2}{du^2} + 3 \tanh u \frac{d}{du} + (k^2 - 1) \operatorname{sech}^2 u + \left( \frac{9}{4} + \gamma^2 \right) \right] y_k = 0$$

- O(4,1) invariant CTBD state:  $y_k = v_{k\gamma} = v_{k\gamma}^*(-u)$

$$v_{k\gamma}(u) \equiv H c_{k\gamma} (\operatorname{sech} u)^{k+1} (1 - i \sinh u)^k F \left( \frac{1}{2} + i\gamma, \frac{1}{2} - i\gamma; k+1; \frac{1 - i \sinh u}{2} \right)$$

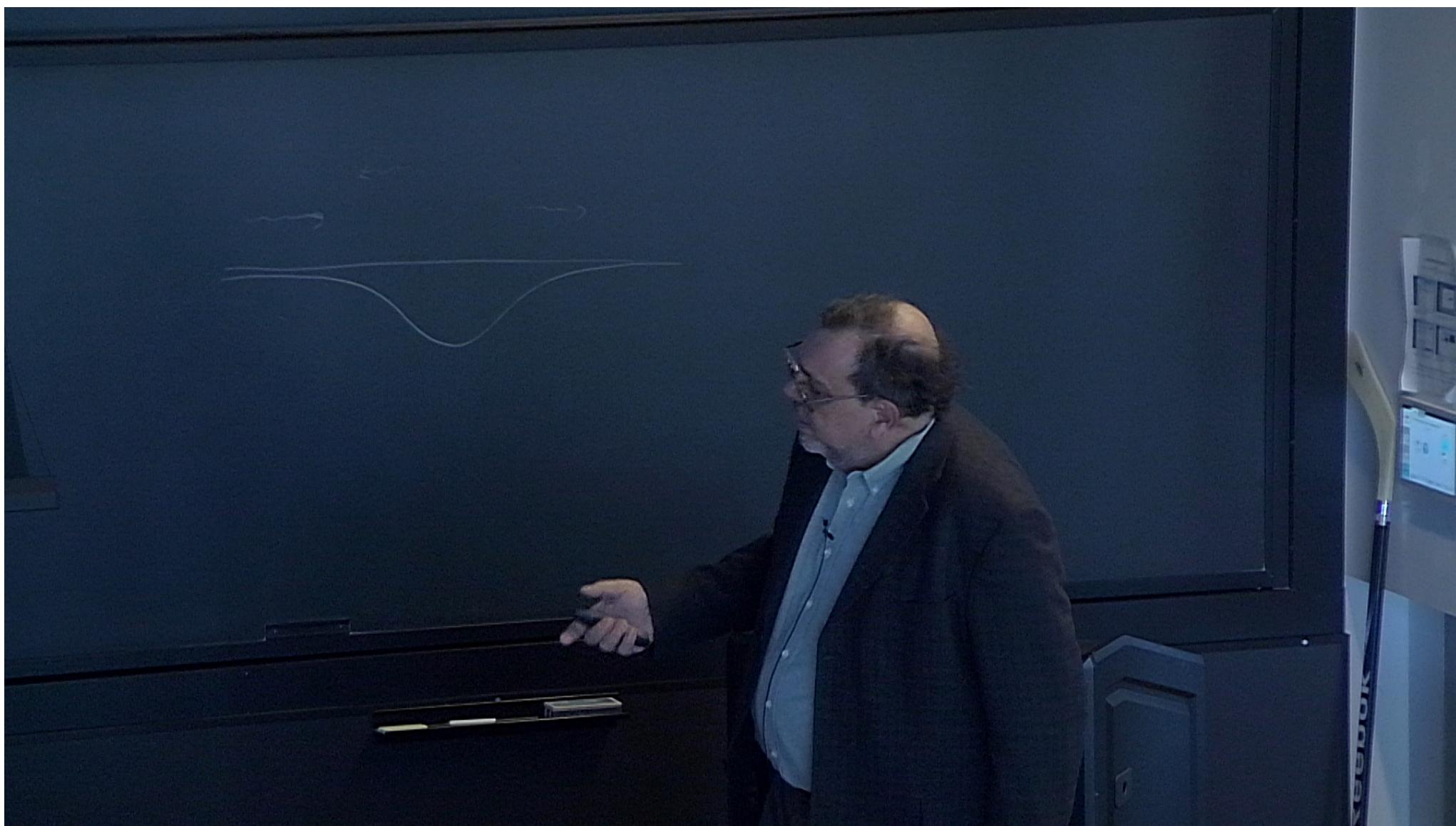
- Time Dependent Oscillator Modes  $f_{k\gamma} = a^{\frac{3}{2}} y_k \quad F_{k\gamma} = a^{\frac{3}{2}} v_{k\gamma}$

- Over the Barrier Scattering  $\left[ -\frac{d^2}{du^2} + \mathcal{U}_k(u) \right] f_{k\gamma} = \gamma^2 f_{k\gamma}$

- Negative Potential  $\mathcal{U}_k(u) \equiv - \left( k^2 - \frac{1}{4} \right) \operatorname{sech}^2 u$

- Scattering Solutions  $f_{k\gamma} \sim e^{\mp i\gamma u}$  as  $|u| \rightarrow \infty \quad \gamma = \sqrt{\frac{m^2}{H^2} - \frac{1}{4}}$

- Vacuum defined by  $m^2 - i\epsilon$  (Feynman/Schwinger not Euclidean)



# In/Out de Sitter Scattering Solns.

- In Soln.  $f_{k\gamma(+)} \Big|_{u<0} \sim (\cosh u)^{\frac{1}{2}} Q_{-\frac{1}{2}-i\gamma}^{-k}(i \sinh u)$   
 $\rightarrow \boxed{\text{const. } e^{-i\gamma u}} \quad u \rightarrow -\infty$

- Out Soln.  $f_{k\gamma}^{(+)} \Big|_{u>0} \sim (\cosh u)^{\frac{1}{2}} Q_{-\frac{1}{2}+i\gamma}^{-k}(i \sinh u)$   
 $\rightarrow \boxed{\text{const. } e^{-i\gamma u}} \quad u \rightarrow +\infty$

- Bogoliubov Transf.  
**In to CTBD**  $\begin{pmatrix} f_{k\gamma(+)} \\ f_{k\gamma(-)} \end{pmatrix} = \begin{pmatrix} A_{k\gamma}^{in} & B_{k\gamma}^{in} \\ B_{k\gamma}^{in*} & A_{k\gamma}^{in*} \end{pmatrix} \begin{pmatrix} F_{k\gamma} \\ F_{k\gamma}^* \end{pmatrix}$

- Out to CTBD**  $\begin{pmatrix} f_{k\gamma}^{(+)} \\ f_{k\gamma}^{(-)} \end{pmatrix} = \begin{pmatrix} A_{k\gamma}^{out} & B_{k\gamma}^{out} \\ B_{k\gamma}^{out*} & A_{k\gamma}^{out*} \end{pmatrix} \begin{pmatrix} F_{k\gamma} \\ F_{k\gamma}^* \end{pmatrix}$  **CTBD**

- In and Out Related by Time Reversal  $A_{k\gamma}^{out} = A_{k\gamma}^{in*} \quad B_{k\gamma}^{out} = B_{k\gamma}^{in*}$
- CTBD State is Time Symmetric - Halfway Between – Contains Particles w.r.t. either the In or Out Vacua

# Particle Creation ‘Events’ in Real Time

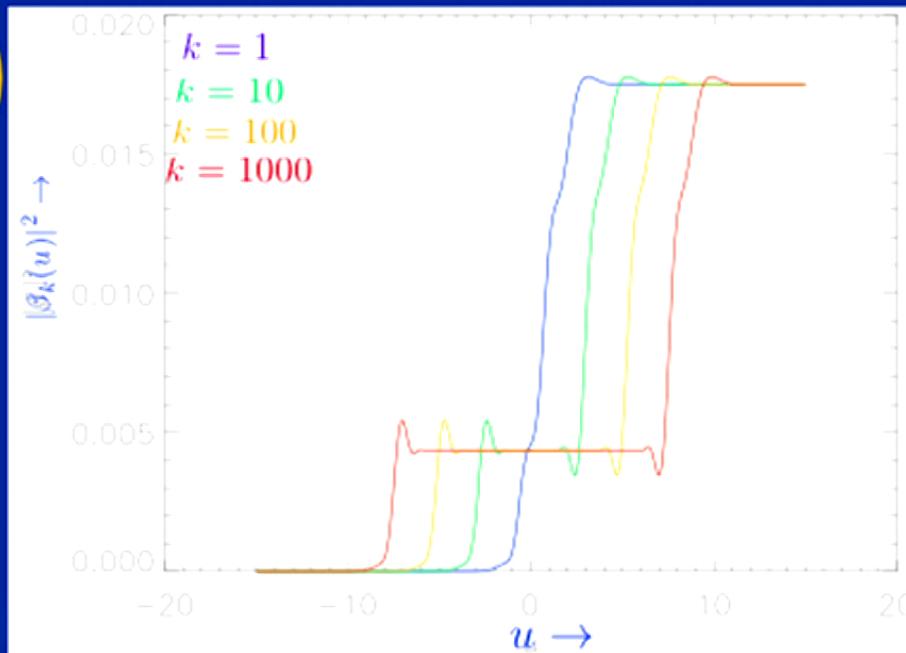
- Adiabatic (WKB) Frequency  $W_k \simeq \left[ \left( k^2 - \frac{1}{4} \right) \operatorname{sech}^2 u + \gamma^2 \right]^{\frac{1}{2}}$
  - Adiabatic Mode Fn.
  - Time Dependent Bogoliubov Transformation
- $$\begin{pmatrix} f_k \\ f_k^* \end{pmatrix} = \begin{pmatrix} \alpha_k & \beta_k \\ \beta_k^* & \alpha_k^* \end{pmatrix} \begin{pmatrix} \tilde{f}_k \\ \tilde{f}_k^* \end{pmatrix}$$

- Now Two Distinct Creation ‘Events’

$$\Delta_1 |\beta_k|^2 = |B_k^{in}|^2 = \frac{1}{e^{2\pi\gamma} - 1}$$

$$\Delta_{tot} |\beta_k|^2 = |B_{k\gamma}^{tot}|^2 = \operatorname{csch}^2(\pi\gamma)$$

Magnitude indep. of  $k$   
But time of each event depends on  $k$



$$P_{\text{phys}} = \frac{k}{a}$$



# Stress-Energy Grows Relativistically in Contracting Phase

$$\varepsilon_R \simeq \frac{1}{2\pi^2 a^3} \sum_{k=K_\gamma(u)}^{K_\gamma(u_0)} k^2 \varepsilon_k^N \Delta \mathcal{N}_{1k\gamma}$$

$$\sim \frac{K_\gamma^4(u_0)}{a^4(u)} \sim \frac{\sinh^4 u_0}{a^4(u)}$$

$$K_\gamma(u_0) < k < K_\gamma(u)$$

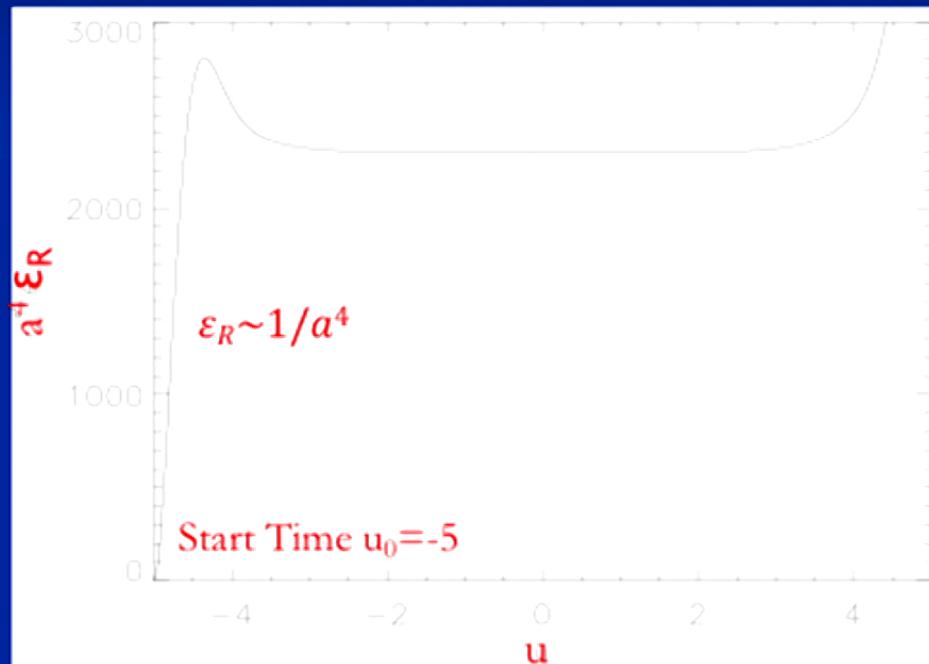
$$K_\gamma(u) = \sqrt{\gamma^2 \sinh^2 u + \frac{1}{4}}$$

Unbounded if started at earliest times

$$u_0 \rightarrow -\infty$$

UV / IR

Finite Set of  $k$  Modes undergo particle creation & contribute at Finite Times

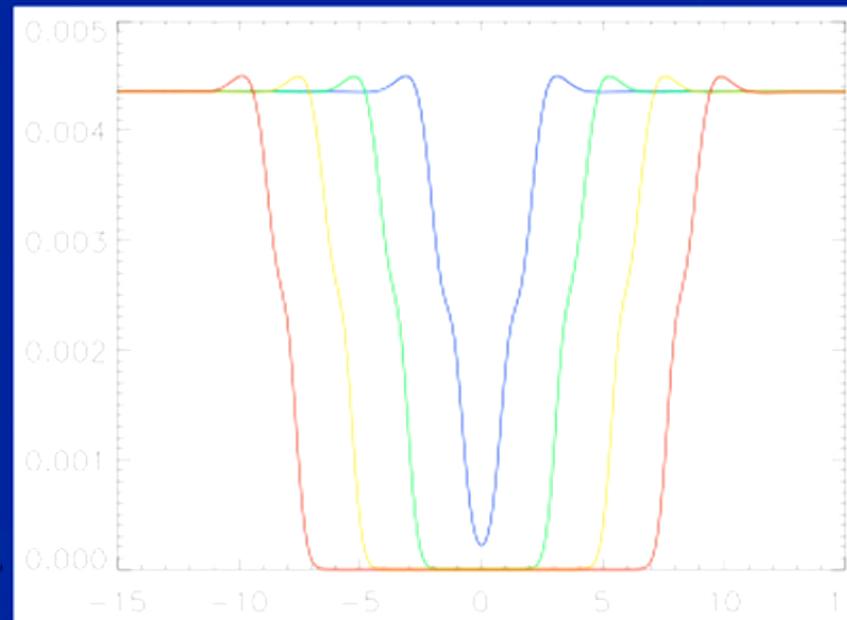


Shows explicitly why de Sitter space is unstable to Particle Creation and Subsequent Blueshifting to Ultra-Relativistic Energies over Long Times

## Bunch-Davies dS Invariant State in $S^3$ Sections

- State of maximum  $O(4,1)$  symmetry for massive fields
- Green Fn. can be obtained by Euclidean Continuation from  $S^4$
- Stress-Energy Tensor  $\langle T^a_b \rangle = p \delta^a_b = -\rho \delta^a_b = \text{const.}$   
Apparently “Nothing happens”
- Actually there is fine tuned equal and opposite time reversed coherent annihilation & creation in each  $k$  mode
- Analog of a time reversal invariant state in constant  $E$  field
- Imag. Part of Eff. Action  $\Gamma = 0$  for such a state
- Contains particles at any time

Not ground state or vacuum  
What happens if perturbed?



# Summary of Part I

- De Sitter Space  $O(4,1)$  Bunch-Davies State is **unstable** to Pair Creation
- In/Out States Fixed by  $m^2 - i\epsilon$  in **Real Time**, break **T** symmetry
- Decay Rate for Massive Field – Expanding Sections

$$\Gamma_{dS} = \frac{\kappa^3 \gamma^3 H^4}{4\pi^2} \ln(1 + |B_\gamma|^2)$$

$$\gamma = \sqrt{\frac{m^2}{H^2} - \frac{1}{4}}$$

$$|B_\gamma|^2 = (e^{2\pi\gamma} - 1)^{-1}$$

- Analog of Schwinger Effect in Constant **E** Field
- In contracting phase relativistic blueshifting of particles dominates  $\epsilon_R$
- Adiabatic Particles created robust, remain, verified by turning off dS
- But Sensitivity to Infrared Horizon at Late Times
- Realistic cosmology requires massless fields/backreaction/interactions
- **Intrinsic Quantum Mechanism for Relaxing  $\Lambda_{eff}$  Exists**

**Question: What is the Relevant Effective Field Theory ?**

## Answer: EFT Required by Conformal Anomaly

- Assuming ***Equivalence Principle***

Universal Coupling to Matter through Metric  $g_{ab}$

- Only two strictly ***relevant*** operators ( $R, \Lambda$ )
- Einstein's General Relativity is the (***almost*** unique) EFT
- But EFT = General Relativity + Quantum Corrections
- Semi-classical Einstein Eqs. ( $m \ll k \ll M_{Pl}$ ):

$$G_{ab} + \Lambda g_{ab} = 8\pi G \langle T_{ab} \rangle$$

- Imply a quantum Trace or ***Conformal Anomaly***:

$$\langle T_a^a \rangle = b C^2 + b' (E - \frac{2}{3} \square R) + b'' \square R$$

$$F = C_{abcd} C^{abcd} = C^2 \quad E = R_{abcd} R^{abcd} - 4R_{ab} R^{ab} + R^2$$

- Massless Poles***  $\Rightarrow$  New (marginally) relevant operator in gravitational sector with ***macroscopic*** effects

# Conformal Anomaly EFT

- Stress-Energy Tensor has Anomalous Trace

$$\langle T_a^a \rangle = bF + b' \left( E - \frac{2}{3} \square R \right) + b'' \square R$$

$$E \equiv {}^*R_{abcd} {}^*R^{abcd} = R_{abcd} R^{abcd} - 4R_{ab} R^{ab} + R^2$$

$$F \equiv C_{abcd} C^{abcd} = R_{abcd} R^{abcd} - 2R_{ab} R^{ab} + \frac{1}{3} R^2$$

$$b' = -\frac{1}{360} \frac{1}{16\pi^2} (N_s + 11N_F + 62N_V)$$

- Non-Local Anomaly Effective Action

$$S_{anom}[g] = \frac{1}{2} \int d^4x \sqrt{-g} \int d^4x' \sqrt{-g'} \left( \frac{E}{2} - \frac{\square R}{3} \right)_x \Delta_4^{-1}(x, x') \left[ bF + b' \left( \frac{E}{2} - \frac{\square R}{3} \right) \right]_{x'}$$

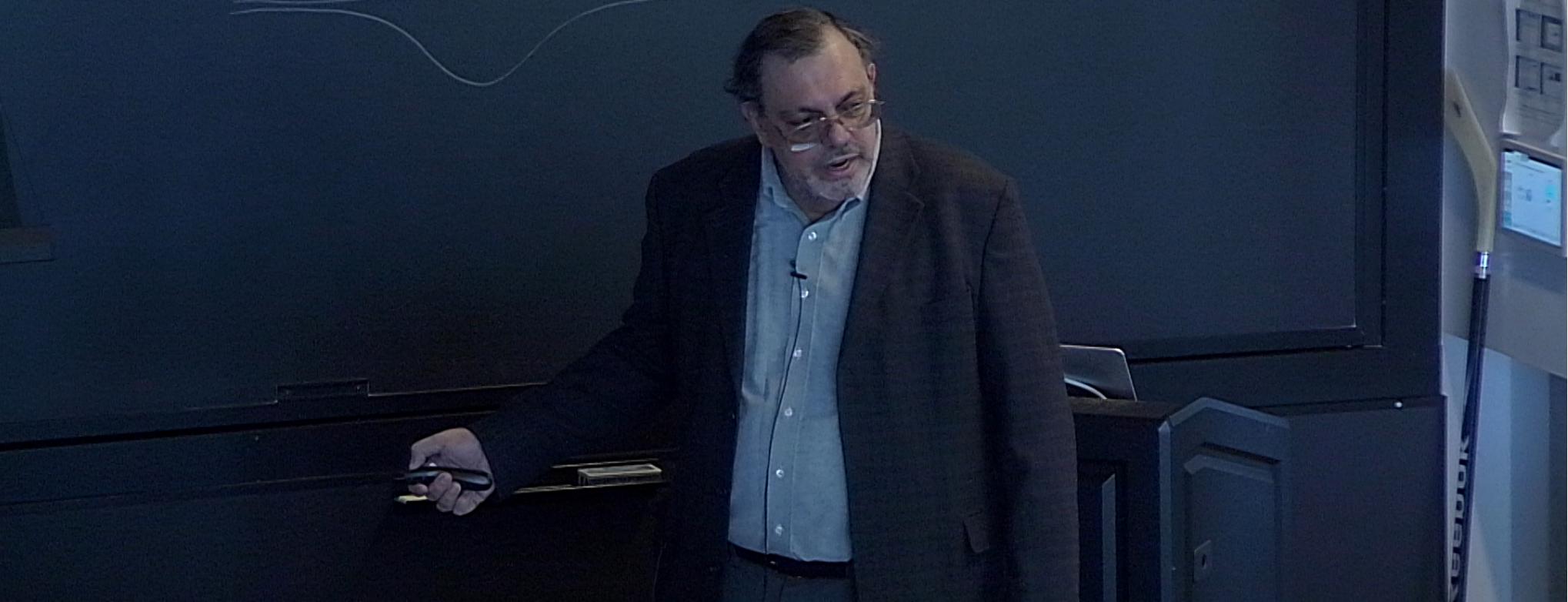
- Made Local by Introducing a Scalar Conformalon  $\varphi$

$$S_{anom}[g; \varphi] = \frac{b'}{2} \int d^4x \sqrt{g} [ -\varphi \Delta_4 \varphi + (E - \frac{2}{3} \square R + \frac{b}{b'} C^2) \varphi ]$$

$$\Delta_4 \varphi = \frac{1}{2} (E - \frac{2}{3} \square R + \frac{b}{b'} C^2)$$

- EFT Modifies GR in infrared—Conformal Factor of Metric Becomes Dynamical:  $\Lambda_{eff}$  can change

$$t_{\text{phys}} = \frac{k}{\alpha} e^{-iS_M - iS_g(\beta)}$$



# Anomaly & de Sitter Non-Invariance

- $\Delta_4 \equiv \square^2 + 2R^{ab}\nabla_a\nabla_b - \frac{2}{3}R\square + \frac{1}{3}(\nabla^a R)\nabla_a$

- In de Sitter Space

$$\Delta_4 \varphi|_{dS} = -\square(-\square + 2H^2)\varphi = \frac{1}{2}E - \frac{1}{3}\square R = 12H^4 > 0$$

- Soln. necessarily breaks  $O(4,1)$  de Sitter Invariance
- Factorization of the Propagator

$$\Delta_4^{-1}|_{dS} = \frac{1}{2H^2} [(-\square)^{-1} - (-\square + 2H^2)^{-1}]$$

- No de Sitter invariant inverse of  $\square$  or  $\Delta_4$  exists (IR)
- General Spatially  $O(4)$  Invariant Soln. in Closed  $S^3$  Sections

$$\varphi_1(u) = 2\ln(\cosh u) + c_0 + c_1 \sin^{-1}(\tanh u) + c_2 \operatorname{sech}^2 u + c_3 \tanh u \operatorname{sech} u$$

- Leads to Anomaly

$$T_{ab} = \frac{1}{\sqrt{-g}} \frac{\delta S_{anom}[g, \varphi_1]}{\delta g^{ab}} \Big|_{dS}$$

Stress Tensor BD Energy Density

$$\varepsilon = -6b'H^4 + \frac{2b'}{a^4} (c_1^2 - c_2^2 - c_3^2 + 4)$$

Arbitrarily Sensitive to initial perturbation in Blueshift (contracting phase)

# Anomaly Stress Tensor in de Sitter Space

- Semi-Classical de Sitter with  $O(4,1)$  Invariant BD State

$$R_{ab} - \frac{R}{2}g_{ab} + \Lambda g_{ab} = 8\pi G_N \langle T_{ab} \rangle_R \rightarrow 8\pi G_N T_{ab}[\varphi_{BD}]$$

- In spatially flat FRW coordinates

$$ds^2|_{dS} = -d\tau^2 + e^{2H\tau} d\vec{x}^2$$

- Solve for Conformal Scalar in de Sitter Space

$$\Delta_4 \varphi|_{dS} = -\square(-\square + 2H^2) \varphi_{BD} = \frac{E_{dS}}{2} = 12H^4$$

- Self-Consistent de Sitter Bunch-Davies State Soln.

$$\varphi_{BD} = 2 \ln a = 2H\tau \quad (\text{conformal to flat})$$

- de Sitter Invariant Stress Tensor for BD State of QFT of any spin

$$T_{ab}[\varphi_{BD}] = -\frac{2}{\sqrt{-g}} \frac{\delta}{\delta g^{ab}} S_{anom}[g; \varphi_{BD}] = 6b' H^4 g_{ab}$$

- Trace  $-R + 4\Lambda = 8\pi G_N b' E_{dS} \Rightarrow H^2 = \frac{\Lambda}{3} + \frac{8\pi G_N}{3} b' H^4$

# Linearized Scalar Metric Perturbations

- Scalar Conformalon Gravitational Wave in de Sitter Space

$$v \sim \frac{1}{a^2} \exp\left(ik \cdot \vec{x} \pm \frac{ik}{a}\right) \quad \text{Blueshift Divergent } a \rightarrow 0$$

(scalar conformal waves in conformally flat de Sitter)

- Correspond to Metric Scalar Perturbations

$$g_{ab} = g_{ab}|_{as} + \delta g_{ab} \quad \varphi = \varphi_{BD} + \delta\varphi$$

$\delta g_{\tau\tau} = -2\Delta$        $\delta_{\tau\tau} = \delta_{\tau\mu} = \delta_{\mu\mu}$

- Non-trivial Soln of  $\tau\tau$  and  $\tau i$  constraints require

$$\delta R^\tau_\tau = -\frac{16\pi G_N}{3} \frac{\vec{\nabla}^2}{a^2} v \quad \delta R^\tau_i = -\frac{16\pi G_N}{3} \vec{\nabla}_i \left( \frac{\partial}{\partial \tau} + 2H \right) v$$

$\frac{\partial v}{\partial \tau} = \frac{v''}{v} = \frac{\partial \ln v}{\partial \tau} = \frac{\partial^2 \ln v}{\partial \tau^2} = \frac{\vec{\nabla}^2 v}{a^2}$   
 and  $\delta R^\tau_i = 0$  with  $v' = \left( \frac{v''}{\partial \tau^2} + H \frac{\partial v}{\partial \tau} - \frac{v'''}{a^2} \right) v \neq 0$

- Sourced by Conformalon (via  $v$ ) in Linearized Einstein Eqs.

$$\Upsilon_A + \Upsilon_C = -\frac{16\pi G_N b'}{3} v$$

$$\frac{\vec{\nabla}^2}{a^2} (\Upsilon_A - \Upsilon_C) = \frac{16\pi G_N H b'}{a^2} \frac{\partial}{\partial \tau} (a^2 v)$$

Compare to

flat space

# Linearized Scalar Metric Perturbations

- Scalar Conformalon Gravitational Wave in de Sitter Space

$$v \sim \frac{1}{a^2} \exp\left(i\vec{k} \cdot \vec{x} \pm \frac{ik}{a}\right) \quad \text{Blueshift Divergent } a \rightarrow 0$$

- Correspond to Metric Scalar Perturbations

$$\begin{aligned} \delta g_{\tau\tau} &= -2A \\ \delta g_{\tau i} &= a \partial_i B \\ \delta g_{ij} &= 2a^2 (\eta_{ij} C + \partial_i \partial_j D) \end{aligned}$$

- For Gauge Invariant Scalars

$$\Upsilon_A \equiv A + \partial_\tau(aB) - \partial_\tau(a^2 \partial_\tau D)$$

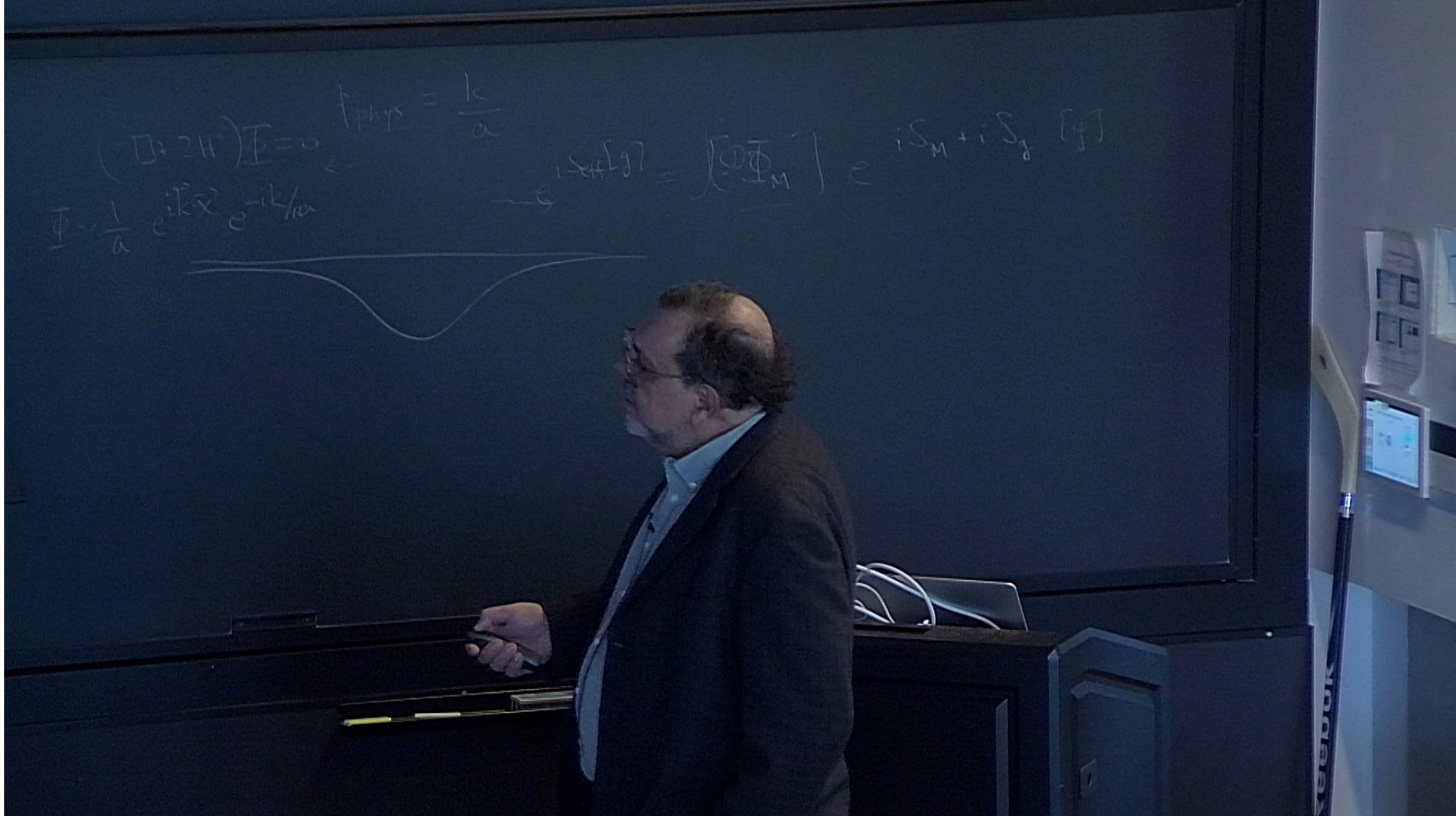
$$\Upsilon_C \equiv C + (\partial_\tau a)B - a(\partial_\tau a)(\partial_\tau D)$$

- Sourced by Conformalon (via  $v$ ) in Linearized Einstein Eqs.

$$\boxed{\begin{aligned} \Upsilon_A + \Upsilon_C &= -\frac{16\pi G_N b'}{3} v \\ \frac{\vec{\nabla}^2}{a^2} (\Upsilon_A - \Upsilon_C) &= \frac{16\pi G_N H b'}{a^2} \frac{\partial}{\partial \tau} (a^2 v) \end{aligned}}$$

Compare to  
flat space

$$(-\nabla^2 + 2H)\Psi = 0$$
$$\Psi \sim \frac{1}{a} e^{ik\bar{x}^2} e^{-ik/a}$$
$$t_{\text{phys}} = \frac{k}{a}$$
$$e^{iS_M[\Psi]} = \langle \Psi | \bar{\Psi} \rangle e^{i(S_M + S_S)[\Psi]}$$



## De Sitter Space is Also ‘Static’

- Static Coordinates

$$T = H^{-1} \sqrt{1 - H^2 r^2} \sinh(Ht)$$

$$W = H^{-1} \sqrt{1 - H^2 r^2} \cosh(Ht)$$

$$X^i = r n^i, \quad i = 1, 2, 3$$

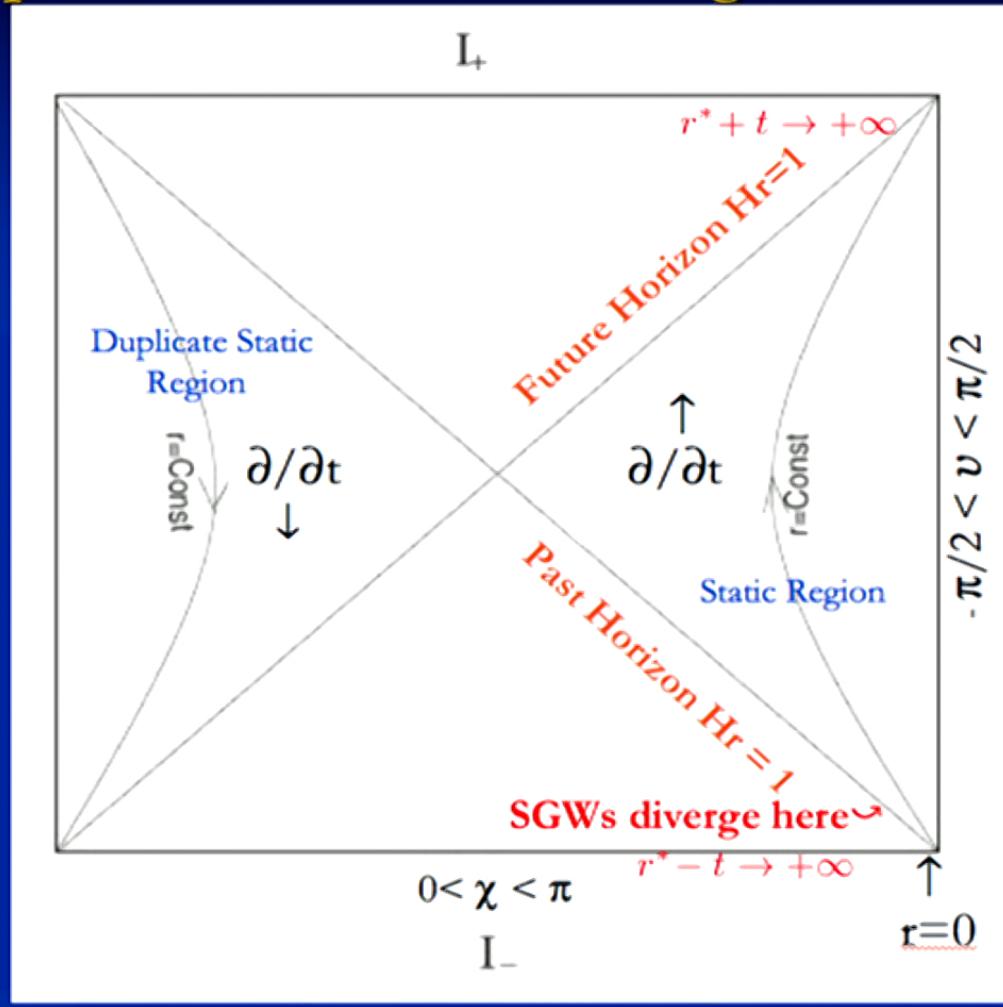
$$ds^2 = -f(r) dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2$$

$$f(r) \equiv 1 - H^2 r^2$$

- Similar to Static Gauge of Constant Electric Field
- Covers only the One Quarter Manifold with  $T + W > 0, W > 0$
- **Horizon** at  $r = 1/H$  breaks spatial homogeneity & translations

# De Sitter Space Carter-Penrose Diagram

- Scalar Gravitational Wave Solns. Diverge on Past Horizon
- By Time Reversal there are also solns. that diverge on the Future Horizon



# Scalar GWs in dS Static Coordinates

- In Static Coordinates Infinite Blueshift at Horizon  $r \rightarrow H^{-1}$

$$ds^2 = -f dt^2 + \frac{dr^2}{f} + r^2 d\Omega^2 \quad f(r) = 1 - H^2 r^2$$

- Linear perturbations from general solution of

$$\Delta_4|_{dS} \varphi_{\omega\ell m} = \square(\square - 2H^2) \varphi_{\omega\ell m} = 0$$

- Regularity at the origin selects only 2 of the 4 possible solns.
- Only 1 Soln. Projected by the Linearized Einstein Constraints

$$v_{\omega\ell m}^{(\pm)} = \left[ \frac{1}{f} \left( \frac{\partial^2}{\partial t^2} - 2H \frac{\partial}{\partial t} \right) - \frac{f}{r^2} \left( r^2 \frac{\partial}{\partial r} \right) + \frac{L^2}{r^2} \right] \varphi_{\omega\pm iH, \ell m} \neq 0$$

Generically Diverges on the de Sitter Horizon  $r \rightarrow H^{-1}$

$$v_{\omega\ell m}^{(\pm)} = e^{-(i\omega \mp H)t} Y_{\ell m}(\hat{n}) \frac{1}{Hr\sqrt{1-H^2r^2}} Q_\ell^{i\omega/H}\left(\frac{1}{Hr}\right)$$

$$\rightarrow Y_{\ell m}(\hat{n}) e^{H(r^* \pm t)} \left\{ \alpha_{-\omega, \ell} e^{-i\omega(t+r^*)} + \alpha_{\omega, \ell} e^{-i\omega(t-r^*)} \right\} \quad r^* \pm t \rightarrow +\infty$$

**There is no soln. finite on both Future & Past Horizon**

# Scalar GWs in dS Static Coordinates

- Gauge Invariant Gravitational Potentials

$$\Upsilon_A + \Upsilon_C \propto v_{\omega\ell m}^{(\pm)}$$

$$\Upsilon_A - \Upsilon_C \propto \varphi_{\omega\pm iH,\ell m}$$

- Diverge on Either Past or Future Horizon

$$r \rightarrow H^{-1} \quad r^* \equiv \frac{1}{2H} \ln \left( \frac{1+Hr}{1-Hr} \right) \rightarrow \infty$$

- Correspond to steady energy flux into/out of the static patch
- No soln., finite on both horizons with no net energy flux into/out of horizon  $\rightarrow$  Backreaction must occur on horizon scale
- Static solns. are also all divergent on the horizon

$$(\Upsilon_A + \Upsilon_C)_{static} \propto \frac{c_1}{f} + \frac{c_2}{Hrf}$$

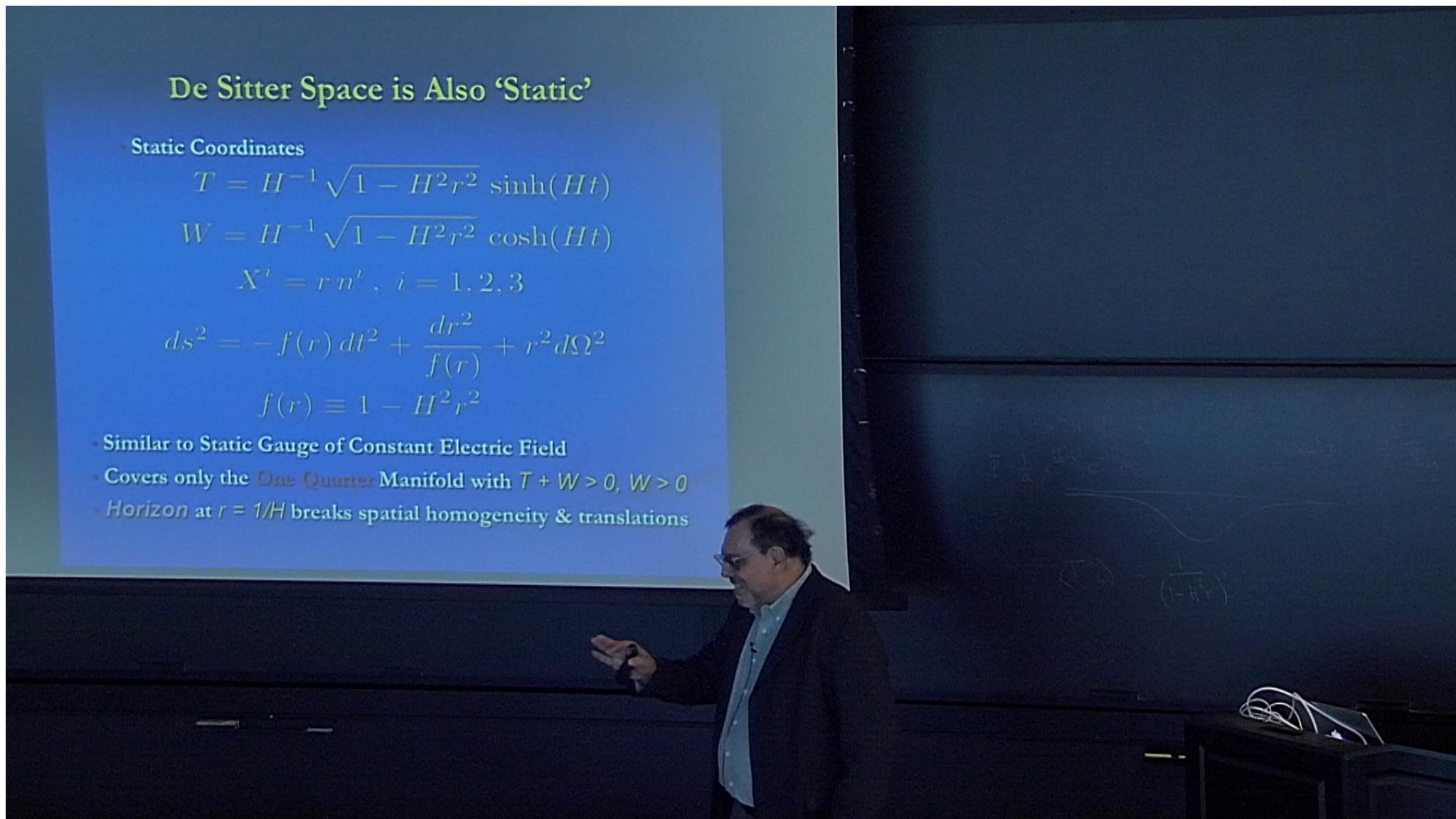
De Sitter Invariance necessarily broken by linear perturbations

## De Sitter Space is Also ‘Static’

- ## • Static Coordinates

$$T = H^{-1} \sqrt{1 - H^2 r^2} \sinh(Ht)$$

- Similar to Static Gauge of Constant Electric Field
  - Covers only the One Quarter Manifold with  $T + W > 0, W > 0$
  - Horizon at  $r = 1/H$  breaks spatial homogeneity & translations



## Cosmological Horizon Modes

- $\delta\langle T_{\mu\nu}\rangle$  in de Sitter space contains state-dependent contributions from  $S_{\text{anom}}$  of conformal scalar  $\phi$  (coherent state variations)
- This is a new relevant scalar degree of freedom in cosmology
- The relevant linearized scalar modes satisfy a **second order** wave eq.
- GR + QFT Anomaly: **Dynamical SGWs** of Metric—Not Inflaton
- **Grow unbounded** close to the de Sitter horizon  $r_H = H^{-1}$
- Static soln for Difference of Gauge Invariant Potentials

$$\Upsilon_C - \Upsilon_A = 8\pi G H^2 b' \left[ \frac{c_1}{Hr} \ln \left( \frac{1 - Hr}{1 + Hr} \right) + \frac{c_2}{Hr} \ln f \right]$$

- Logarithmic behavior as  $r \rightarrow r_H$  indicates a weight  $w=0$  conformal field just as required for the scale invariant HZ Spectrum



## Conformal Invariance on dS Horizon (dS/CFT)

- Realization of Conformal Invariance in dS in Static Coordinates

$$ds^2 = -f(r) dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2 \equiv f(r) ds_{opt}^2$$
$$f(r) \equiv 1 - H^2 r^2$$

showing the de Sitter horizon at  $r = r_H = H^{-1}$ ,  $f(r) = 0$

- $ds_{opt}^2$  is called the 'optical' metric
- On static  $t = \text{const.}$  slices in Poincare coordinates it is

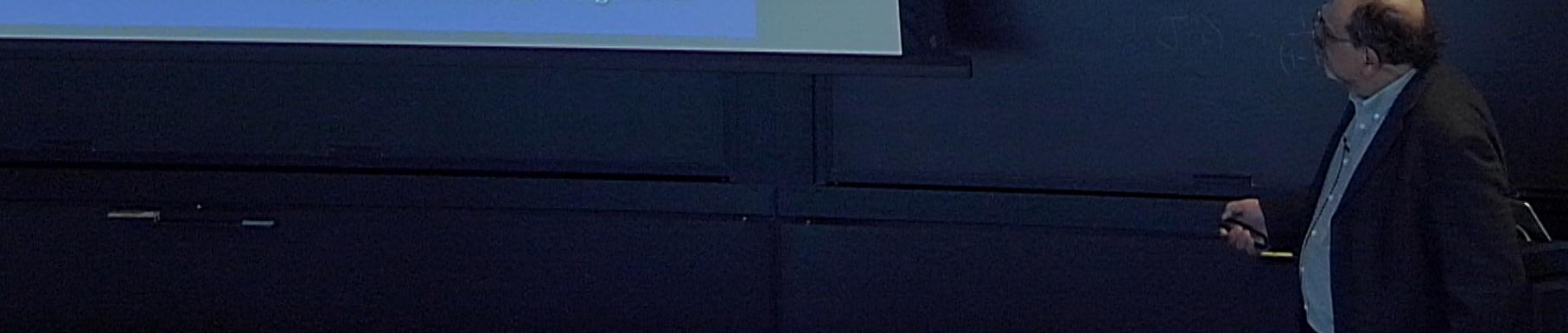
$$\bar{x} \equiv \frac{r n_x}{1 - H r n_z} \quad ds_{opt}^2 = d\ell_L^2 = \frac{1}{H^2 \bar{z}^2} (d\bar{z}^2 + d\bar{x}^2 + d\bar{y}^2)$$
$$\bar{y} \equiv \frac{r n_y}{1 - H r n_z}$$
$$\bar{z} \equiv \frac{r_n f^{\frac{1}{2}}}{1 - H r n_z}$$

Lobachevsky (Euclidean AdS<sub>3</sub>) Space  
- Conformal Behavior expected at the  
Horizon Boundary  $\bar{z} \rightarrow 0$ ,  $r \rightarrow r_H$



## Conformal Symmetry: dS/CFT

- de Sitter group SO(4,1) has 10 Killing vectors (isometries)  
decomposing on the de Sitter horizon sphere into
  - 3 rotations
  - 3 conformal transformations  $\times 2 = 6$
  - 1 time translation
- The subgroup SO(3,1) is the Lorentz group of null rays, and the conformal group of  $S^2$  of the horizon sphere
  - 3 rotations
  - 3 conformal transformations (above)
- Any SO(4,1) de Sitter invariant Green's fn. is SO(3,1) conformally invariant on the de Sitter horizon (dS/CFT) eg.  
$$\Delta_4^{-1}(x, x') \rightarrow -\frac{1}{16\pi^2} \ln(1 - \hat{n} \cdot \hat{n}') + c_0 \propto \Delta_2^{-1}(\hat{n}, \hat{n}')$$
- Dimension/conformal weight 0 field correlator = logarithm



## Conformal Invariance on Horizon Sphere

- Invariant Distance on Lobachevsky Space is

$$d^2(x, x') = \frac{1}{4\bar{z}\bar{z}'} \left[ (\bar{z} - \bar{z}')^2 + (\bar{x} - \bar{x}')^2 + (\bar{y} - \bar{y}')^2 \right]$$
$$\rightarrow \frac{1 - \hat{n} \cdot \hat{n}'}{2\sqrt{f(r)f(r')}}$$

- Conformal Weight  $w$  Correlation Fn. on  $S^2$  Horizon Sphere

$$G(x, x'; w) \propto [d(x, x')]^{-2w} \propto (1 - \hat{n} \cdot \hat{n}')^{-w}$$

- Conformal Weight  $w = 0$  is a logarithm

$$G(\hat{n}, \hat{n}'; w=0) \propto \ln(1 - \hat{n} \cdot \hat{n}') + c_0 \propto \sum_{\ell=1}^{\infty} \sum_{m=-\ell}^{\ell} \frac{Y_{\ell m}(\hat{n}) Y_{\ell m}(\hat{n}')}{\ell(\ell+1)} + c_0$$
$$\ell(\ell+1)c_{\ell} = 6c_2$$

$\Rightarrow$  Harrison-Zel'dovich Angular Power Spectrum

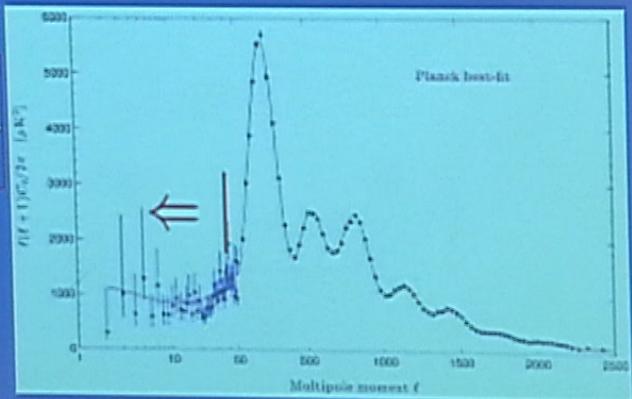
## Conformal Invariance on dS Horizon & CMB

$$\left\langle \frac{\delta T}{T}(\hat{n}) \frac{\delta T}{T}(\hat{n}') \right\rangle \propto \left\langle (\Upsilon_A - \Upsilon_C)(\hat{n})(\Upsilon_A - \Upsilon_C)(\hat{n}') \right\rangle \propto \ln(1 - \hat{n} \cdot \hat{n}')$$

has conformal weight  $w = 0$  scale invariant Harrison-Zel'dovich Spectral Index

$$n_s = 1 \Rightarrow \\ \ell(\ell + 1)c_\ell = \text{const.} \\ \ell \lesssim 40$$

Plank Best-Fit  
⇒



CMB Anisotropy can come from Conformal Horizon Fluctuations

### 3-pt. Bispectrum Correlator on Horizon Sphere

- Conformal Ward Identities on Horizon Sphere determine

$$G_3(\hat{n}_1, \hat{n}_2, \hat{n}_3; w) = \frac{a_3(w)}{[(1 - \hat{n}_1 \cdot \hat{n}_2)(1 - \hat{n}_2 \cdot \hat{n}_3)(1 - \hat{n}_1 \cdot \hat{n}_3)]^{\frac{w}{2}}}$$

- As  $w \rightarrow 0$  appropriate for HZ scale invariance this becomes

$$G_3(\hat{n}_1, \hat{n}_2, \hat{n}_3; 0) = C_3 [\ln(1 - \hat{n}_1 \cdot \hat{n}_2) + \ln(1 - \hat{n}_2 \cdot \hat{n}_3) + \ln(1 - \hat{n}_1 \cdot \hat{n}_3)] + c$$

completely separable and a **different shape from slow roll inflation** (not well parametrized by  $f_{\text{NL}}$ )

- If this non-Gaussian Bispectrum  $G_3$  is observed in the CMB it implies the temperature fluctuations arise at or near the dS horizon  $S^2$ , related to dS/CFT conformal invariance
- Non spatially homogeneous & isotropic cosmology possible
- Need to consider generic position within dS horizon sphere

## Tensor GW Spectral Index & Conformal Invariance

Conformal Invariance also determines the Tensor Correlator

$$G_{ijkl}^{(2)}(\vec{x}, \vec{x}') = \langle h_{ij}(\vec{x}) h_{kl}(\vec{x}') \rangle \sim P_{ijkl}^{(2)}(\vec{x}, \vec{x}') |\vec{x} - \vec{x}'|^{-n_T}$$
$$\sim \int d^3 \vec{p} e^{i\vec{p} \cdot (\vec{x} - \vec{x}')} \hat{P}_{ijkl}^{(2)}(\vec{p}) |\vec{p}|^{n_T - 3}$$

in terms of the spin-2 transverse, traceless tensor projector

$$\hat{P}_{ijkl}^{(2)}(\vec{p}) = \frac{1}{2} (\Theta_{ik}\Theta_{jl} + \Theta_{il}\Theta_{jk}) - \frac{1}{3} \Theta_{ij}\Theta_{kl}$$
$$\Theta_{ij}(\vec{p}) \equiv \delta_{ij} - \frac{p_i p_j}{|\vec{p}|^2}$$

- Since 3D scalars and tensors are different parts of the same 4D metric tensor perturbation of dS space  $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$  their conformal weights must be equal – and spectral indices give

$$n_T = n_S - 1 \approx 0 \Leftarrow \text{a prediction of dS/CFT Conformal Invariance}$$

## Dynamical Vacuum Energy

- Conformal part of the metric,  $g_{ab} = e^{2\sigma} g_{ab}$  - constrained --frozen--by trace of Einstein's eq.  $R=4\Lambda$  becomes dynamical and can fluctuate due to  $\varphi$  in EFT
- Fluctuations of  $\varphi \sim 2\sigma$  describe a conformally invariant phase of gravity in 4D
- In this conformal phase  $\Lambda$  flows to zero fixed point
- The Quantum Phase Transition to this phase characterized by 'melting' of the scalar condensate  $\Lambda$  at horizon
- $\Lambda_{\text{eff}}$  a dynamical state dependent condensate described by 3-form gauge potential & 4-form field strength coupled to the anomaly scalar conformalon  $\varphi$  at the horizon

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[JCAP 09:024 \(2012\)](#)

## New Horizons in Cosmology

- Einstein's GR receives Quantum Corrections relevant at macroscopic Distances & near Event Horizons
  - A new scalar degree of freedom in EFT of Gravity required by the Conformal/Trace Anomaly
  - Scalar Gravitational Waves predicted in both flat and de Sitter
  - Linear response in de Sitter space confirms dS Instability & Relevance of the Conformal Anomaly to Cosmology
  - $\Lambda_{\text{eff}}$  is not a constant but a dynamical condensate which can change due to scalar fluctuations which couple strongly at horizon
  - Can give rise to the CMB fluctuations (without an inflaton)
  - Tests of Conformal Hypothesis in CMB Non-Gaussianity & Tensor Primordial GW Spectrum
  - Observed dark energy of our Universe may be a residual macroscopic finite size effect depending on  $(\mathbb{R})$  boundary conditions at the cosmological horizon scale
- Space/Time Dependent Dark Energy Cosmology Possible

