

Title: TBA

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Abstract: <p>TBA</p>



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Perimeter.key — 編集済み

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AdS from Tensor networks: Optimization of Path Integral

Phys.Rev. D95 (2017): hep-th/1609.04645
Phys.Rev.Lett.119, 071602 (2017): hep-th/1703.00456
JHEP 1711 (2017): hep-th/1706.07056
JHEP07(2018)086: hep-th/1804.01999
hep-th/1812.xxxx

Collaboration with
A.Bhattacharyya, P. Caputa, S. Das, N. Kundu, T. Takayanagi, K.
Umemoto, K. Watanabe

Masamichi Miyaji

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The slide is a Keynote presentation titled "Perimeter.key". It features a sidebar on the left containing seven numbered notes (1 through 7) with small preview images. The main content area contains the title "AdS from Tensor networks: Optimization of Path Integral", several arXiv references, a "Collaboration with" section listing names, and the name "Masamichi Miyaji" followed by the affiliation "Yukawa Institute for Theoretical Physics, Kyoto University".





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Motivation

- AdS/Tensor Network conjecture [Swingle]

Tensor Network Bulk timeslice in AdS

The diagram illustrates the AdS/CFT correspondence. On the left, a 'Tensor Network' is shown as a circular arrangement of nodes connected by lines, forming a grid-like structure. On the right, a 'Bulk timeslice in AdS' is shown as a cylinder with a red elliptical cross-section. A double-headed arrow between them is labeled 'Equivalent!'.

- This conjecture explains RT formula.
- Useful tool to draw quantum information theory from AdS/CFT, such as Fidelity Susceptibility and Entanglement of Purification etc



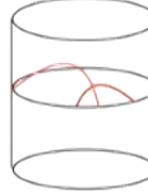
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Problems & Questions

- Continuum limit
 - General formalism including interaction?
 - Known only for free theories [Haegeman et al.]
 - Lack of precise correspondence between metric and TN.
- Time dependence
 - No Cauchy slice on which all HRT surfaces live.





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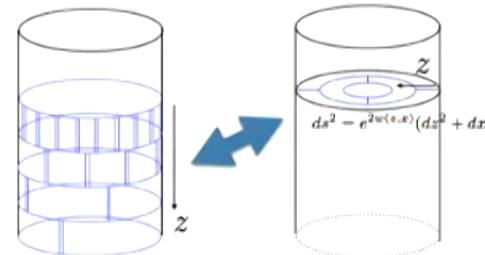


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Outline

- Optimization of path integral :
 - Path integral representation of **Continuous** tensor network.
 - Bulk metric from path integral and **optimization**.
 - Relation to complexity.
 - Entanglement wedge cross section.





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A screenshot of a Mac OS X desktop showing a Keynote presentation titled "Perimeter.key". The main slide area is blank. On the left, a slide outline shows slide 5, "Tensor Network and AdS/CFT", highlighted with a blue selection bar. The slide numbers 1 through 7 are visible along the left edge. The top menu bar includes "表示", "スケルトン", "スライドを追加", "再生", "Keynote Live", and various tool icons like 表, グラフ, テキスト, 図形, メディア, コメント, 共同制作, フォーマット, アニメーション, and 音楽. The status bar at the bottom shows "Pirsa: 18120013" and "Page 6/28".

1. Tensor Network and AdS/CFT

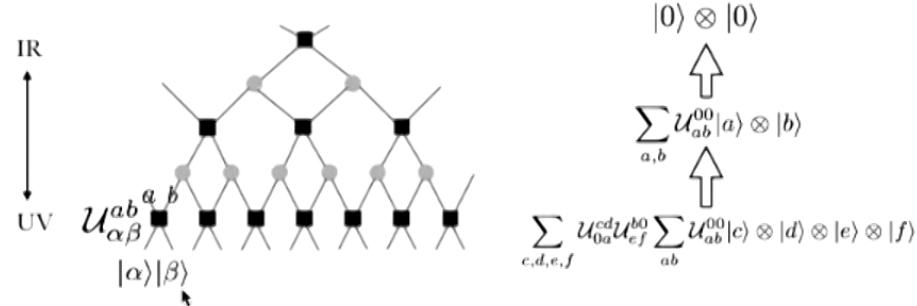


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Tensor Network [White][Vidal]

Efficient representation of ground state wave function.



- RG flow in real space: flow of coarse grained states.
- Grounds state gets simplified and loses short range entanglement.



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AdS/Tensor Network proposal

Entanglement entropy of both tensor network state and vacuum state of holographic CFT are given by **the area of minimal surface**.

Conjecture [Swingle]

→ **Tensor network = Timeslice of bulk spacetime in AdS/CFT**

Timeslice of AdS_{d+1} Tensor network



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2. AdS from optimization

- Path integral representation of **Continuous** tensor network.
- Bulk metric from path integral and “**optimization**”.
- Relation to complexity.



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Deforming Euclidean Path Integral

[M.M. Takayanagi, Watanabe][S. Caputa, Kundu, M.M. Takayanagi, Watanabe]

In Euclidean path integral, ground state wave function is

$$\langle \phi_0 | \Omega_{vac} \rangle \propto \lim_{T \rightarrow \infty} \langle \phi_0 | e^{-T H} | \phi_B \rangle$$
$$\propto \int \mathcal{D}\phi(x, \tau) e^{-\int_{-\infty}^{-\epsilon} d\tau \int dx \mathcal{L}_E[\phi(x, \tau)]}$$
$$\phi(x, -\epsilon) = \phi_0(x)$$

We will deform this path integral!

$\phi(x, -\epsilon) = \phi_0(x)$

$\phi(x, \tau = -\infty) = \phi_B(x)$

Effective lattice spacing is position independent.



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Deforming Euclidean Path Integral

We want to identify Euclidean time as RG scale direction.

→ Set UV cut off position dependent.

Lattice spacing: ϵ z

UV cut off: $1/\epsilon$ $1/z$

So that we can reproduce change of lattice spacing of real space RG.



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Liouville Action

For 2d CFT, path integral changes **only** up to overall constant from Weyl anomaly.

$$Z[\phi_0(x)] = \int_{z>\epsilon} \mathcal{D}\phi(x, z) e^{-\int_{\epsilon}^{\infty} dz \int dx \mathcal{L}_E[\phi(x, z)]}$$

Weyl transformation

$$\phi(x, z = \epsilon) = \phi_0(x)$$
$$= e^{-S_L[w]} \int_{z>\epsilon} \mathcal{D}\phi(x, z) e^{-\int_{\epsilon}^{\infty} dz \int dx \mathcal{L}_E[\phi(x, z), w(x, z)]} = e^{-S_L[w]} Z[\phi_0(x), w]$$

$\phi(x, z = \epsilon) = \phi_0(x)$

Independent from ϕ_0

The wave function is invariant!

Liouville action: $S_L[w] = \frac{c}{24} \int_{\epsilon}^{\infty} dz \int dx \left[(\partial w)^2 + e^{2w} - \frac{1}{\epsilon^2} \right]$ [Polyakov]



Continuous Tensor Network from Path Integral

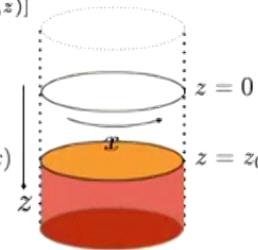
Define a flow of states $(|\Omega_{z_0}\rangle)_{z_0 > 0}$

$$\langle \phi_0 | \Omega_{z_0} \rangle \propto \int_{z > z_0} \mathcal{D}\phi(x, z) e^{-\int_{z_0}^{\infty} dz \int dx \mathcal{L}_E[\phi(x, z), w(x, z)]}$$

$$\phi(x, z_0) = \phi_0(x)$$

$$(\epsilon < z_0 < \infty)$$

$$\phi(x, z_0) = \phi_0(x)$$



We dropped modes with energy bigger than $1/z_0$ in coarse grained state $|\Omega_{z_0}\rangle$

This realizes RG flow of continuous Tensor Network!

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Continuous Tensor Network

Interpolation from vacuum state to product state.

$$|\Omega_{z_0}\rangle = \tilde{P} e^{i \int_0^\infty \hat{K}(z=\epsilon e^{-u})} |\Omega_\infty\rangle$$

$|\Omega_\infty\rangle$: unentangled trivial state $\frac{1}{z}$: RG energy scale
 $|\Omega_0\rangle$: ground state

ex) Free scalar theory [Haegeman, Osborne, Verschelde and Verstraete]

$$a_k |\Omega_{vac}\rangle = 0 \xrightarrow{\text{Vacuum}} (a_k + a_{-k}^\dagger) |B\rangle = 0 \xrightarrow{\text{Bogoliubov transformation}} \text{Boundary state (Dirichlet)}$$

$$\hat{K}(z) := \int dk \Gamma[|k|z] (a_k^\dagger a_{-k}^\dagger - a_k a_{-k}) \quad \Gamma[z] = 1 \ (z < 1) \\ \text{Otherwise } 0$$

$$(\cosh \frac{u}{2} a_k + \sinh \frac{u}{2} a_k^\dagger) |\Omega_{z=\epsilon e^{-u}}\rangle = 0 \quad (|k| < \frac{e^u}{\epsilon} = \frac{1}{z})$$

$$(a_k + a_k^\dagger) |\Omega_{z=\epsilon e^{-u}}\rangle \approx 0 \quad (|k| > \frac{e^u}{\epsilon} = \frac{1}{z})$$



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Complexity function

- Liouville action indeed counts the number of local unitaries in tensor network.

$$S_L[w] = \frac{c}{24} \int_{\epsilon}^{\infty} dz \int dx \left[(\partial w)^2 + e^{2w} - \frac{1}{\epsilon^2} \right]$$

Density of scale transformations [Czech]

Density of transformations that change entanglement

- When Liouville action is minimum, tensor network is “optimized”.



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Proposal

The geometry

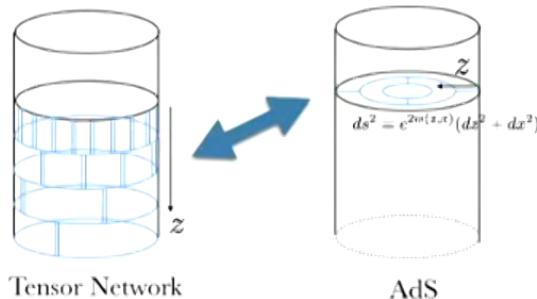
$$ds^2 = e^{2w(z,x)}(dz^2 + dx^2)$$

Weyl factor

is the metric of the time slice in the bulk when the Liouville action

$$S_L[w]$$

is minimum, so that the tensor network is “optimized”.



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Proposal

- More generally, we minimize

$$e^{S[w]} = \frac{\int_{\phi|_{z=\epsilon}=\phi_0} \mathcal{D}\phi e^{-S[\phi,w]}}{\int_{\phi|_{z=\epsilon}=\phi_0} \mathcal{D}\phi e^{-S[\phi]}}$$

in various examples.

Vacuum state

- Liouville action is minimum, when the metric

$$ds^2 = e^{2w(z,x)}(dz^2 + dx^2)$$

is pure AdS. For instance

$$e^{2w} = \frac{\epsilon^2}{z^2}$$



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Examples

- Excited state

Consider state excited by primary operator of dimension h at infinite past. Effect of Weyl transformation

$$ds^2 = \frac{dz^2 + dx^2}{\epsilon^2} \quad \Rightarrow \quad ds^2 = e^{2w}(dz^2 + dx^2)$$

to Liouville action is

$$S_L[w] - h \cdot w(z = -\infty) \quad (h \ll c)$$


Conical deficit geometry

The solution is conical deficit geometry, with deficit angle

$$2\pi(1 - \sqrt{1 - \frac{24h}{c}}) \quad (h \ll c)$$



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Examples

Thermofield double state ($t=0$)

$$|\Psi_{TFD}\rangle = \frac{1}{\sqrt{Z(\beta)}} \sum_i e^{-\beta E_i} |E_i\rangle \otimes |E_i\rangle$$

$$\langle \tilde{\psi}_1, \tilde{\psi}_2 | \Psi_{TFD} \rangle = \int_{-\beta/4 < z < \beta/4} \mathcal{D}\psi(x, z) e^{-S_{CFT}}$$

$$\psi(x, -\beta/4) = \bar{\psi}_1(x)$$

$$\psi(x, \beta/4) = \bar{\psi}_2(x)$$

We get Liouville action again,

$$S_L[w] = \frac{c}{24} \int_{-\beta/4}^{\beta/4} dz \int dx \left[(\partial w)^2 + e^{2w} - \frac{1}{c^2} \right]$$

We get time slice of BTZ at $t=0$, with correct temperature, after optimization.

$$ds^2 = \frac{8\pi^2}{\beta^2} \cdot \frac{dz^2 + dx^2}{\cos^2(\frac{2\pi z}{\beta})}$$



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Examples

Relevant deformation [Bhattacharyya, Caputa, Das, Kundu, Miyaji, Takayanagi]

$$S_E + \int d\tau_E dx \lambda_0(x) O(\tau_E, x)$$

We assume we have functional

$$\lambda[z, x, w(z, x)]$$

for each $w(z, x)$, such that

$$e^{S_L[w, \lambda_0]} := \frac{\int_{z>\epsilon} \mathcal{D}\phi(x, z) e^{- \int_{\epsilon}^{\infty} dz \int dx \mathcal{L}_E[\phi, \lambda[w], w]}}{\int_{z>\epsilon} \mathcal{D}\phi(x, z) e^{- \int_{\epsilon}^{\infty} dz \int dx \mathcal{L}_E[\phi, \lambda_0]}}$$

is independent from $\phi_0(x)$.

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Examples

- Assume small λ_0 , then

$$\lambda[z, x, w] = \lambda_0 e^{(\Delta\phi - 2)w(z, x)} + \mathcal{O}(\lambda_0^2)$$

- Perturbed Liouville action is

$$S_L[w] = \frac{c}{24} \int_{\epsilon}^{\infty} dz \int dx \left[(\partial w)^2 + e^{2w} + \lambda_0^2 e^{(2\Delta - 2)w} - \frac{1}{\epsilon^2} \right] - b e^{1-\Delta} \lambda_0 \int dx$$

↑

- We obtain geometry with cap-off:

$$e^w = \frac{1}{z} \left(1 - \frac{\lambda_0^2}{2(5-2\Delta)} z^{-2\Delta+4} + \dots \right)$$

which is consistent with the result from AdS/CFT prediction.
 [Hung, Myers, Smolkin]

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17 Higher dimensional generalization

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- In higher dimensions, we do not necessarily have nonvanishing conformal anomaly.
- We define complexity functional by (naive) generalization of Liouville action:

$$ds^2 = \frac{1}{\epsilon^2} g_{ab} dx^a dx^b \iff ds^2 = e^{2w} g_{ab} dx^a dx^b$$
$$S_L[w] = \frac{d-1}{16\pi G_N} \int_{\Sigma} \sqrt{g} \left[e^{dw} + e^{(d-2)w} (g^{ab} \partial_a w \partial_b w) + \frac{e^{(d-2)w}}{(d-1)(d-2)} R_g \right]$$

- We can still confirm that this generalized Liouville action reproduces correct metric for pure AdS and massive particle.



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Comparison to complexity = Action proposal

Complexity = Action proposal [Brown, Roberts, Susskind, Swingle, Zhao]

$$\mathcal{C} = \frac{\mathcal{A}}{\pi}$$

\mathcal{A} : Gravity action in WDW patch

Comparison

For pure AdS, metric can written

$$ds^2 = -d\eta^2 + \cos^2 \eta e^{2w(x,y)}(dx^2 + dy^2)$$

Let us pretend $w(x,y)$ is undetermined function, then we can see

$$\boxed{\mathcal{A} \propto S_L} \quad (d > 2)$$

When $d=2$, WDW action vanishes in contrast to Liouville action.
[Reynolds, Ross]



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Entanglement Wedge Cross Section

- Let us consider time slice of entanglement wedge of AB in pure AdS.

The diagram shows a circular boundary representing the RT surface for subsystem AB. A blue curve, labeled 'RT surface for AB', divides the circle into two regions. A red dotted line, labeled 'Entanglement wedge cross section', connects points A and B on the boundary, representing a codimension 2 surface of minimal area.

RT surface for AB

Entanglement wedge cross section

- Area of codimension 2 surface γ which is anchored from RT surface, which has minimal area, is called **entanglement wedge cross section**.
- Entanglement wedge cross section is proposed to be dual to **entanglement of purification of AB**. [Takayanagi, Umemoto][Nguyen, Devalakul, Halbasch, Zalewski, Swingle]

$$E_P[\rho_{AB}] := \underset{\begin{subarray}{c} \text{Tr}_{H_{A'} \otimes H_{B'}} |\Psi\rangle\langle\Psi| = \rho_{AB} \\ |\Psi\rangle \in H_A \otimes H_{A'} \otimes H_B \otimes H_{B'} \end{subarray}}{\text{Min}} S_{|\Psi\rangle}(AA')$$



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Entanglement Wedge Cross Section

[Caputa, M.M., Takayanagi, Umehashi] (To appear)

- We found entanglement wedge cross section can arise from optimization of path integral.
- Consider adjacent intervals $[a, p]$ and $[p, b]$ in CFT
- Add point q , and consider optimization which leaves metric on A and B invariant.

The diagram shows a horizontal line segment with points labeled a , p , b , and q from left to right. There are two red dots on the line: one at a and another at b . The region between a and b is shaded pink. Above the line, there is a label y and below it, a label \tilde{y} . The entire diagram is set against a pink rectangular background.

$$ds^2 = dyd\tilde{y} \xrightarrow{\text{Optimization}} ds^2 = \frac{\epsilon^2}{(\text{Im}\sqrt{\frac{y-a}{b-y}})^2} \cdot \frac{(b-a)^2}{4|b-y|^3|y-a|} dyd\tilde{y}$$



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Entanglement Wedge Cross Section

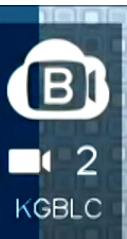
- Weyl transformed wave function on $A\tilde{A}B\tilde{B}$ defines purification of reduced density matrix ρ_{AB} .
(Subclass of purification)
- We compute entanglement entropy of $A\tilde{A}$ on Weyl transformed background

$$S_{A\tilde{A}} = \frac{c}{6} \log \left[\frac{(b-a)(q-p)^2}{2\epsilon(q-a)(q-b)} \right]$$

- Minimization over q of $S_{A\tilde{A}}$ gives entanglement wedge cross section!

$$S_{A\tilde{A}} = \frac{c}{6} \log \left[\frac{2(p-a)(b-p)}{\epsilon(b-a)} \right] = \frac{\text{Area}(\gamma^{\text{EWCS}})}{4G_N}$$

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Entanglement Wedge Cross Section

- This matching works for adjacent intervals in finite temperature system.
- The result is general, and is based only on field theory.
- The calculation **does not** use holography nor large c , and applies to any CFT₂.
- Generalizations to non-adjacent intervals often contain singular Weyl transformation, and we have not found proper ones unfortunately.

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A screenshot of a Mac desktop during a video conference. In the top left corner, there is a video feed of a person with dark hair. Below the video feed, the name "Masamichi Miyaji" is displayed. The main window is a Keynote presentation titled "Perimeter.key" with the subtitle "Perimeter". The slide is titled "Summary" and contains the following bullet points:

- Continuous tensor network from Euclidean path integral and Weyl transformation.
- Identification of Weyl rescaled metric as metric on bulk time slice, via optimization.
 - Reproduced AdS/CFT results in several explicit examples
- Possible identification of Liouville action with WDW action.
- Entanglement wedge cross section from optimization.

The sidebar on the left shows thumbnails for slides 23 through 29. The status bar at the bottom indicates "Pirsa: 18120013" and "Page 28/28".