Title: On the interplay of topology and interactions: Reduced topological classification and fractional Fermi liquid

Date: Dec 11, 2018 11:00 AM

URL: http://pirsa.org/18120022

Abstract: <p dir="ltr"><strong id="docs-internal-guid-f16ff12c-7fff-8004-739e-bd709aac5a2d">This seminar will focus on two cases where the interplay of topology and interactions allows for phases that go beyond simple quasi-particle descriptions. Both models are amenable to sign free auxiliary field quantum Monte Carlo simulations. </p>

<p dir="ltr"><strong id="docs-internal-guid-f16ff12c-7fff-8004-739e-bd709aac5a2d">First, we design a two-dimensional model consisting of four Dirac-fermion layers on the square lattice. The interaction is given by a four-fermion term where each fermion is from a different layer. In the uncorrelated case, the topology is determined by a Z-valued winding number and previous studies, often using the bulk-boundary correspondence and dimensional reduction arguments, predict the reduction to a Z4 classification in the presence of correlations. An adiabatic path between formerly distinct phases has to visit a strongly interacting state that cannot be described on a mean-field level. We study the phase diagram of the full bulk system and find a symmetry broken state separating topological distinct phases. An attempt to frustrate the ordered state introduces a first order phase transition [Fig. 1 (left)]. </p>

<p dir="ltr"><strong id="docs-internal-guid-f16ff12c-7fff-8004-739e-bd709aac5a2d">Second, we consider Dirac electrons on the honeycomb lattice Kondo coupled to spin-1/2 degrees of freedom on the kagome lattice. The interactions between the spins are chosen along the lines of the Balents-Fisher-Girvin model that is known to host a Z2 spin liquid and a fer- romagnetic phase. While in the ferromagnetic phase the Dirac electrons acquire a gap, they remain massless in the Z2 spin liquid phase due to the breakdown of Kondo screening. Since our model has an odd number of spins per unit cell, this phase is a non-Fermi liquid, also called fractionalized Fermi liquid, that violates the conventional Luttinger theorem, which relates the Fermi volume to the particle density in a Fermi liquid. We probe the Kondo breakdown in this non-Fermi liquid phase via conventional observables such as the spectral function, and also by studying the mutual information between the electrons and the spins [Fig. 1 (right)]. </p>

<p dir="ltr"><strong id="docs-internal-guid-f16ff12c-7fff-8004-739e-bd709aac5a2d">Figure 1: Schematic sketch of the phase diagram discussed during the beginning of the seminar (left). Numerical Results consistent with a FL* to magnetic insulator transition as subject of the second half (right). </p>

How can we classify states of matter?

- Landau's paradigm:
  - Local order parameter
- Symmetry protected topological phases:
  - Global order parameter, for example, given by a winding number
  - Extensively categorized **without interactions** (ten-fold way)
  - Symmetries have to be satisfied, for example:
    - Time-reversal, particle-hole or chiral symmetry
    - Mirror or other lattice symmetries
- Intrinsic topological order
  - No protection required
  - Ground state degeneracy
  - Fractionalized topological excitation
  - Examples:
    - Spin liquid
    - Fractional quantum Hall states

Correlations effects? Exotic, strongly interaction phase?

Kondo breakdown: topological order induces fractional Fermi liquid

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Outline

- Method:
  auxiliary field quantum Monte Carlo

- Part I:
  reduced classification of symmetry protected topology

- Part II:
  Kondo breakdown - fractional Fermi liquid

- Conclusion

Blankenbecler, Scalapino and Sugar: PRD 1981
Bercx, Goth, Hofmann, Assaad: SciPost, 2017

Method

alf.physik.uni-wuerzburg.de

- We use our recent **A**lgorithm for **L**attice **F**ermions (**ALF**), a general implementation of the auxiliary field Quantum Monte Carlo (BSS).

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  - free part: single body operators
  - interaction type 1: single body operators coupled to an Ising field with given dynamics

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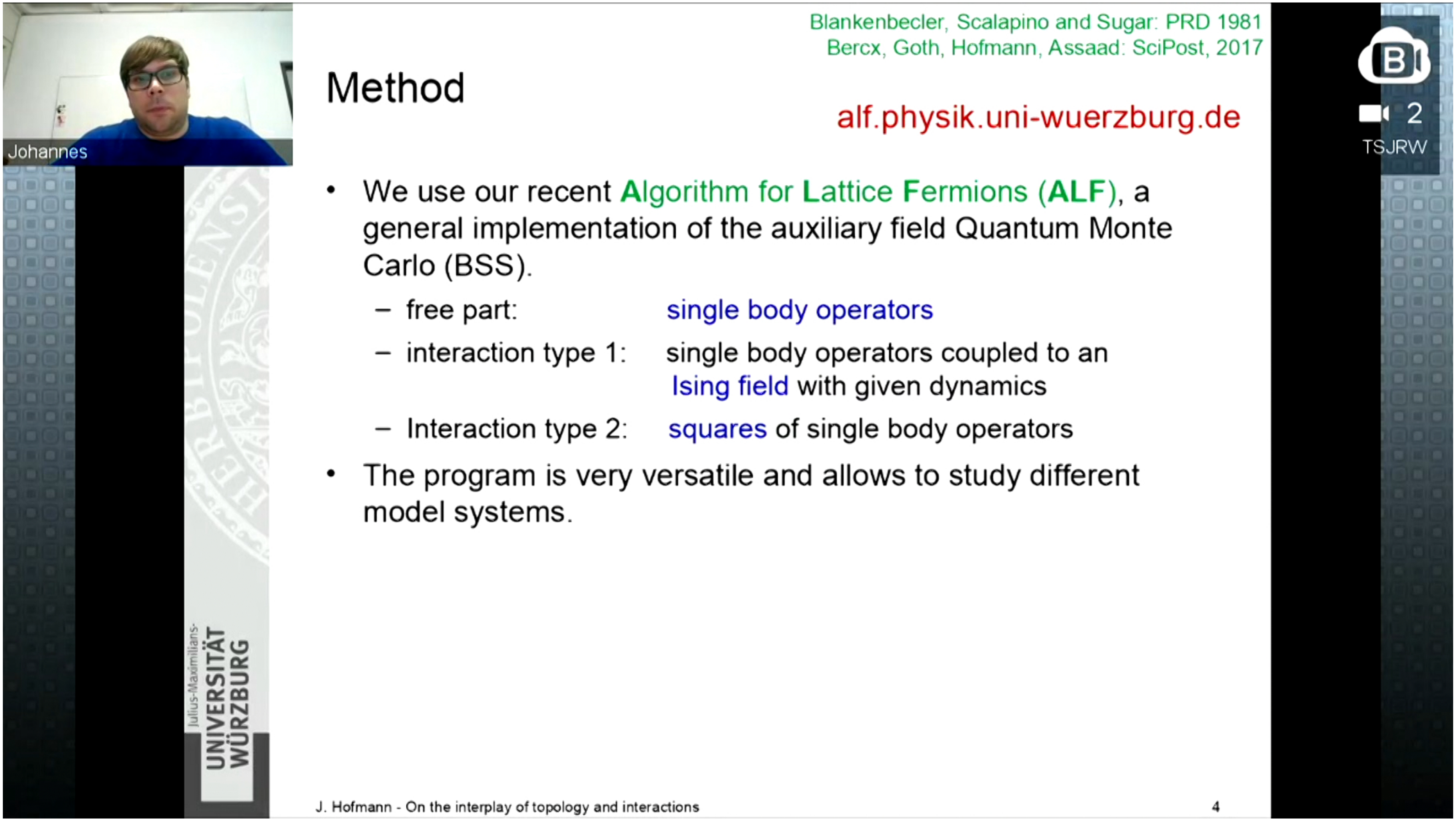
Johannes
Blankenbecler, Scalapino and Sugar: PRD 1981
Bercx, Goth, Hofmann, Assaad: SciPost, 2017
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Blankenbecler, Scalapino and Sugar: PRD 1981
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- We use our recent **A**lgorithm for **L**attice **F**ermions (**ALF**), a general implementation of the auxiliary field Quantum Monte Carlo (BSS).
  - free part: single body operators
  - interaction type 1: single body operators coupled to an Ising field with given dynamics
  - Interaction type 2: squares of single body operators
- The program is very versatile and allows to study different model systems.
- The package includes a finite temperature as well as a projective ground state version.
- It provides access to many oberservables:
  - Static and dynamic correlation function
  - Renyi entropies

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Part I: reduced topological classification

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Symmetry protected topology 101

1D and 2 degrees of freedom:

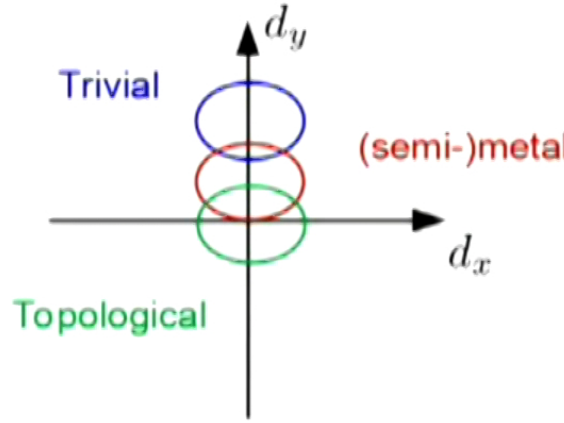
$$H(k) = \vec{d}(k)\vec{\sigma}$$

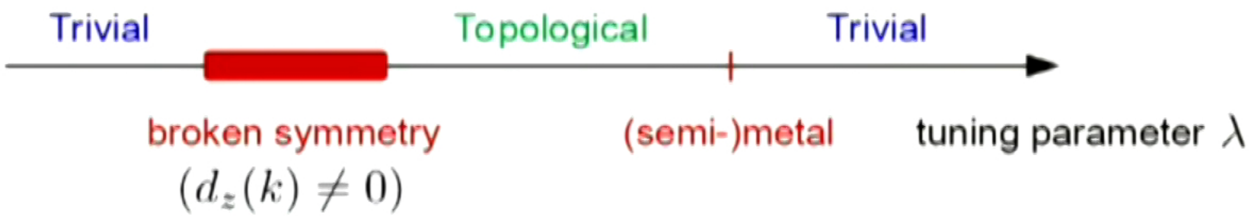
BDI symmetry constraint:

$$d_z(k) = 0$$

Relevant for:

- Kitaev chain (p-wave superconductor)
- Su-Schrieffer-Heger model





Can we adiabatically connect topologically distinct states?

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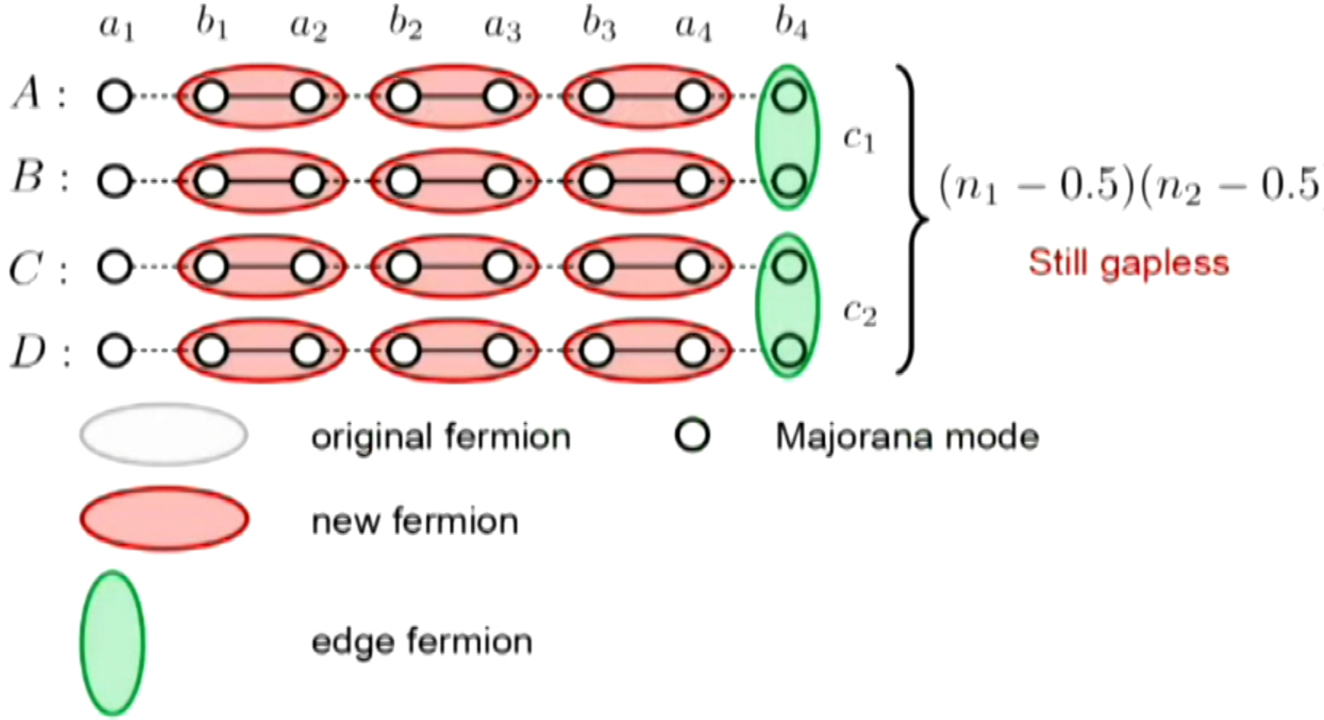
Fidkowski, Kitaev: PRB, 2010

Reduction of topological classification in the interacting Kitaev chain

- L. Fidkowski's and A. Kitaev's original argument:
  - Consider Majorana edge states in multiple chains



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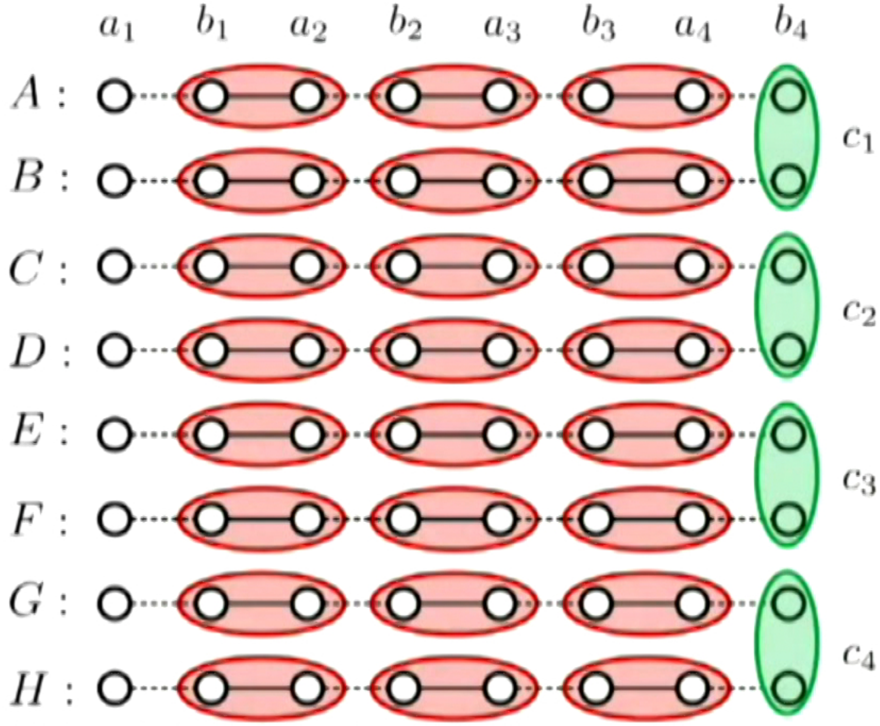

$$(n_1 - 0.5)(n_2 - 0.5)$$

$$(c_1^\dagger\, c_2^\dagger)\vec{\sigma}\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} \times$$

$$(c_3^\dagger\, c_4^\dagger)\vec{\sigma}\begin{pmatrix} c_3 \\ c_4 \end{pmatrix}$$

$$(n_3 - 0.5)(n_4 - 0.5)$$

Unique and gapped groundstate

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Our model in two dimensions

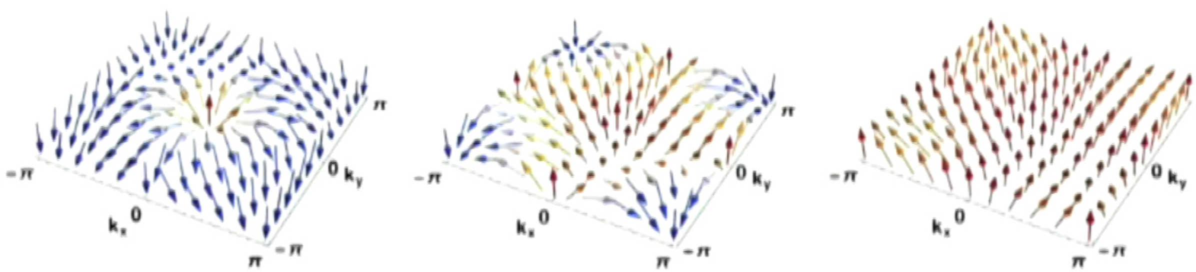
- 2D Dirac Hamiltonian $\sum_{\mathbf{k}} \Psi_{\mathbf{k}}^{\dagger} \begin{pmatrix} H_+(\mathbf{k}) & 0 \\ 0 & H_-(\mathbf{k}) \end{pmatrix} \Psi_{\mathbf{k}}$,

  where $\Psi_{\mathbf{k}}$ is a four-component spinor:

$$H_{\pm}(\mathbf{k}) = t[\sin(k_x)\sigma_x + \sin(k_y)\sigma_y] \pm m(\mathbf{k})\sigma_z$$

$$m(\mathbf{k}) = 2 + \lambda + \cos(k_x) + \cos(k_y)$$

- Two copies of QHE with opposite winding $\mathbb{Z}$ (here: H_+)



-4 Topo. insulator -2 Topo. insulator 0 Trivial insulator λ

$$H_\pm(\mathbf{k}) = t[\sin(k_x)\sigma_x + \sin(k_y)\sigma_y] \pm m(\mathbf{k})\sigma_z$$

$$\sum_{\mathbf{k}} \Psi_{\mathbf{k}}^\dagger \begin{pmatrix} H_+(\mathbf{k}) & 0 \\ 0 & H_-(\mathbf{k}) \end{pmatrix} \Psi_{\mathbf{k}}$$

Interaction

- We consider four copies $(\mathbb{Z} \to \mathbb{Z}_4)$ of the original Dirac Hamiltonian – indexed by A, B, C and D –
- and use the correlation $U \sum_{\mathbf{i}} \Psi_{\mathbf{i},A}^\dagger \gamma_5 \Psi_{\mathbf{i},B} \Psi_{\mathbf{i},D}^\dagger \gamma_5 \Psi_{\mathbf{i},C}$
- The interaction can be transformed into

$$\frac{U}{8} \sum_{\mathbf{i}} \sum_{\alpha=x,y} \left(S_{\mathbf{i}}^{1,\alpha} + S_{\mathbf{i}}^{2,\alpha}\right)^2 - \left(S_{\mathbf{i}}^{1,\alpha} - S_{\mathbf{i}}^{2,\alpha}\right)^2$$

$$S_{\mathbf{i}}^{1/2,\alpha} = (\Psi_{\mathbf{i}}^{A/C\dagger} \Psi_{\mathbf{i}}^{B/D\dagger})\sigma^\alpha \otimes \gamma_5 \begin{pmatrix} \Psi_{\mathbf{i}}^{A/C} \\ \Psi_{\mathbf{i}}^{B/D} \end{pmatrix}$$

- The interaction term has a unique, symmetric and gapped ground state

Morimoto, Furusaki, Mudry: PRB, 2015
Queiroz, Khalaf, Stern: PRL, 2016

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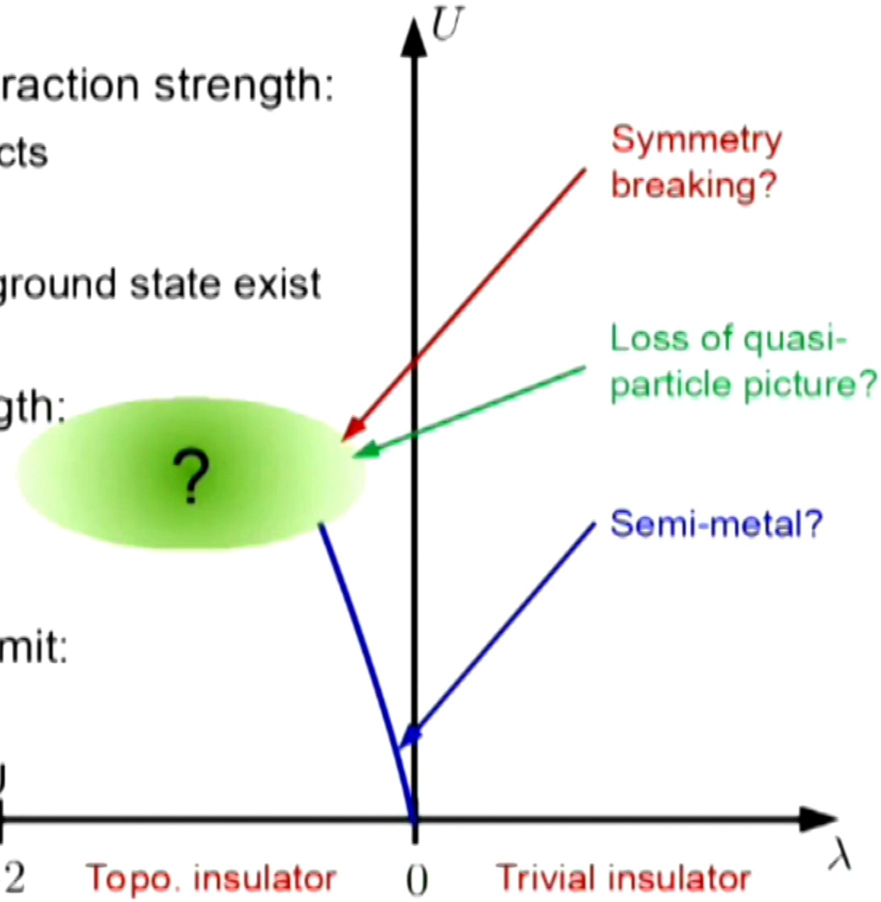
Interaction

$$\frac{U}{8}\sum_{\mathbf{i}}\sum_{\alpha=x,y}\left(S_{\mathbf{i}}^{1,\alpha}+S_{\mathbf{i}}^{2,\alpha}\right)^2-\left(S_{\mathbf{i}}^{1,\alpha}-S_{\mathbf{i}}^{2,\alpha}\right)^2$$

- Limit of infinite interaction strength:
  - identical 0D objects
  - exactly solvable
  - unique, gapped ground state exist
- Intermediate strength:
  - Possible MF
    $M_{\mathbf{i}}^{\alpha}=S_{\mathbf{i}}^{1,\alpha}-S_{\mathbf{i}}^{2,\alpha}$
- Weak interaction limit:
  - gapped insulator
  - stable for small U



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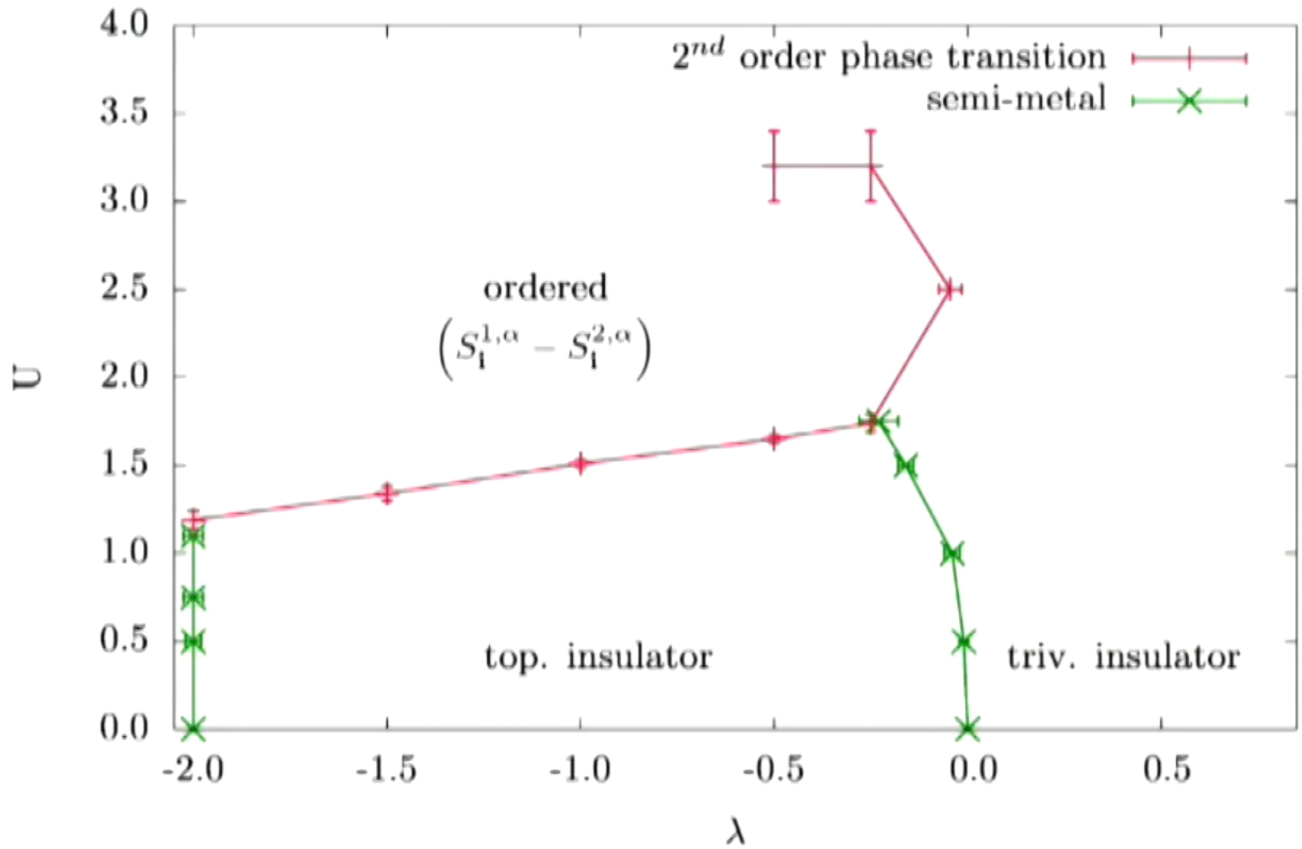
Results: phase diagram



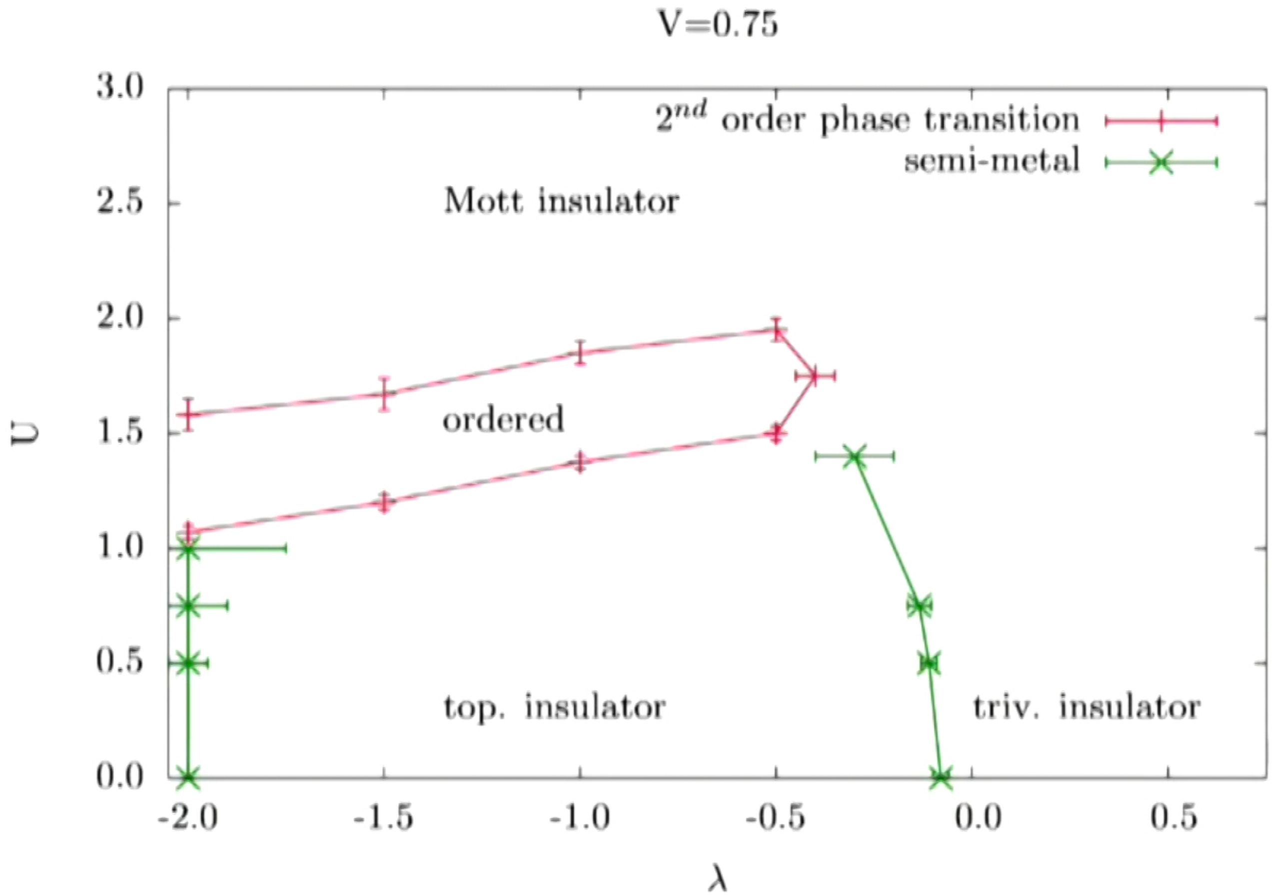
Add frustration to the system

- Introduce the new term $V \sum_{\mathbf{i}} \left(S_{\mathbf{i}}^{1,z} + S_{\mathbf{i}}^{2,z} \right)^2$ with

$$S_{\mathbf{i}}^{1/2,z} = (\Psi_{\mathbf{i}}^{A/C\dagger} \Psi_{\mathbf{i}}^{B/D\dagger}) \sigma^z \begin{pmatrix} \Psi_{\mathbf{i}}^{A/C} \\ \Psi_{\mathbf{i}}^{B/D} \end{pmatrix}$$

- The previous introduced x- and y-components and the new z-component form an **SU(2) algebra**.
- Hence, the new term **frustrates the xy-polarized pseudo-magnetic order**.
- The **large U limit** still has the **same ground state**. It already was a singlet such that its energy does not change. States with non-zero z-component only gain energy, but never decrease it.

Results: phase diagram with weak frustration



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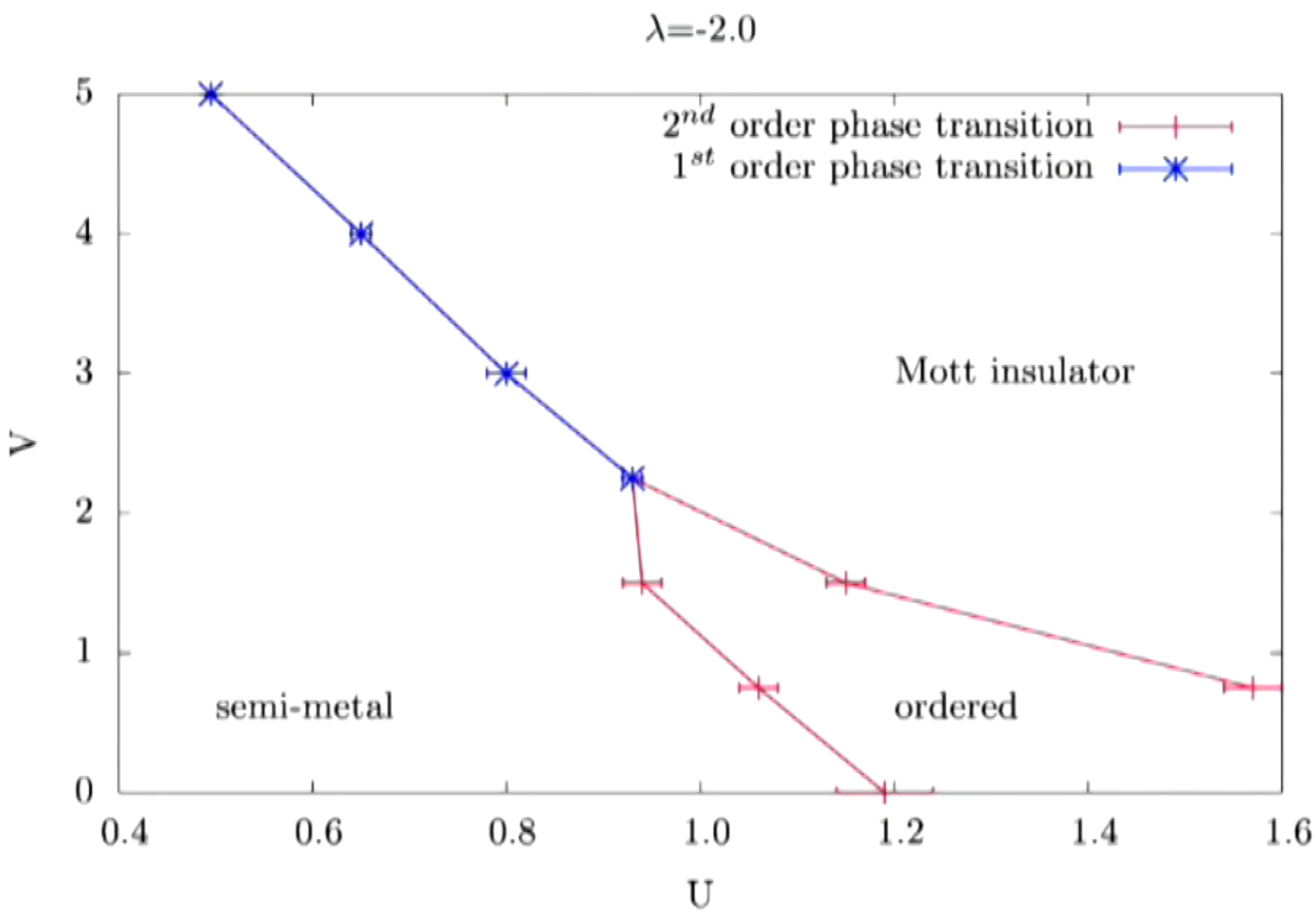
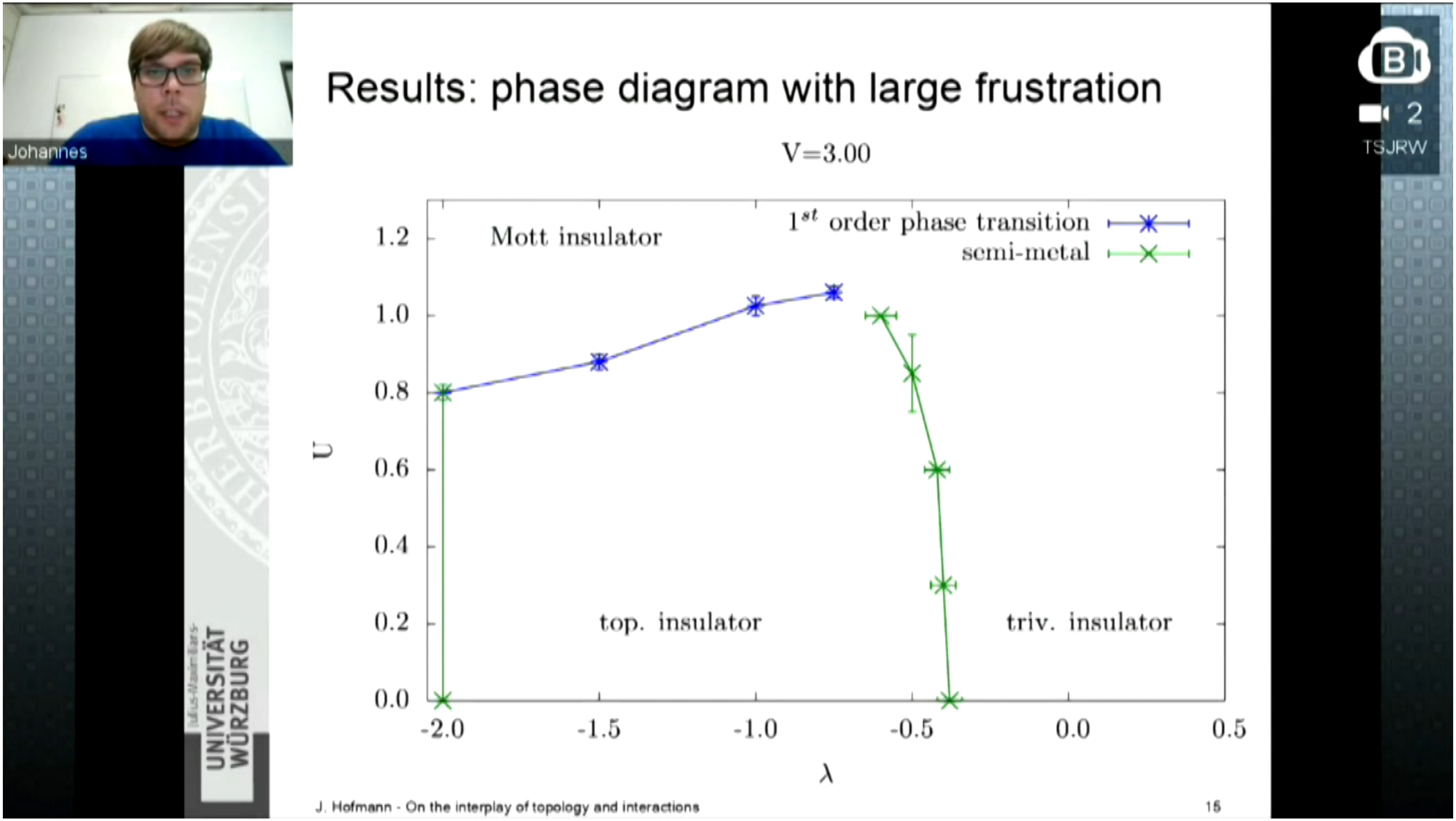
Results: increasing frustration

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Results: phase diagram with large frustration
V=3.00
1^{st} order phase transition
semi-metal
Mott insulator
top. insulator
triv. insulator
U
λ
1.2
1.0
0.8
0.6
0.4
0.2
0.0
-2.0
-1.5
-1.0
-0.5
0.0
0.5
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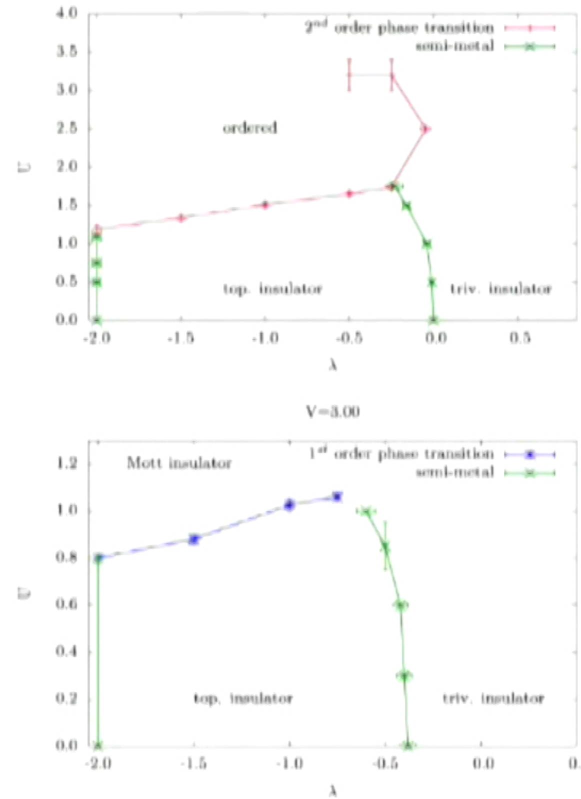
Ayyar, Chandrasekharan: PRD 2015
He, Wu, You, Xu, Meng, Lu: PRB 2016
You, He, Xu, Vishwanath: PRX 2018

Interpretation and future directions

- **No adiabatic path yet**: Gross-Neveu critical points or first order transition.
- Numerical studies: second order phase transitions from massless to **massive Dirac phases without symmetry-breaking fermion bilinears**.
- possible explanation: **symmetric mass generation** (fractionalisation)
- Shift focus from edge state arguments to bulk criticality
- Exciting approach to novel and **exotic quantum criticality in fermion systems**.



Publication in preparation!

Part II: fractional Fermi liquid

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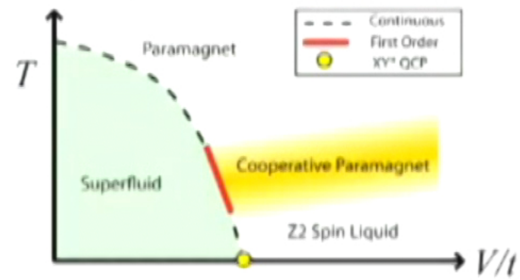
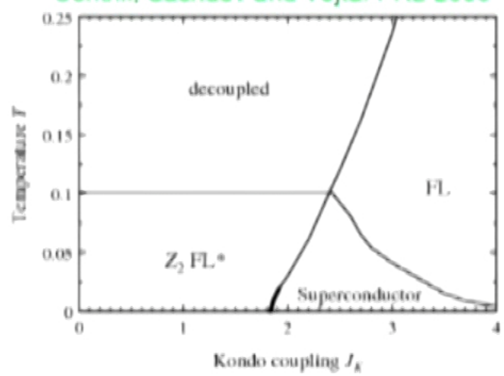
Fractional Fermi liquids

Balents, Fisher and Girvin: PRB 2002
Isakov, Melko and Hastings: Science 2012



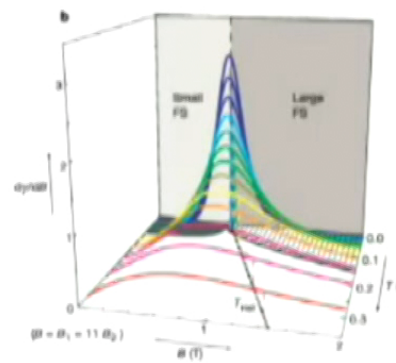
Frustrated quantum magnets can give rise to intrinsic topological order.

Senthil, Sachdev and Vojta: PRL 2003



Spin liquids **Kondo-coupled** to conduction electrons can generate interesting phases like fractionalized Fermi liquid.

Paschen, *et. al.*: letters to nature 2004



Fractional Fermi liquid is one possibility to understand Kondo breakdown: Restoration of "small" Fermi surface

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Fractional Fermi liquids

Balents, Fisher and Girvin: PRB 2002
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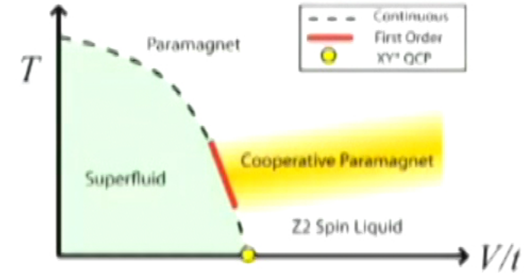

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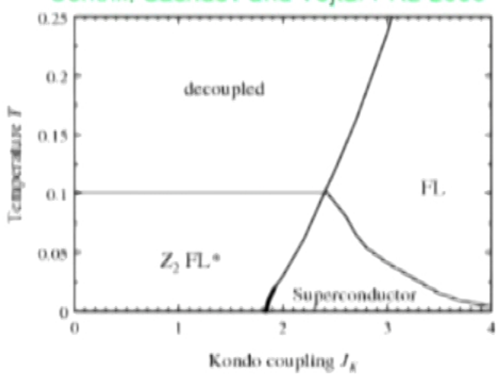

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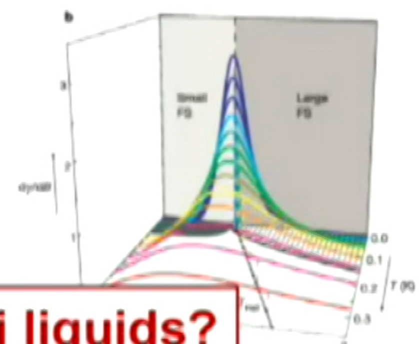


Fractional Fermi liquid is one possibility to understand Kondo breakdown: Restoration of “small” Fermi surface

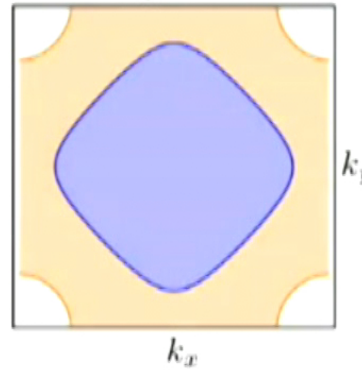
Can we realize fractional Fermi liquids?

Oshikawa: PRL, 2000

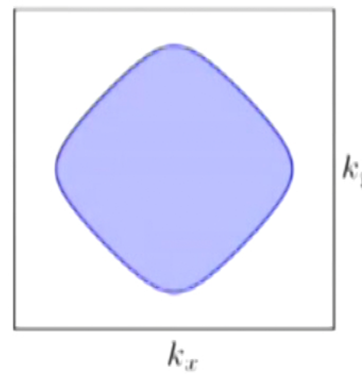
Kondo physics 101

- Kondo physics studies conduction electrons coupled to local moments (spin channel).
- One possible phase is the so called **heavy Fermi liquid** that conserves translation and time-reversal symmetry.
- Here, the **local moments are screened** by the conduction electrons and therefore contribute to the Fermi sea and thus generate a **"large" Fermi surface** (Luttinger's sum rule).
- The restoration of the **"small" Fermi surface** – without symmetry breaking – is known as **Kondo breakdown**.
- This phase **violates Luttinger's theorem** and one explanation thereof is **topological order**.

"large" FS



"small" FS



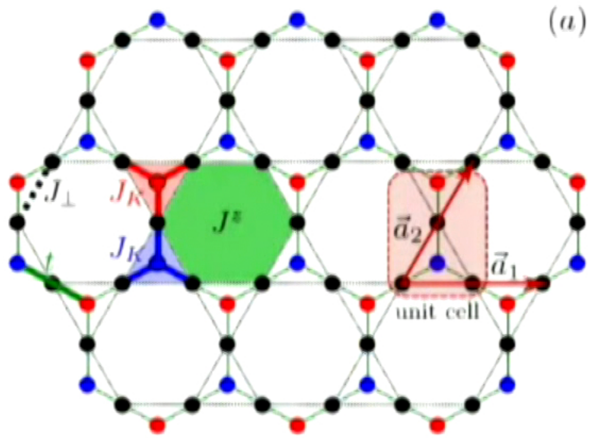
Balents, Fisher and Girvin: PRB 2002

Model definition

- The Hamiltonian consists of two subsystem:
  - ***Balents-Fisher-Girvin*** spin model
  - Conduction electrons on honeycomb (*two Fermi points*)
- The two parts are ***Kondo*** coupled with unusual sign structure

  $J_R^\alpha = ((-1)^R, (-1)^R, 1)$

- Heavy Fermi liquid should have **"large" Fermi surface**.

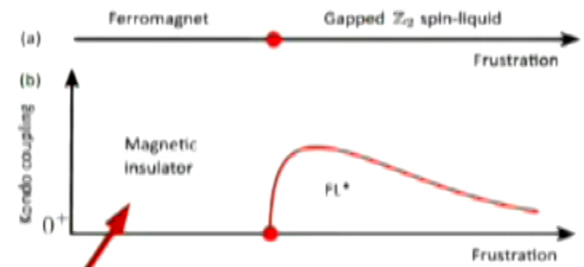


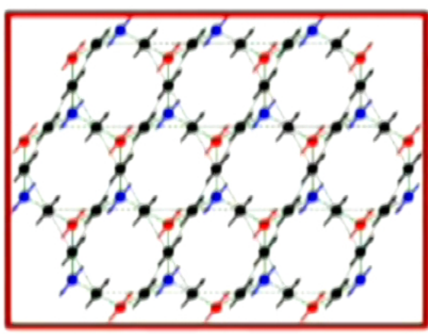
$$
\begin{aligned}
H_{BFG} &= -J_\perp \sum_{\langle i,j\rangle} \left[S_i^+ S_j^- + h.c.\right] + J^z \sum_{\hexagon} \left[\sum_{i\in\hexagon} S_i^z\right]^2 \\
H_c &= -t \sum_{\langle R,R'\rangle,\sigma} c^\dagger_{R,\sigma} c_{R',\sigma} + h.c. \\
H_K &= J_K \sum_{\langle R,j\rangle,\alpha} J_R^\alpha\, c_R^\dagger \sigma^\alpha c_R\, S_j^{\alpha,f}
\end{aligned}
$$

Sketch of expected phase diagram at T=0

- For weak frustration the spin systems orders ferromagnetically.
  - Kondo-coupling generates an AFM mass term for the conduction electrons
    - ⇒ ***magnetic insulator***

Senthil, Sachdev and Vojta: PRL 2003
Grover and Senthil: PRB 2010
Isakov, Melko and Hastings: Science 2012





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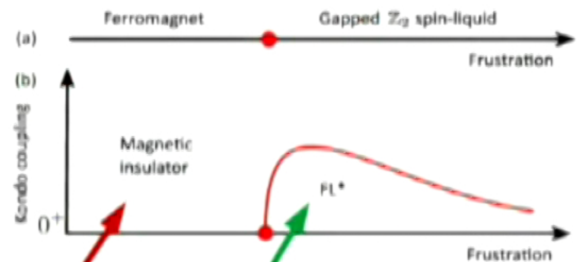
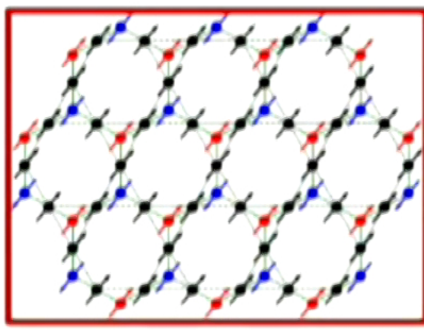
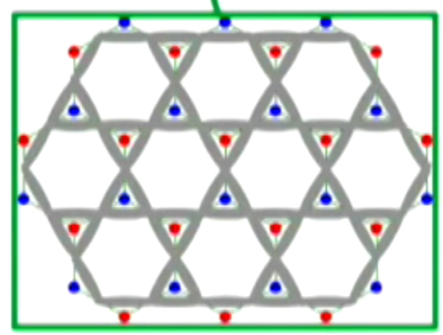
Sketch of expected phase diagram at T=0

Senthil, Sachdev and Vojta: PRL 2003
Grover and Senthil: PRB 2010
Isakov, Melko and Hastings: Science 2012

- For weak frustration the spin systems orders ferromagnetically.
  - Kondo-coupling generates an AFM mass term for the conduction electrons
    $\Rightarrow$ ***magnetic insulator***
- For higher frustration the spin system forms a $\mathbb{Z}_2$ spin liquid.
  - At the transition the Kondo coupling is irrelevant
  - The spin liquid phase itself is gapped and topologically protected
    $\Rightarrow$ ***fractional Dirac semi-metal***

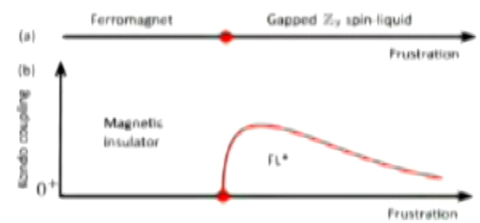


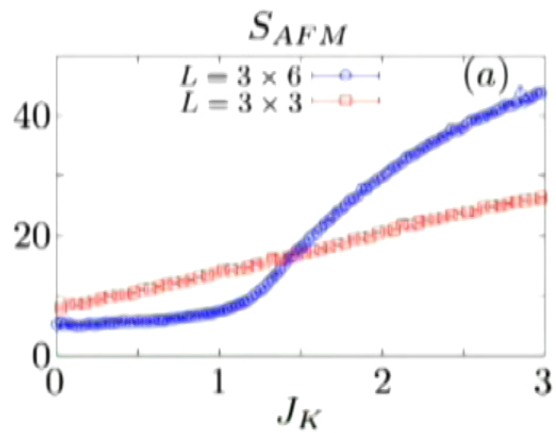




Numerical results

Hofmann, Assaad, Grover: ArXiv, 2018





simulation parameters:

$$\beta t = 12$$
$$J^z/t = 7.5$$

observable definition:

$$S_{AFM} = 1/L \sum_{\boldsymbol{IJ}} \hat{S}^x_{\boldsymbol{I}} \hat{S}^x_{\boldsymbol{J}} + \hat{S}^y_{\boldsymbol{I}} \hat{S}^y_{\boldsymbol{J}}$$

$$\hat{\boldsymbol{S}}_{\boldsymbol{I}} = \sum_{\boldsymbol{i} \in \boldsymbol{I}} \hat{\boldsymbol{S}}^f_{\boldsymbol{i}} + \sum_{\boldsymbol{x} \in \boldsymbol{I}} (-1)^{\boldsymbol{x}} \hat{\boldsymbol{S}}^c_{\boldsymbol{x}}$$

Weak Kondo coupling: structure factor decreases with system size, hence **no magnetic order**.

Stronger interactions: signal grows with system size, suggesting **magnetic order**.

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Numerical results

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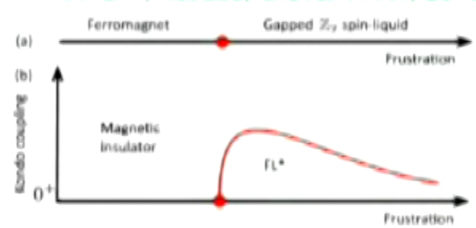


Observable:

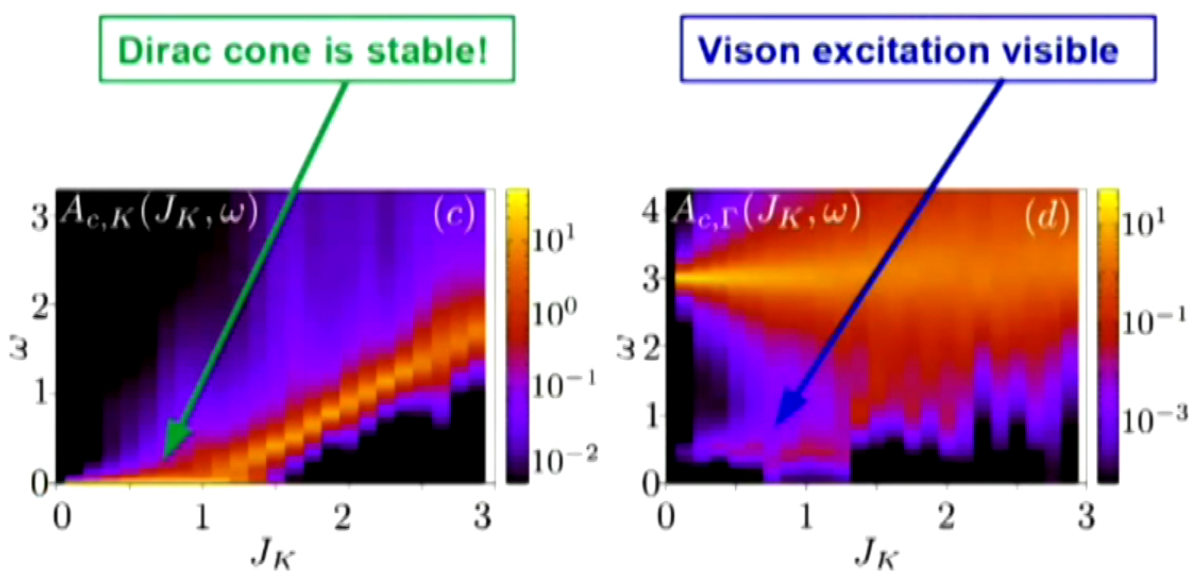
spectrum of the conduction electrons at Dirac point K and the center Γ

simulation parameters:

$\beta t = 12$

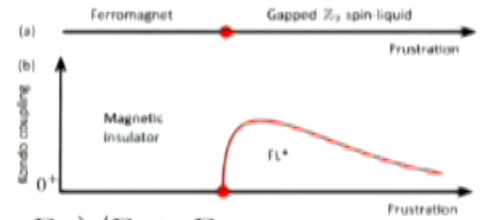
$J^z/t = 7.5$

$L \times L = 3 \times 6$



Hofmann, Assaad, Grover: ArXiv, 2018

Numerical results



simulation parameters:

$\beta t = 12$

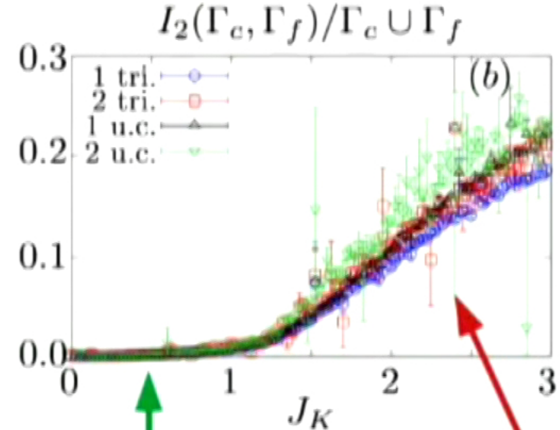
$J^z/t = 7.5$

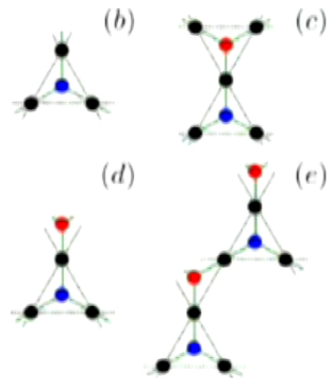
$L \times L = 3 \times 6$

observables:

Renyi mutual information

normalized to sites of patch





Effectively vanishing entanglement. Decoupled low-energy physics.

Volume law: Considerable Kondo coupling

$$I_2(\Gamma_c, \Gamma_f) \equiv S_2(\Gamma_c \cup \Gamma_f) - S_2(\Gamma_c) - S_2(\Gamma_f)$$

$$S_2(\Gamma) \equiv -\ln[\mathrm{Tr}_\Gamma(\rho_\Gamma^2)]$$

Numerical results $(J^z/t = 7.5)$

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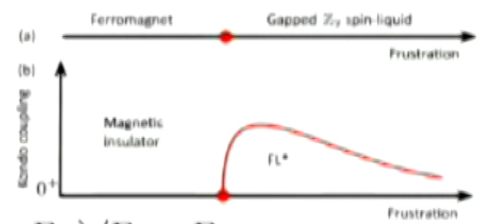

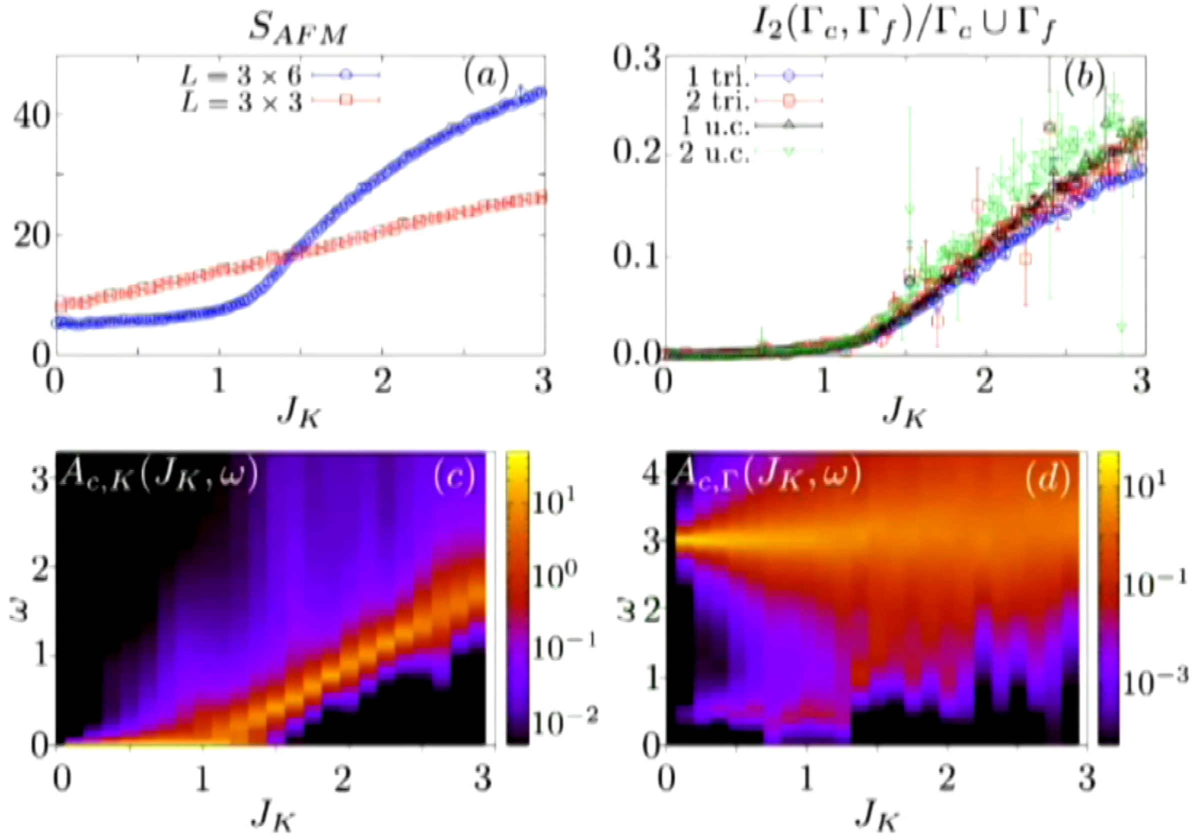

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Challenges and future directions

- First-time realization of **fractional Fermi liquid** in an unbiased, numerically exact simulation
- The main challenge: long auto-correlation times
  - Need for improved sampling schemes
  - More favorable designer model with equivalent phases
- In the **future:**
  study the **phase transition** on large lattices
  - test irrelevance of Kondo coupling (perturbative argument)
  - Critical point described by deconfined $\mathbb{Z}_2$ gauge theory
- Study Kondo-coupled **deconfined quantum-critical points**
  - Natural extension to U(1) gauge theory

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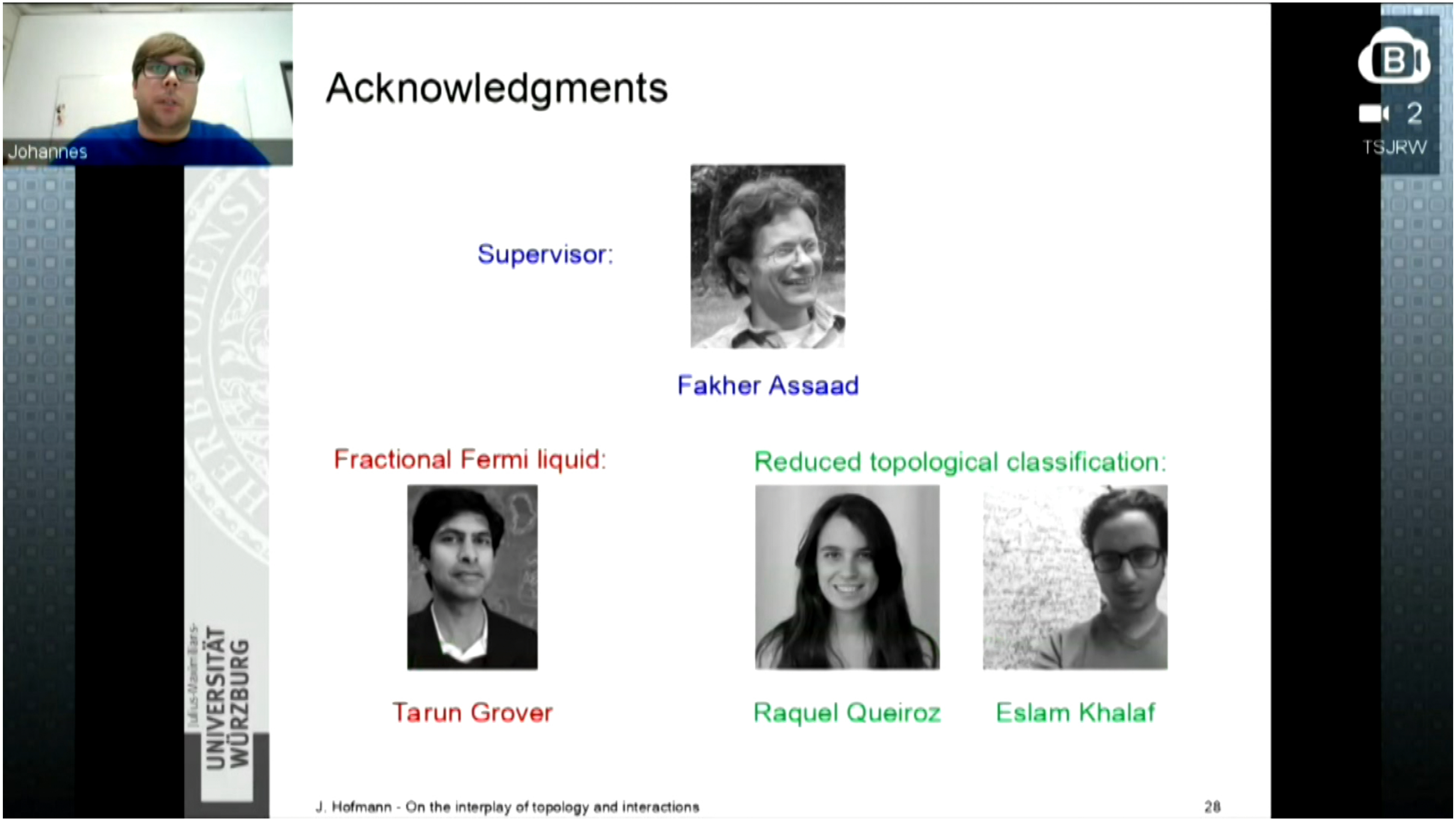
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Conclusion

- *Numerical advances* in solvable models and general implementation **ALF**: Access to complicated models, exotic states, and phase transitions
- **Reduced topological classification**: No adiabatic path, but *symmetry breaking* and *first order phase transitions*
- **Fractionalized Fermi liquid**: First time unbiased realization
- **Future projects**:
  - Redesign from **bulk criticality**
  - Study the **phase transition**
  - Kondo-coupled **deconfined quantum critical points**