Title: On the Brink of Fractionalization - Yichen Hu

Date: Dec 04, 2018 03:30 PM

URL: http://pirsa.org/18120014

Abstract: <p>Systems of strongly interacting particles can give rise to topological phases beyond non-interacting limit. Although unique features of strongly interacting topological phases, such as fractionalization of quantum degrees of freedom, have important applications in quantum information processing, these topological phases are still far from experimental realizations. In this talk, by presenting constructions of two strongly interacting topological phases, I will argue the key mechanism of their realizations is to add interactions near topological phase transitions. I will first introduce a model of interacting Majorana fermions that describes a superconducting phase with Fibonacci topological order. Then I will show that a correlated fluid of electrons and holes, dubbed fractional excitonic insulator phase, can exhibit a fractional quantum Hall effect at zero magnetic field. I will present physical evidence and conjecture that this phase can be realized in a higher angular momentum excitonic paired system in the presence of interactions.   </p>

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On the Brink of Fractionalization

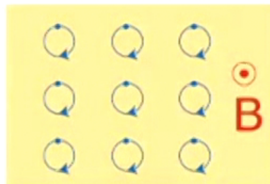
Yichen Hu

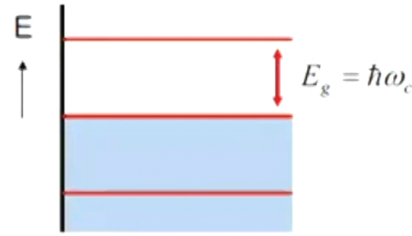
Department of Astronomy and Physics
University of Pennsylvania

Perimeter Institute, Dec 4th 2018

Band Paradigm

- Fractional quantum Hall state





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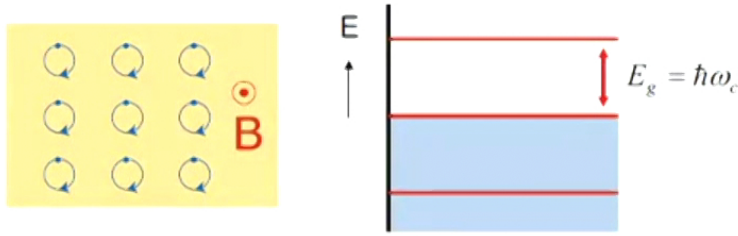
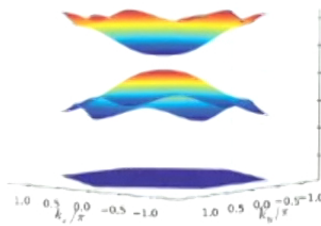


- Fractional Chern insulator



Tang, et al. '11 ; Neupert et al. '11
Sun et al. '11 ; Regnault, Bernevig '11

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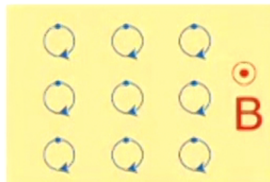

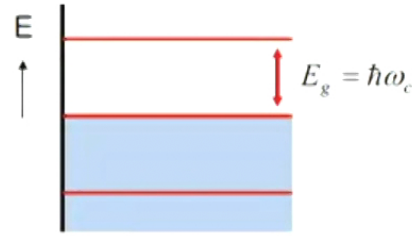

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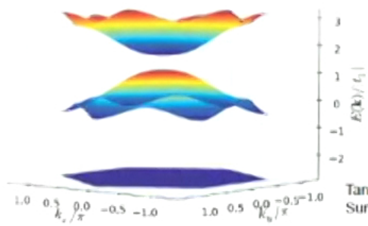

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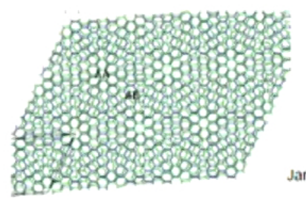
- Twisted bilayer graphene



Jarillo-Herrero et al. '18

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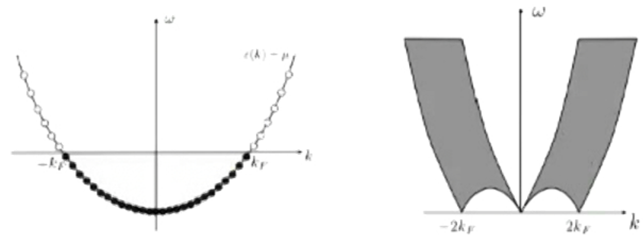
lternative?

- 1D: Luttinger liquid



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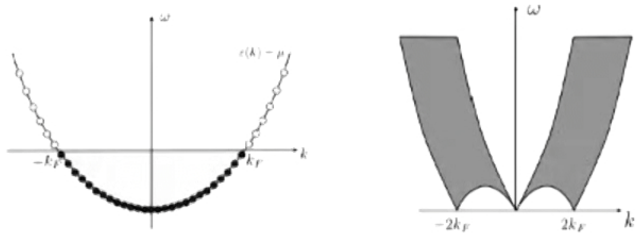
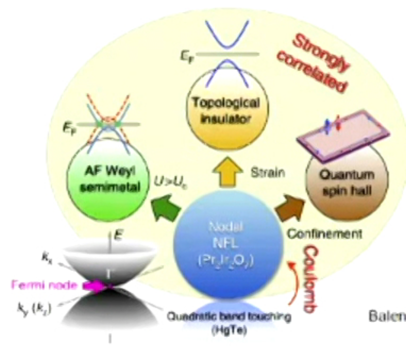


- 2D,3D: Quadratic Fermi node



Balents, Shin et al. '15

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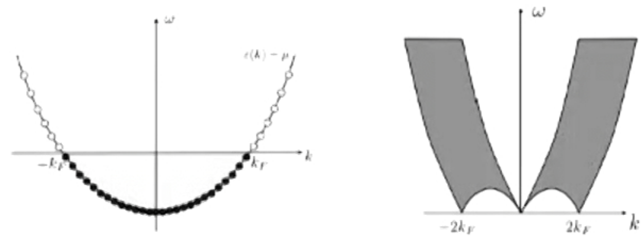

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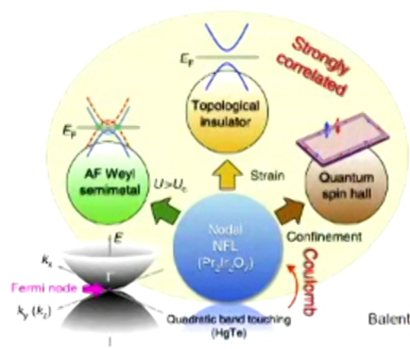

Balents, Shin et al. '15

- Interactions near topological phase transitions=Strongly interacting topological phases?

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Fibonacci Topological Superconductor

- Majorana fermions in topological SC
- Fibonacci anyons from interacting Majorana fermions
- Network construction

Fractional Excitonic Insulator

- Chern insulator as $p_x + ip_y$ excitonic insulator
- Fractional excitonic insulator phase
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rana fermions in topological SC

- $\gamma_i = \gamma_i^\dagger$ and $\{\gamma_i, \gamma_j\} = 2\delta_{ij}$.

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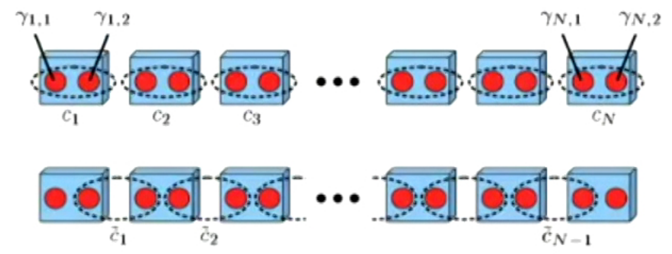
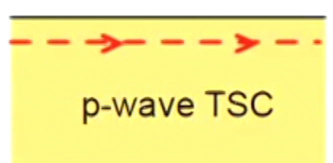
- $\gamma_i = \gamma_i^\dagger$ and $\{\gamma_i, \gamma_j\} = 2\delta_{ij}$.
  - a Bogoliubov quasiparticle$(e + h)$ in superconductor.
- Majorana bound states on the edge of topological SC
  - 1D Kitaev chain Kitaev, '01
  - 2D p-wave topological superconductor Read and Green, '00





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rana fermions in topological SC

g Dirac fermion

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- $c_{Dirac} = 1 = \frac{1}{2} + \frac{1}{2}$ with $c_{MF} = \frac{1}{2}$

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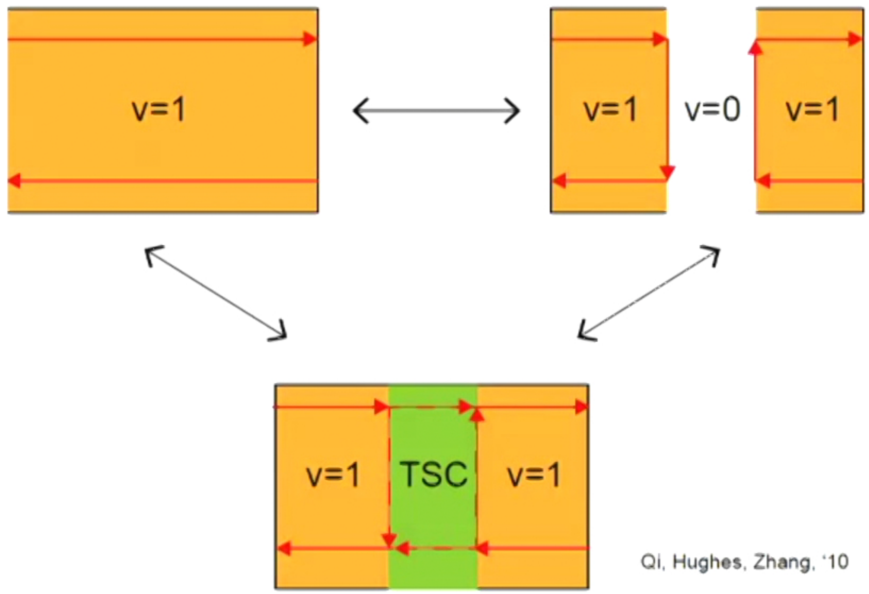
- Splitting of integer quantum Hall transition



Qi, Hughes, Zhang, '10

nacci anyons from interacting Majorana fermions

ological order

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- Anyon type:
  1. Vacuum $\mathbb{I}$
  2. Fibonacci anyon τ
- Fusion rule:
  $\tau \times \tau = 1 + \tau$
- Ground state degeneracy: ϕ^{N_τ}, $\phi = \frac{1+\sqrt{5}}{2}$
- Braiding Fib anyons allows for universal quantum computation
- $\mathbb{Z}_3$ parafermion CFT:
  1. $\nu = \frac{12}{5}$ FQH state Read and Rezayi, '98
  2. Trench construction($\nu = \frac{2}{3}$ FQH+SC) Mong et al. '14; Vaezi '14

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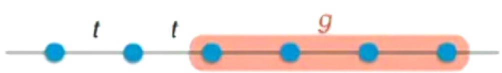
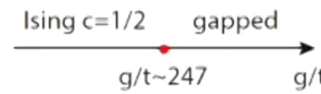
d Coset

- Tri-critical Ising in 1D interacting Majorana fermion chain

$$H = \sum_i -it\gamma_i\gamma_{i+1} + g\gamma_i\gamma_{i+1}\gamma_{i+2}\gamma_{i+3}$$





Franz, Affleck et al. ' 15

Anyon types

$\mathbb{I}$	σ	σ'	ϵ	ϵ'	ϵ''
0	3/80	7/16	1/10	3/5	3/2

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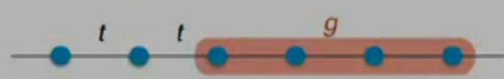

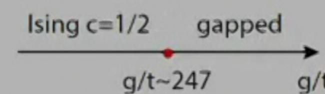

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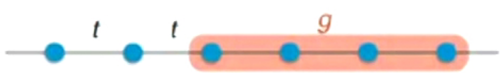

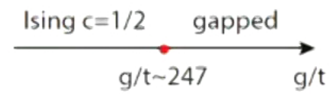

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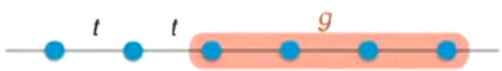

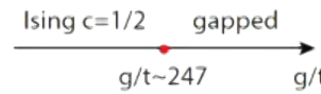

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- $SO(7)_1$ coset Shatashvili and Vafa, '95

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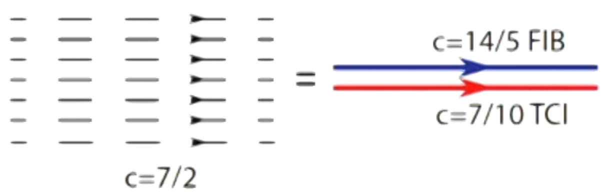

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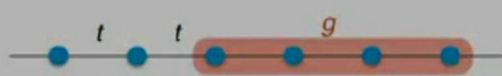

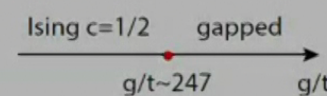

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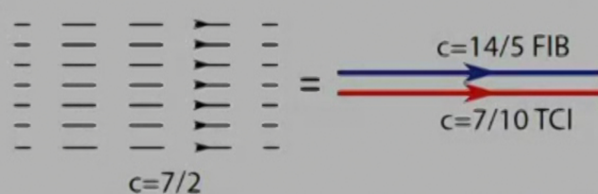

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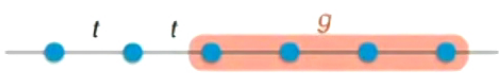

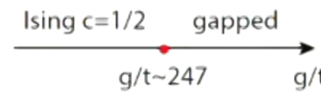

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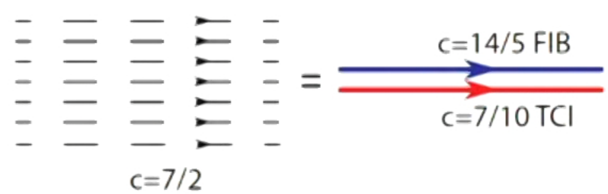

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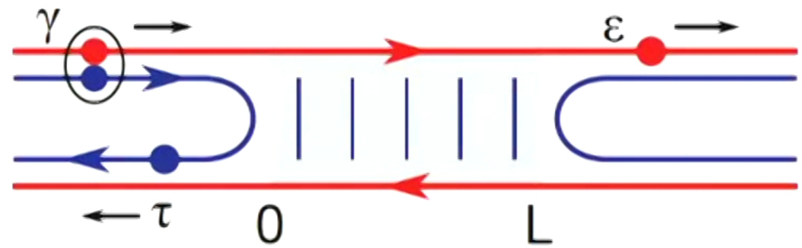
nacci anyons from interacting Majorana fermions

g Majorana fermion

- Gap out the Fibonacci sector



$$H = -\frac{iv}{2}\sum_{a=1}^{7}(\gamma_{aR}\partial_x\gamma_{aR} - \gamma_{aL}\partial_x\gamma_{aL}) + \lambda\sum_{A=1}^{14} J_R^A J_L^A$$

$(G_2)_1$ currents*

$$J_{L/R}^A = M_{ab}^A \gamma_a^{L/R}\gamma_b^{L/R}$$

$$\gamma = \tau \times \epsilon$$

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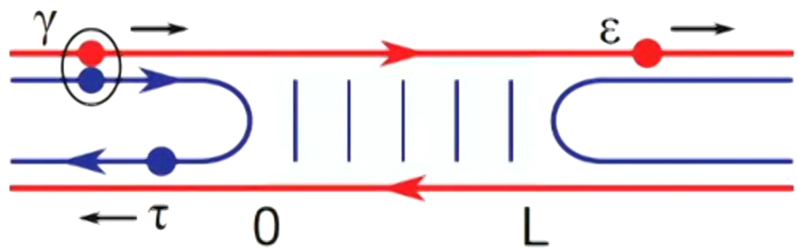

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Fibonacci Topological Superconductor

- Majorana fermions in topological SC
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- Network construction

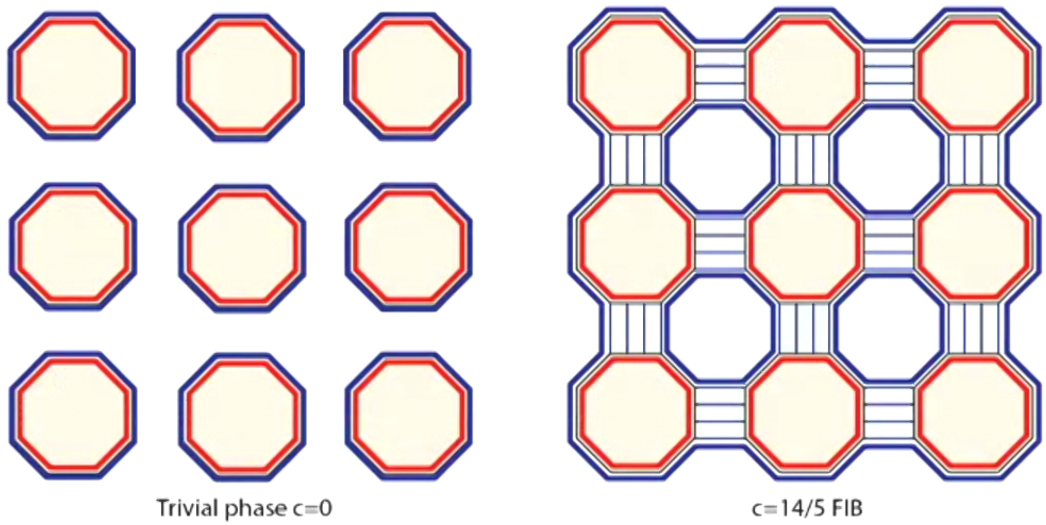
Fractional Excitonic Insulator

- Chern insulator as $p_x + ip_y$ excitonic insulator
- Fractional excitonic insulator phase
- Higher angular momentum band inversion

ork Construction

cci phase

- Islands of $n = 7$ topological SCs



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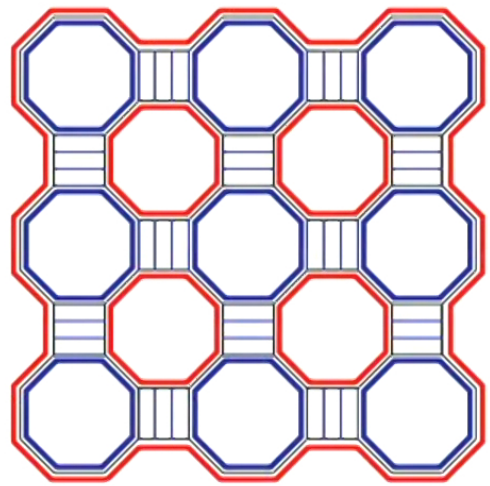
...work Construction

...bonacci phase

- "Particle-hole" conjugate of the Fibonacci phase
  $$c = 7/2 - 14/5 = 7/10$$



c=7/10 TCI

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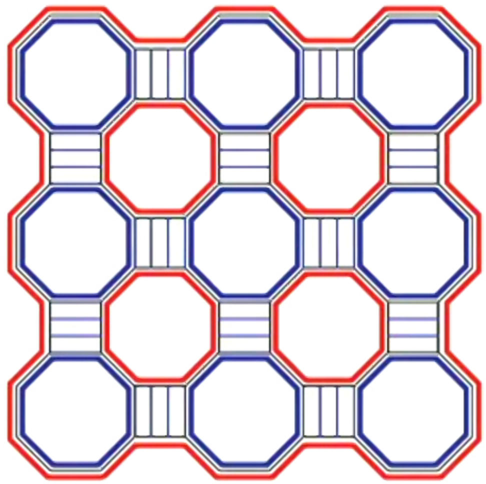
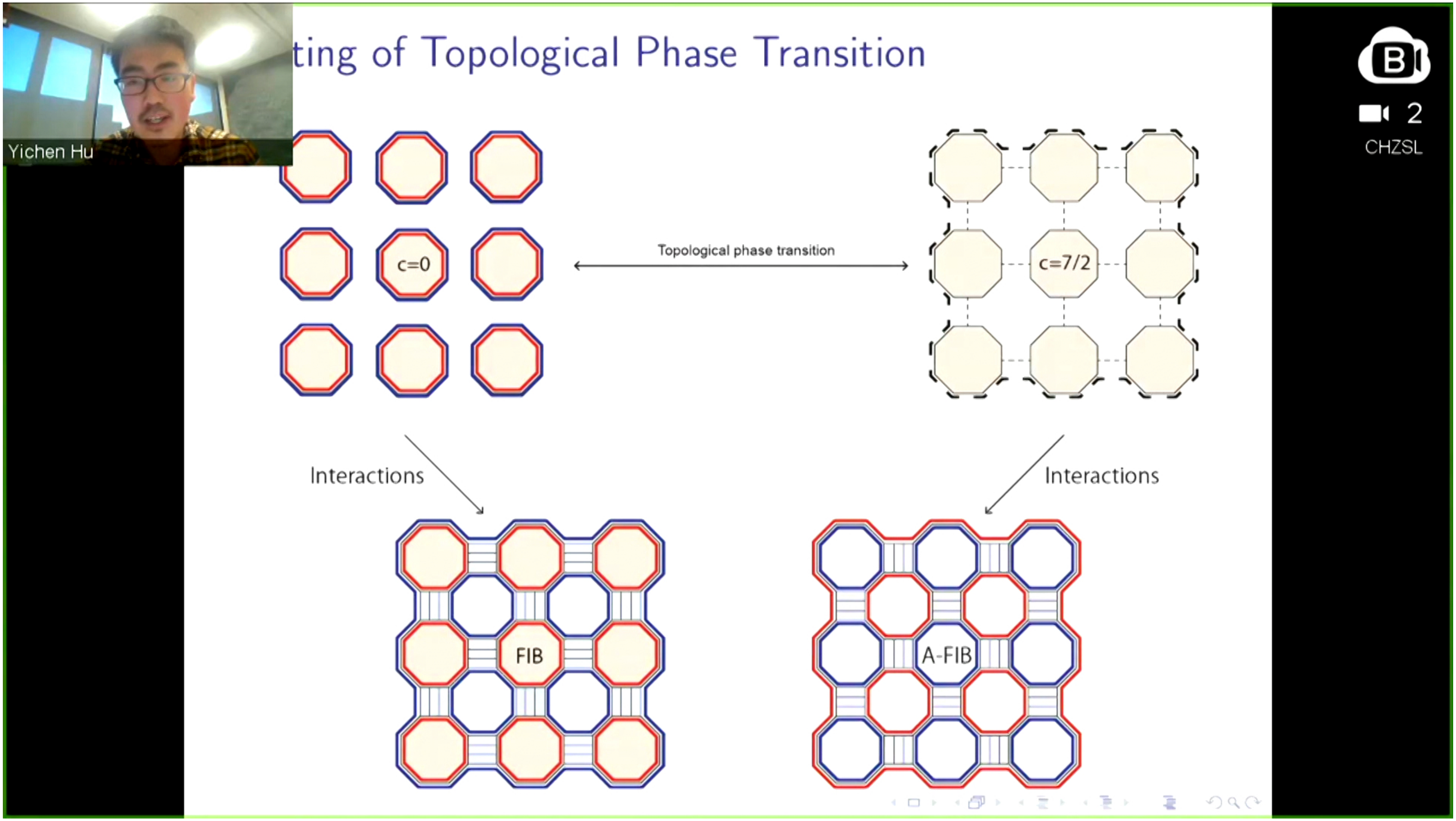


c=7/10 TCI

- TCI=FIB$\times$Ising

	$\mathbb{I}$	σ	ψ
$\mathbb{I}$	$\mathbb{I}$	σ'	ϵ''
τ	ϵ	σ	ϵ'


ting of Topological Phase Transition
Yichen Hu
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c=0
Topological phase transition
c=7/2
Interactions
Interactions
FIB
A-FIB

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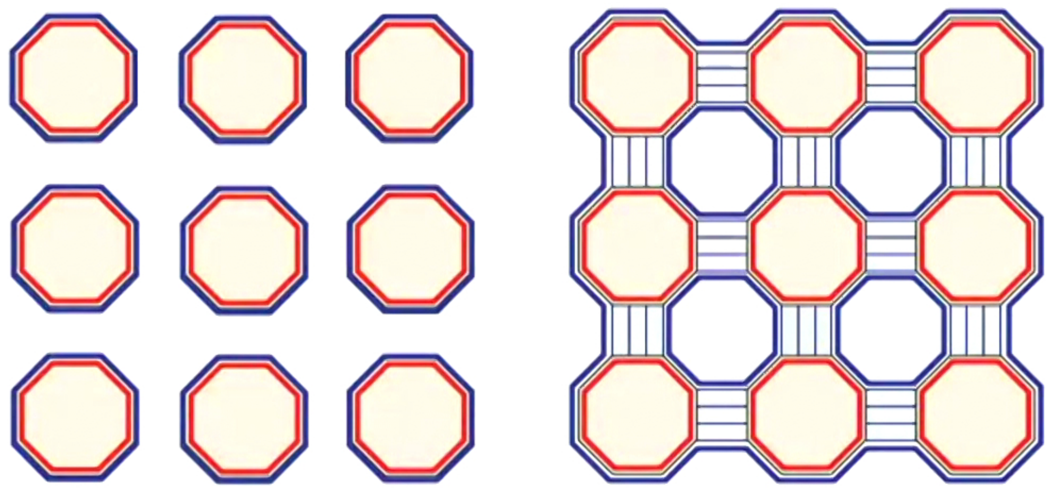

Trivial phase c=0

c=14/5 FIB

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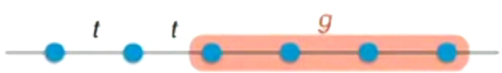

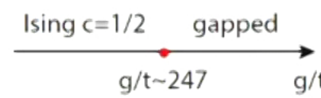

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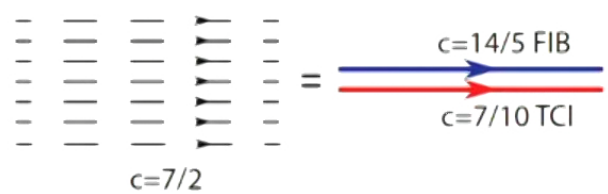

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- Fractional excitonic insulator phase
- Higher angular momentum band inversion

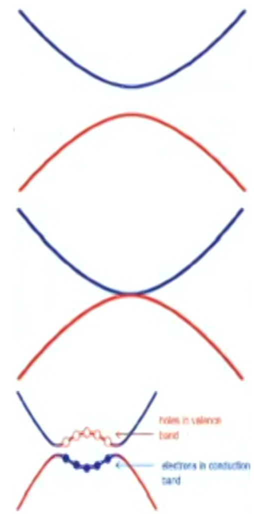
n insulator

, excitonic insulator

amiltonian

$$\mathcal{H} = \sum_{\mathbf{k}} \epsilon_{\mathbf{k}} (\psi^{\dagger}_{e\mathbf{k}} \psi_{e\mathbf{k}} + \psi^{\dagger}_{h\mathbf{k}} \psi_{h\mathbf{k}}) + \Delta_{\mathbf{k}} \psi^{\dagger}_{e\mathbf{k}} \psi^{\dagger}_{h-\mathbf{k}} + h.c.,$$

with $\epsilon_{\mathbf{k}} = (k^2 + m_D)/2; \quad \Delta_{\mathbf{k}} = v(k_x + ik_y).$



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n insulator

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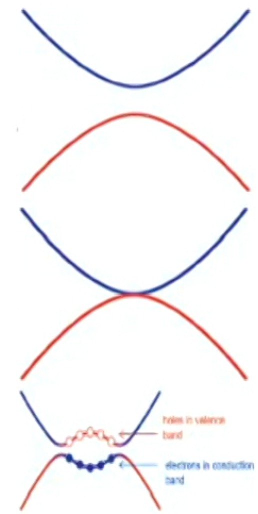

"BCS" ground state Read and Green '00

$$|\psi_{m=1}\rangle = \prod_{\mathbf{k}} (u_{\mathbf{k}} + v_{\mathbf{k}} \psi^{\dagger}_{e\mathbf{k}} \psi^{\dagger}_{h-\mathbf{k}}) |0\rangle,$$

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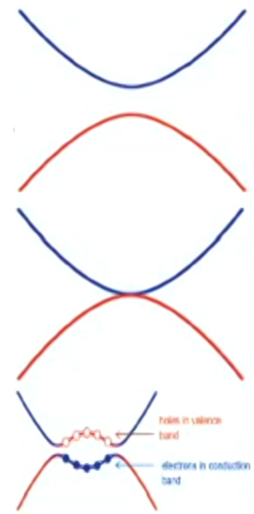

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Pairing function

$$g_{\mathbf{k}} = \frac{v_{\mathbf{k}}}{u_{\mathbf{k}}} \rightleftarrows g(z = x + iy \to \infty) = \begin{cases} m_D > 0 & e^{-|z|/\xi} \\ m_D < 0 & \frac{1}{z} \end{cases}$$

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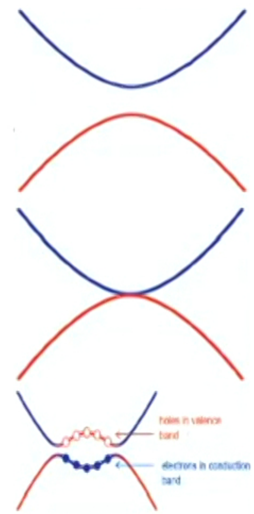

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p-wave excitonic condensate

$$|\psi_{m=1}\rangle = \sum_{N=1}^{\infty} \frac{f^N}{N!} |\psi^N_{m=1}\rangle \propto e^{\int dz dw g(z-w) \psi^\dagger_e(z) \psi^\dagger_h(w)} |0\rangle$$
$$m_D = -v^2 \to \psi^N_{m=1}(\{z_i, w_j\}) = \det[g(\mathbf{z} - \mathbf{w})]$$
$$= \frac{\prod_{i<i'}(z_i - z_{i'}) \prod_{j<j'}(w_j - w_{j'})}{\prod_{i,j}(z_i - w_j)}$$

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tional excitonic insulator

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Wavefunction

$$\psi_m^N(\{z_i, w_j\}) = \frac{\prod_{i<i'}(z_i - z_{i'})^m \prod_{j<j'}(w_j - w_{j'})^m}{\prod_{i,j}(z_i - w_j)^m}$$

(see also Dubail and Read '15)

$$|\psi_m\rangle = \sum_N \frac{f^N}{N!} |\psi_m^N\rangle \text{ with } f = \frac{v}{2\pi}.$$

Similar to Laughlin(or Halperin) type wavefunction, except:

- No Gaussian factor
- Singular denominator

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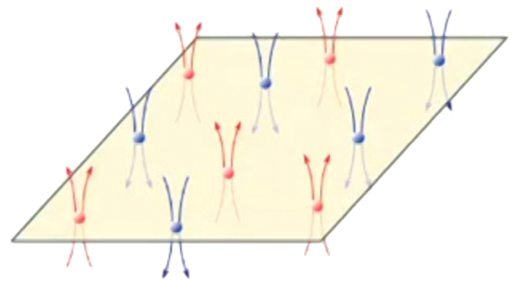
Composite fermion theory



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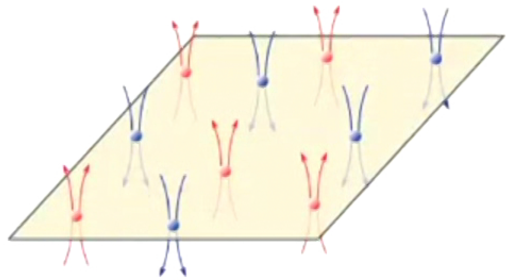

Singular gauge transformation

$$\nabla \times \mathbf{a} = 2\pi(m-1)(\psi_e^\dagger \psi_e - \psi_h^\dagger \psi_h)$$

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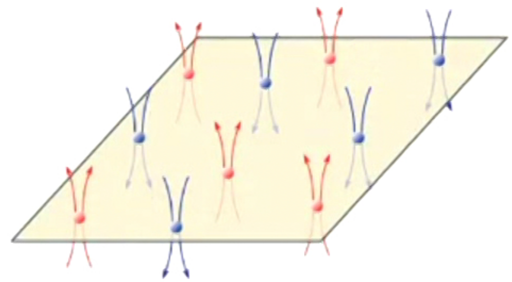

Singular gauge transformation

$$\nabla \times \mathbf{a} = 2\pi(m-1)(\psi_e^\dagger \psi_e - \psi_h^\dagger \psi_h)$$

Mean field

If $\langle \rho_e \rangle = \langle \rho_h \rangle$, then $B_{av} = 0$.
Composite fermions form Chern insulator $(C = 1) \rightleftarrows$ original fermions form a FQH fluid with $\sigma_{xy} = \frac{1}{m}\frac{e^2}{h}$.

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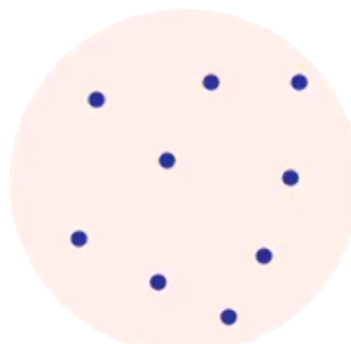
lasma analogy for Laughlin's wavefunction

$$\langle \psi_m | \psi_m \rangle = \text{partition function for classical plasma.}$$

$$-\beta V = \sum_{i<j} 2m q_i q_j \log |z_i - z_j| / \xi.$$

$$\prod_{i<j} (z_i - z_j)^m e^{-\sum_i |z_i|^2 / 4 l_B^2} \qquad \begin{cases} m \lesssim 11 & \text{fluid(screening plasma)} \\ m \gtrsim 11 & \text{Wigner crystal} \end{cases}$$



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Plasma analogy (see also Dubail and Read '15)

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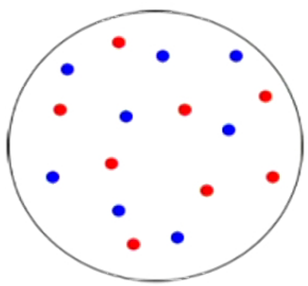

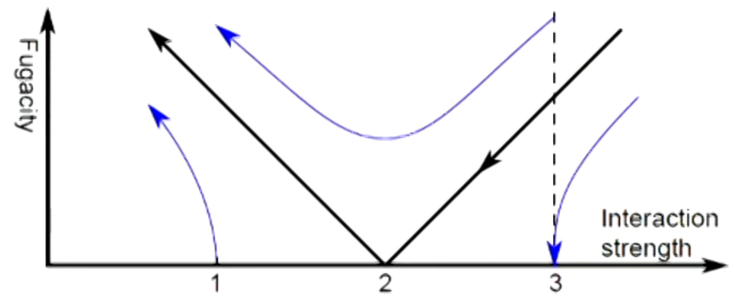

tional excitonic insulator

we shall do just the opposite - start with a ground state and solve r the Hamiltonian, matching questions to answers as in the show *Jeopardy*"

Exact Question to the Answer Arovas and Girvin, '92

Define

$$Q_e(z) = 2\partial_{z^*}\psi_e - v_m(\partial_z - ia)^{m-1}\psi_h^\dagger$$

$$Q_h(z) = 2\partial_{z^*}\psi_h - v_m(\partial_z + ia)^{m-1}\psi_e^\dagger$$

with $v_m = \frac{2\pi f}{(m-1)!}$ and $a(z) = m\int d^2u \frac{\rho(u)}{i(z-u)}$; $\quad \rho = \psi_e^\dagger\psi_e - \psi_h^\dagger\psi_h$.

$$Q_{e(h)}\,|\psi_m\rangle = 0$$

$$\mathcal{H}_m = \frac{1}{2}\int d^2\mathbf{z}\left[Q_e^\dagger(z)Q_e(z) + Q_h^\dagger(z)Q_h(z)\right].$$

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Non-interacting limit($a = 0$)

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Non-interacting limit($a = 0$)

- $m = 1$, $\mathcal{H}_1^{a=0}$ is the $(p + ip)$ excitonic paired system.

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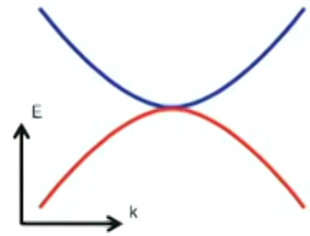
tional excitonic insulator

d" zeros

Hamiltonian



$$\mathcal{H}_m^{a=0} = \sum_{\mathbf{k}} \epsilon_{\mathbf{k}} (\psi_{e\mathbf{k}}^\dagger \psi_{e\mathbf{k}} + \psi_{h\mathbf{k}}^\dagger \psi_{h\mathbf{k}}) + \Delta_{\mathbf{k}} \psi_{e\mathbf{k}}^\dagger \psi_{h-\mathbf{k}}^\dagger + h.c.,$$
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Wavefunction

$$g(z \to \infty) \propto \frac{1}{z^m}$$
$$\phi_m^N(\{z_i, w_j\}) = \det\left[g(z_i - w_j)\right] = \frac{\prod_{i<i'}(z_i - z_{i'}) \prod_{j<j'}(w_j - w_{j'}) P(\{z_i, w_j\})}{\prod_{i,j}(z_i - w_j)^m}$$

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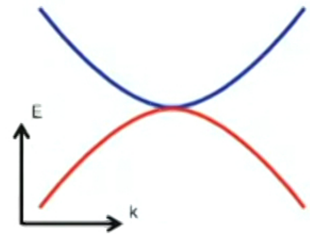

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Conjecture

Short-range repulsive interaction is effective at putting "wasted" zeros on top of particles.

$$\frac{\prod_{i<i'}(z_i - z_{i'}) \prod_{j<j'}(w_j - w_{j'}) P(\{z_i, w_j\})}{\prod_{i,j}(z_i - w_j)^m} \to \frac{\prod_{i<i'}(z_i - z_{i'})^m \prod_{j<j'}(w_j - w_{j'})^m}{\prod_{i,j}(z_i - w_j)^m}$$

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$_{j'})P(\{z_i, w_j\})$

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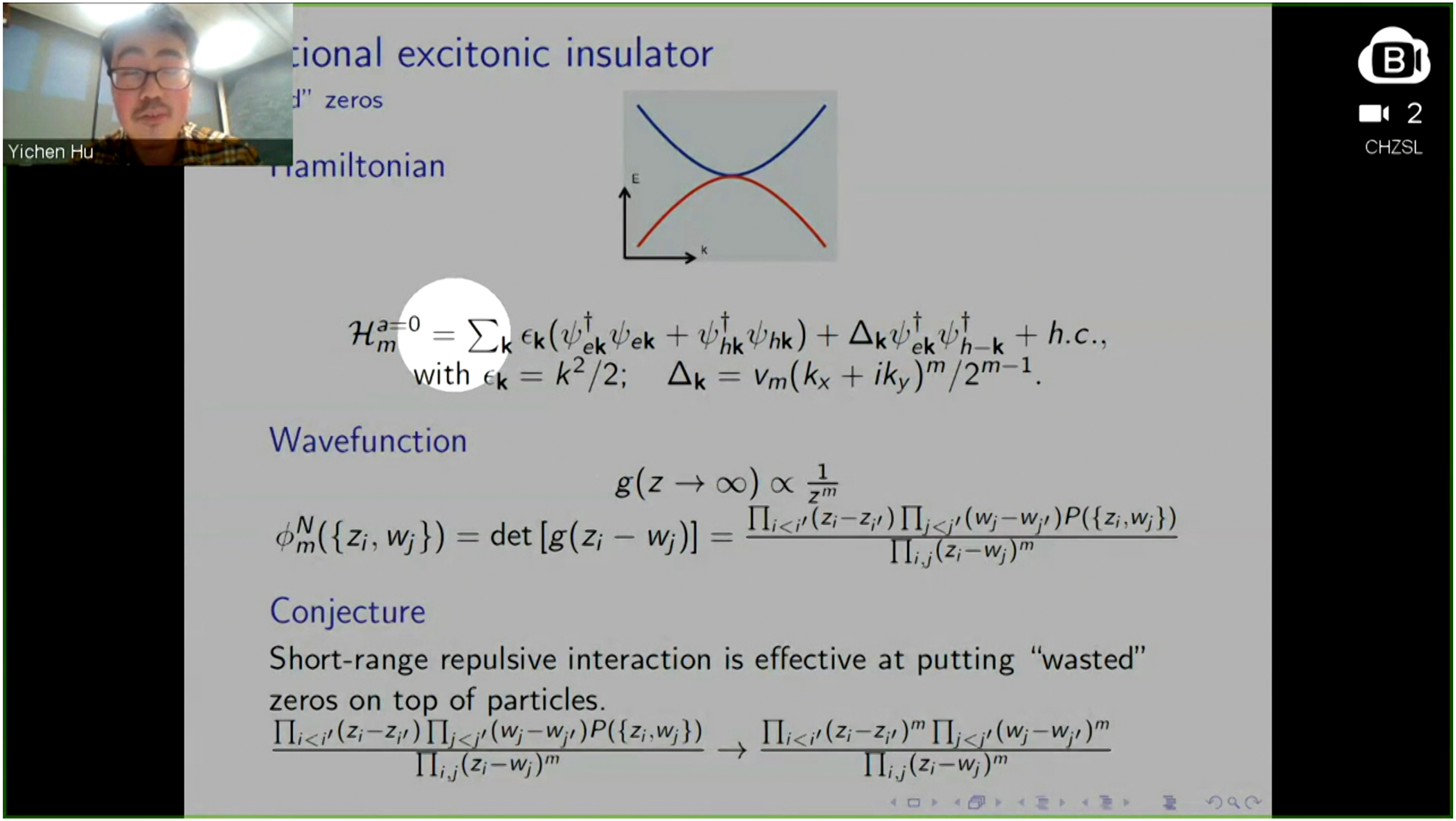
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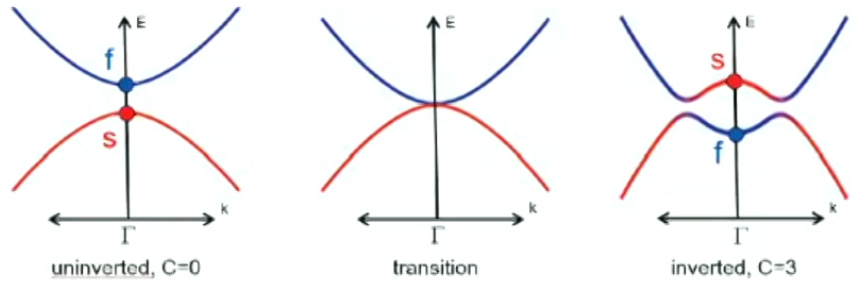

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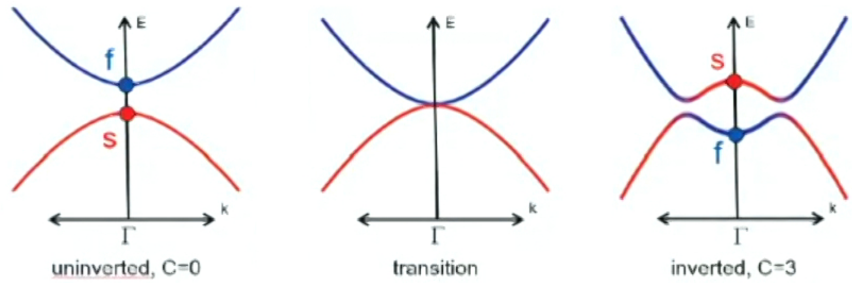

- C_6 rotational symmetry
- Invert bands differ in angular momentum by 3 at Γ
- Chern number changes by 3 at transition
- Quadratic band touching → interactions are important
- Target for band structure engineering (see arXiv:1809.10665)

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Summary

- Interactions near topological phase transitions=Strongly interacting topological phases?
- the Fibonacci phase in topological superconducting systems
- Fractional excitonic insulator phase

Future directions

- Phase diagram
- Materials with higher angular momentum inversion.
- Extensions to other strongly interacting topological phases.

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