Title: TBA

Date: Dec 12, 2018 09:30 AM

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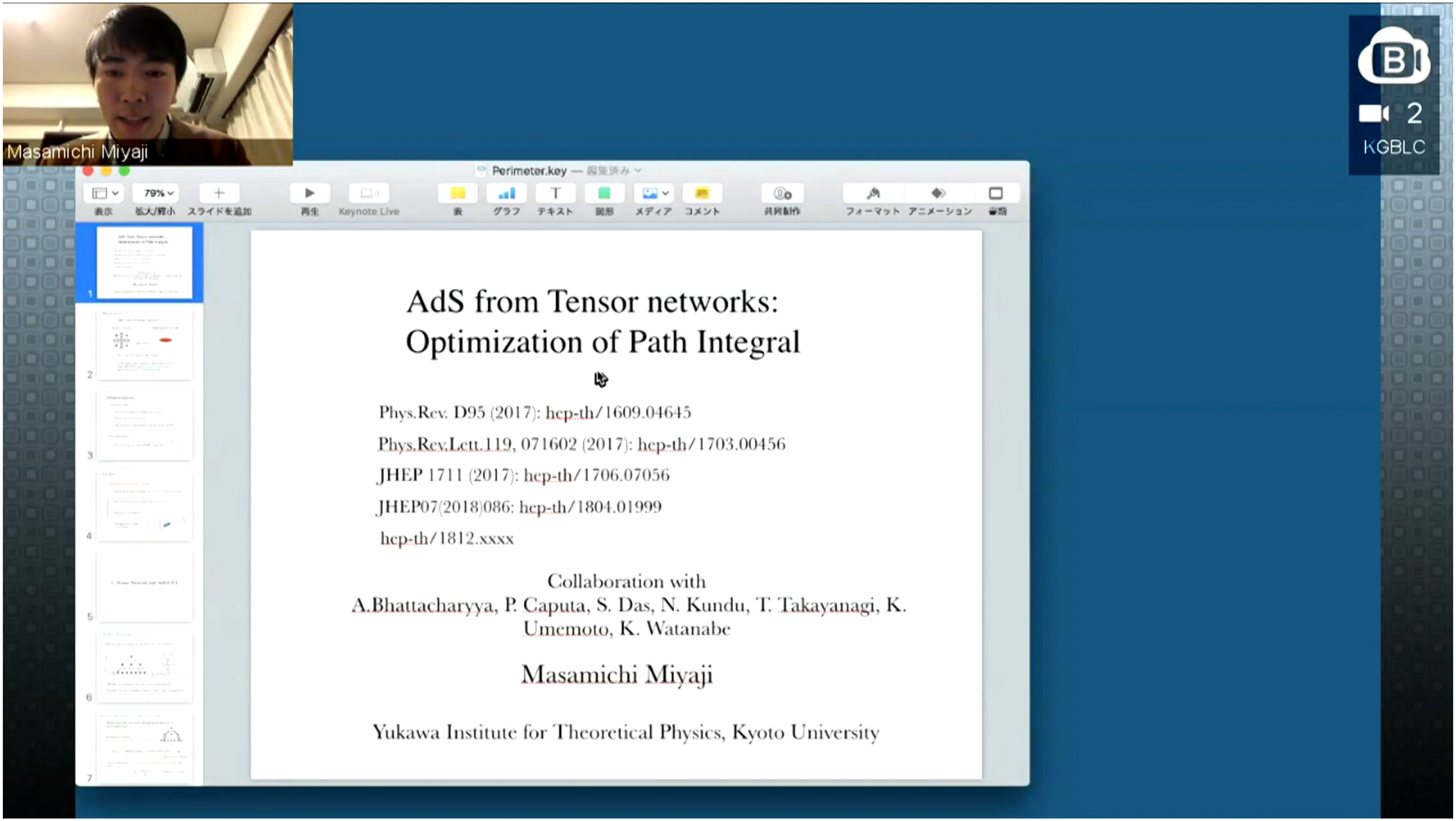
Abstract: <p>TBA</p>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AdS from Tensor networks:
Optimization of Path Integral
Phys.Rev. D95 (2017): hep-th/1609.04645
Phys.Rev.Lett.119, 071602 (2017): hep-th/1703.00456
JHEP 1711 (2017): hep-th/1706.07056
JHEP07(2018)086: hep-th/1804.01999
hep-th/1812.xxxx
Collaboration with
A.Bhattacharyya, P. Caputa, S. Das, N. Kundu, T. Takayanagi, K. Umemoto, K. Watanabe
Masamichi Miyaji
Yukawa Institute for Theoretical Physics, Kyoto University
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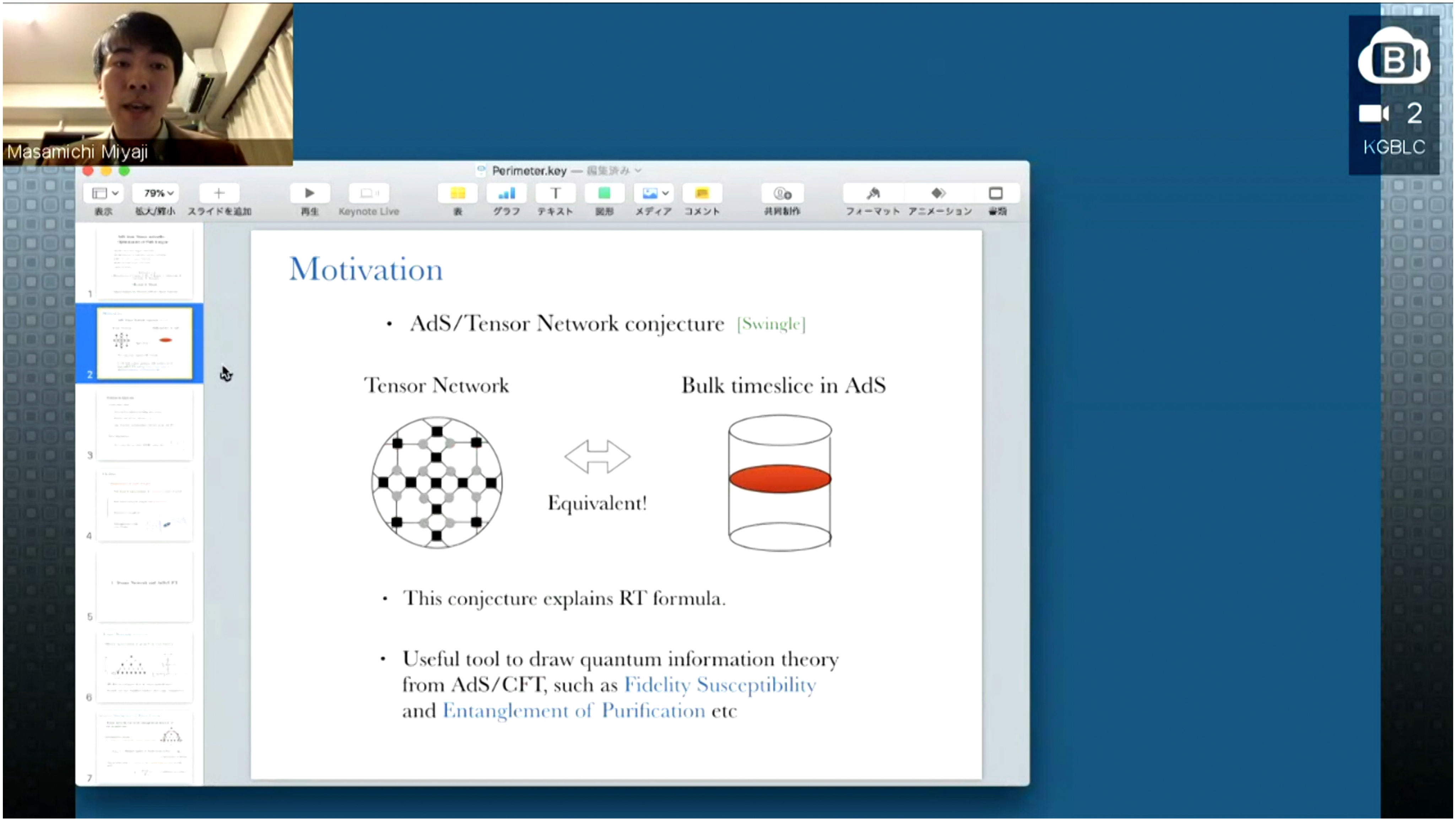
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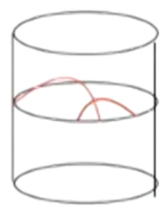
Motivation
• AdS/Tensor Network conjecture [Swingle]
Tensor Network
Bulk timeslice in AdS
Equivalent!
• This conjecture explains RT formula.
• Useful tool to draw quantum information theory from AdS/CFT, such as Fidelity Susceptibility and Entanglement of Purification etc

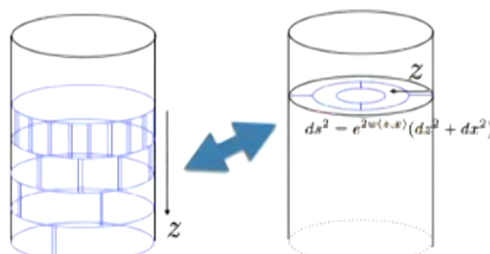
Problems & Questions

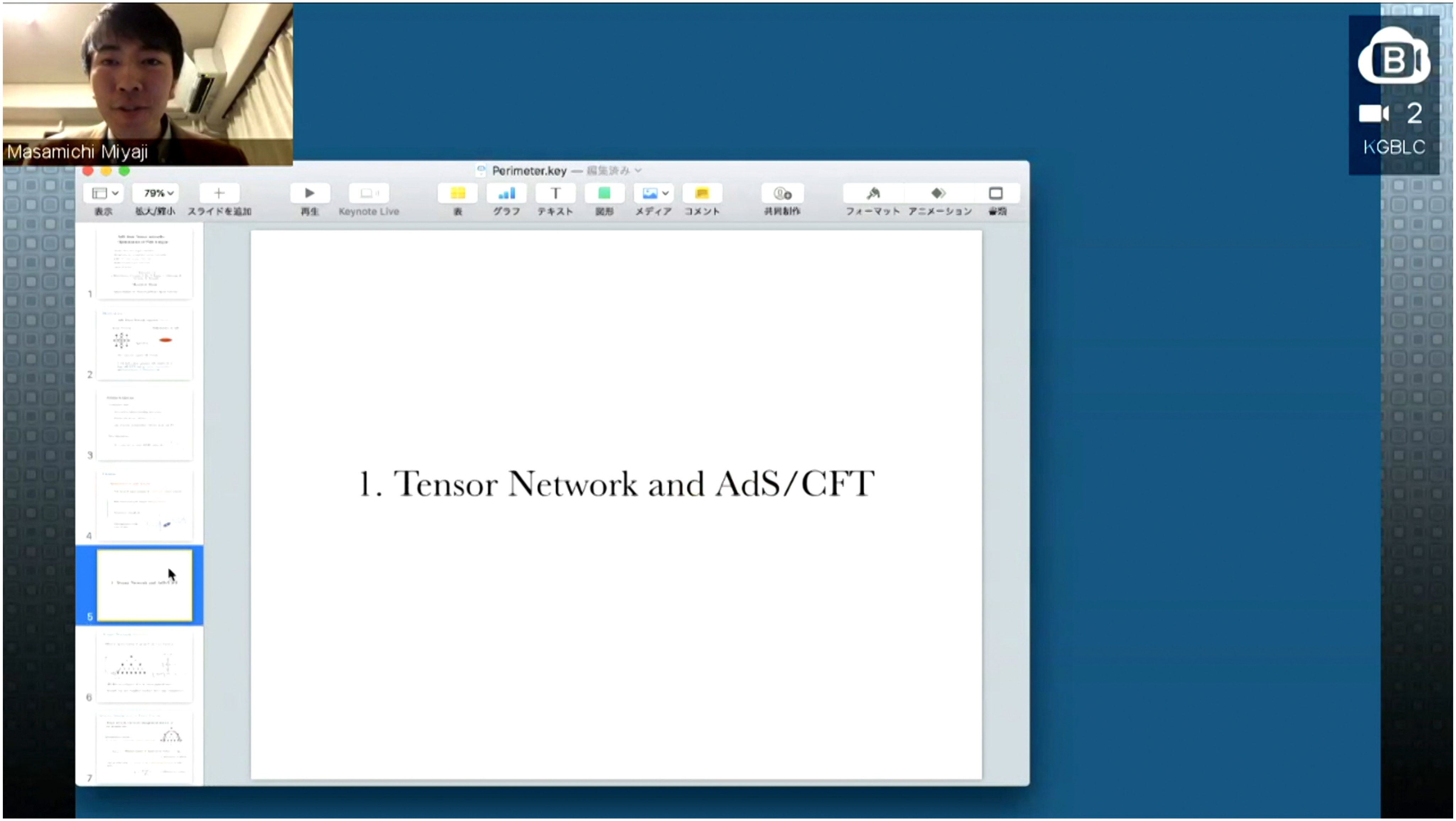
- Continuum limit
  - General formalism including interaction?
  - Known only for free theories [Haegeman et al.]
  - Luck of precise correspondence between metric and TN.
- Time dependence
  - No Cauchy slice on which all HRT surfaces live.



Outline

- Optimization of path integral :
  - Path integral representation of Continuous tensor network.
  - Bulk metric from path integral and optimization.
  - Relation to complexity.
  - Entanglement wedge cross section.

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表示
79%
拡大/縮小
スライドを追加
再生
Keynote Live
表
グラフ
テキスト
図形
メディア
コメント
共同制作
フォーマット
アニメーション
書類
1. Tensor Network and AdS/CFT
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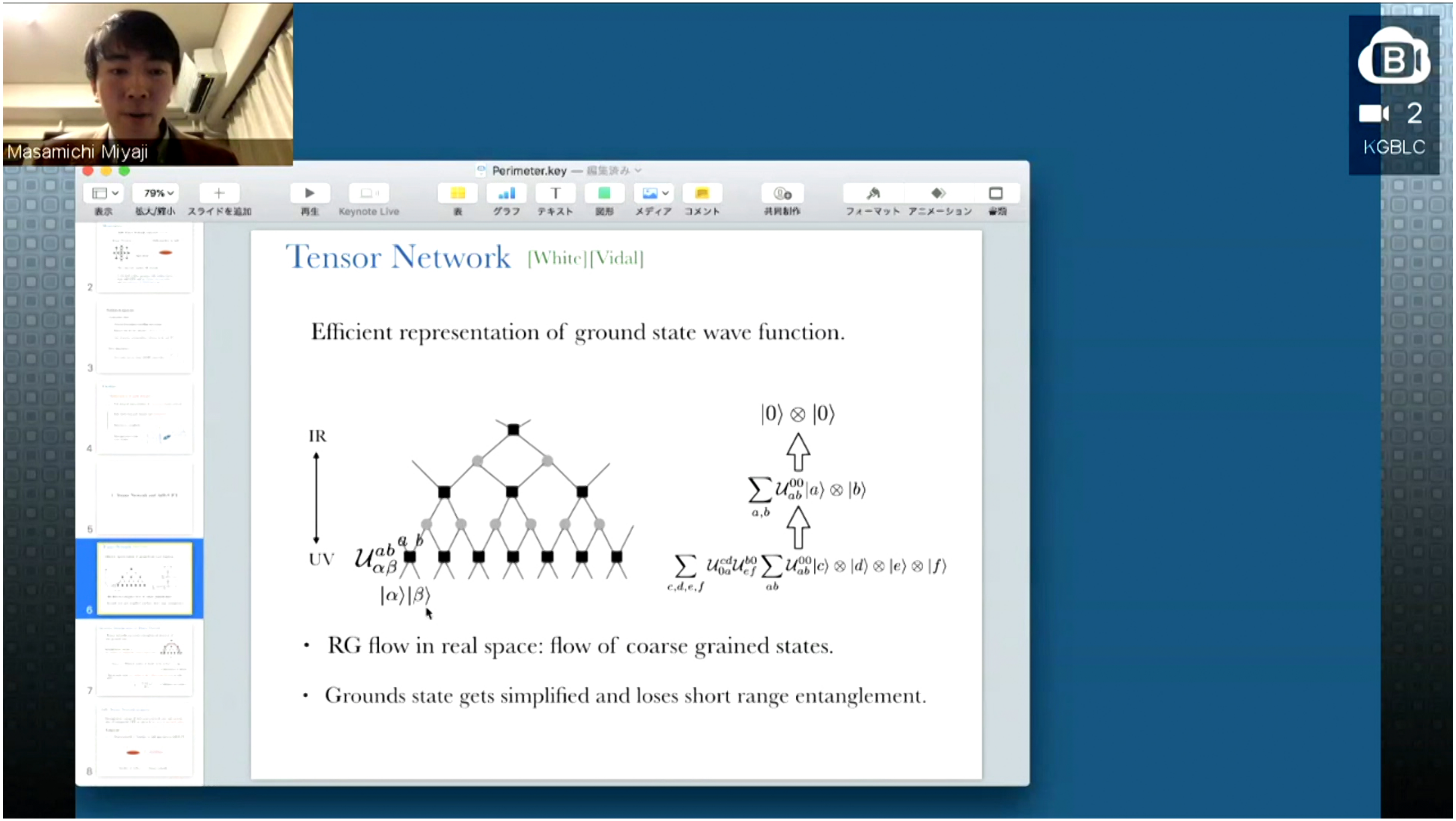
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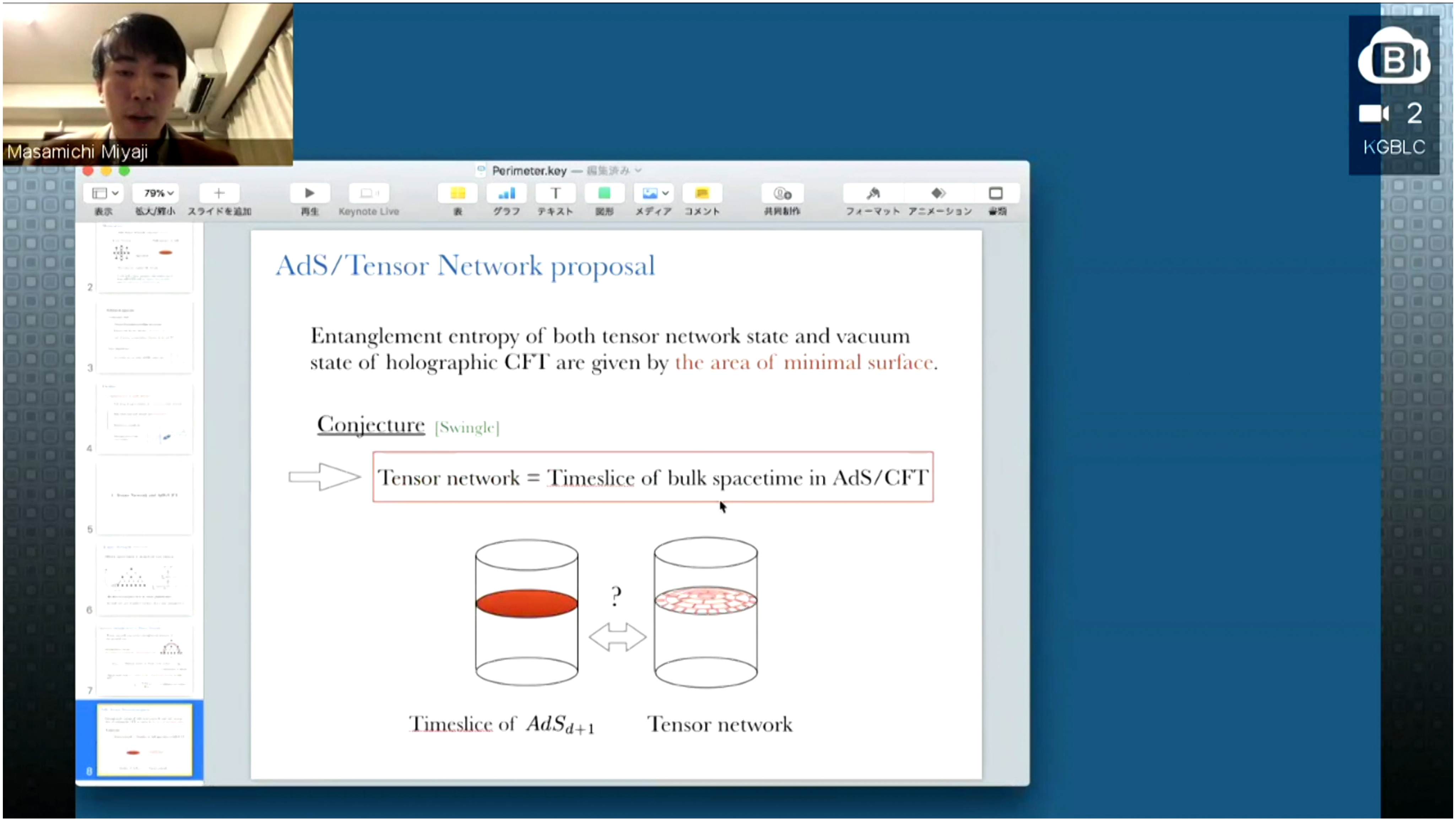
Tensor Network [White][Vidal]
Efficient representation of ground state wave function.
IR
UV
$\mathcal{U}^{ab}_{\alpha\beta}$ a b
$|\alpha\rangle|\beta\rangle$
$|0\rangle \otimes |0\rangle$
$\sum_{a,b} \mathcal{U}^{00}_{ab}|a\rangle \otimes |b\rangle$
$\sum_{c,d,e,f} \mathcal{U}^{cd}_{0a}\mathcal{U}^{b0}_{ef} \sum_{ab} \mathcal{U}^{00}_{ab}|c\rangle \otimes |d\rangle \otimes |e\rangle \otimes |f\rangle$
• RG flow in real space: flow of coarse grained states.
• Grounds state gets simplified and loses short range entanglement.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AdS/Tensor Network proposal
Entanglement entropy of both tensor network state and vacuum state of holographic CFT are given by the area of minimal surface.
Conjecture [Swingle]
Tensor network = Timeslice of bulk spacetime in AdS/CFT
?
Timeslice of AdS_{d+1}
Tensor network
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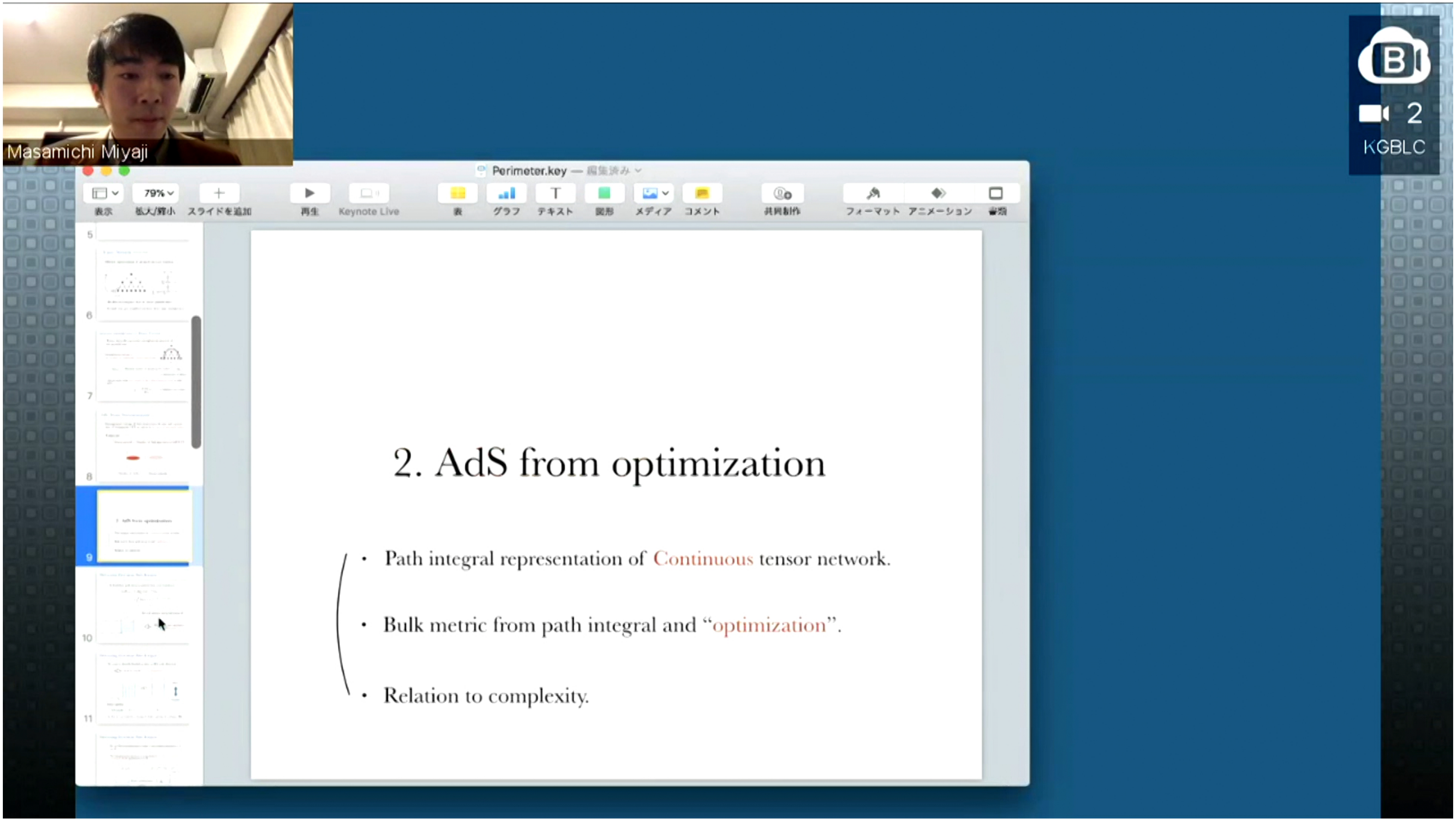
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2. AdS from optimization
• Path integral representation of Continuous tensor network.
• Bulk metric from path integral and "optimization".
• Relation to complexity.
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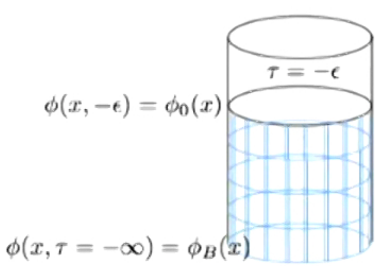
Deforming Euclidean Path Integral

[M.M, Takayanagi, Watanabe][Caputa, Kundu, M.M, Takayanagi, Watanabe]

In Euclidean path integral, ground state wave function is

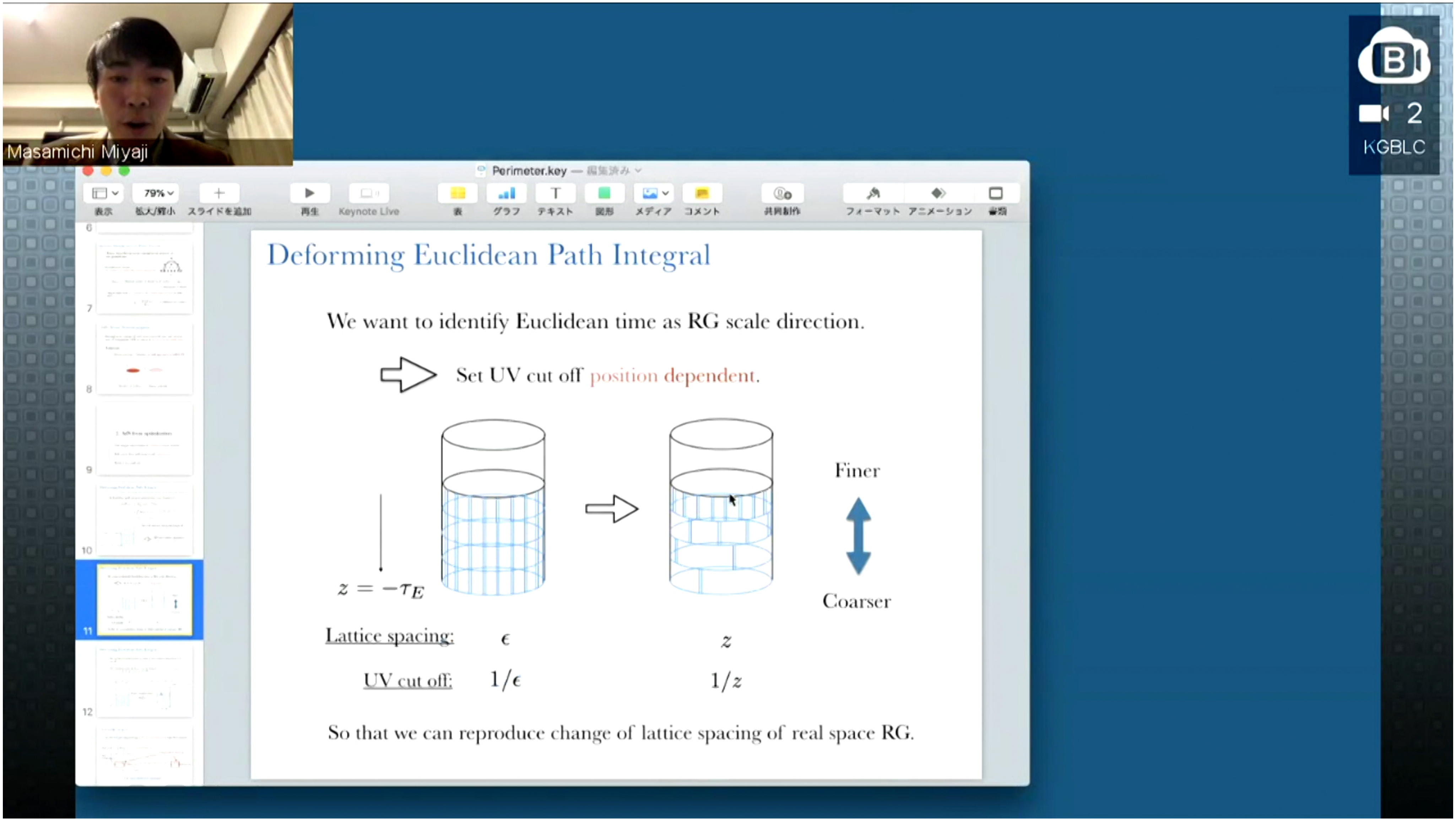
$$\langle \phi_0 | \Omega_{vac} \rangle \propto \lim_{T \to \infty} \langle \phi_0 | e^{-TH} | \phi_B \rangle$$

$$\propto \int_{\substack{\tau < -\epsilon \\ \phi(x,-\epsilon) = \phi_0(x)}} \mathcal{D}\phi(x,\tau)\; e^{-\int_{-\infty}^{-\epsilon} d\tau \int dx \mathcal{L}_E[\phi(x,\tau)]}$$



We will deform this path integral!

Effective lattice spacing is position independent.



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Deforming Euclidean Path Integral
We want to identify Euclidean time as RG scale direction.
Set UV cut off position dependent.
$z = -\tau_E$
Finer
Coarser
Lattice spacing:
ϵ
z
UV cut off:
$1/\epsilon$
$1/z$
So that we can reproduce change of lattice spacing of real space RG.
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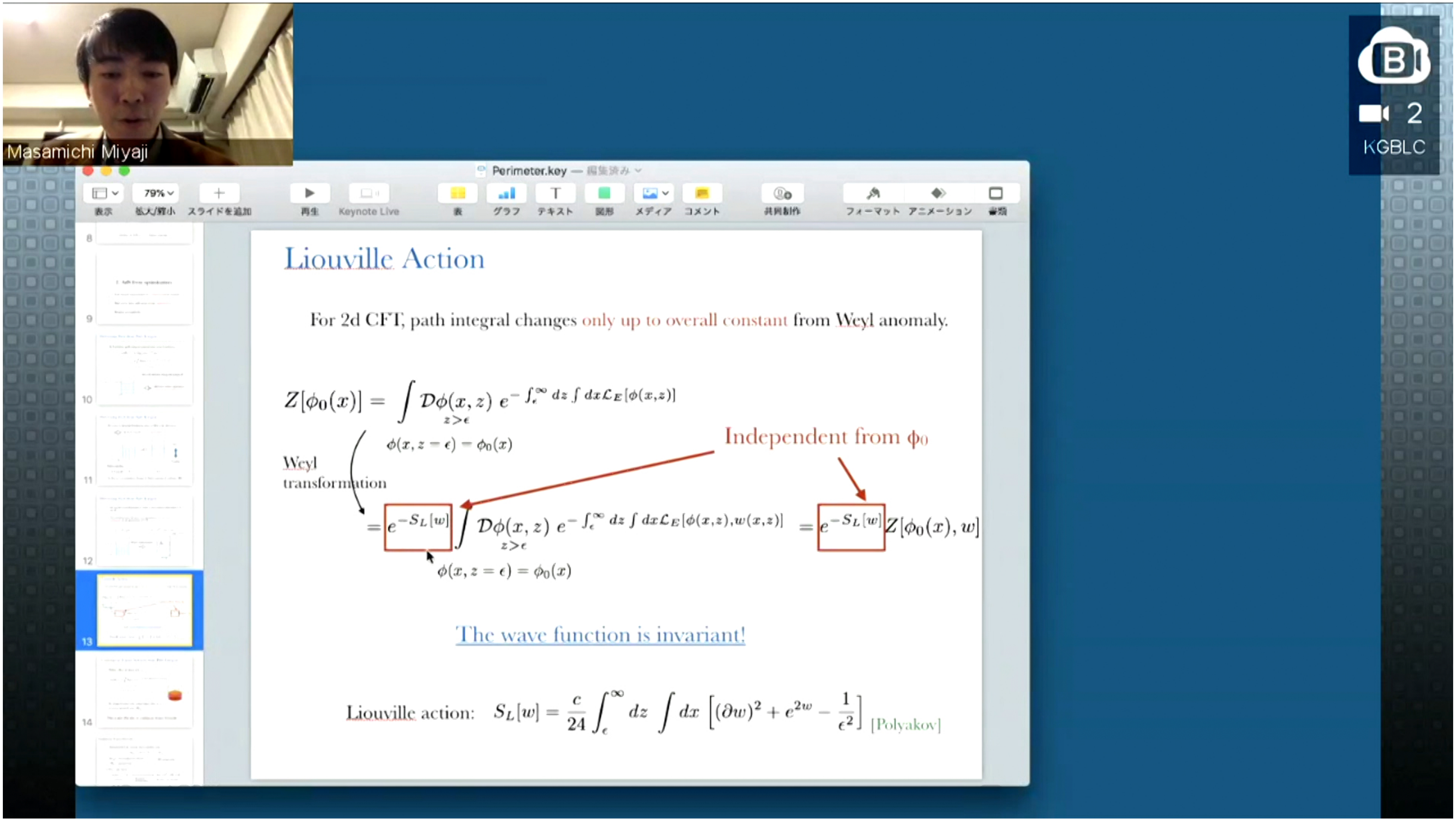
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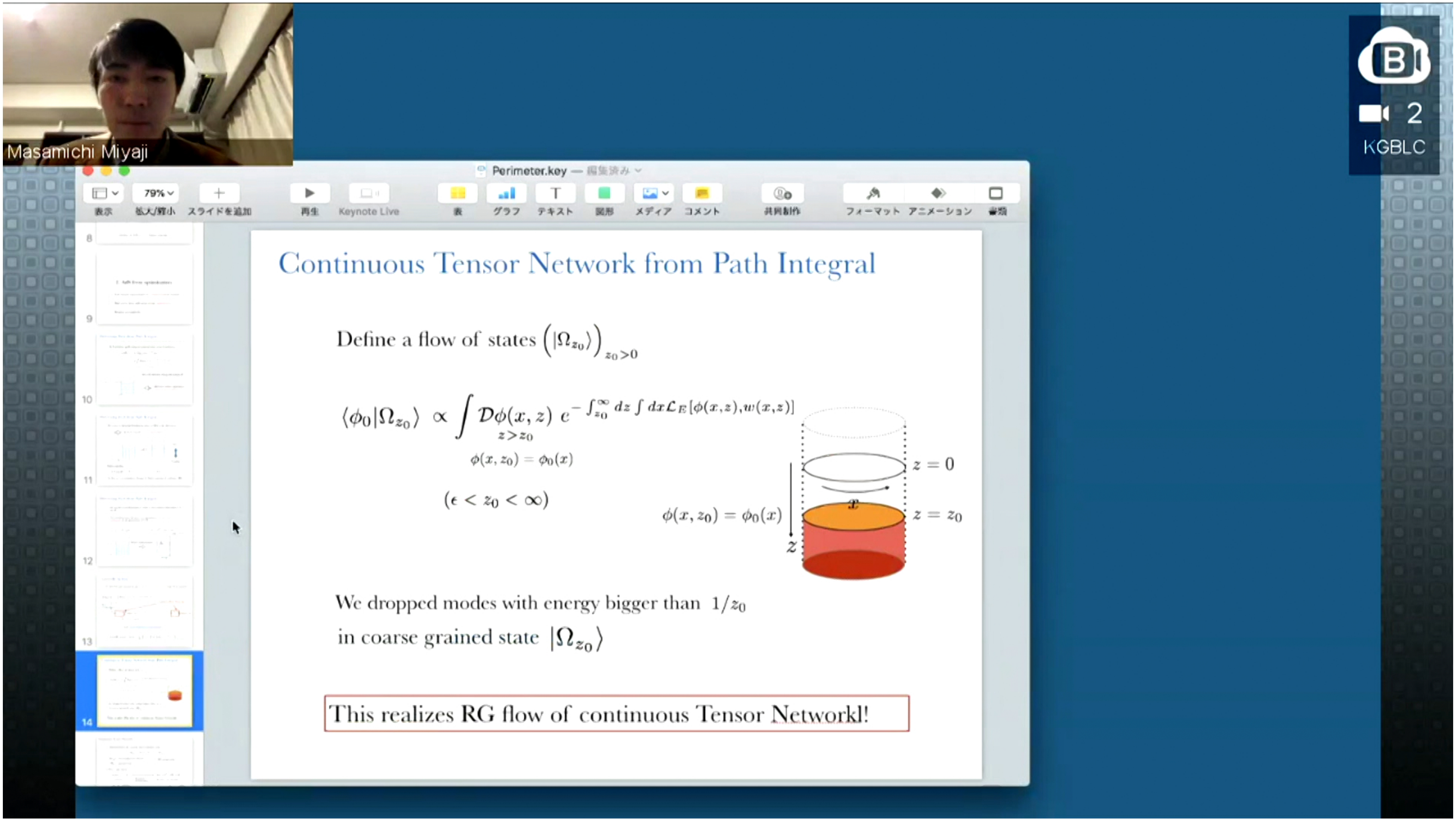
Liouville Action
For 2d CFT, path integral changes only up to overall constant from Weyl anomaly.
$Z[\phi_0(x)] = \int_{z>\epsilon} \mathcal{D}\phi(x,z)\, e^{-\int_\epsilon^\infty dz \int dx \mathcal{L}_E[\phi(x,z)]}$
$\phi(x, z=\epsilon) = \phi_0(x)$
Weyl transformation
Independent from ϕ_0
$= e^{-S_L[w]} \int_{z>\epsilon} \mathcal{D}\phi(x,z)\, e^{-\int_\epsilon^\infty dz \int dx \mathcal{L}_E[\phi(x,z), w(x,z)]} = e^{-S_L[w]} Z[\phi_0(x), w]$
$\phi(x, z=\epsilon) = \phi_0(x)$
The wave function is invariant!
Liouville action: $S_L[w] = \frac{c}{24}\int_\epsilon^\infty dz \int dx \left[(\partial w)^2 + e^{2w} - \frac{1}{\epsilon^2}\right]$ [Polyakov]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Continuous Tensor Network from Path Integral
Define a flow of states $\left(|\Omega_{z_0}\rangle\right)_{z_0>0}$
$\langle\phi_0|\Omega_{z_0}\rangle \propto \int_{\substack{z>z_0 \\ \phi(x,z_0)=\phi_0(x)}} \mathcal{D}\phi(x,z)\, e^{-\int_{z_0}^{\infty} dz \int dx \mathcal{L}_E[\phi(x,z),w(x,z)]}$
$(\epsilon < z_0 < \infty)$
$z = 0$
x
$\phi(x,z_0) = \phi_0(x)$
$z = z_0$
z
We dropped modes with energy bigger than $1/z_0$
in coarse grained state $|\Omega_{z_0}\rangle$
This realizes RG flow of continuous Tensor Networkl!

Continuous Tensor Network

Interpolation from vacuum state to product state.

$$|\Omega_{z_0}\rangle = \tilde{P} e^{i\int_0^{\infty} \hat{K}(z=\epsilon e^{-u})} |\Omega_\infty\rangle$$

$|\Omega_\infty\rangle$: unentangled trivial state

$|\Omega_0\rangle$: ground state

$\frac{1}{z}$: RG energy scale

ex) Free scalar theory [Haegeman, Osborne, Verschelde and Verstraete]

$$a_k|\Omega_{vac}\rangle = 0 \longrightarrow (a_k + a^\dagger_{-k})|B\rangle = 0$$

Vacuum — Bogoliubov transformation — Boundary state (Dirichlet)

$$\hat{K}(z) := \int dk\Gamma[|k|z]\left(a^\dagger_k a^\dagger_{-k} - a_k a_{-k}\right)$$

$\Gamma[z] = 1 \ (z < 1)$
Otherwise 0

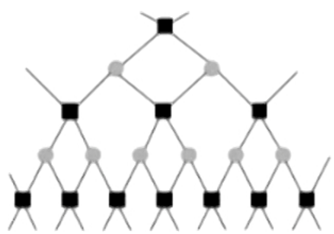
$$(\cosh\frac{u}{2} a_k + \sinh\frac{u}{2} a^\dagger_k)|\Omega_{z=\epsilon e^{-u}}\rangle = 0 \qquad (|k| < \frac{e^u}{\epsilon} = \frac{1}{z})$$

$$(a_k + a^\dagger_k)|\Omega_{z=\epsilon e^{-u}}\rangle \approx 0 \qquad (|k| > \frac{e^u}{\epsilon} = \frac{1}{z})$$

Complexity function

- Liouville action indeed counts the number of local unitaries in tensor network.

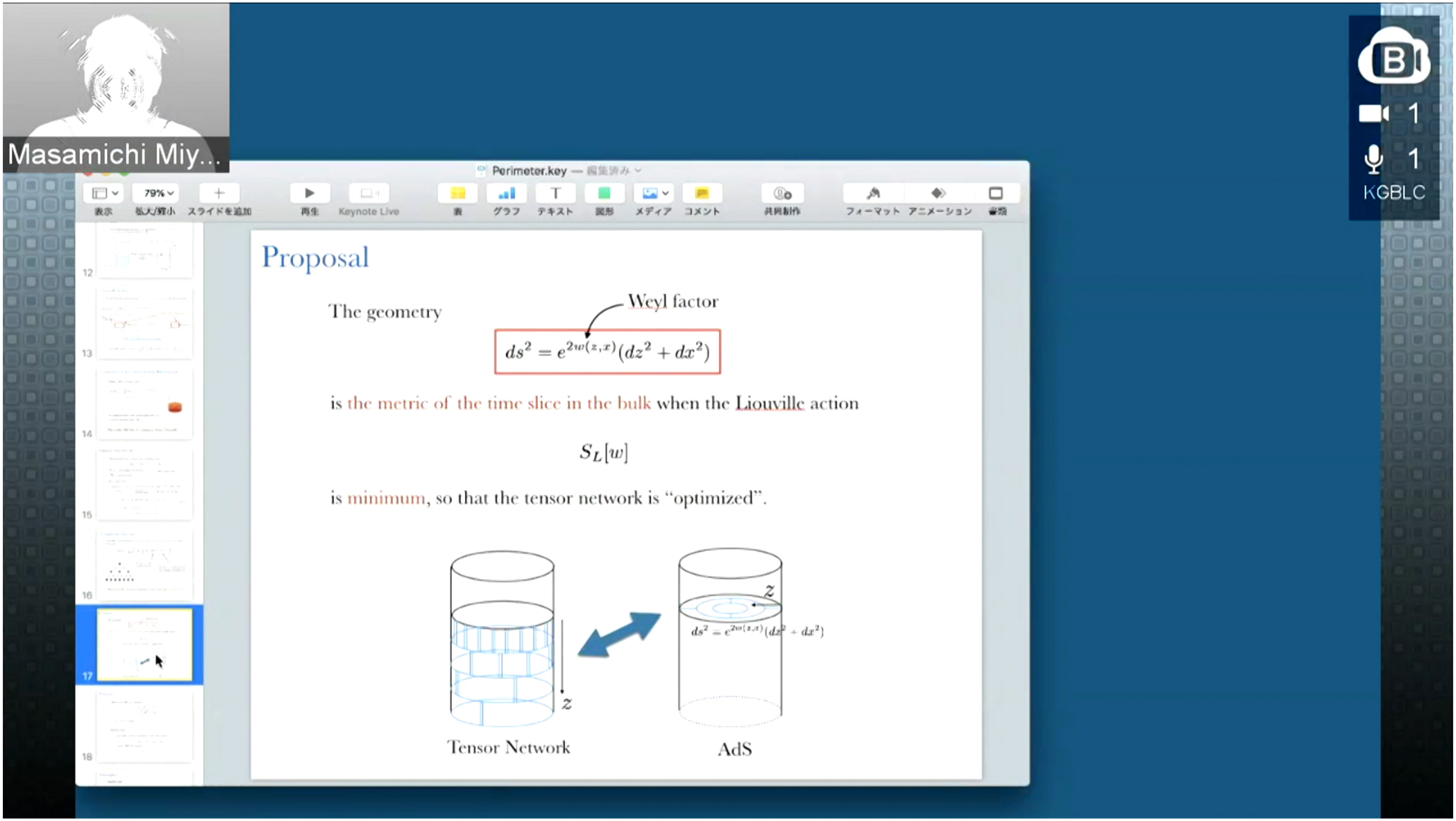
$$S_L[w] = \frac{c}{24}\int_{\epsilon}^{\infty} dz \int dx \left[(\partial w)^2 + e^{2w} - \frac{1}{\epsilon^2}\right]$$



Density of scale transformations [Czech]

Density of transformations that change entanglement

- When Liouville action is minimum, tensor network is "optimized".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Proposal
The geometry
Weyl factor
$ds^2 = e^{2w(z,x)}(dz^2 + dx^2)$
is the metric of the time slice in the bulk when the Liouville action
$S_L[w]$
is minimum, so that the tensor network is "optimized".
z
Tensor Network
AdS
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Proposal

- More generally, we minimize

$$e^{S[w]} = \frac{\int_{\phi|_{z=\epsilon}=\phi_0} \mathcal{D}\phi \quad e^{-S[\phi,w]}}{\int_{\phi|_{z=\epsilon}=\phi_0} \mathcal{D}\phi \quad e^{-S[\phi]}}$$

in various examples.

Vacuum state

- Liouville action is minimum, when the metric

$$ds^2 = e^{2w(z,x)}(dz^2 + dx^2)$$

is pure AdS. For instance

$$e^{2w} = \frac{\epsilon^2}{z^2}$$

Examples

- Excited state

Consider state excited by primary operator of dimension h at infinite past. Effect of Weyl transformation

$$ds^2 = \frac{dz^2 + dx^2}{\epsilon^2} \quad \Rightarrow \quad ds^2 = e^{2w}(dz^2 + dx^2)$$

to Liouville action is

$$S_L[w] - h \cdot w(z = -\infty) \qquad (h << c)$$

Conical deficit geometry

The solution is conial deficit geometry, with deficit angle

$$2\pi(1 - \sqrt{1 - \frac{24h}{c}}) \qquad (h << c)$$

Examples

Thermodield double state (t=0)

$$|\Psi_{TFD}\rangle = \frac{1}{\sqrt{Z(\beta)}} \sum_i e^{-\beta E_i} |E_i\rangle \otimes |E_i\rangle$$

$$\langle \tilde{\psi}_1, \tilde{\psi}_2 | \Psi_{TFD} \rangle = \int_{-\beta/4 < z < \beta/4} \mathcal{D}\psi(x,z) \; e^{-S_{CFT}}$$

$$\psi(x, -\beta/4) = \tilde{\psi}_1(x)$$

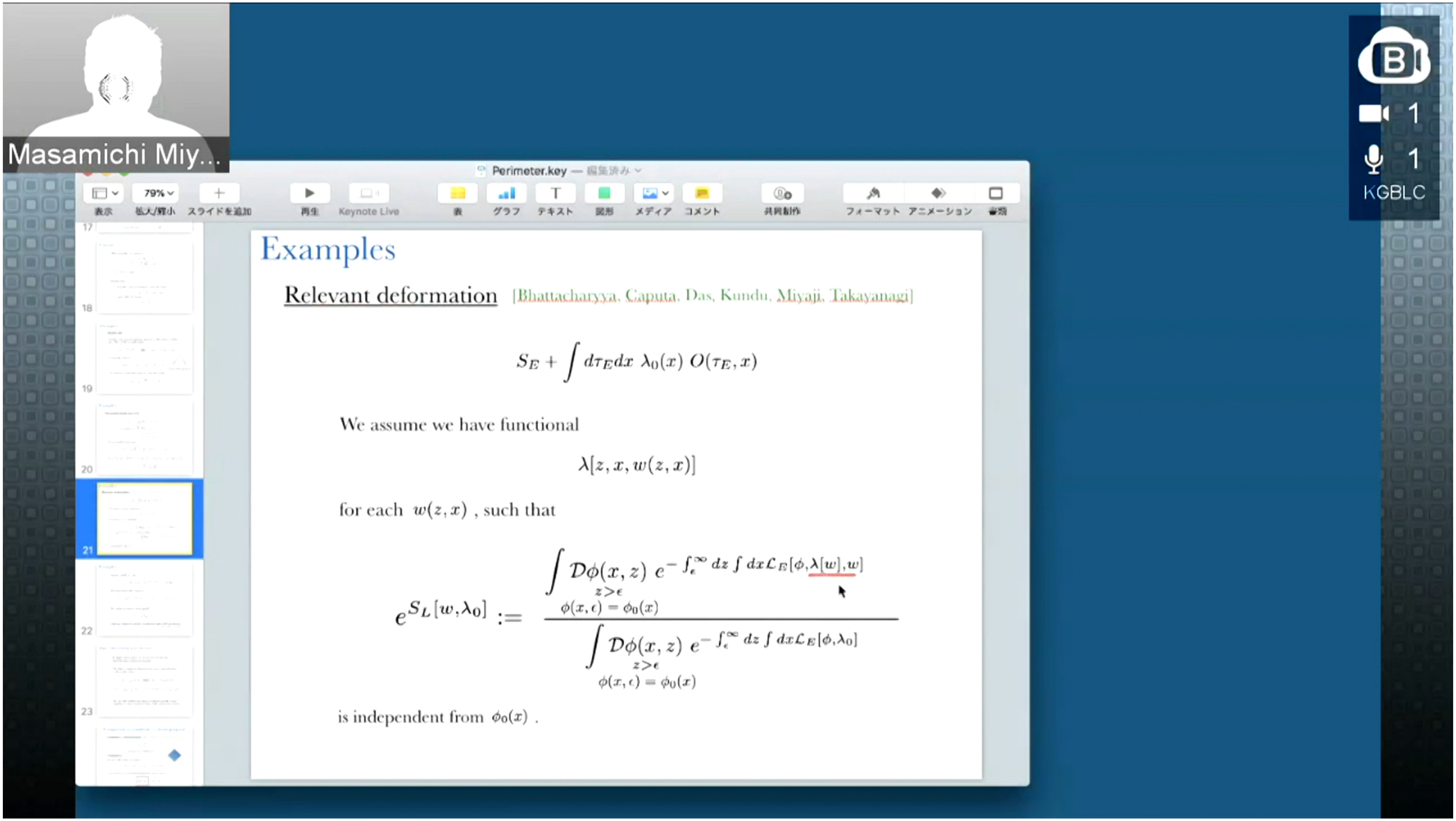
$$\psi(x, \beta/4) = \tilde{\psi}_2(x)$$

We get Liouville action again,

$$S_L[w] = \frac{c}{24} \int_{-\beta/4}^{\beta/4} dz \int dx \left[(\partial w)^2 + e^{2w} - \frac{1}{\epsilon^2} \right]$$

We get time slice of BTZ at t=0, with correct temperature, after optimization.

$$ds^2 = \frac{8\pi^2}{\beta^2} \cdot \frac{dz^2 + dx^2}{\cos^2(\frac{2\pi z}{\beta})}$$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Examples
Relevant deformation [Bhattacharyya, Caputa, Das, Kundu, Miyaji, Takayanagi]
$S_E + \int d\tau_E dx\ \lambda_0(x)\ O(\tau_E, x)$
We assume we have functional
$\lambda[z, x, w(z,x)]$
for each $w(z,x)$, such that
$e^{S_L[w,\lambda_0]} := \frac{\int_{z>\epsilon,\ \phi(x,\epsilon)=\phi_0(x)} \mathcal{D}\phi(x,z)\ e^{-\int_\epsilon^\infty dz \int dx \mathcal{L}_E[\phi,\lambda[w],w]}}{\int_{z>\epsilon,\ \phi(x,\epsilon)=\phi_0(x)} \mathcal{D}\phi(x,z)\ e^{-\int_\epsilon^\infty dz \int dx \mathcal{L}_E[\phi,\lambda_0]}}$
is independent from $\phi_0(x)$.

Examples

- Assume small λ_0 , then

$$\lambda[z,x,w] = \lambda_0 \; e^{(\Delta_{\mathcal{O}}-2)w(z,x)} + \mathcal{O}(\lambda_0^2)$$

- Perturbed Liouville action is

$$S_L[w] = \frac{c}{24}\int_\epsilon^\infty dz \int dx \left[(\partial w)^2 + e^{2w} + \lambda_0^2 \, e^{(2\Delta-2)w} - \frac{1}{\epsilon^2}\right] - b\epsilon^{1-\Delta}\lambda_0 \int dx$$

- We obtain geometry with cap-off:

$$e^w = \frac{1}{z}\left(1 - \frac{\lambda_0^2}{2(5-2\Delta)} z^{-2\Delta+4} + \cdots\right)$$

which is consistent with the result from AdS/CFT prediction.

[Hung, Myers, Smolkin]

Higher dimensional generalization

- In higher dimensions, we do not necessarily have nonvanishing conformal anomaly.

- We define complexity functional by (naive) generalization of Liouvelle action:

$$ds^2 = \frac{1}{\epsilon^2} g_{ab} dx^a dx^b \Longrightarrow ds^2 = e^{2w} g_{ab} dx^a dx^b$$

$$S_L[w] = \frac{d-1}{16\pi G_N} \int_\Sigma \sqrt{g} \left[e^{dw} + e^{(d-2)w} (g^{ab} \partial_a w \partial_b w) + \frac{e^{(d-2)w}}{(d-1)(d-2)} R_g \right]$$

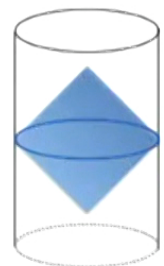
- We can still confirm that this generalized Liouville action reproduces correct metric for pure AdS and massive particle.

Comparison to complexity = Action proposal

Complexity = Action proposal [Brown, Roberts, Susskind, Swingle, Zhao]

$$\mathcal{C} = \frac{\mathcal{A}}{\pi}$$

$\mathcal{A}$: Gravity action in WDW patch



Comparison

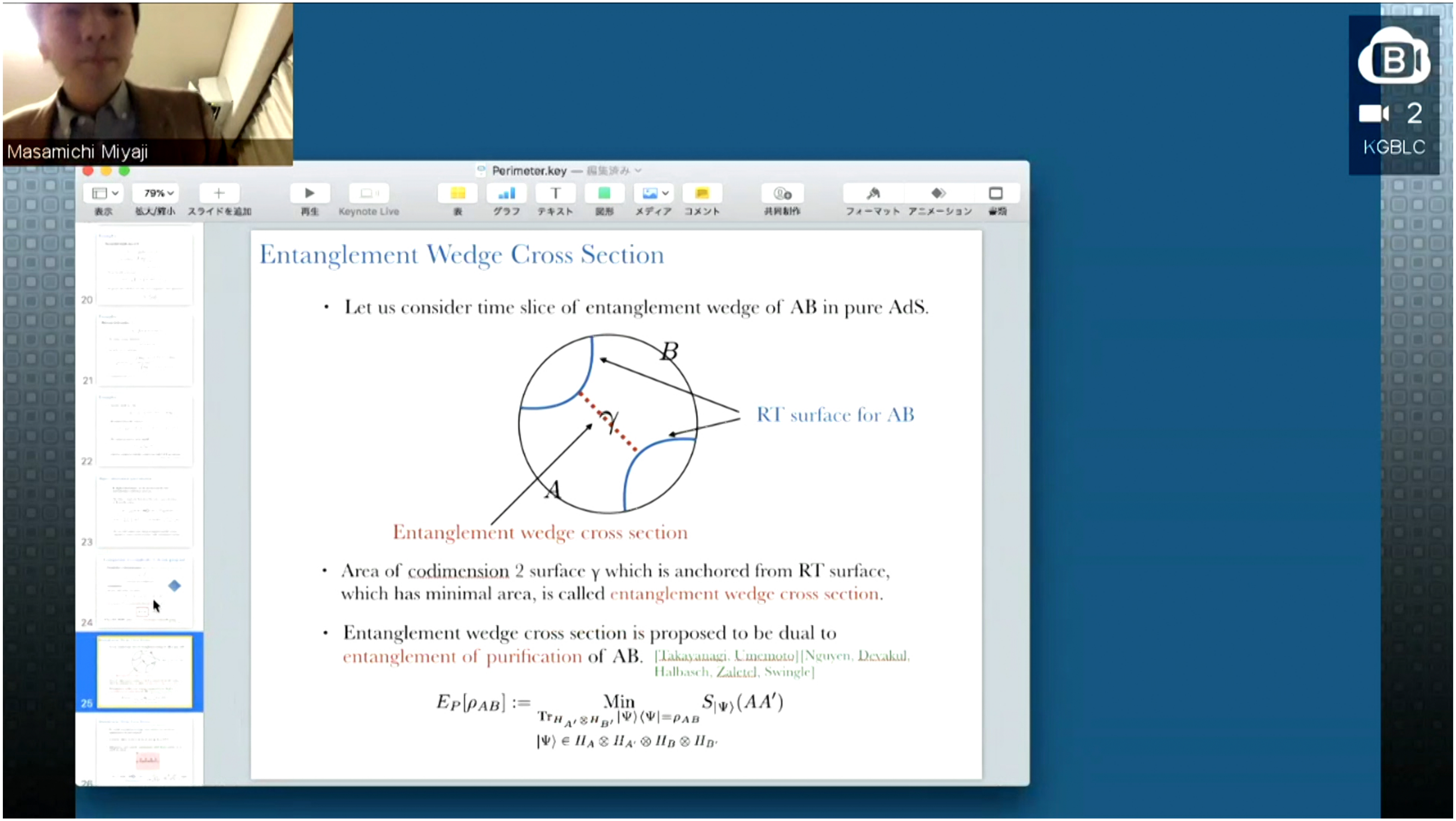
For pure AdS, metric can written

$$ds^2 = -d\eta^2 + cos^2\eta e^{2w(x,y)}(dx^2 + dy^2)$$

Let us pretend $w(x,y)$ is undetermined function, then we can see

$$\boxed{\mathcal{A} \propto S_L} \quad (d > 2)$$

When d=2, WDW action vanishes in contrast to Liouville action.
[Reynolds, Ross]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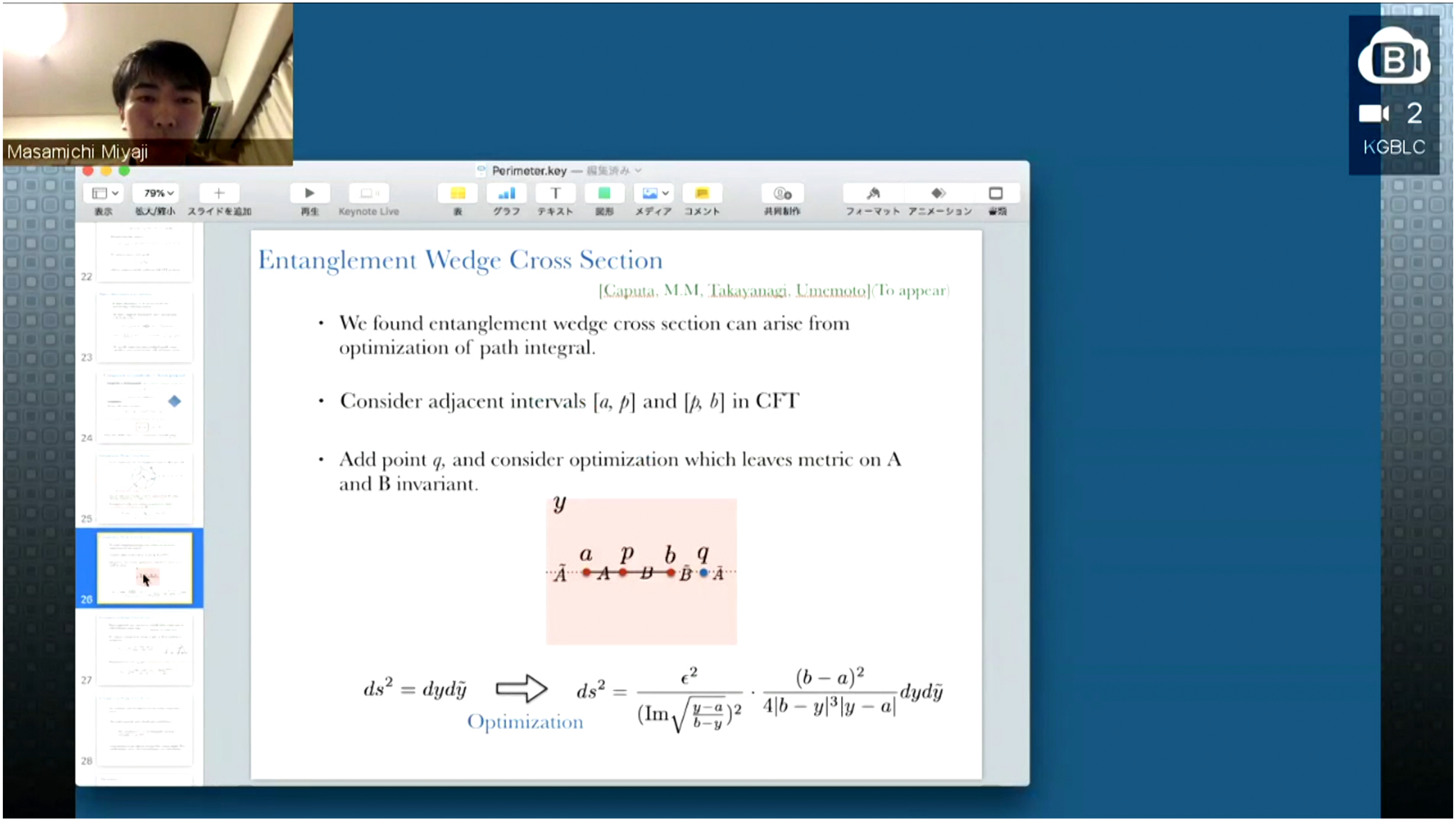
Entanglement Wedge Cross Section
• Let us consider time slice of entanglement wedge of AB in pure AdS.
B
RT surface for AB
γ
A
Entanglement wedge cross section
• Area of codimension 2 surface γ which is anchored from RT surface, which has minimal area, is called entanglement wedge cross section.
• Entanglement wedge cross section is proposed to be dual to entanglement of purification of AB. [Takayanagi, Umemoto][Nguyen, Devakul, Halbasch, Zaletel, Swingle]
$E_P[\rho_{AB}] := \underset{\substack{\mathrm{Tr}_{H_{A'}\otimes H_{B'}}|\Psi\rangle\langle\Psi|=\rho_{AB} \\ |\Psi\rangle \in H_A\otimes H_{A'}\otimes H_B\otimes H_{B'}}}{\mathrm{Min}} S_{|\Psi\rangle}(AA')$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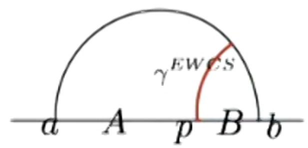
Entanglement Wedge Cross Section
[Caputa, M.M, Takayanagi, Umemoto](To appear)
• We found entanglement wedge cross section can arise from optimization of path integral.
• Consider adjacent intervals $[a, p]$ and $[p, b]$ in CFT
• Add point q, and consider optimization which leaves metric on A and B invariant.
y
a p b q
$\tilde{A}$ A B $\tilde{B}$ $\tilde{A}$
$ds^2 = dyd\tilde{y}$
Optimization
$ds^2 = \frac{\epsilon^2}{(\mathrm{Im}\sqrt{\frac{y-a}{b-y}})^2} \cdot \frac{(b-a)^2}{4|b-y|^3|y-a|} dyd\tilde{y}$

Entanglement Wedge Cross Section

- Weyl transformed wave function on $A\tilde{A}B\tilde{B}$ defines purification of reduced density matrix ρ_{AB}.
  (Subclass of purification)

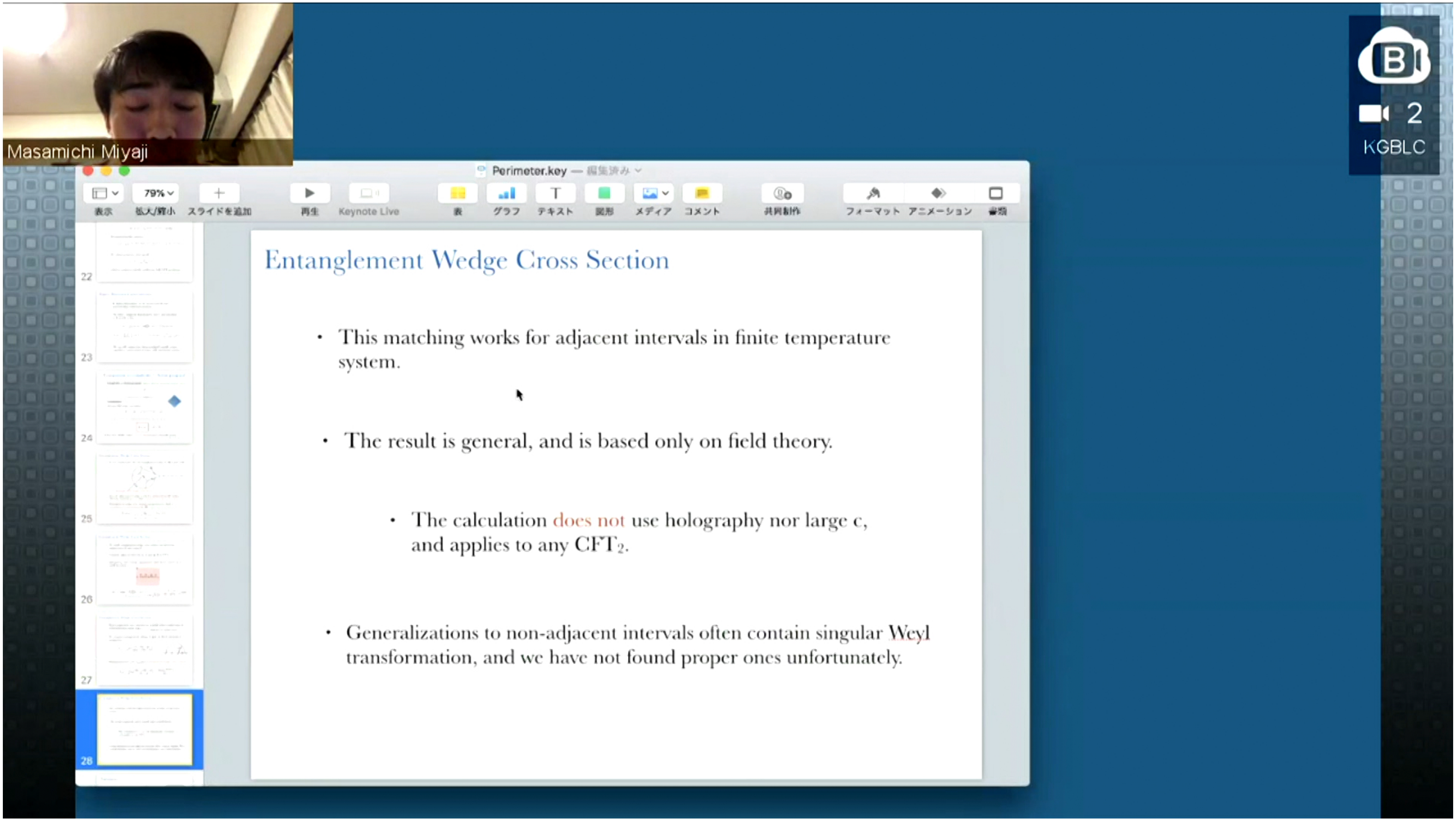
- We compute entanglement entropy of $A\tilde{A}$ on Weyl transformed background

$$S_{A\tilde{A}} = \frac{c}{6}\log\left[\frac{(b-a)(q-p)^2}{2\epsilon(q-a)(q-b)}\right]$$



- Minimization over q of $S_{A\tilde{A}}$ gives entanglement wedge cross section!

$$S_{A\tilde{A}} = \frac{c}{6}\log\left[\frac{2(p-a)(b-p)}{\epsilon(b-a)}\right] = \frac{\text{Area}(\gamma^{EWCS})}{4G_N}$$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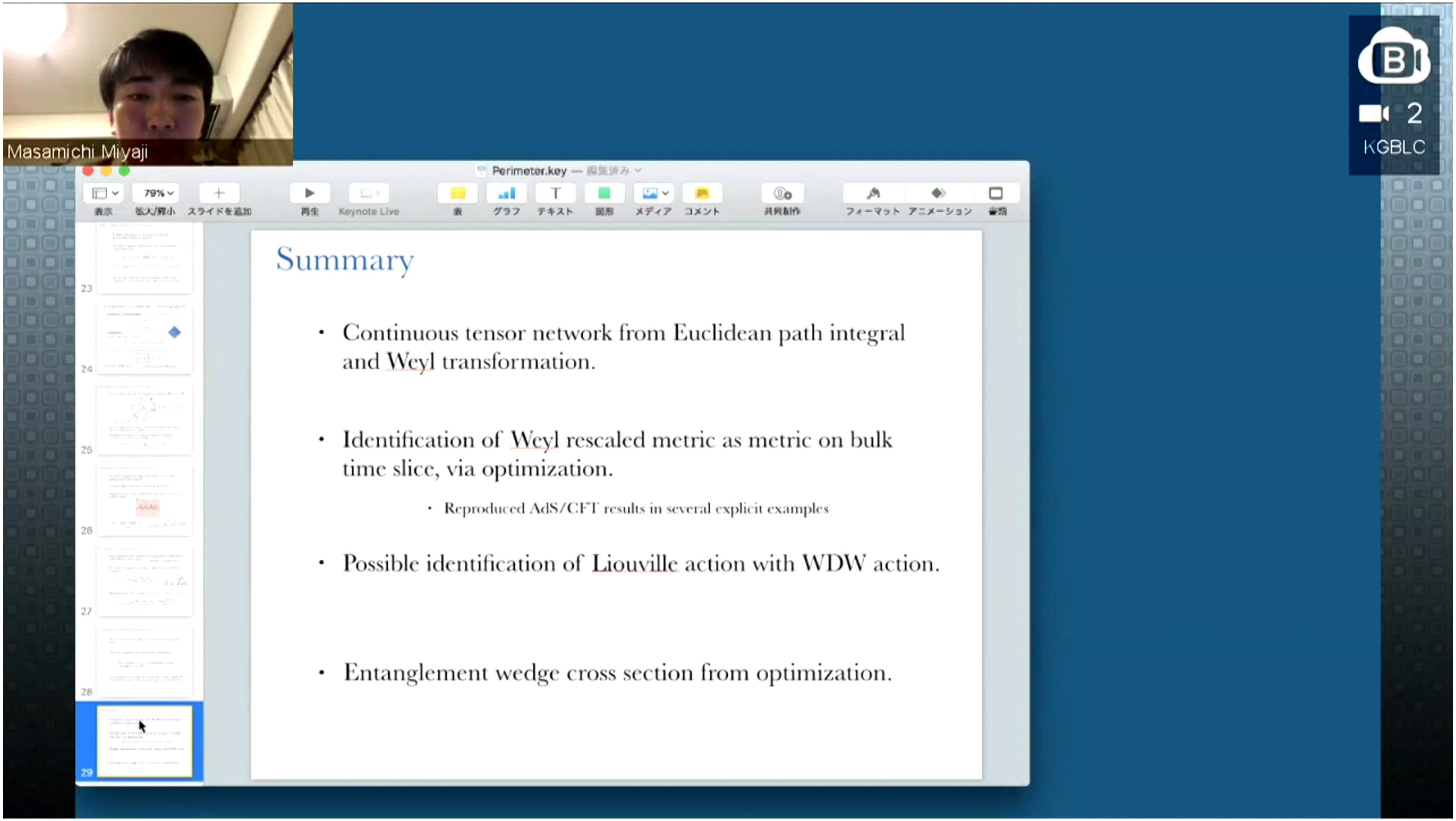
Entanglement Wedge Cross Section
• This matching works for adjacent intervals in finite temperature system.
• The result is general, and is based only on field theory.
• The calculation does not use holography nor large c, and applies to any CFT_2.
• Generalizations to non-adjacent intervals often contain singular Weyl transformation, and we have not found proper ones unfortunately.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Summary
Continuous tensor network from Euclidean path integral and Weyl transformation.
Identification of Weyl rescaled metric as metric on bulk time slice, via optimization.
Reproduced AdS/CFT results in several explicit examples
Possible identification of Liouville action with WDW action.
Entanglement wedge cross section from optimization.