Title: Renormalization and Effective Field Theory - Lecture 11

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Abstract:

Chern-Simons

$$\mathcal{E}(M^3) = \Omega^\bullet(M^3) \otimes \mathfrak{g}[1]$$

$$S_{CS} = \frac{1}{2}\int \langle \alpha, d\alpha \rangle + \frac{1}{6}\int \langle \alpha, [\alpha, \alpha] \rangle \quad \text{for } \alpha \in \mathcal{E}(M^3)$$

Gauge fix

Choose Lagrangian $\mathcal{L} \subseteq \Omega^\bullet(M^3) \otimes \mathfrak{g}[1]$

$$\int_{\mathcal{L}} e^{S_{CS}/\hbar}$$

Require $\langle \alpha, d\alpha \rangle$ is a non-degen. quadratic form on $\mathcal{L}$

Hodge Theory

$d^*: \Omega^k(M) \longrightarrow \Omega^{k-1}(M)$

[in local coords

$d^* \sim g_{ij} \frac{\partial}{\partial x_i} \iota_{\partial x_j}$

($\iota_{\partial x_j}$ removes dx_j)]

$d^* = * d *$

$\Delta = [d, d^*]$

$= d d^* + d^* d$

ory

$(M) \longrightarrow \Omega^{k-1}(M)$

words

$i \dfrac{\partial}{\partial x_i} \; (dx_j$

removes dx_j

$d^* = *d*$

$\Delta = [d, d^*]$

$= dd^* + d^*d$

M compact:

$$\Omega^*(M) = \mathrm{Im}\, d^* \oplus \mathrm{Im}\, d \oplus \mathcal{H} \qquad \mathcal{H} = \mathrm{Ker}\, \Delta$$

$\mathcal{H} \simeq H^*(M)$ finite dimensional

We will integrate over

$$\mathrm{Im}\, d^\vee \subseteq \mathrm{Im}\, d \oplus \mathrm{Im}\, d^\vee = \mathcal{H}^\perp$$

Left with: an "effective measure" on space of sol^{n}s to the EOM.

Tangent space of this is $\mathcal{H} \otimes \mathfrak{g}[1]$

If $\alpha \in \mathrm{Im}\, d^{\circ} \otimes g(1)$

$\int \alpha \, d\alpha$ is a non-degenerate quadratic form

α a 1-form in $\mathrm{Im}\, d^{\circ}$

$$\alpha = \sum \alpha^i \, dx_i$$

Then $d^{\circ}\alpha = \sum g_{ij} \frac{\partial}{\partial x_j} \alpha_i = 0$

Lorenz gauge

finite dimens

Heat Kernel

Define $K_t(x,y) \in \Omega^3(M \times M)$

So that:

1) $\partial_t K_t(x,y) + \Delta_x K_t(x,y) = 0$

2) $K_0(x,y) = \delta_{x=y}$

$$K_t(x,y) = \sum_{|I|+|J|=3} f_{I,J}(x,y,t)\, dx^I\, dy^J$$

With flat metric

$$K_t = e^{-\|x-y\|^2/t}\, t^{-3/2}\, d(x_1 - y_1)\, d(x_2 - y_2)\, d(x_3 - y_3)$$

With flat metric

$$K_t = e^{-\|x-y\|^2/t}\, t^{-3/2} \left(d(x_1-y_1)\, d(x_2-y_2)\, d(x_3-y_3) \right)$$

$$-\quad \Delta_x K_t = \Delta_y K_t$$

$$-(d_x+d_y) K_t = 0$$

$$\text{As, } \partial_t\left[(d_x+d_y)K_t\right] = (d_x+d_y)\Delta_x K_t = \Delta_x (d_x+d_y) K_t$$

$$[[d, d'], d] = 0$$

$$\partial_t \quad t^{-3/2} \left(\delta(x_1 - y_1)\, \delta(x_2 - y_2)\, \delta(x_3 - y_3) \right)$$

Heat eqn

Initial condition

$$(d_x + d_y)\, \Delta_x K_t = \Delta_x (d_x + d_y) K_t$$

$$[[d, d'], d] = 0$$

Propagator

$P \in \underline{\mathrm{Im}\, d^\dagger} \otimes \mathrm{Im}\, d^\dagger$

is $P(x,y) = \int_0^\infty d_x^\dagger \, K_t(x,y)$

Key properties

1) $d_x^\dagger P(x,y) = 0$ and $d_y^\dagger P(x,y) = 0$ ✓

2) If $\alpha \in \mathrm{Im}\, d^\dagger$, then $d_x^\dagger \int P(x,y)\,\alpha(y) = \alpha(x)$

$$d_x \int_y \int_0^\infty d_x^* K_t(x,y) \wedge \alpha(y)$$

$$= \int_0^\infty \int_y \left([d_x, d_x^*] - d_x^* d_x \right) K_t(x,y) \wedge \alpha(y)$$

$$= -\int_y d_x^* d_x K_t(x,y) \alpha(y) - \int_0^\infty \int_y \partial_t K_t(x,y) \alpha(y)$$

$$d_x \int_y \int_0^\infty d_x^* \, K_t(x,y) \wedge \alpha(y)$$

$$= \int_0^\infty \int_y \left([d_x, d_x^*] - d_x^* d_x \right) K_t(x,y) \wedge \alpha(y)$$

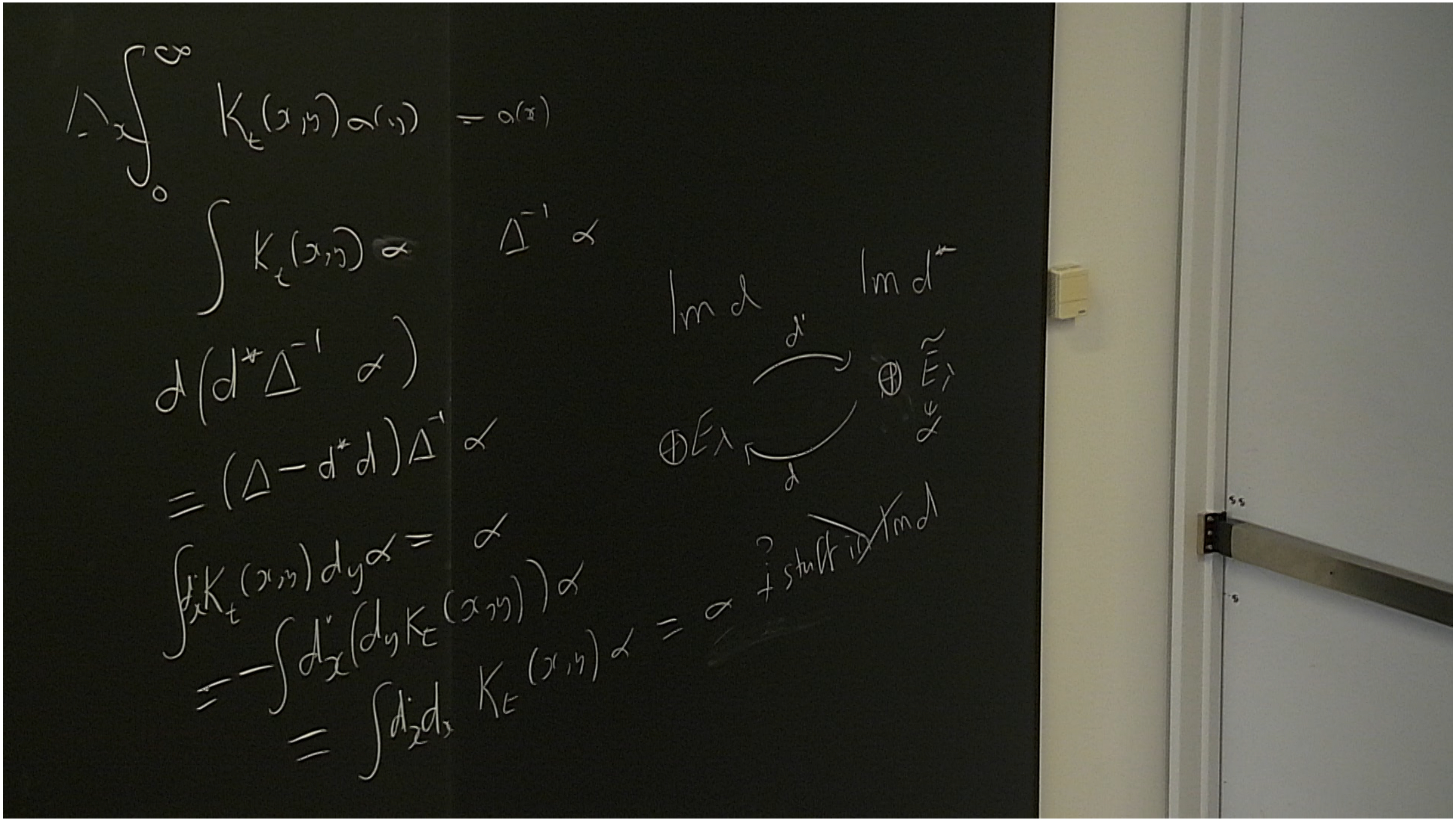
$$= \boxed{-\int_y d_x^* d_x \, K_t(x,y)\, \alpha(y)} - \int_0^\infty \int_y \partial_t K_t(x,y)\, \alpha(y)$$

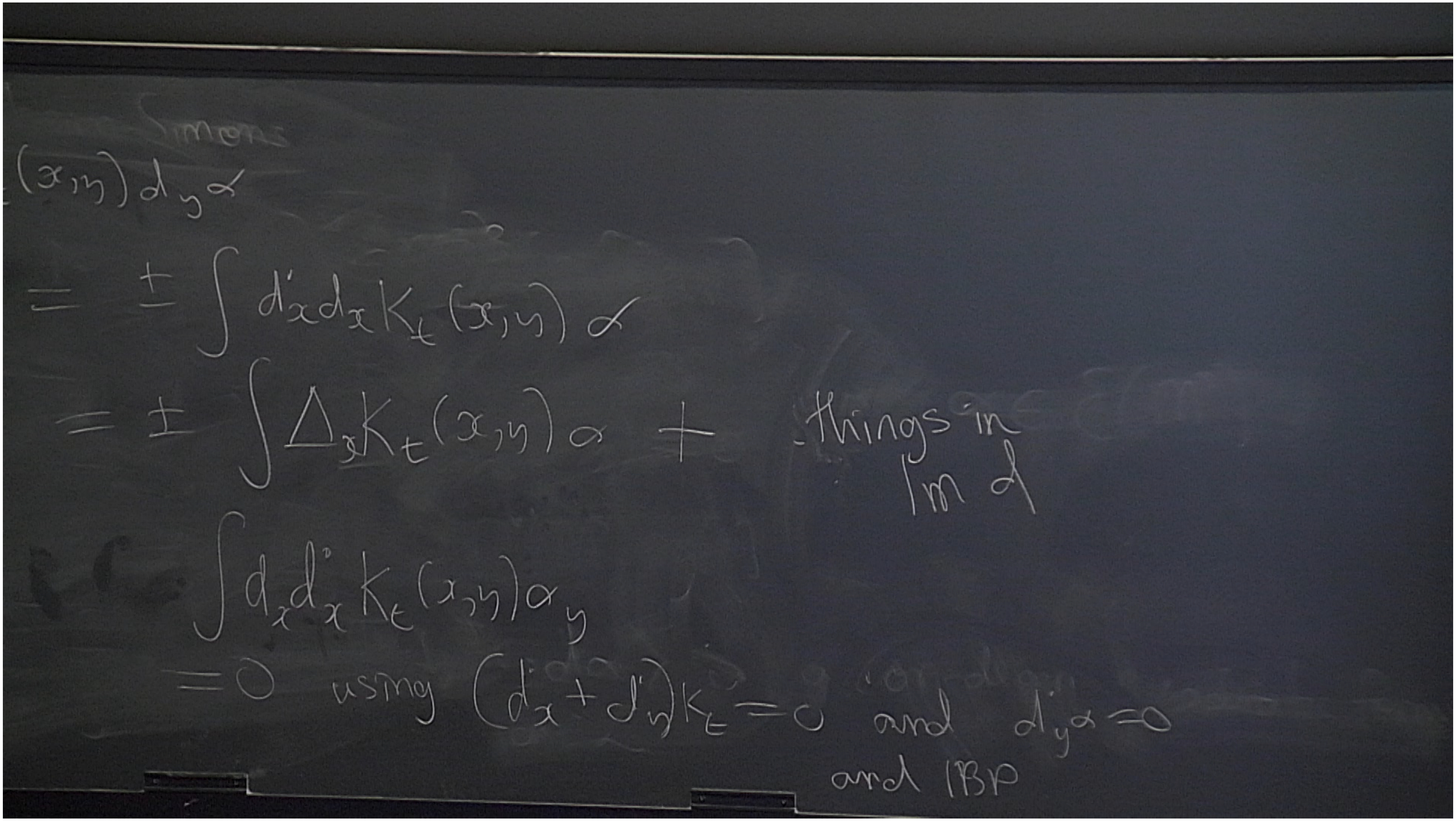
$\rightarrow$ problematic?

$K_0 = \delta_{x=y}$

and K_∞ = proj. onto

$= \int \delta_{x,y}\, \alpha(y) = \alpha(x)$ ✓


$\Delta_x \int_0^\infty K_t(x,y)\,\alpha(y) = \alpha(x)$
$\int K_t(x,y)\,\alpha \qquad \Delta^{-1}\alpha$
$d(d^*\Delta^{-1}\alpha)$
$= (\Delta - d^*d)\,\Delta^{-1}\alpha$
$\int d_x K_t(x,y)\, d_y \alpha = \alpha$
$= -\int d_x^*(d_y K_t(x,y))\,\alpha$
$= \int d_x^* d_y\, K_t(x,y)\,\alpha = \alpha \overset{?}{+} \text{stuff in } \cancel{\mathrm{Im}\, d}$
$\mathrm{Im}\, d \qquad \mathrm{Im}\, d^*$
$\oplus E_\lambda \underset{d}{\overset{d^*}{\rightleftarrows}} \oplus \tilde{E}_\lambda$
ψ
α


Simons
$(x,y)\, d_y \alpha$
$= \pm \int d_x^* d_x K_t(x,y)\, \alpha$
$= \pm \int \Delta_x K_t(x,y)\, \alpha$ + things in Im d
$\int d_x d_x^* K_t(x,y)\, \alpha_y$
$= 0$ using $(d_x^* + d_y^*) K_t = 0$ and $d_y^* \alpha = 0$
and IBP

Propagator

$$P(\varepsilon, L) = \int_{\varepsilon}^{L} d_x^{\cdot} K_t \, dt$$

We can use the same Feynman rules as before

- Vertex $\int \alpha^3$
- edge : put $P(\varepsilon, L) = \int_{t=\varepsilon}^{L} d_x^{\cdot} K_t$

$$W(P(\varepsilon, L), I_{CS})(\alpha)$$

$$= \sum_{\text{conn graphs}} \hbar^{\#\text{loops}} \cdot \text{amplitude}$$

fn of $\alpha \in \Omega^*(M^3) \otimes \mathfrak{g}[1]$

<u>Just as before</u>

$\exists!$ set of counter terms, "purely singular" as a fn of ε

So $W(P(\varepsilon, L), I^{CS} - I^{CT}(\varepsilon))$ has an $\varepsilon \to 0$ limit

View

$P(\varepsilon, L)$ as in $\Omega^2(m \times m)$

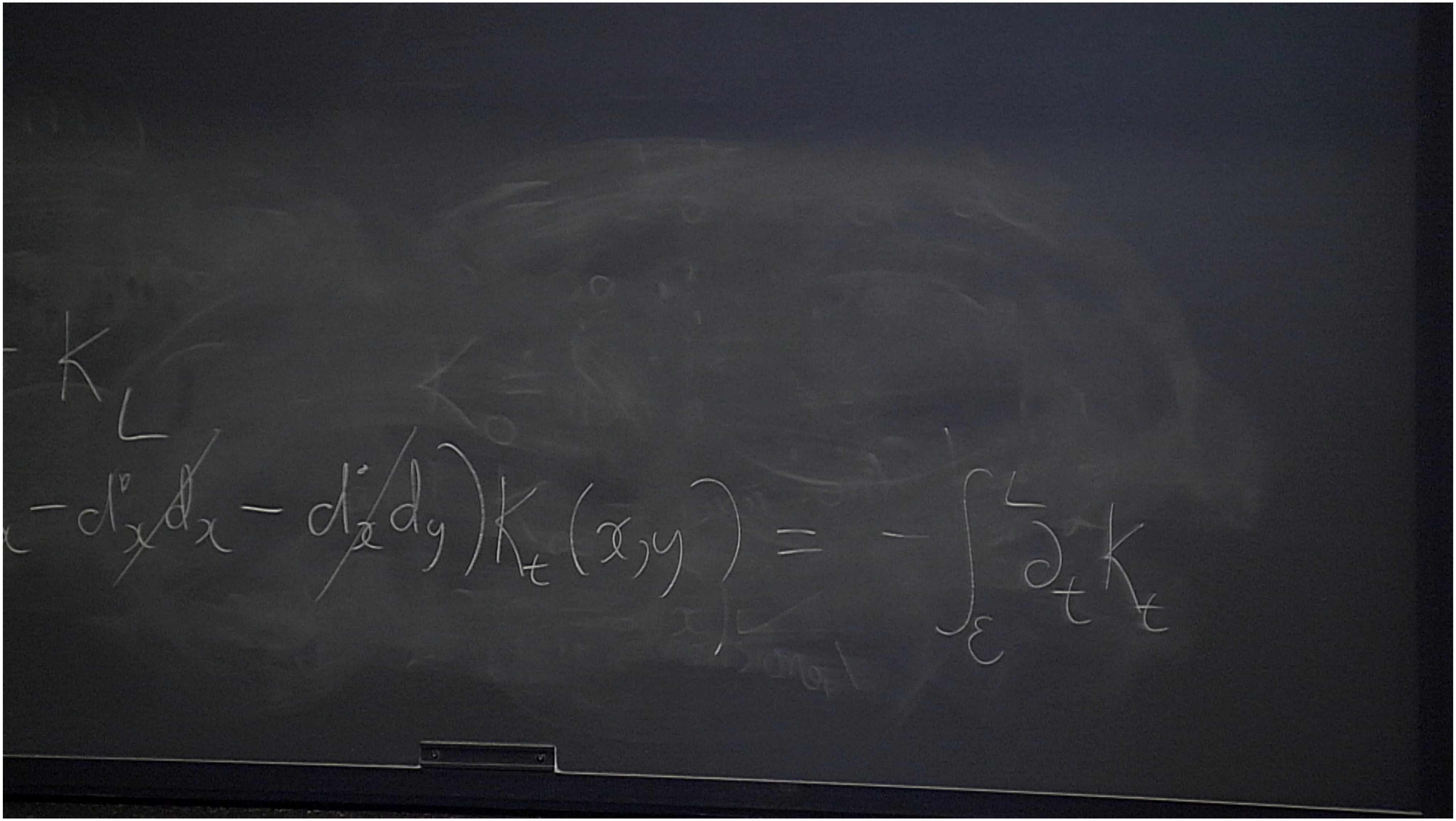
Then:

KEY FORMULA

$$(d_x + d_y) P_L(\varepsilon, L) = -K_\varepsilon + K_L$$

As,

$$(d_x + d_y) \int_\varepsilon d_x K_t(x,y) = \int_\varepsilon^L (\Delta_x - \cancel{d_x^* d_x} - \cancel{d_x^* d_y}) K_t(x,y)$$


K_L
$-d^\dagger_x d_x - d^\dagger_x d_y) K_t(x,y) = -\int_\varepsilon^L \partial_t K_t$

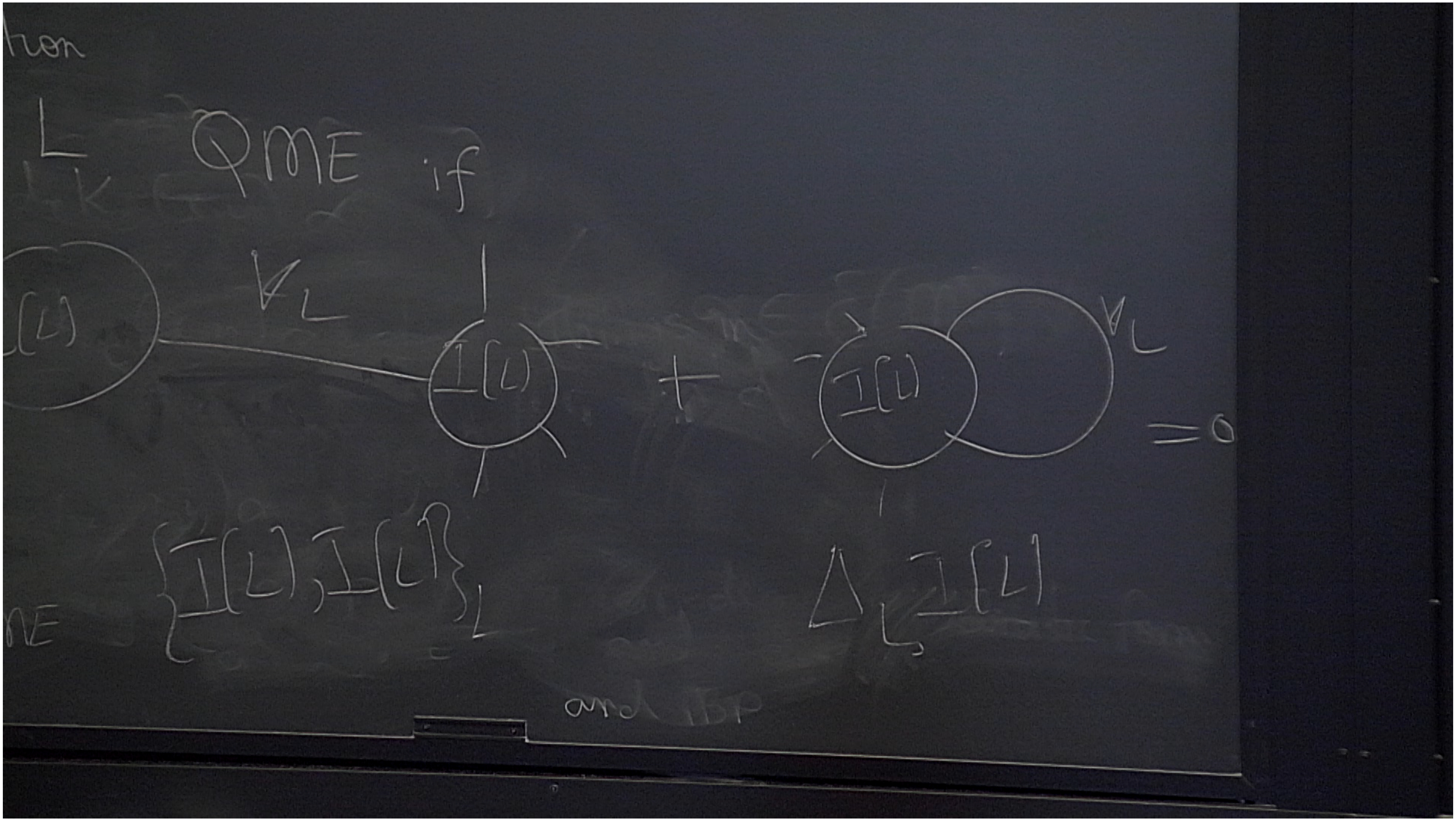
We say some interaction $I[L]$ satisfies scale L QME if

$d_{dR} I[L] + I[L] \overset{K_L}{—} I[L] +$

d_{dR} $I[L]$

$L = 0$ this is ordinary QME

$\{I[L], I[L]\}_L$

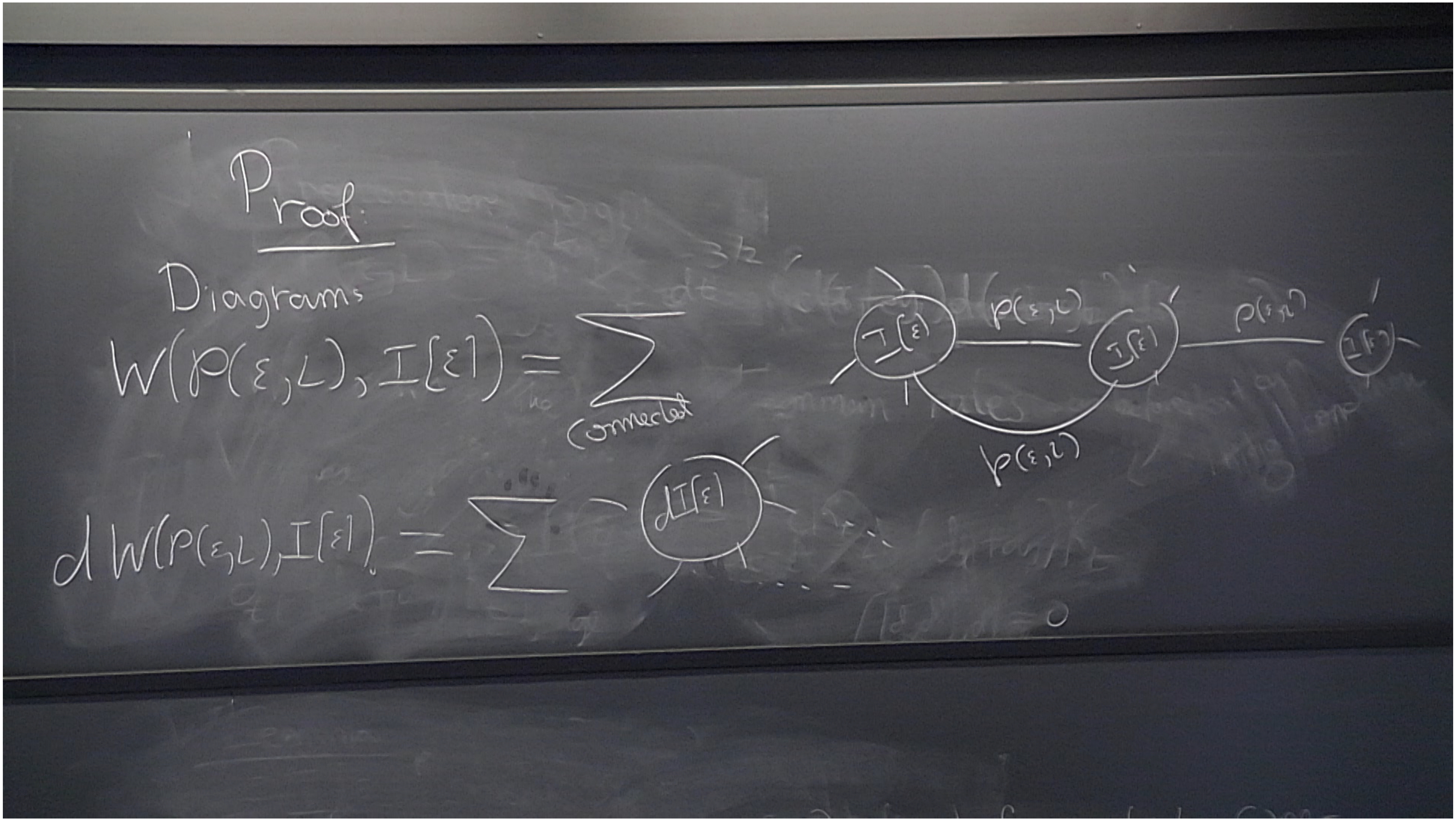

QME if
∀L
I(L)
+
I(L)
∀L
=0
{I(L), I(L)}L
ΔL I(L)
and

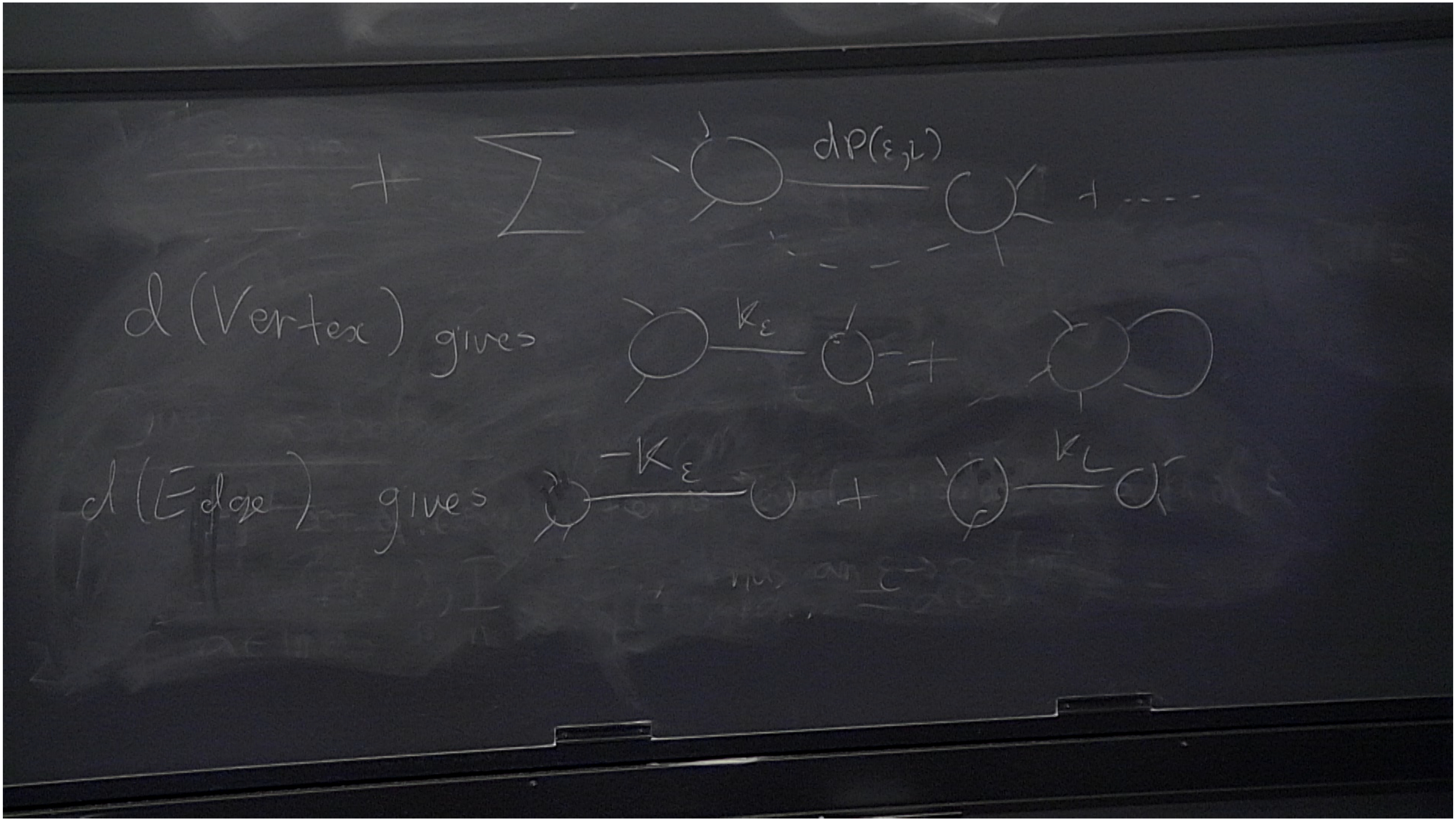
Lemma

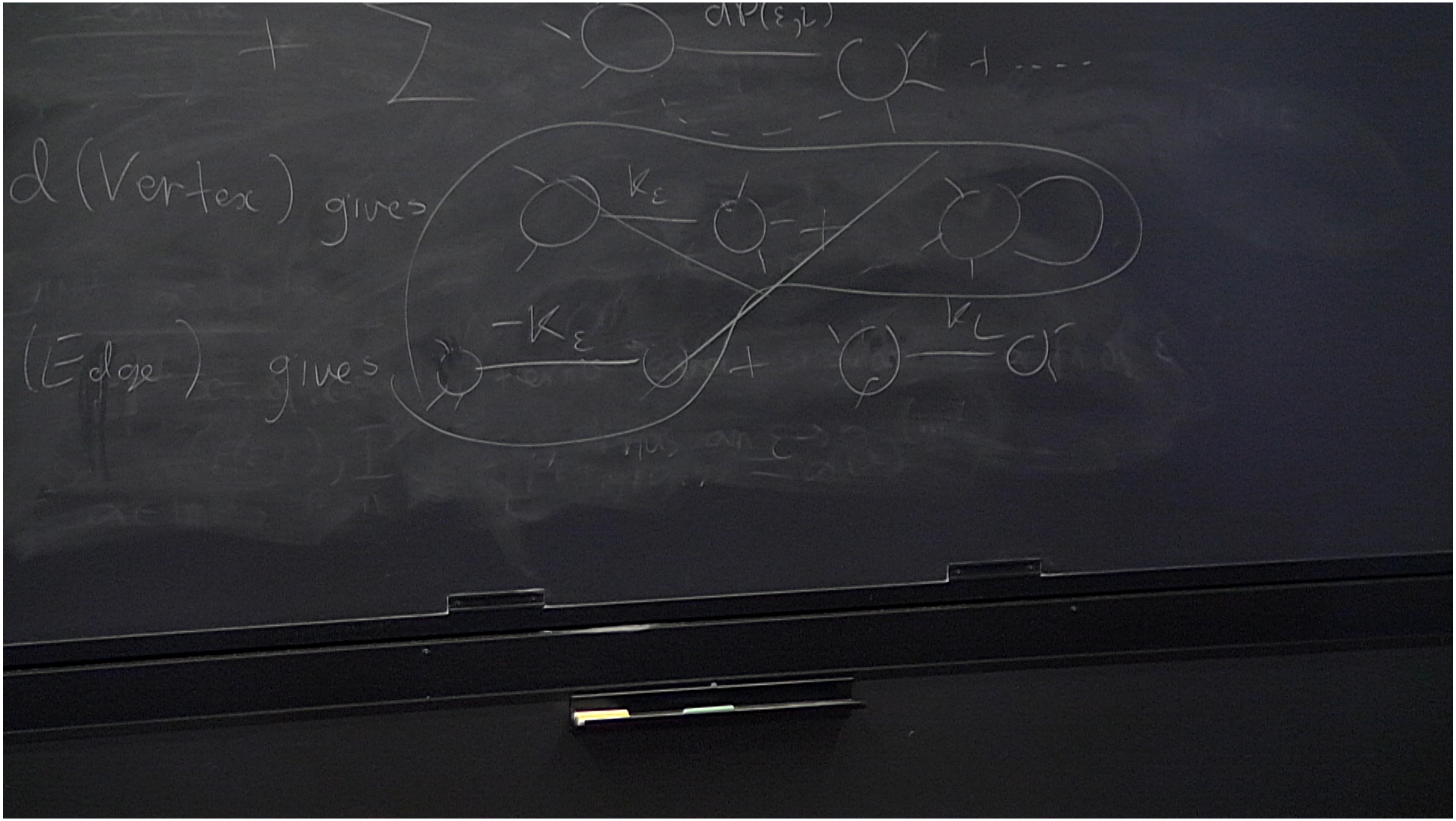
If $\underline{I}[\varepsilon]$ satisfies scale ε QME

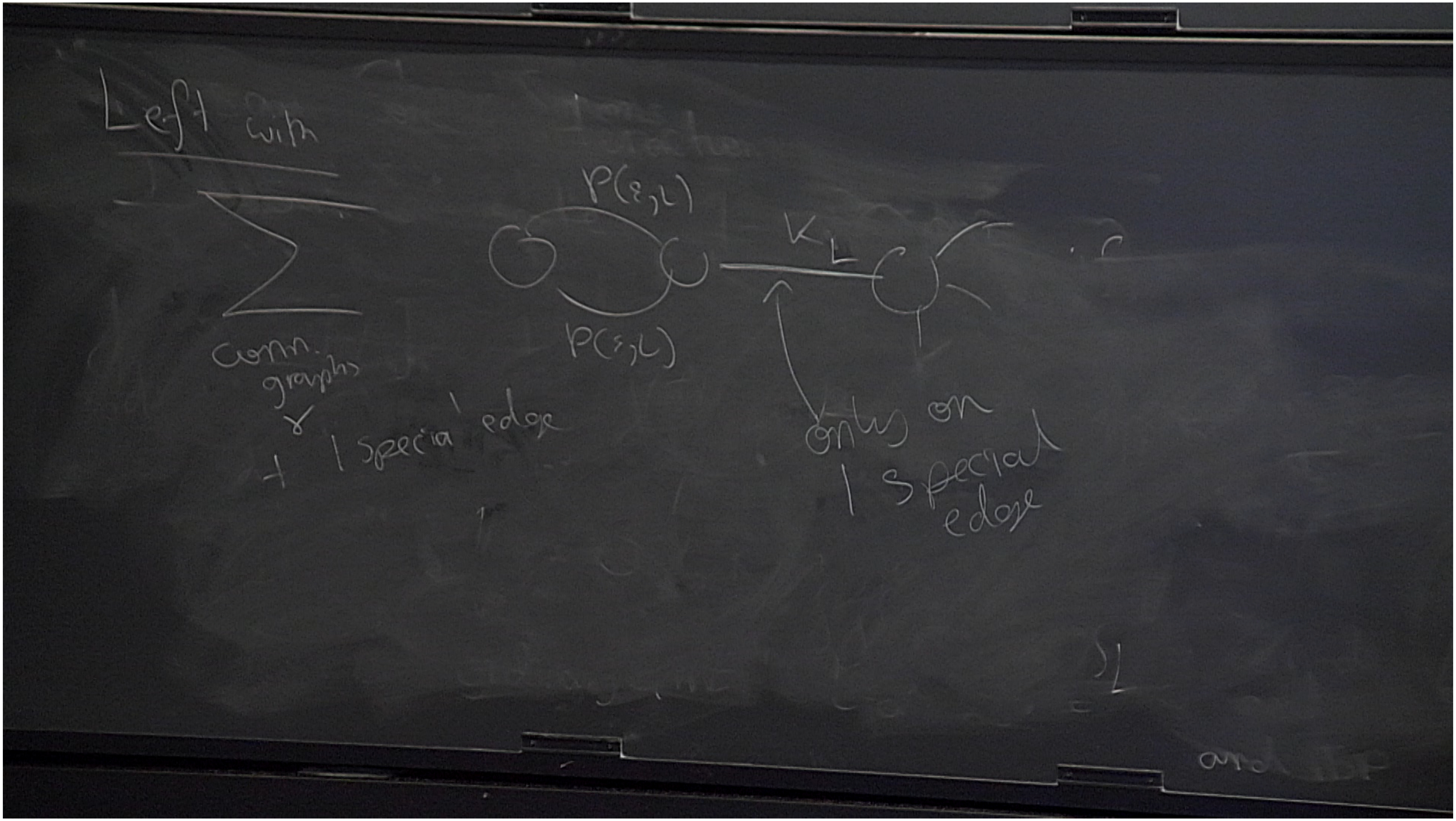
Then $W(\rho(\varepsilon,L), \underline{I}[\varepsilon])$ satisfies scale L QME

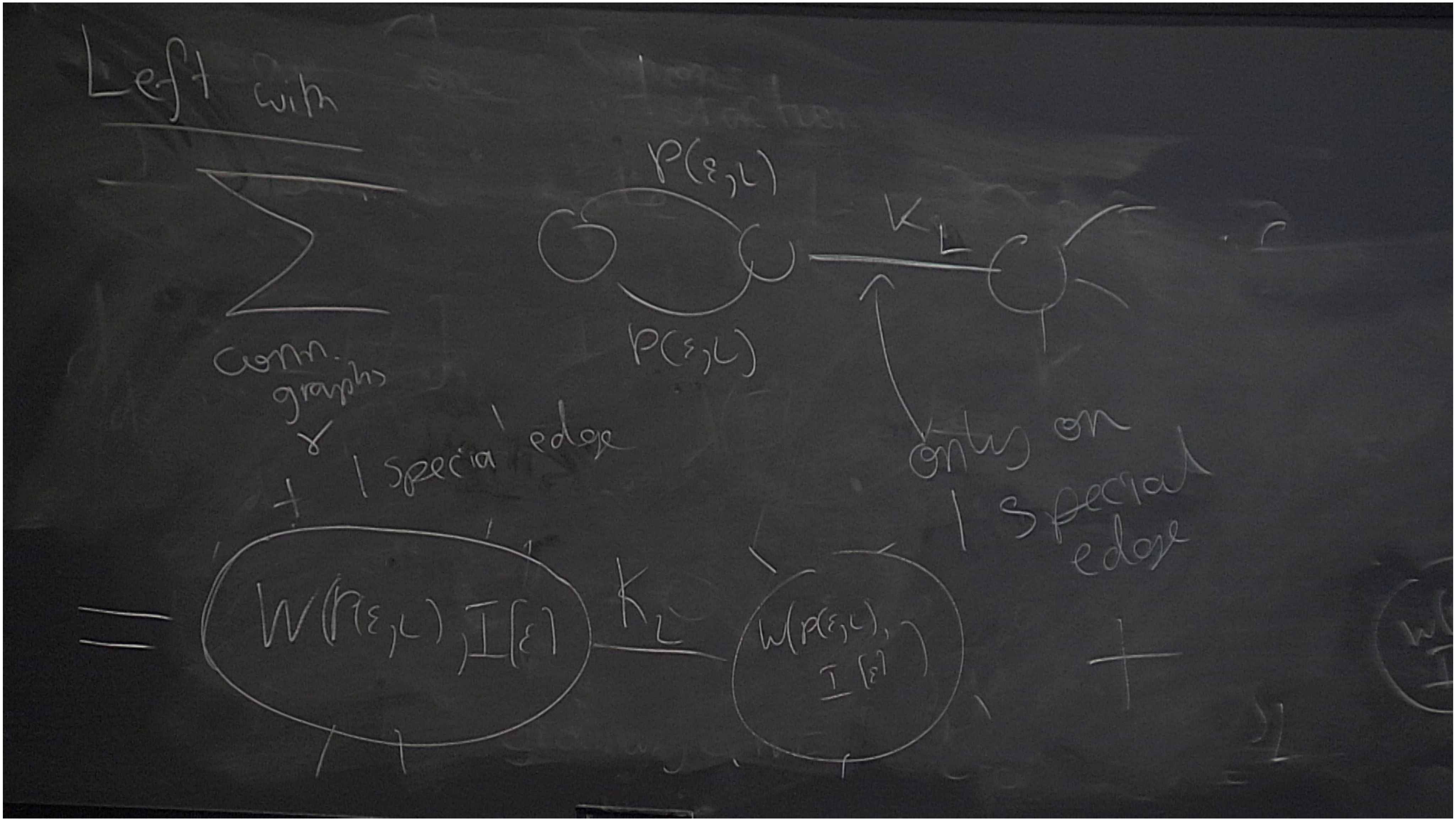
RG flow

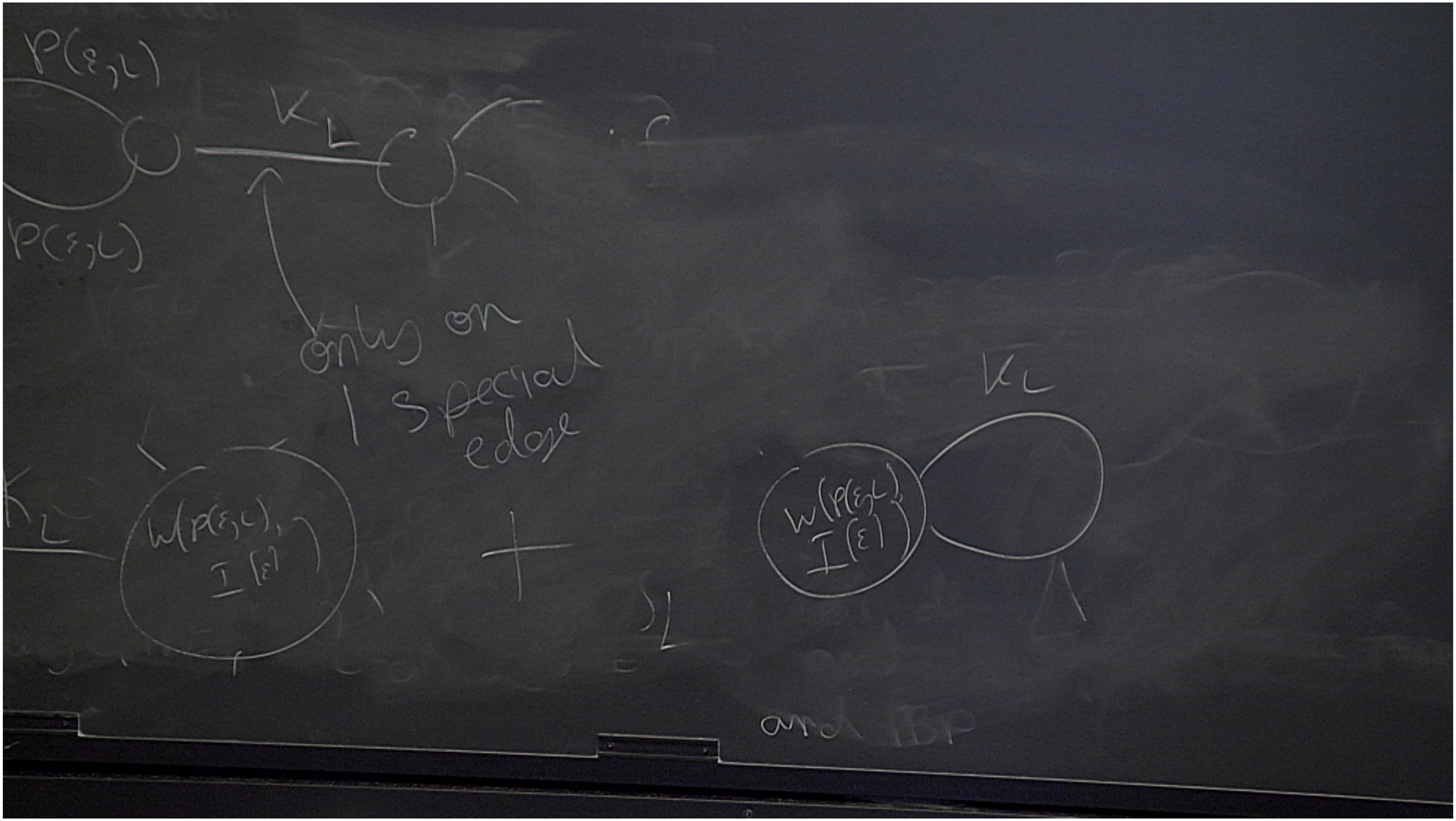

Proof:
Diagrams
W(P(ε,L), I[ε]) = Σ connected
I[ε]
I[ε]
I[ε]
P(ε,L)
P(ε,L)
P(ε,L)
dW(P(ε,L), I[ε]) = Σ
dI[ε]


+ Σ
dP(ε,L)
+ ----
d(Vertex) gives
Kε
+
d(Edge) gives
−Kε
+
KL


d (Vertex) gives
(Edge) gives
K_ε
$-K_\varepsilon$


Left with
∑
conn. graphs
+ 1 'special' edge
P(ε,L)
P(ε,L)
K_L
only on
1 special
edge


Left with
$\sum$
conn. graphs γ
+ 1 special edge
$P(\varepsilon, L)$
$P(\varepsilon, L)$
K_L
only on 1 special edge
$= W(P(\varepsilon,L), I[\varepsilon]) \underline{K_L} W(P(\varepsilon,L), I[\varepsilon])$
+


P(ε,L)
K_L
P(ε,L)
only on
1 special
edge
K_L
W(P(ε,L),
I[ε]
+
S_L
K_L
W(P(ε,L),
I[ε]
and