Title: Quantum Cosmology with Matter Time - A toy model for quantum gravity

Date: Dec 06, 2018 02:30 PM

URL: http://pirsa.org/18120015

Abstract: <p>The path integral approach to quantum gravity has given us many interesting results in the semiclassical regime. However, the canonical path integral for General Relativity still poses a variety of conceptual and computational challenges. To probe the deep quantum regime, Path Integral Monte Carlo (PIMC) techniques are being used successfully in approaches like Causal Dynamical Triangulations and Causal Set Theory. </p>

<p>In this talk I will discuss how we can apply PIMC techniques to make the canonical Path Integral amenable to computation using matter-time. I will present results from PIMC simulations of homogeneous & isotropic cosmologies, and discuss the extension of this technique to anisotropic as well as inhomogeneous settings. </p>

Quantum Cosmology with Matter Time

A toy model for Quantum Gravity

Masooma Ali
University of New Brunswick

Path Integral for Quantum Gravity

$$\int_{\mathcal{G}} \mathcal{D}[g]\mathcal{D}[\phi]e^{iS}$$

- Action – Einstein-Hilbert, BF, something else
- Paths – what should be included?
- Measure – not easy to define, gauge invariant?, convergence?
- Discretization – various approaches: spin foams, causal sets, CDT

3

Path Integral for Quantum Gravity

- **Amplitude** [Hawking & Gibbons (1993), Feldbrugge et al (2016 - 2018), Speziale et. al(2006 – 2009), Ashtekar et al(2010)]
- **Solutions to WDW equation** [Hartle & Hawking , Vilenkin(1982-1987), Feldbrugge et al (2016)]
- **Expectation values of observables** [Ambjorn et al (2005), Ambjorn et al (2008-2009), Rovelli (2006), Bianchi et al(2009)]
- **Phase Transitions / Phase structure** [Ambjorn et al (2004), Surya(2012), Bahr & Steinhaus(2016), Delcamp & Dittrich(2017)]

Canonical Path Integral

$$\int \mathcal{D}[\pi]\mathcal{D}[q]\mathcal{D}N\mathcal{D}N^a \exp\{iS_{ADM} + ...\}$$

- Formally the integrals over the Lapse and Shift impose the constraints
  - Computationally difficult to implement
  - Notion of dynamics?
- Calculate the propagator, expectation values of observables, wavefunctions
  - Can do this using Path Integral Monte Carlo; how do we wick rotate?

We can fix a gauge. We can use matter

GR with Dust

- Consider GR coupled to dust:

$$S = \int d^4x\sqrt{-g}R - \frac{1}{2}\int d^4x\sqrt{-g}M\left(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + 1\right)$$

- The ADM action is:

$$S = \int dtd^3x\ \left[\tilde{\pi}^{ab}\dot{q}_{ab} + p_\phi\dot{\phi} - N\mathcal{H} - N^a\mathcal{C}_a\right]$$

with:

$$\mathcal{H} = \mathcal{H}_G + \mathcal{H}_D$$
$$C_a = C_a^G + C_a^D$$

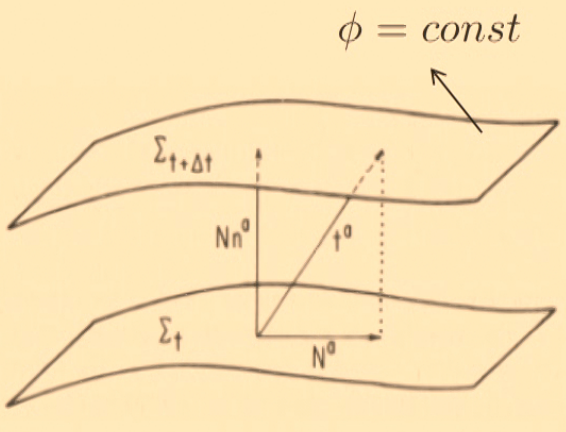
Dust time [Husain & Pawlowski (2012)]

$$\lambda \equiv \phi - t = 0$$

$$N = 1$$

$$H_{physical} = -p_\phi = \int d^3x \, (\mathcal{H}_G)$$

$$S_{GF} = \int dt d^3x \left(\pi^{ab}\dot{q}_{ab} - \mathcal{H}_G - N^a\mathcal{C}_a\right)$$



- The Hamiltonian constraint is linear in p_ϕ
- The gauge condition is second class with the Hamiltonian constraint
- $\det\{\mathcal{H}, \lambda\} = 1$
- $\mathcal{C}_a^D = 0$
- The shift is still arbitrary

All is well classically.

- Linear order in perturbation theory - two graviton modes and a non-propagating scalar mode. [Ali et. al.(2016)]
- For Bianchi IX, we recover the usual Mixmaster dynamics with the Kasner transition law in the asymptotic limit. [Ali & Husain (2017)]
- Bonus - transition law with the dust degree of freedom. Explicit decoupling of dust from the geometry in the asymptotic limit.

Path Integral in Dust time

$$\int \mathcal{D}[q]\mathcal{D}[\pi]\mathcal{D}N^a \exp\left\{ i \int dt d^3x \left(\pi^{ab}\dot{q}_{ab} - \mathcal{H}_p - N^a C_a\right)\right\}$$

Model: FLRW + Dust + Cosmological Constant

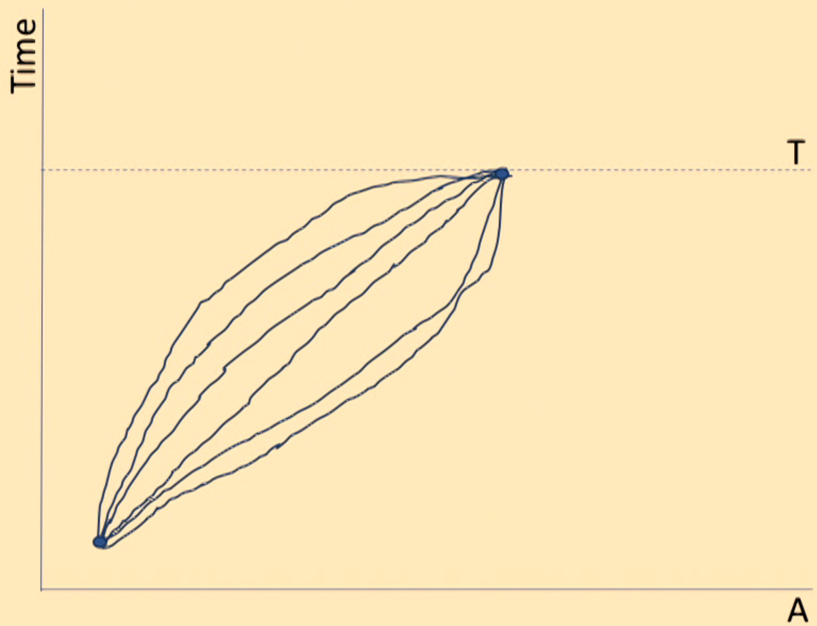
$$S_{GF} = \int dt \left(\frac{\dot{A}^2}{2} + \frac{\Lambda}{2}A^2 - kA^{2/3} \right)$$

- Dynamics of spatially flat FLRW maps to that of an oscillator. [Ali et. al.(2018)]
- Singularity avoidance for all values of the cosmological constant.
- Closed/Open FLRW quantum theory is not analytically tractable, but we can use Monte Carlo methods.

$$Z \xrightarrow{t \to it} \int \mathcal{D}A\, e^{-S_E(A)} \qquad \text{with} \qquad S_E = \int_0^T d\tau \left[\frac{\dot{A}^2}{2} - \frac{\Lambda}{2} A^2 + k A^{2/3} \right]$$

- The path integral is unconditionally convergent for $\Lambda < 0, k > 0$
- The path integral is convergent for $T \leq \frac{\pi}{\sqrt{\Lambda}}$ when $\Lambda > 0, k > 0$ [Carreau et. al(1990)]
- We use the Metropolis algorithm to sample the distribution $\pi(A) = \frac{e^{-S_E(A)}}{\int \mathcal{D}A' e^{-S_E(A')}}$
- Using these samples we compute the ground state wavefunction, study fluctuations around classical paths and compute the "No Boundary wavefunctions"

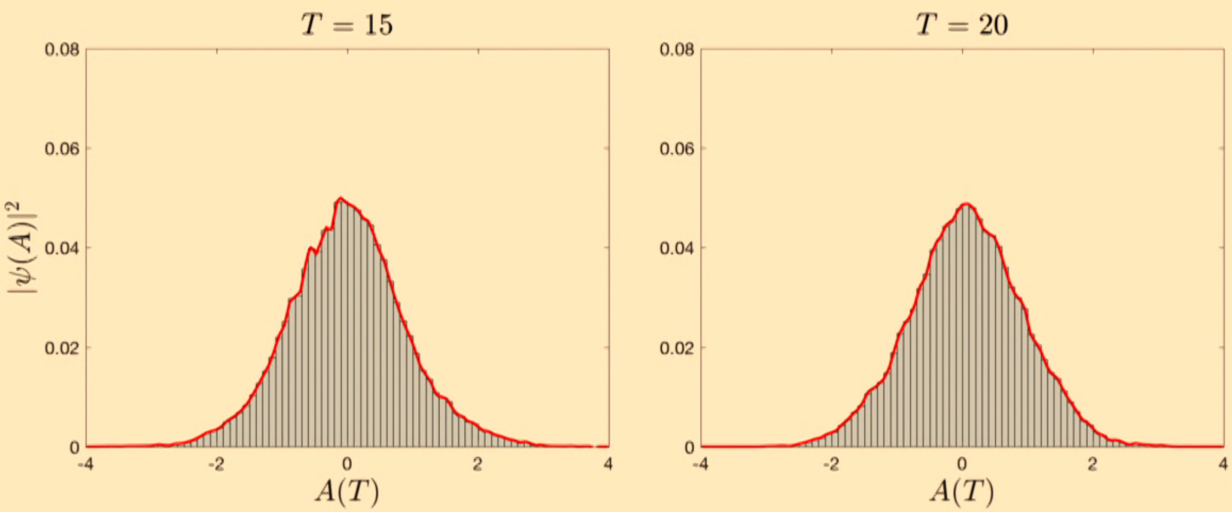
Calculating Expectation Values



$$\langle \hat{O} \rangle = \frac{\int \mathcal{D}x \, O \, e^{\frac{i}{\hbar} S}}{\int \mathcal{D}x e^{\frac{i}{\hbar} S}}$$

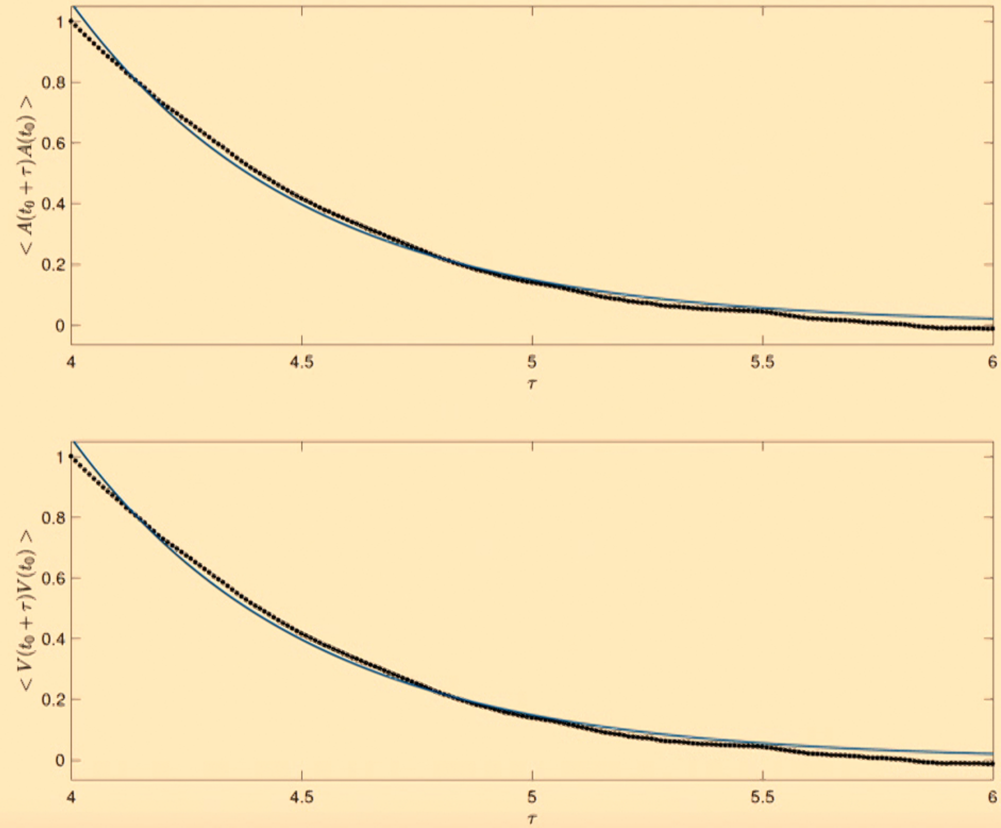
- Sample paths with end points fixed
- Calculate observables by averaging

$\Lambda < 0, k > 0$



- The Euclidean action is positive definite
- A unique ground state exists
- We can calculate correlation functions

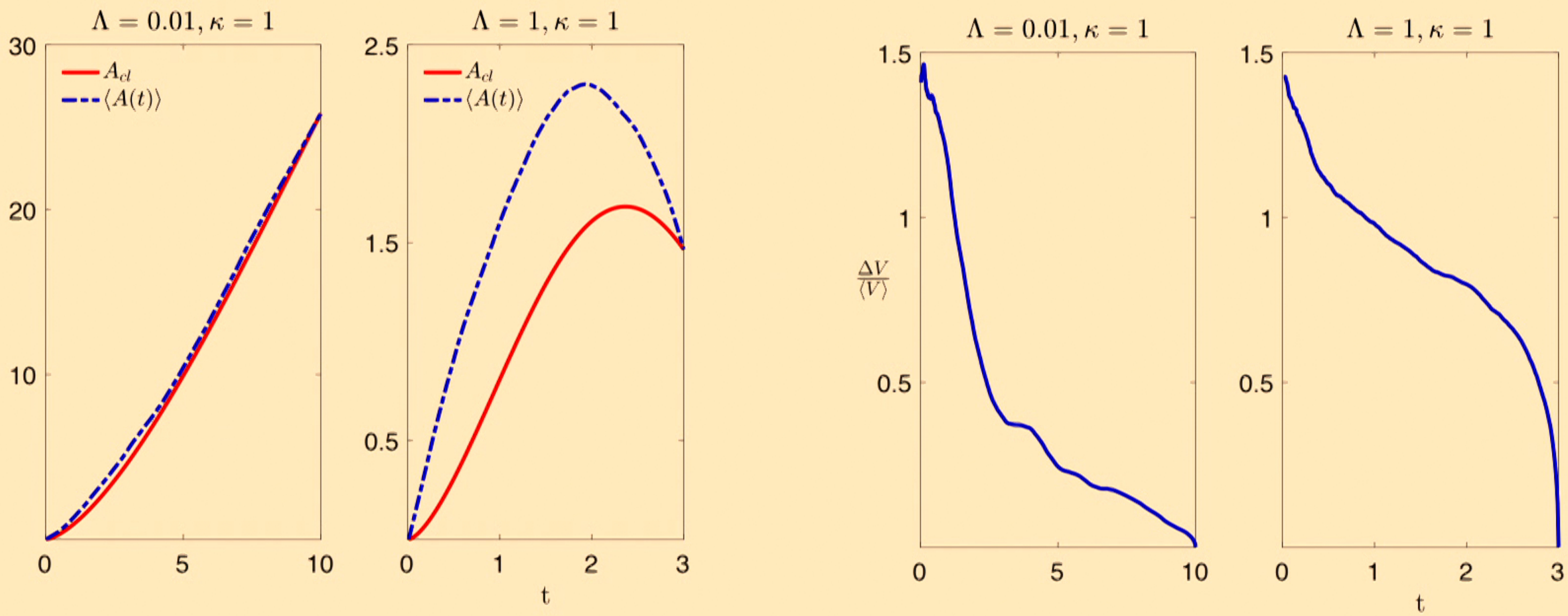
$$\Lambda < 0, k > 0$$



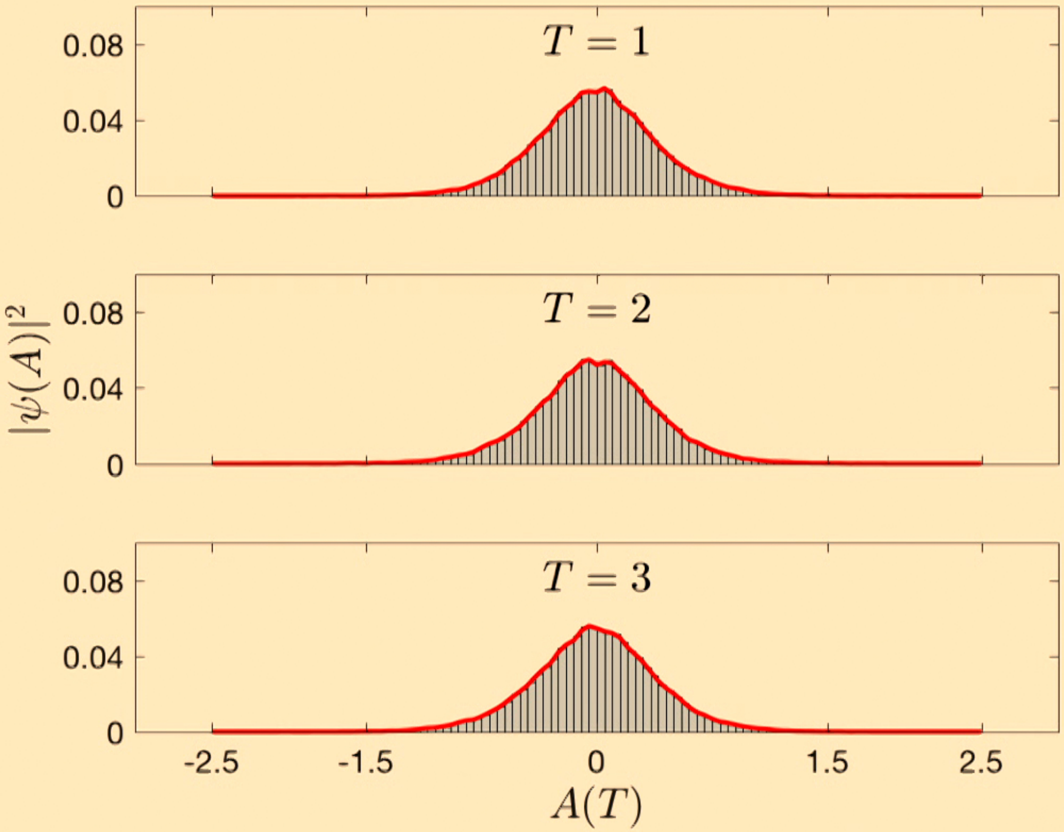
Two point Correlation functions

$\Lambda > 0, k > 0$

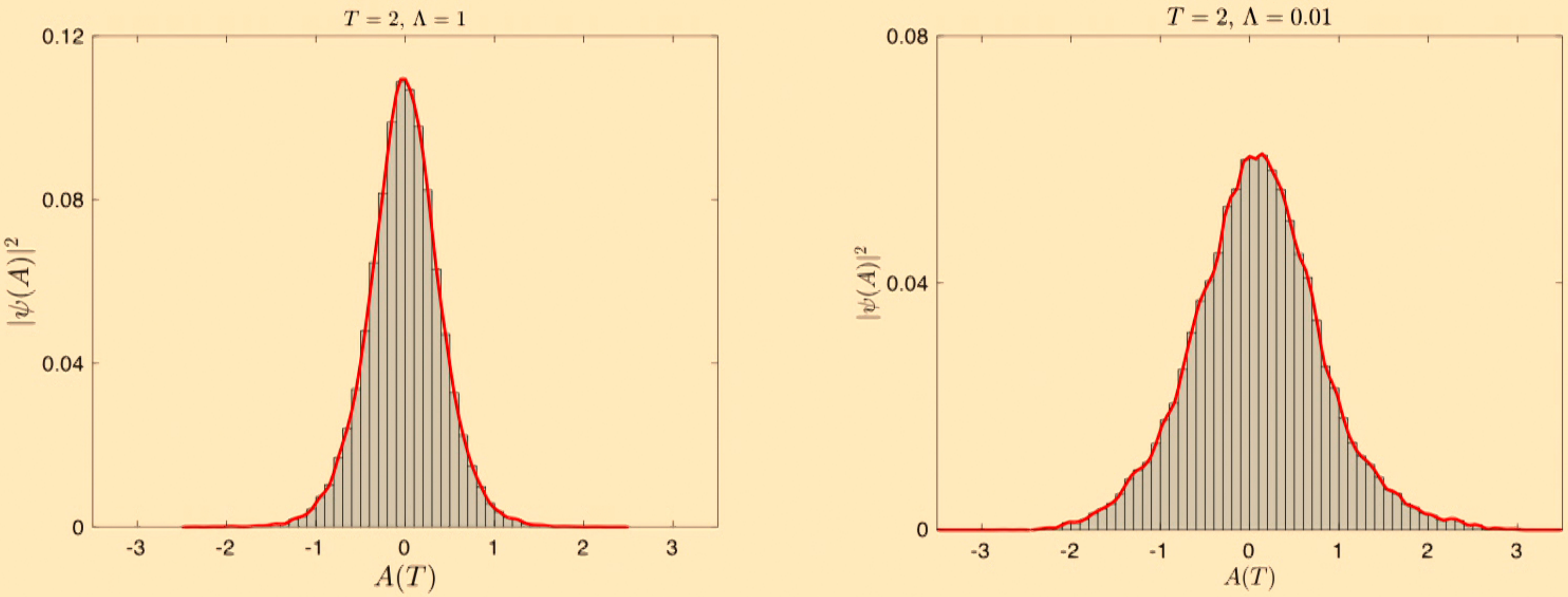
Fluctuations around classical paths:



No Boundary Wavefunctions



No Boundary Wavefunctions



Higher probability of larger universes with smaller cosmological constant

17

Summary

- Fixing a time gauge before quantization is very useful since it gives a notion of dynamics and also simplifies technical issues
- The dynamics of spatially flat FLRW + cosmological constant + dust can be mapped to that of an oscillator
- A unique ground state exists for closed FLRW + negative cosmological constant + dust.
- Positive cosmological constant - Higher probability of large universes for smaller cosmological constants
- Fluctuations around classical paths dampen as volume increases

Caveats

- Not all four geometries are allowed
- The quantum theory is a theory of 3-geometries
- Dust is not a physical fundamental matter field

Next Steps – Anisotropic Cosmologies

- For Bianchi I, positivity of dust density guarantees positivity of the Euclidean action
- No unique ground state
- No emergence of classical paths
- Need to modify the sampling algorithm to deal with high rejection rates
- Spatial curvature adds a potential term to the action and efficient sampling gets more difficult

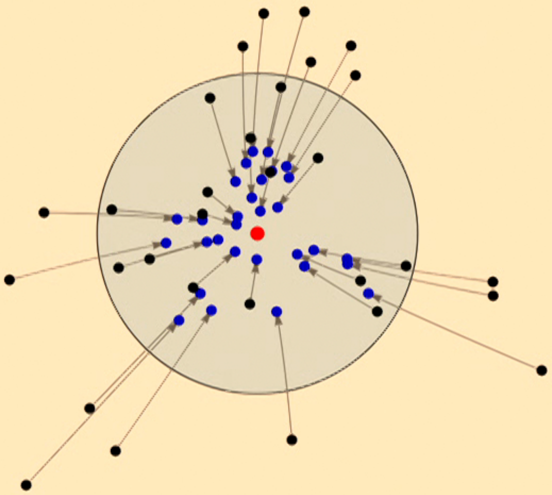
Next Steps – Gowdy Models

- Simple 2D field theories which are classically well studied
- There is diffeomorphism constraint to deal with. Gauge fix before MC or impose constraint on paths?
- Test bed to study MC techniques generalizable to higher dimensions
- Test bed for looking at the spherically symmetric sector

Next Steps – Monte Carlo on Lefschetz thimbles [Alexandru et al(2016)]

- Complexify the coordinates: $x \to \xi$ ➡ $\langle O \rangle = \dfrac{\langle e^{iS_I} O \rangle_R}{\langle e^{iS_I} \rangle_R}$

- Define thimble with flow equation : $\dfrac{d\xi}{d\tau} = \dfrac{\partial \bar{S}}{\partial \xi}, \quad \xi(0) = \phi$



- Curvature of the thimble is unknown

- Can sample points close to the critical point and flow them

$$\langle O(\xi) \rangle = \frac{\langle O(\xi(\phi))\Phi(\phi) \rangle_S}{\langle \Phi(\phi) \rangle_S}$$

$$S = S_R - \ln|\det J|$$