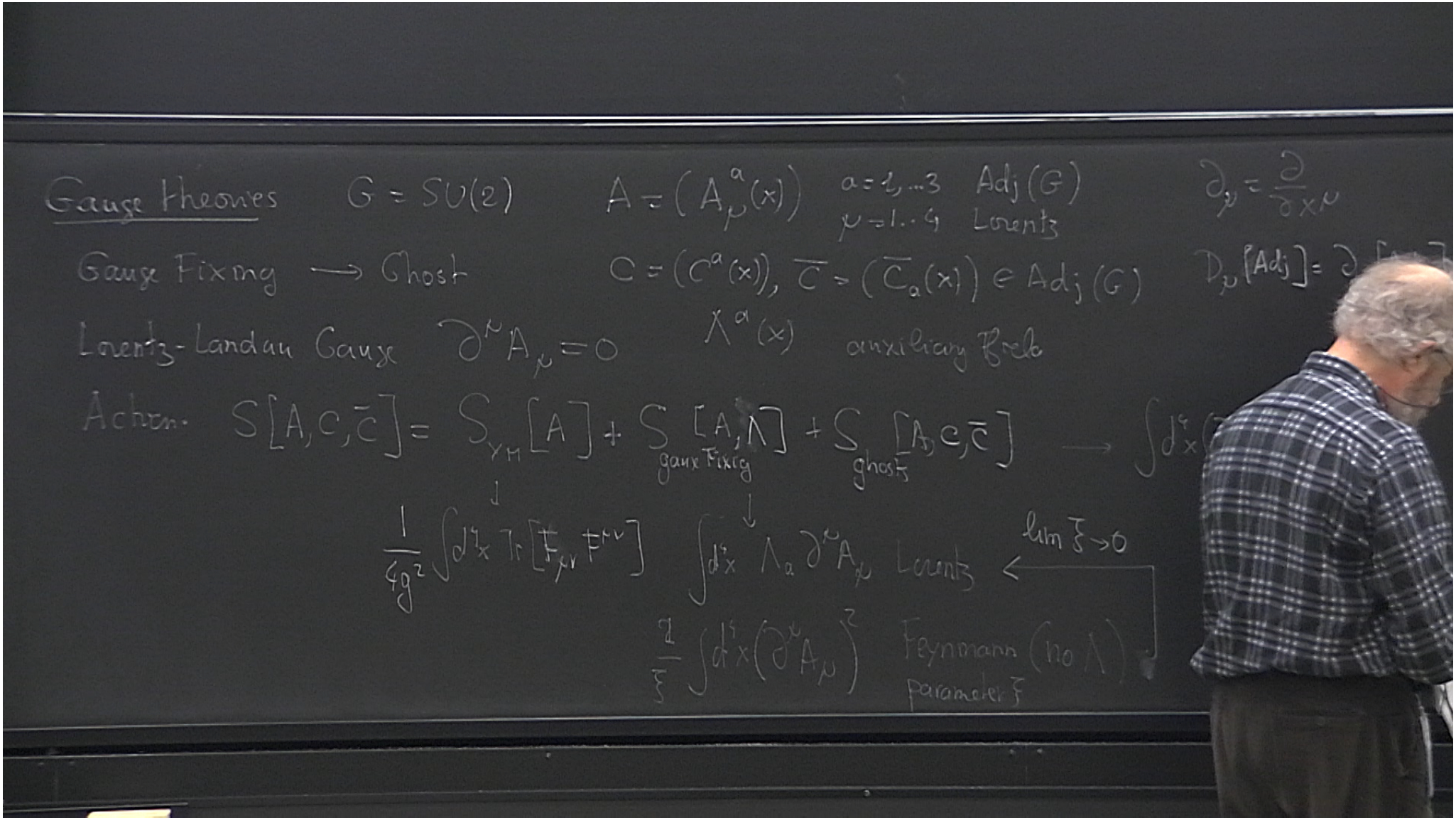
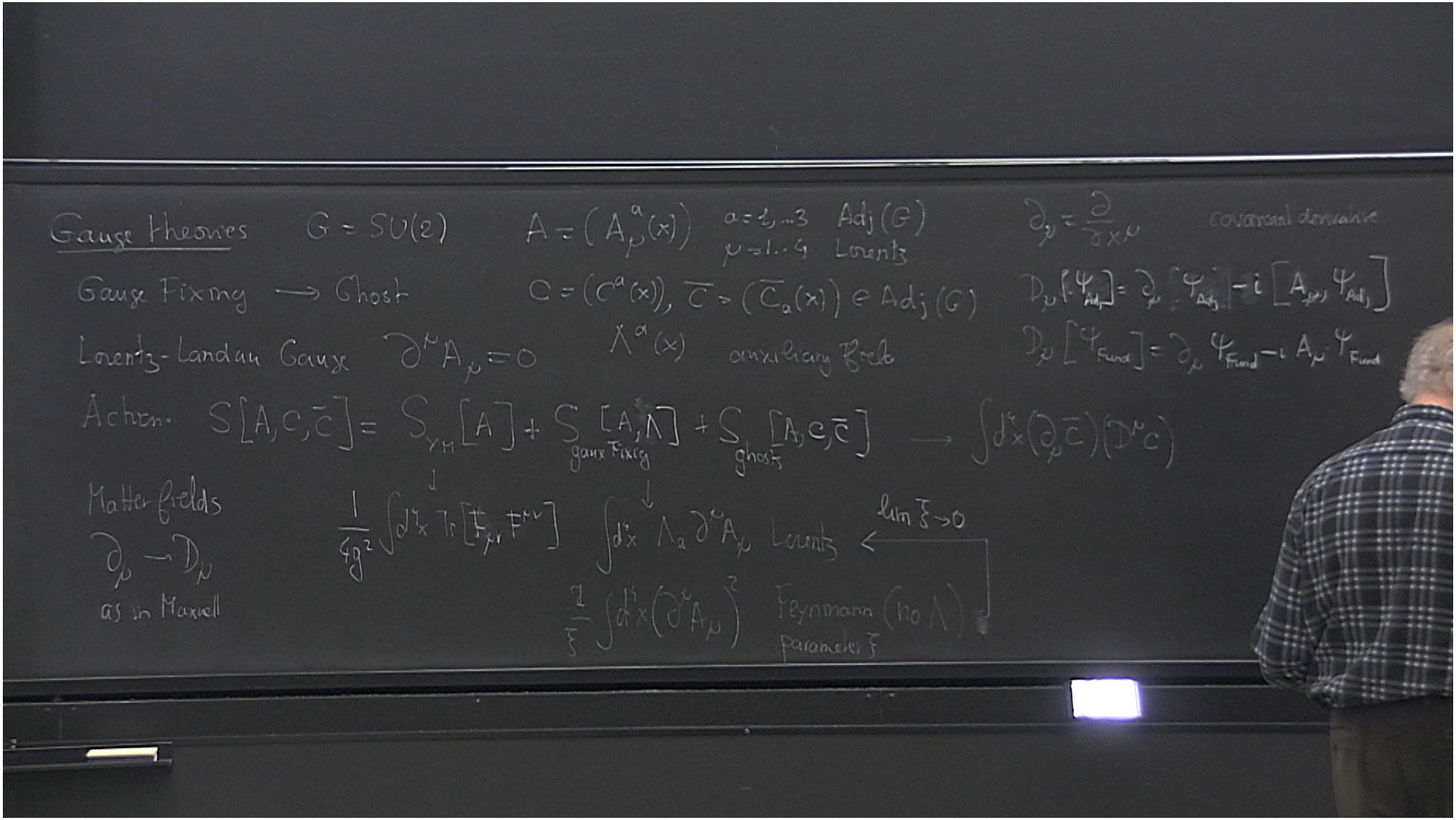
Title: PSI 2018/2019 - Quantum Field Theory I - Lecture 15

Date: Dec 06, 2018 10:00 AM

URL: http://pirsa.org/18120002

Abstract:


Gauge theories
G = SU(2)
A = (A^a_μ(x))
a = 1,...3 Adj(G)
μ = 1..4 Lorentz
∂_μ = ∂/∂x^μ
Gauge Fixing → Ghost
C = (C^a(x)), C̄ = (C̄_a(x)) ∈ Adj(G)
D_μ[Adj] = ∂
Lorentz-Landau Gauge ∂^μ A_μ = 0
Λ^a(x) auxiliary Fields
Action. S[A,C,C̄] = S_YM[A] + S_GaugeFixing[A,Λ] + S_ghosts[A,C,C̄] → ∫d^4x
1/4g^2 ∫d^4x Tr[F_μν F^μν]
∫d^4x Λ_a ∂^μ A_μ Lorentz
lim ξ→0
1/ξ ∫d^4x (∂^μ A_μ)^2 Feynmann (no Λ) parameter ξ


Gauge theories G = SU(2) A = (A_μ^a(x)) a = 1,...3 Adj(G) μ = 1..4 Lorentz
∂_μ = ∂/∂x^μ covariant derivative
Gauge Fixing → Ghost C = (C^a(x)), C̄ = (C̄_a(x)) ∈ Adj(G)
D_μ[Ψ_Adj] = ∂_μ Ψ_Adj − i [A_μ, Ψ_Adj]
Lorentz-Landau Gauge ∂^μ A_μ = 0 Λ^a(x) auxiliary field
D_μ[Ψ_Fund] = ∂_μ Ψ_Fund − i A_μ Ψ_Fund
Action S[A,C,C̄] = S_YM[A] + S_gauge Fixing[A,Λ] + S_ghosts[A,C,C̄] → ∫d^4x (∂_μ C̄)(D^μ C)
Matter fields
∂_μ → D_μ
as in Maxwell
1/4g² ∫d^4x Tr[F_μν F^μν]
∫d^4x Λ_a ∂^μ A_μ Lorentz
lim ξ→0
1/ξ ∫d^4x (∂^μ A_μ)^2 Feynmann (no Λ) parameter ξ

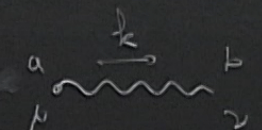
$\partial_\mu = \frac{\partial}{\partial x^\mu}$ covariant derivative

$D_\mu [\psi_{Adj}] = \partial_\mu [\psi_{Adj}] - i [A_\mu, \psi_{Adj}]$

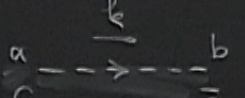
$D_\mu [\psi_{Fund}] = \partial_\mu \psi_{Fund} - i A_\mu \psi_{Fund}$

$\int d^4x (\partial_\mu \bar{c})(D^\mu c)$

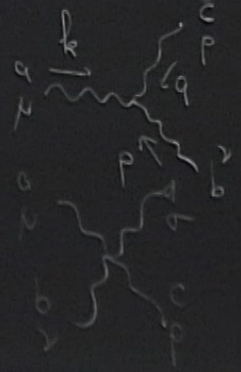
Perturbative theory rescale $A \to g A$ $\eta_{\mu\nu} = (-1, +\,+\,+)$ metric



$= \delta_{ab} \frac{-i}{k^2 - i\epsilon} \left(\eta_{\mu\nu} + (\xi - 1) \frac{k_\mu k_\nu}{k^2} \right)$

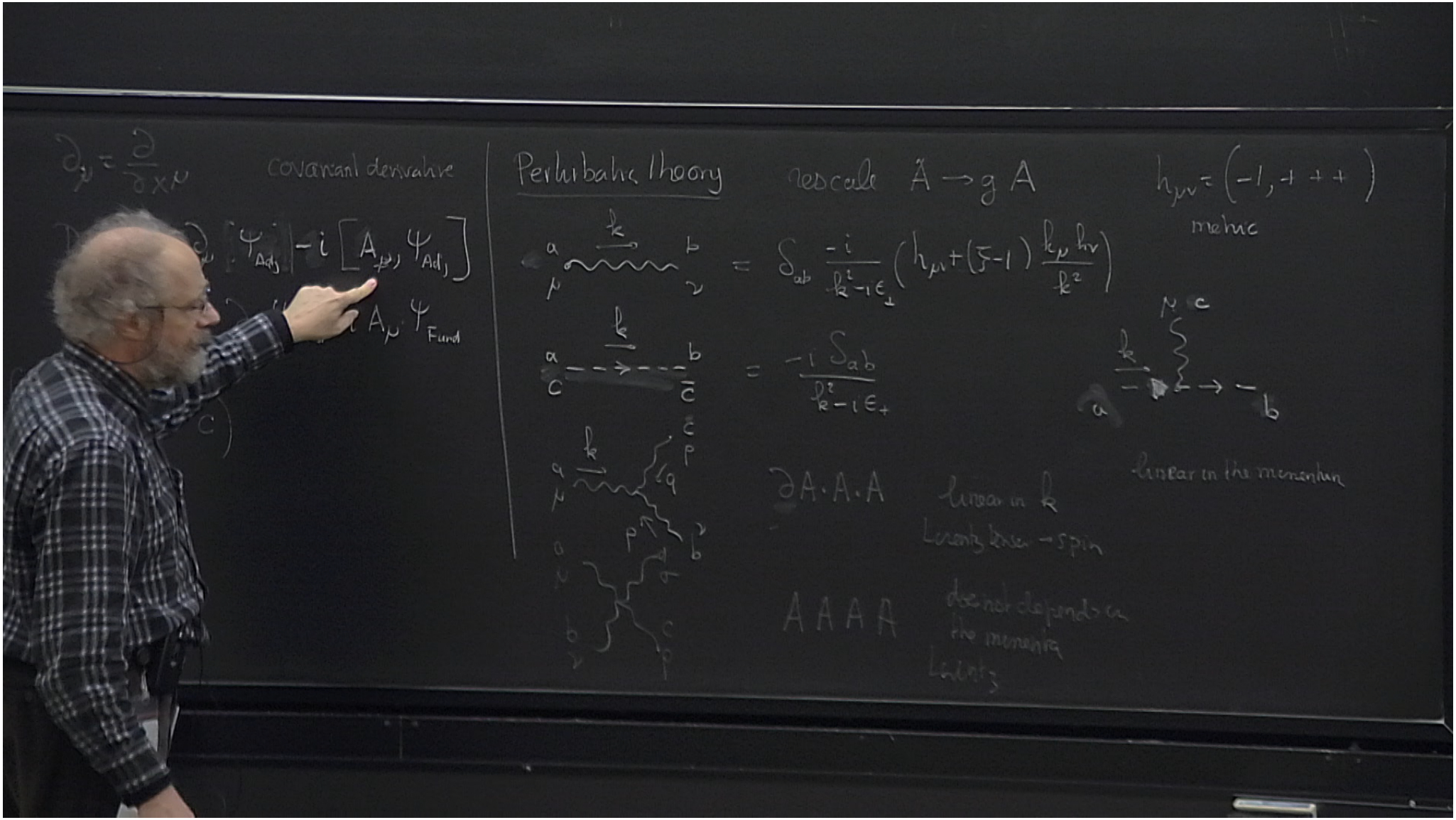


$= \frac{-i \delta_{ab}}{k^2 - i\epsilon}$



$\partial A \cdot A \cdot A$ linear in k Lorentz index - spin

$AAAA$ does not depends on the momenta Lorentz


covariant derivative
Perturbative theory
rescale
metric
linear in the momentum
linear in k
Lorentz tensor - spin
AAAA

Radiative corrections: 1 loop . . .

Renormalizable properties

$[A_\mu] \sim \text{length}^{-1}$

$[F_{\mu\nu}] \sim \text{length}^{-2}$

$d = 4 \quad [g] \sim \text{length}^{0}$

Radiative corrections. 1 loop . . .

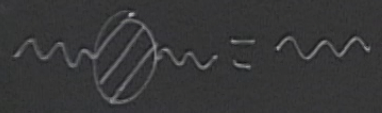
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$\log \Lambda$ UV divergences



Radiative corrections: 1 loop . . .

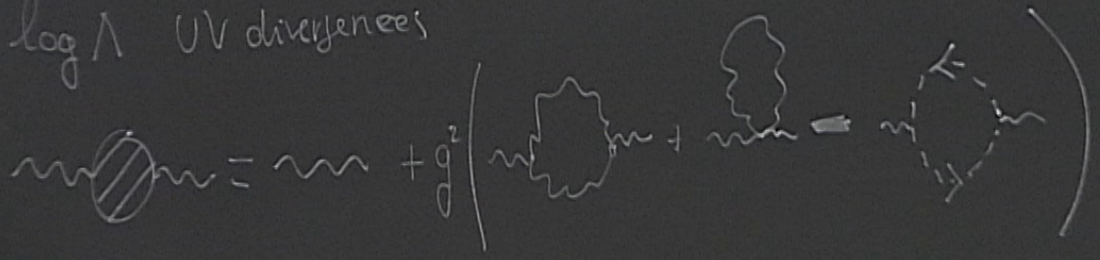
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$\log \Lambda$ UV divergences



Radiative corrections. 1 loop . . .

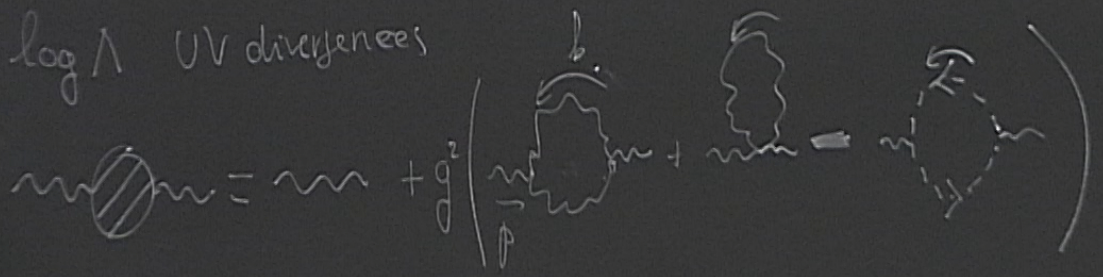
Renormalizatica properties

$[A_\mu] \sim \text{length}^{-1}$

$[F_{\mu\nu}] \sim \text{length}^{-2}$

$d = 4 \quad [g] \sim \text{length}^{0} \int d^4k \, \frac{1}{k^2} \frac{1}{k^2} k^2 + p^\mu p^\nu \frac{1}{k^2 k^2}$

$\log \Lambda$ UV divergences



Radiative corrections. 1 loop . . .

Renormalizatica properties

$[A_\mu] \sim \text{length}^{-1}$

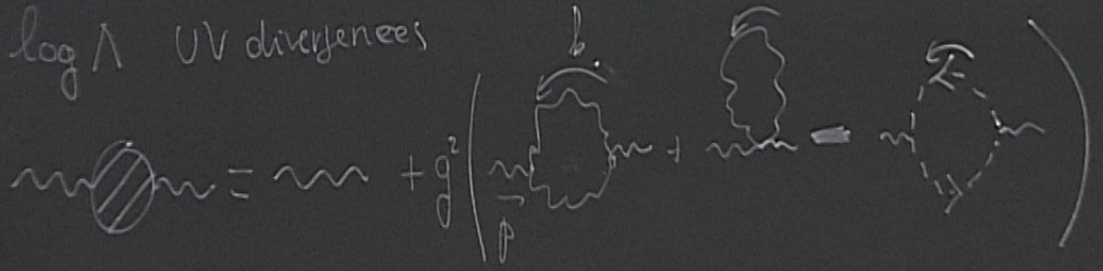
$[F_{\mu\nu}] \sim \text{length}^{-2}$

$d = 4 \quad [g] \sim \text{length}^{0}$

$\log\Lambda$ UV divergences

Λ^2 divergence $\quad p^\mu p^\nu \log\Lambda$

$$\int_{1<|k|<\Lambda} d^4k \; \frac{1}{k^2}\frac{1}{k^2} k^2 + p^\mu p^\nu \frac{1}{k^2 k^2}$$



Radiative corrections. 1 loop . . .

Renormalizatica properties

$[A_\mu] \sim \text{length}^{-1}$

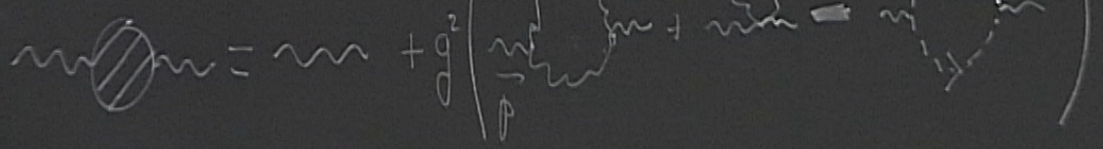
$[F_{\mu\nu}] \sim \text{length}^{-2}$

$d = 4 \quad [g] \sim \text{length}^0$

$\log \Lambda$ UV divergences

Λ^2 divergence cancells

$$\int_{|k|<\Lambda} d^4k \; \frac{1}{k^2}\frac{1}{k^2} k^2 \qquad p^\mu p^\nu \log\Lambda + p^\mu p^\nu \frac{1}{k^2 k^2}$$



Radiative corrections. 1 loop . . .

Renormalizatica properties

$[A_\mu] \sim \text{length}^{-1}$

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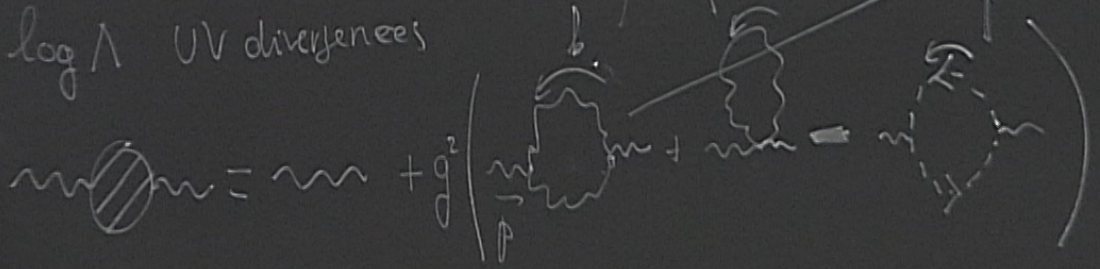
$d = 4 \quad [g] \sim \text{length}^0$

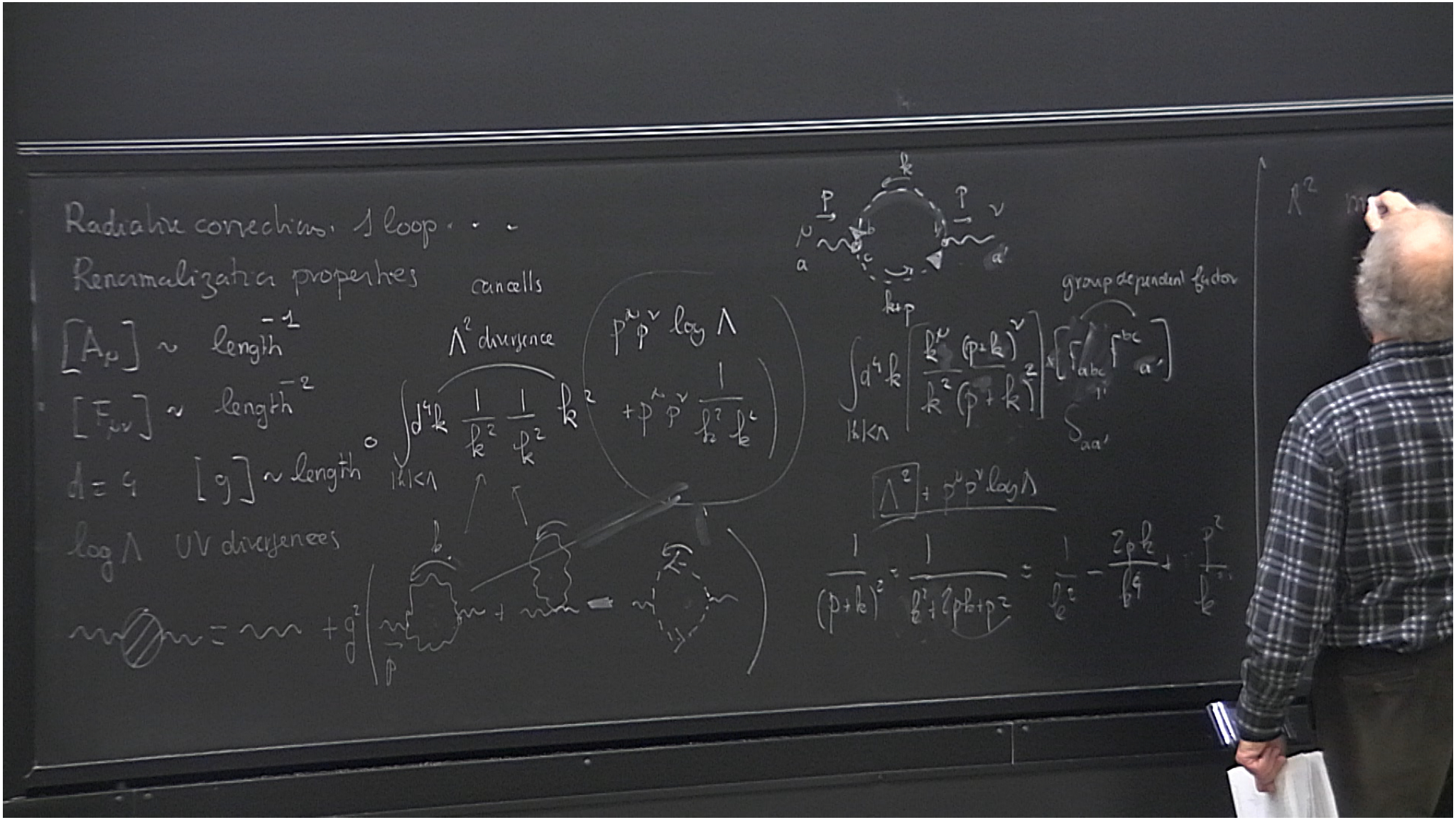
$\log \Lambda$ UV divergences

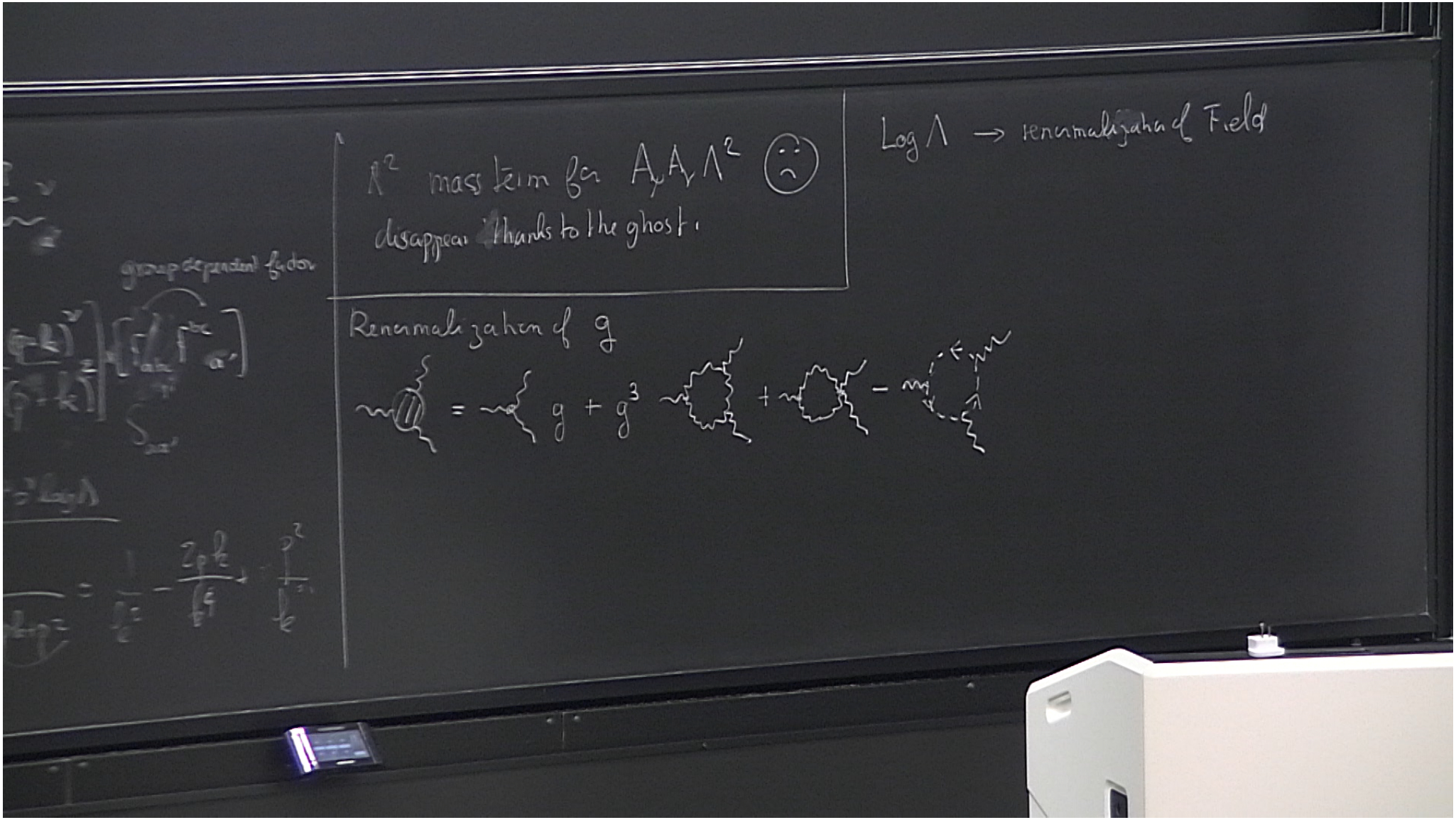
$\int_{|k|<1} d^4k \, \frac{1}{k^2} \frac{1}{k^2} k^2$ — Λ^2 divergence — cancells

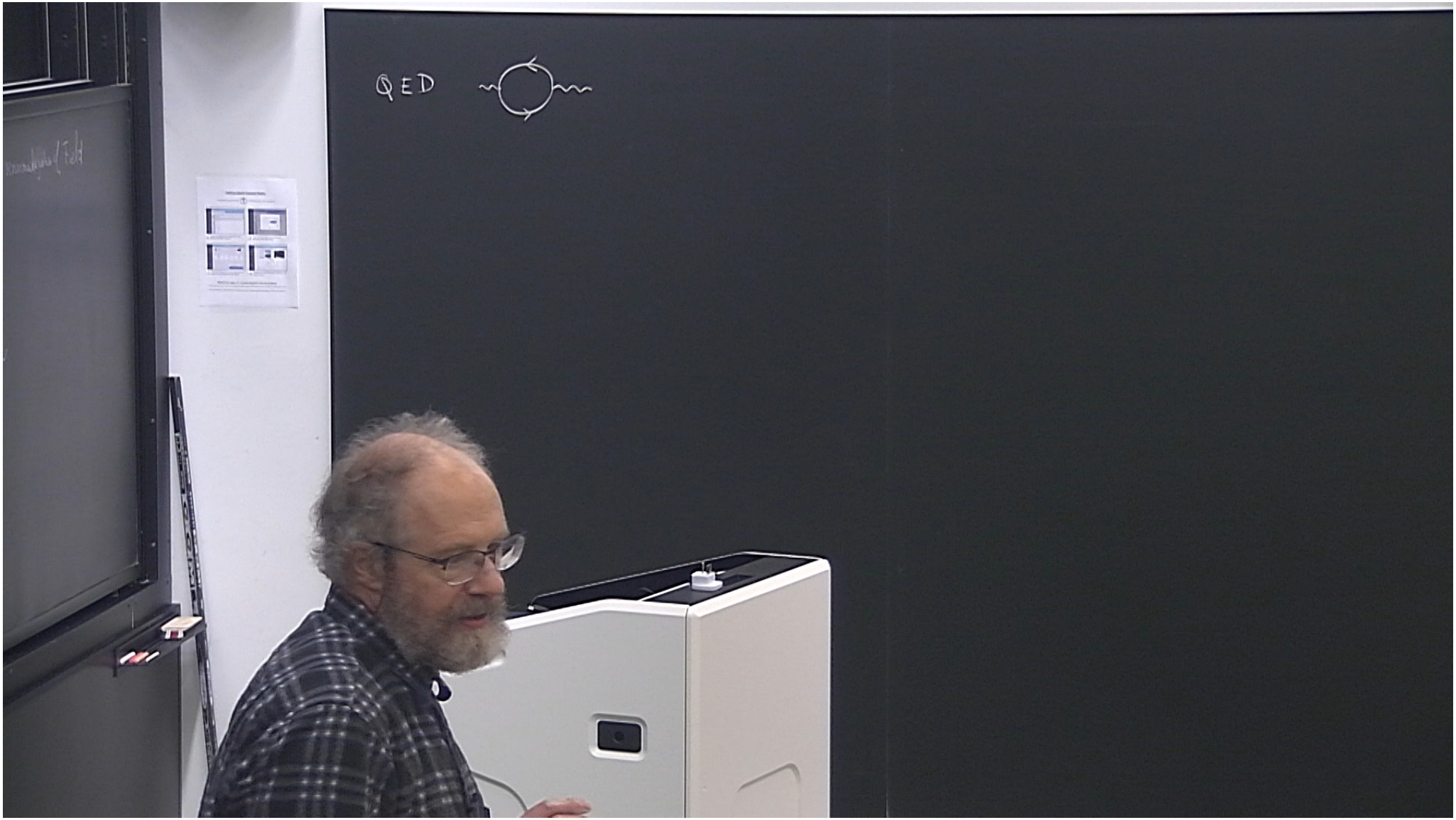
$p^\mu p^\nu \log \Lambda + p^\mu p^\nu \frac{1}{k^2} \frac{1}{k^2}$

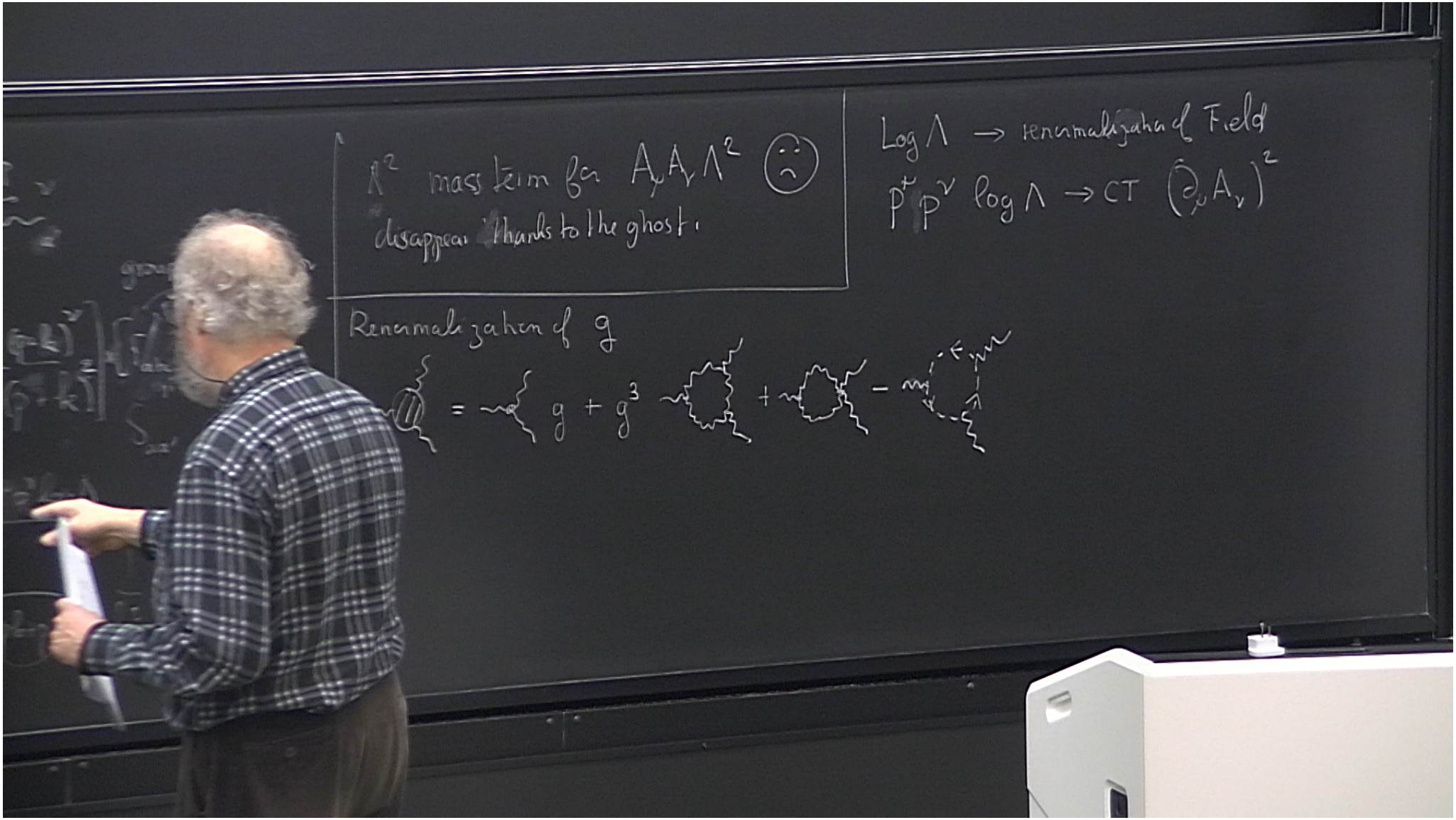





Radiative corrections: 1 loop . . .
Renormalization properties
$[A_\mu] \sim \text{length}^{-1}$
$[F_{\mu\nu}] \sim \text{length}^{-2}$
$d = 4 \quad [g] \sim \text{length}^0$
$\log\Lambda$ UV divergences
cancells
Λ^2 divergence
$\int_{|k|<\Lambda} d^4k \frac{1}{k^2}\frac{1}{k^2} k^2$
$p^\mu p^\nu \log\Lambda$
$+ p^\mu p^\nu \frac{1}{p_2^2 k^2}$
group dependent factor
$\int_{|k|<\Lambda} d^4k \left[\frac{k^\mu (p+k)^\nu}{k^2 (p+k)^2}\right] \left[\Gamma_{abc}\Gamma^{bc}_{a'}\right] S_{aa'}$
$\Lambda^2 + p^\mu p^\nu \log\Lambda$
$\frac{1}{(p+k)^2} = \frac{1}{k^2 + 2pk + p^2} = \frac{1}{k^2} - \frac{2pk}{k^4} + \frac{p^2}{k^4}$


group dependent factor
k^2 mass term for $A_\nu A_\nu \Lambda^2$
disappear thanks to the ghost.
Log Λ → renormalization of Field
Renormalization of g
g + g³


QED


Λ^2 mass term for $A_\nu A_\nu \Lambda^2$
disappear thanks to the ghost.
$\log\Lambda \rightarrow$ renormalization of Field
$p^\nu p^\nu \log\Lambda \rightarrow$ CT $(\partial_\nu A_\nu)^2$
Renormalization of g
$= g + g^3$

Λ^2 mass term for $A_\nu A_\nu \Lambda^2$ ☹ disappear thanks to the ghost.

$\mathrm{Log}\,\Lambda \rightarrow$ renormalization of Field

$p^\mu p^\nu \log\Lambda \rightarrow$ CT $(\partial_\mu A_\nu)^2$

linear term in the action

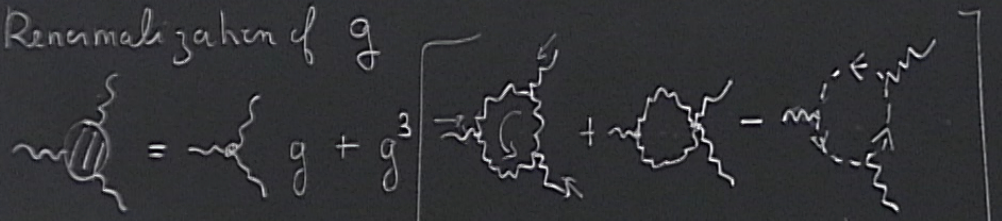
YM

potential linear $\Lambda \rightarrow 0$

$+ \quad p^\mu \log\Lambda$

CT $A\,A\,\partial A$

Renormalization of g



$$\int d^4k \frac{1}{(k^2)^3}\left(k\,b\,k + p\cdot k\,k\right)$$

Λ^2 mass term for $A_\mu A_\nu \Lambda^2$ ☹
disappear thanks to the ghost.

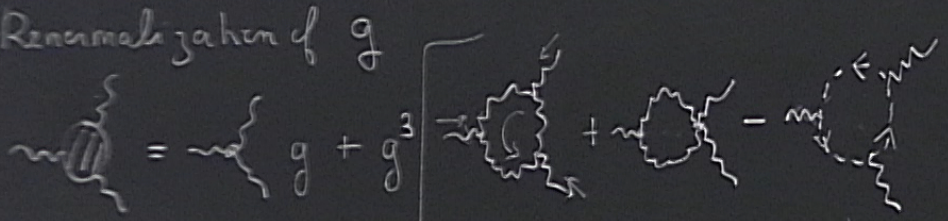
$\log\Lambda \rightarrow$ renormalization of Field

$p^\mu p^\nu \log\Lambda \rightarrow$ CT $(\partial_\mu A_\nu)^2$

linear term in the action

YM

Renormalization of g



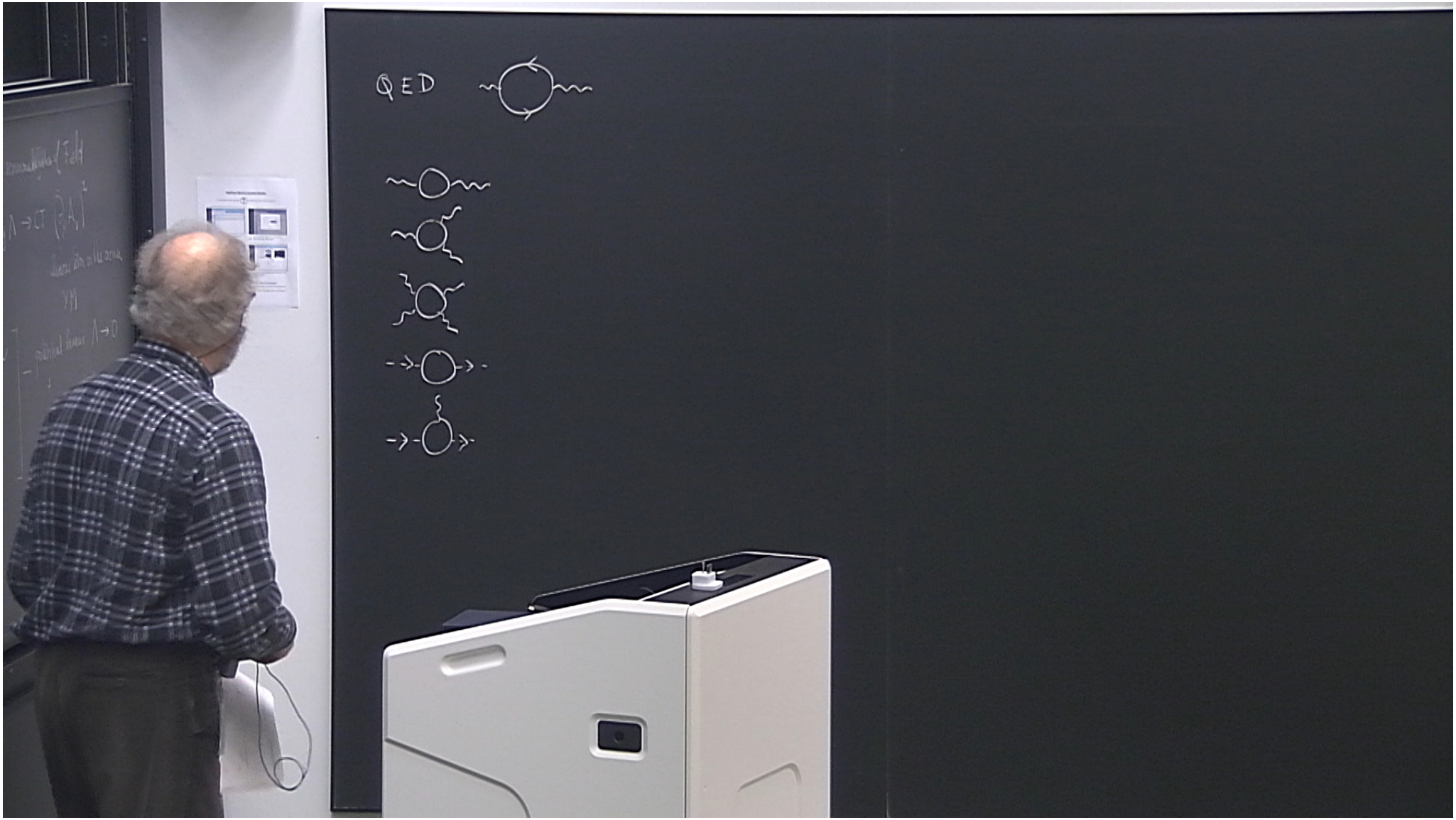
$= g + g^3$ [ ] − potential linear $\Lambda \rightarrow 0$

$+ \; p^\mu \log\Lambda$

CT $A\,A\,\partial A$

$\int d^4k \frac{1}{(k^2)^3}\left(k\,k\,k + p\cdot k\,k\right)$

$\Rightarrow$ renormalization of g


QED

as in Maxwell $\frac{1}{2\xi}\int d^4x\,(\partial^\mu A_\mu)^2$ Feynmann parameter ξ (no Λ)

Gauge theory is renormalizable

1 Loop 't Hooft, Weltman '72

dimensional regularization $d = 4-\epsilon$ $\frac{1}{\epsilon} \leftrightarrow \log\Lambda$

gauge invariant

as in Maxwell $\frac{1}{2\xi}\int d^4x\,(\partial^\mu A_\mu)^2$ Feynmann parameter ξ (no Λ)

Gauge theory is renormalizable

- 1 Loop 't Hooft, Veltman '72
  dimensional regularization $d = 4-\epsilon$ $\frac{1}{\epsilon} \leftrightarrow \log\Lambda$
  gauge invariant
- all orders proofs Lee-Zinn-Justin
- BRST symmetry

as in Maxwell $\frac{1}{2\xi}\int d^4x (\partial^\mu A_\mu)^2$ Feynmann parameter ξ (no Λ)

Gauge theory is renormalizable

- 1 Loop 't Hooft, Veltman '72
  dimensional regularization $d = 4-\epsilon$ $\frac{1}{\epsilon} \leftrightarrow \log \Lambda$
  gauge invariant
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- 1 loop calculation

as in Maxwell $\frac{1}{2\xi}\int d^4x\,(\partial^\mu A_\mu)^2$ Feynmann parameter ξ (no Λ)

Gauge theory is renormalizable

- 1 Loop 't Hooft, Veltman '72
  dimensional regularization $d = 4-\epsilon$ $\frac{1}{\epsilon} \leftrightarrow \log\Lambda$
  gauge invariant
- all orders proofs Lee-Zinn-Justin
- BRST symmetry
- 1 loop calculation : Background Gauge Fixing

PS:

$\frac{1}{2\xi}\int d^4x\,(\partial^\mu A_\mu)^2$ Feynmann parameter ξ (no Λ)

$\frac{1}{\epsilon} \leftrightarrow \log\Lambda$

Renormalization of the gauge coupling constant g
(+ redefinition of the field)

$\frac{1}{2\xi}\int d^4x\,(\partial^\mu A_\mu)^2$ Feynmann parameter ξ $(\log\Lambda)$

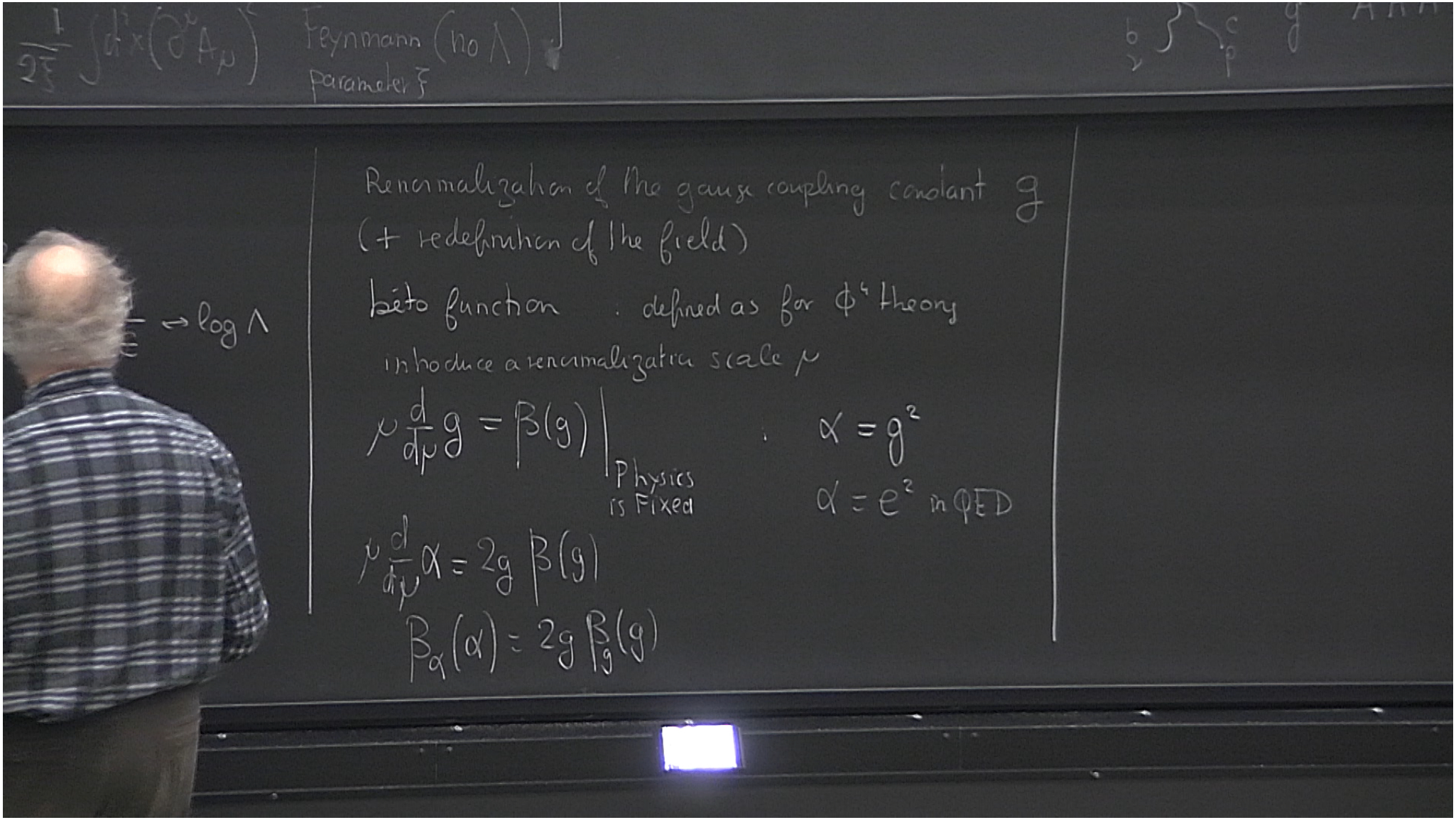
$\frac{1}{\epsilon} \to \log\Lambda$

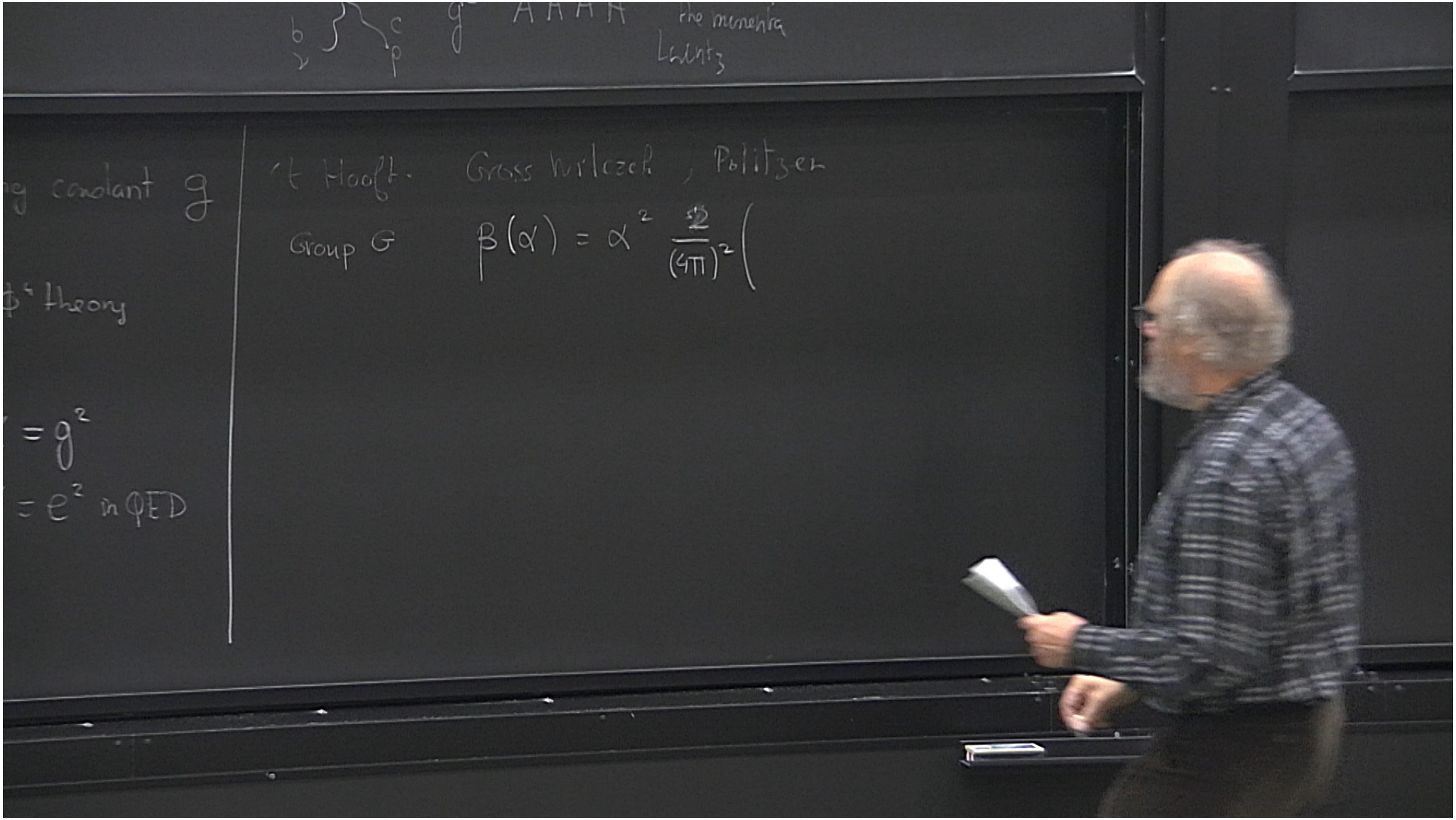
Renormalization of the gauge coupling constant g
(+ redefinition of the field)

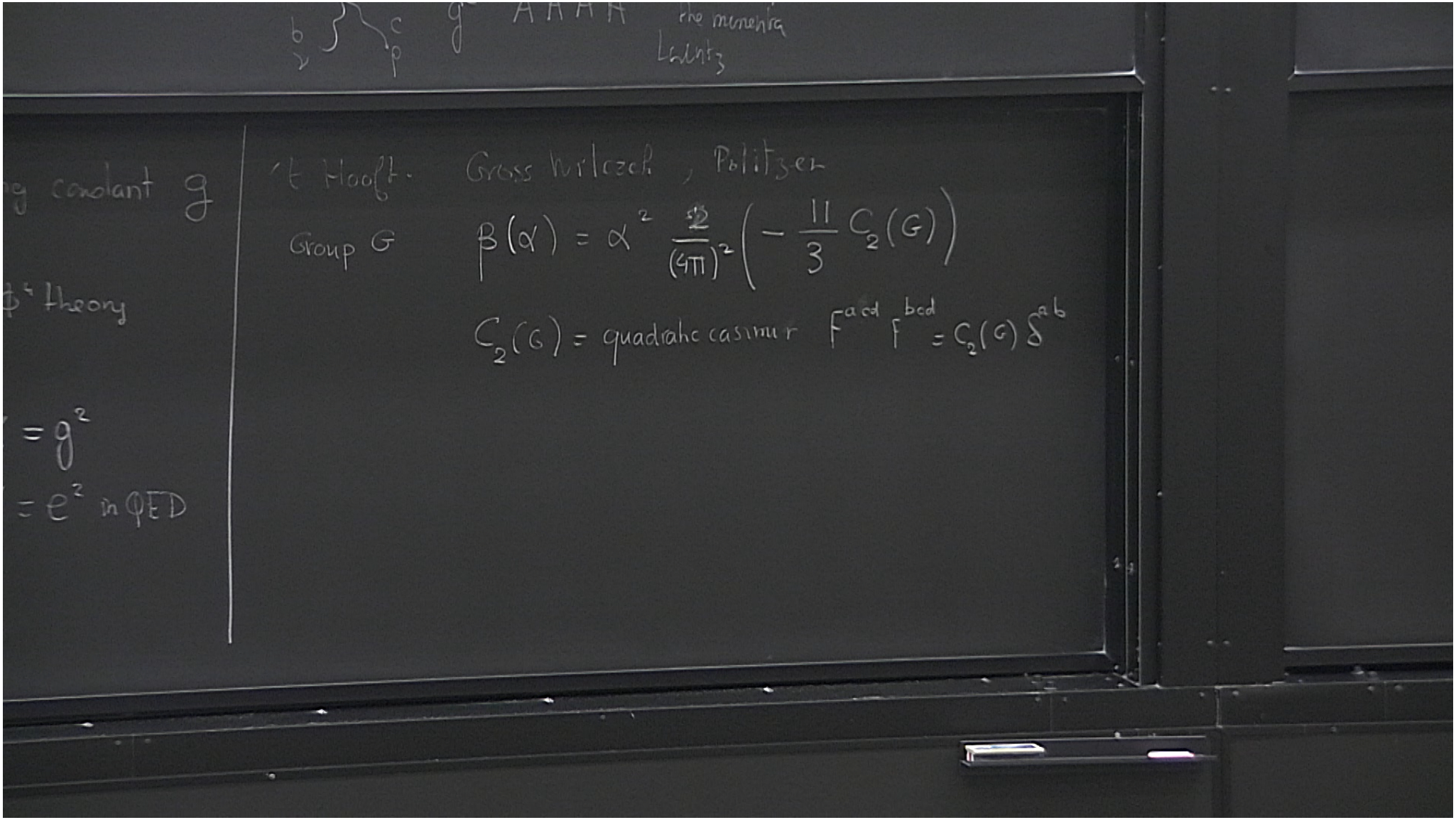
beta function : defined as for ϕ^4 theory

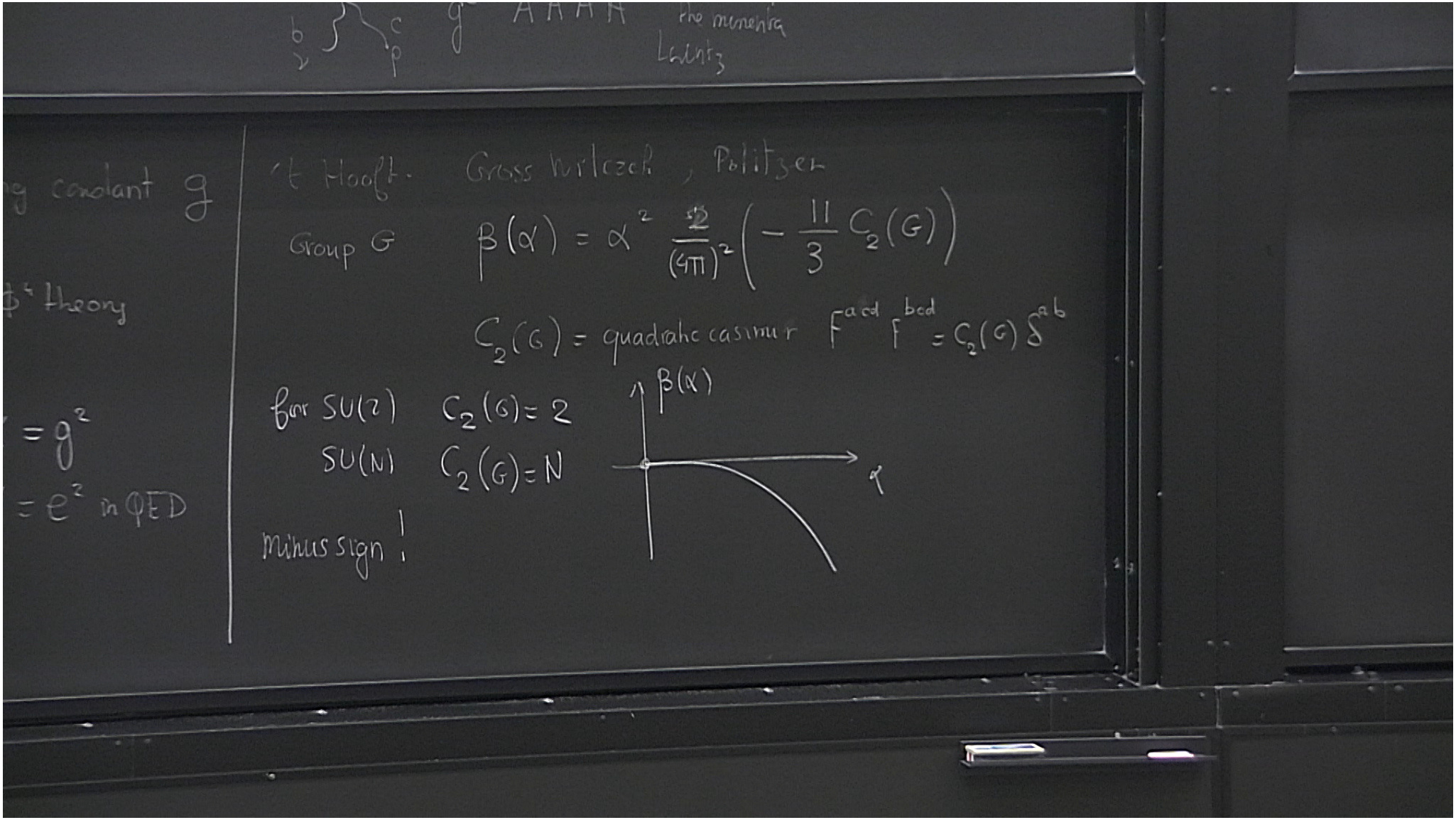
introduce a renormalization scale μ

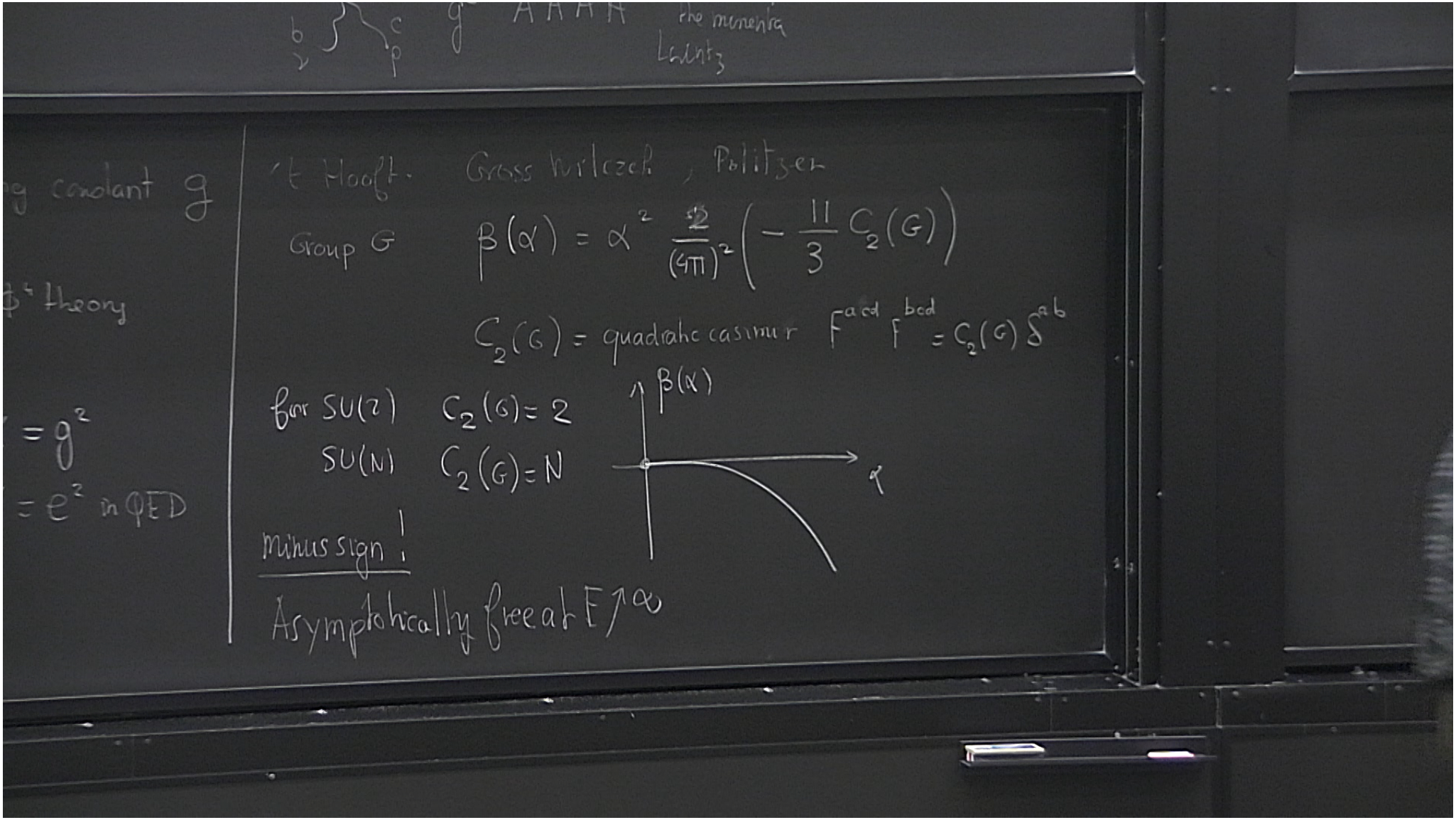
$$\mu\frac{d}{d\mu}g = \beta(g)\Big|_{\text{Physics is Fixed}}$$

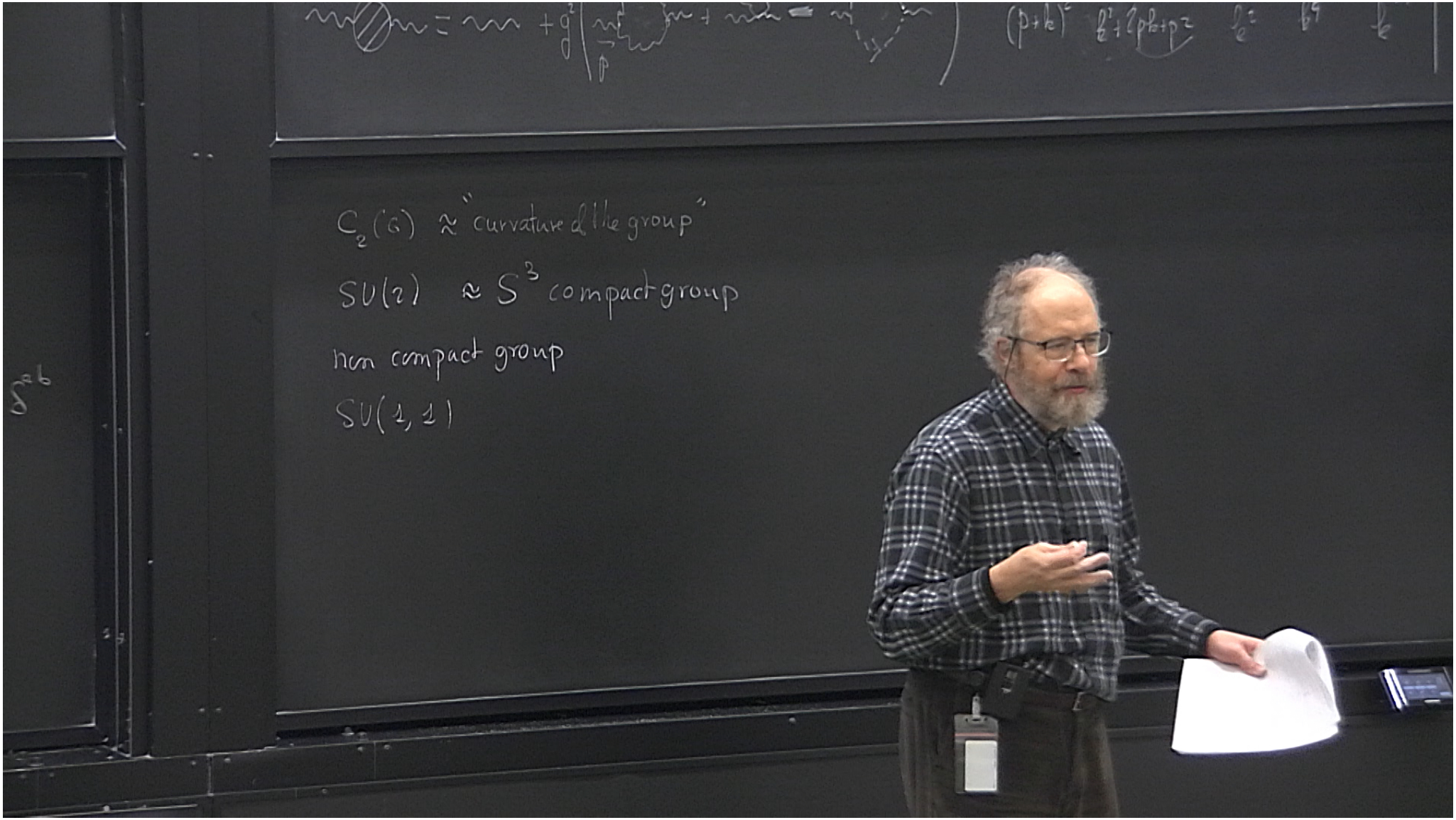

Feynmann parameter ξ
Renormalization of the gauge coupling constant g
(+ redefinition of the field)
beta function : defined as for ϕ^4 theory
introduce a renormalization scale μ
$\mu \frac{d}{d\mu} g = \beta(g)\Big|_{\text{Physics is Fixed}}$: $\alpha = g^2$
$\alpha = e^2$ in QED
$\mu \frac{d}{d\mu} \alpha = 2g\,\beta(g)$
$\beta_\alpha(\alpha) = 2g\,\beta_g(g)$

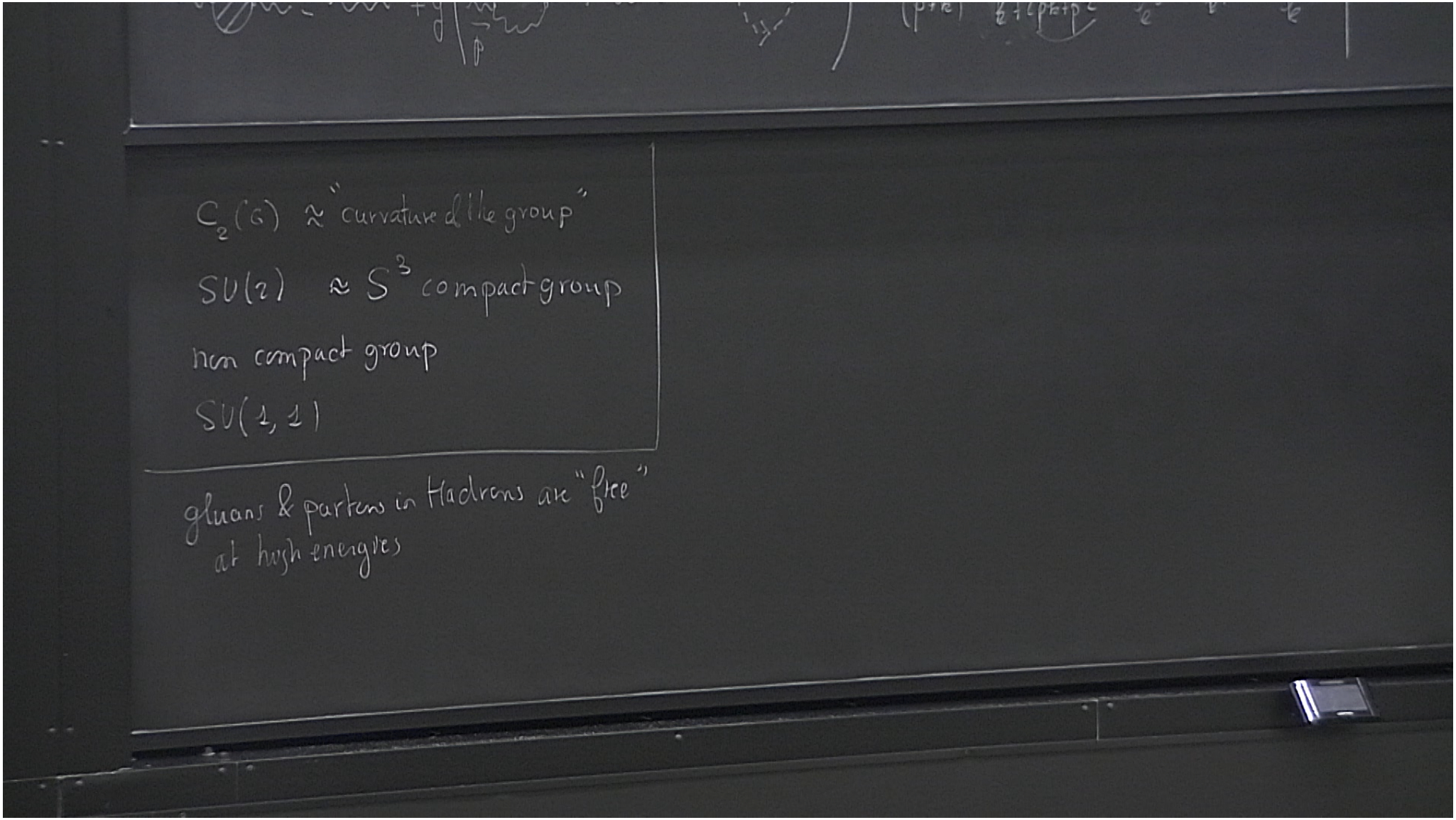

the numerator
Lecture 3
't Hooft. Gross Wilczek, Politzer
Group G
$\beta(\alpha) = \alpha^2 \frac{12}{(4\pi)^2}($
$= g^2$
$= e^2$ in QED


g coulant g
ϕ^4 theory
$= g^2$
$= e^2$ in QED
't Hooft. Gross Wilczek, Politzer
Group G
$\beta(\alpha) = \alpha^2 \frac{\Omega}{(4\pi)^2}\left(-\frac{11}{3} C_2(G)\right)$
$C_2(G)$ = quadratic casimir $F^{acd} F^{bcd} = C_2(G)\delta^{ab}$


g constant g
ϕ^4 theory
$= g^2$
$= e^2$ in QED
't Hooft. Gross Wilczek, Politzer
Group G
$\beta(\alpha) = \alpha^2 \frac{\Omega}{(4\pi)^2}\left(-\frac{11}{3}C_2(G)\right)$
$C_2(G)$ = quadratic casimir $F^{acd}F^{bcd} = C_2(G)\delta^{ab}$
for SU(2) $C_2(G) = 2$
SU(N) $C_2(G) = N$
minus sign!
$\beta(\alpha)$
α


't Hooft. Gross Wilczek, Politzer
Group G
$\beta(\alpha) = \alpha^2 \frac{12}{(4\pi)^2}\left(-\frac{11}{3}C_2(G)\right)$
$C_2(G)$ = quadratic casimir $F^{acd}F^{bcd} = C_2(G)\delta^{ab}$
for SU(2) $C_2(G) = 2$
SU(N) $C_2(G) = N$
$\beta(\alpha)$
α
minus sign!
Asymptotically free at $E \nearrow \infty$


$C_2(G) \approx$ "curvature of the group"
$SU(2) \approx S^3$ compact group
non compact group
$SU(1,1)$


$C_2(G)$ ≈ "curvature of the group"
$SU(2) \approx S^3$ compact group
non compact group
$SU(1,1)$
gluons & partons in Hadrons are "free" at high energies

$C_2(G) \approx$ "curvature of the group"

$SU(2) \approx S^3$ compact group

non compact group

$SU(1,1)$

gluons & partons in Hadrons are "free" at high energies

'70's $\Rightarrow$ H.E Physics

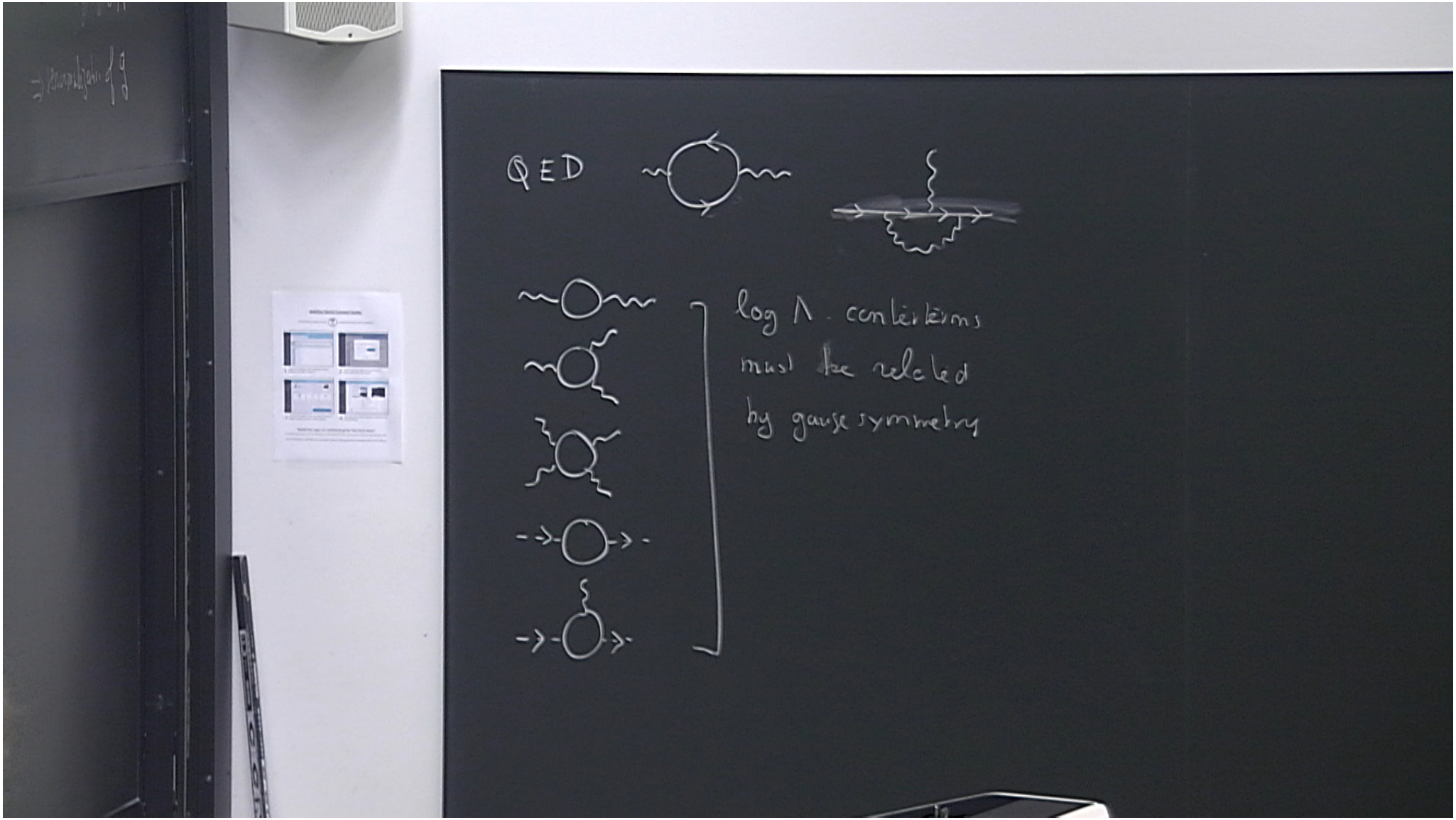
Consistency of YM theories

YM ≠ QED or ϕ^4

$\alpha(E)$ $\qquad E\frac{d\alpha}{dE} = -b_2\,\alpha^2 \qquad b_2 > 0$

$\alpha_0 = \alpha(E=1)$, $\qquad \alpha(E) = \frac{\alpha_0}{1 + b_2\,\alpha_0 \log E}$

$E \to \infty \qquad \sim \frac{1}{\log E} \searrow 0$


QED
log Λ counterterms
must be related
by gauge symmetry

Lecture 3

't Hooft - Gross Wilczek, Politzer

Matter

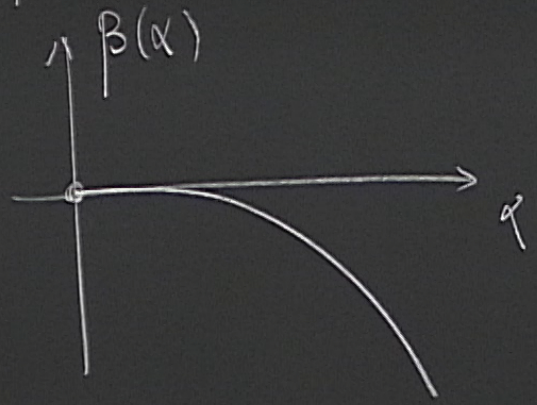
Group G

$$\beta(\alpha) = \alpha^2 \frac{+2}{(4\pi)^2}\left(-\frac{11}{3}C_2(G) + \right.$$

$C_2(G)$ = quadratic casimir $\quad f^{acd} f^{bcd} = C_2(G)\,\delta^{ab}$

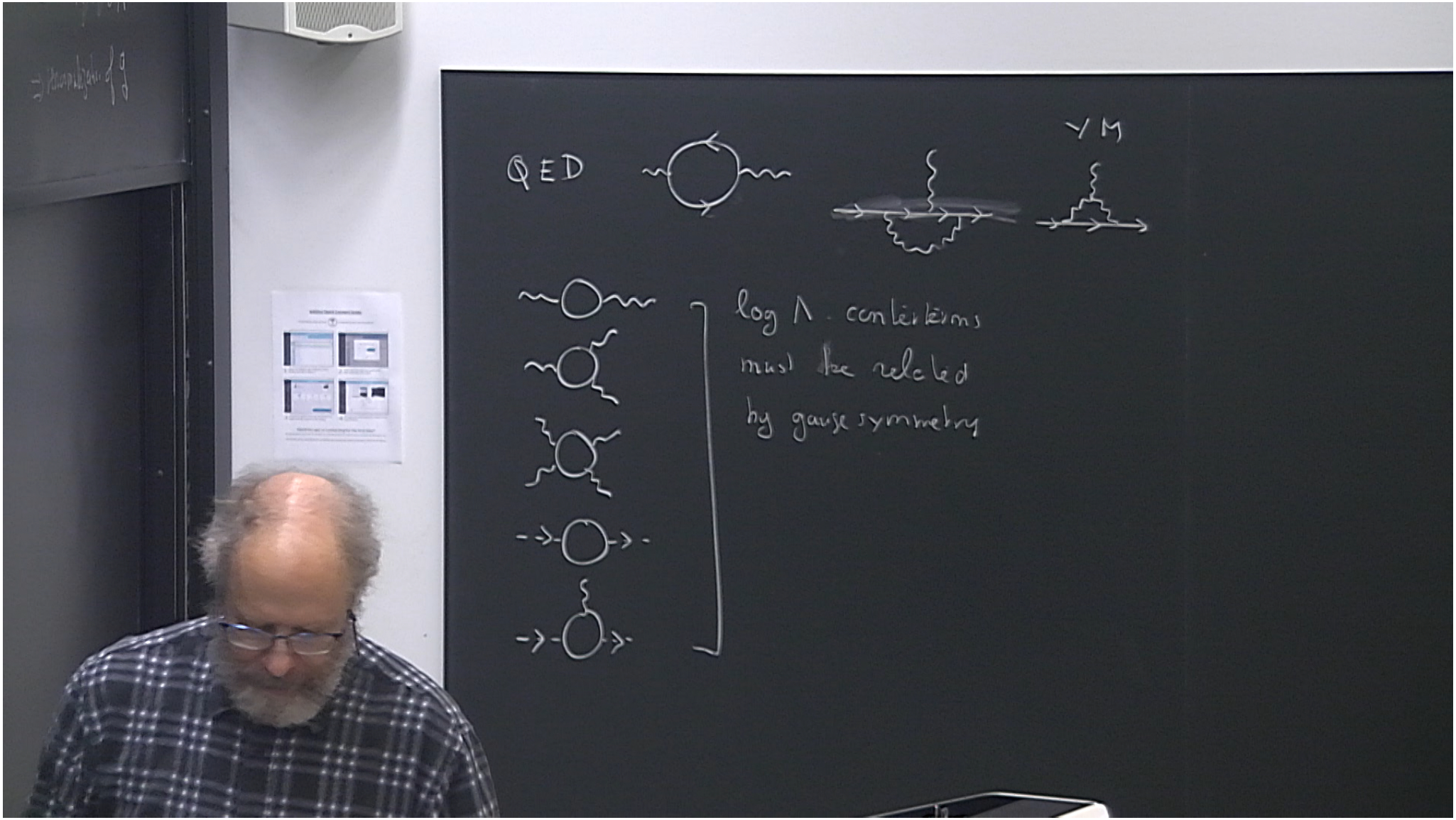
for SU(2) $C_2(G) = 2$

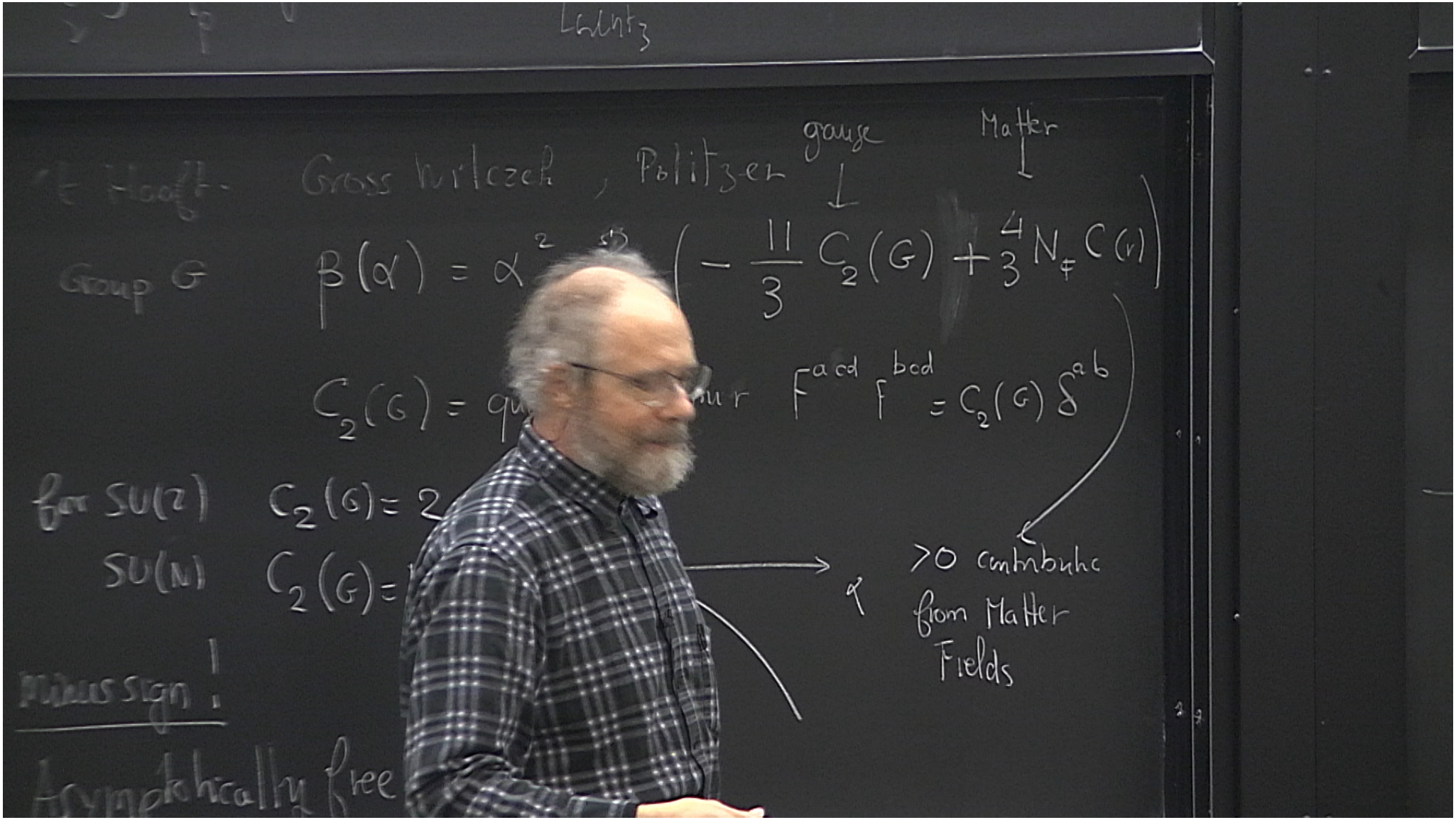
SU(N) $C_2(G) = N$



Minus sign !

Asymptotically free at $E \uparrow \infty$


QED
YM
log Λ counterterms
must be related
by gauge symmetry


Gross Wilczek, Politzer
gauge
Matter
Group G
$\beta(\alpha) = \alpha^2 \left(-\frac{11}{3} C_2(G) + \frac{4}{3} N_f C(r) \right)$
$C_2(G) = \text{qu}$ ur $F^{acd} F^{bcd} = C_2(G) \delta^{ab}$
for SU(2) $C_2(G) = 2$
SU(N) $C_2(G) =$
Minus sign !
asymptotically free
>0 contribution from Matter Fields

Consistency of YM theories

YM ≠ QED or ϕ^4

$\alpha(E)$ $\qquad E\frac{d\alpha}{dE} = -b_2\,\alpha^2 \qquad b_2 > 0$

$\alpha_0 = \alpha(E=1)$, $\qquad \alpha(E) = \frac{\alpha_0}{1 + b_2\,\alpha_0 \log E}$

$E \nearrow \infty \qquad \sim \frac{1}{\log E} \searrow 0$

Still true for

QCD $\quad SU(3)$ + Quarks

WS model $\quad SU(2) \times U(1) \times U(1)$ (up to the Higgs field)

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introduce a regulator $E < \Lambda$

at scale Λ, $\alpha_{BARE} \simeq \alpha(\Lambda) \sim$ log

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at scale Λ, $\alpha_{BARE} \simeq \alpha(\Lambda) \sim \frac{1}{\log \Lambda} \ll 1$

$\Lambda \to \infty$ continuum limit can be controlled

Mathematical Physics $\Longrightarrow$ Proof YM exists



YM theories

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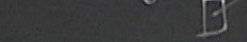
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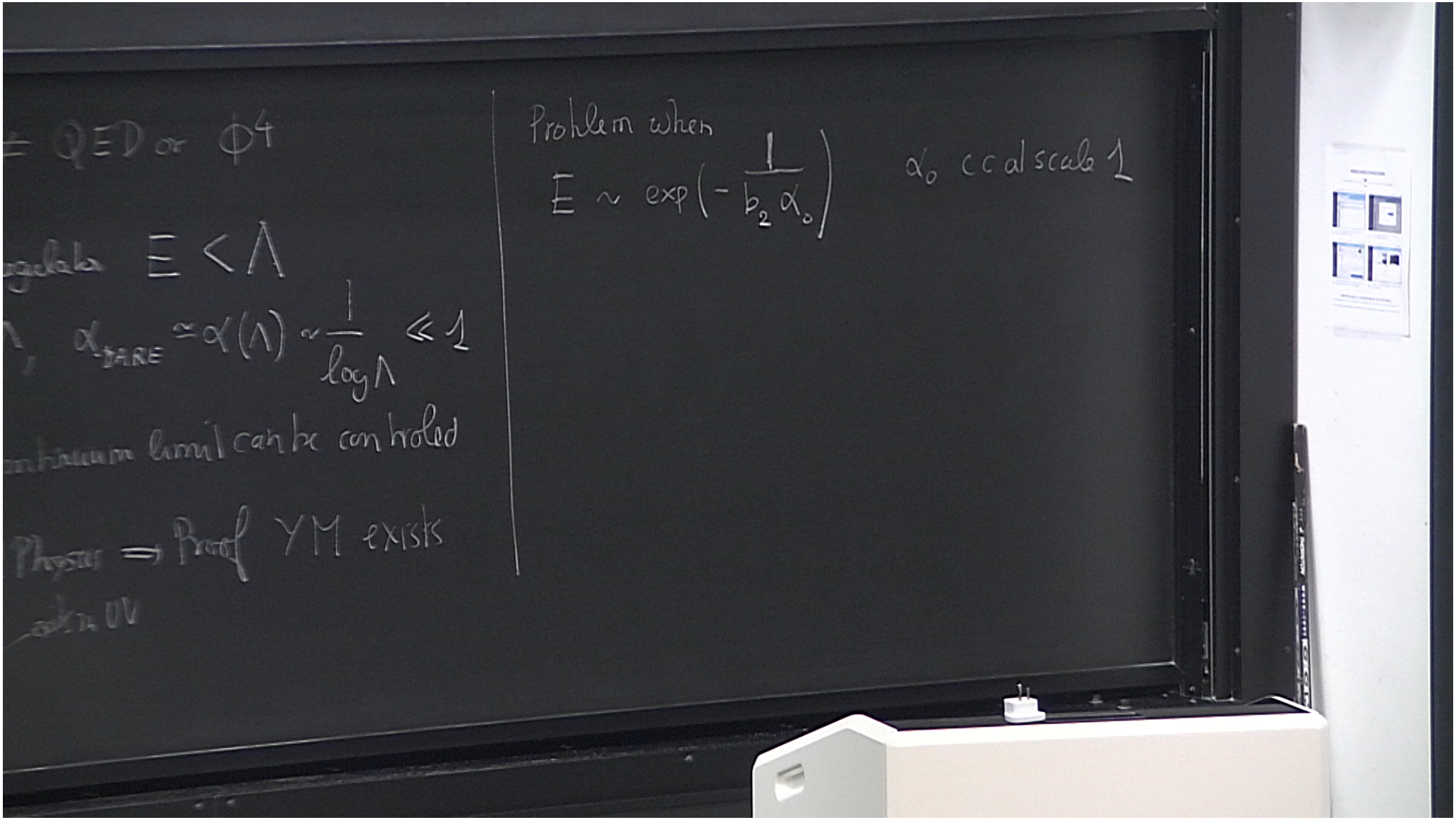
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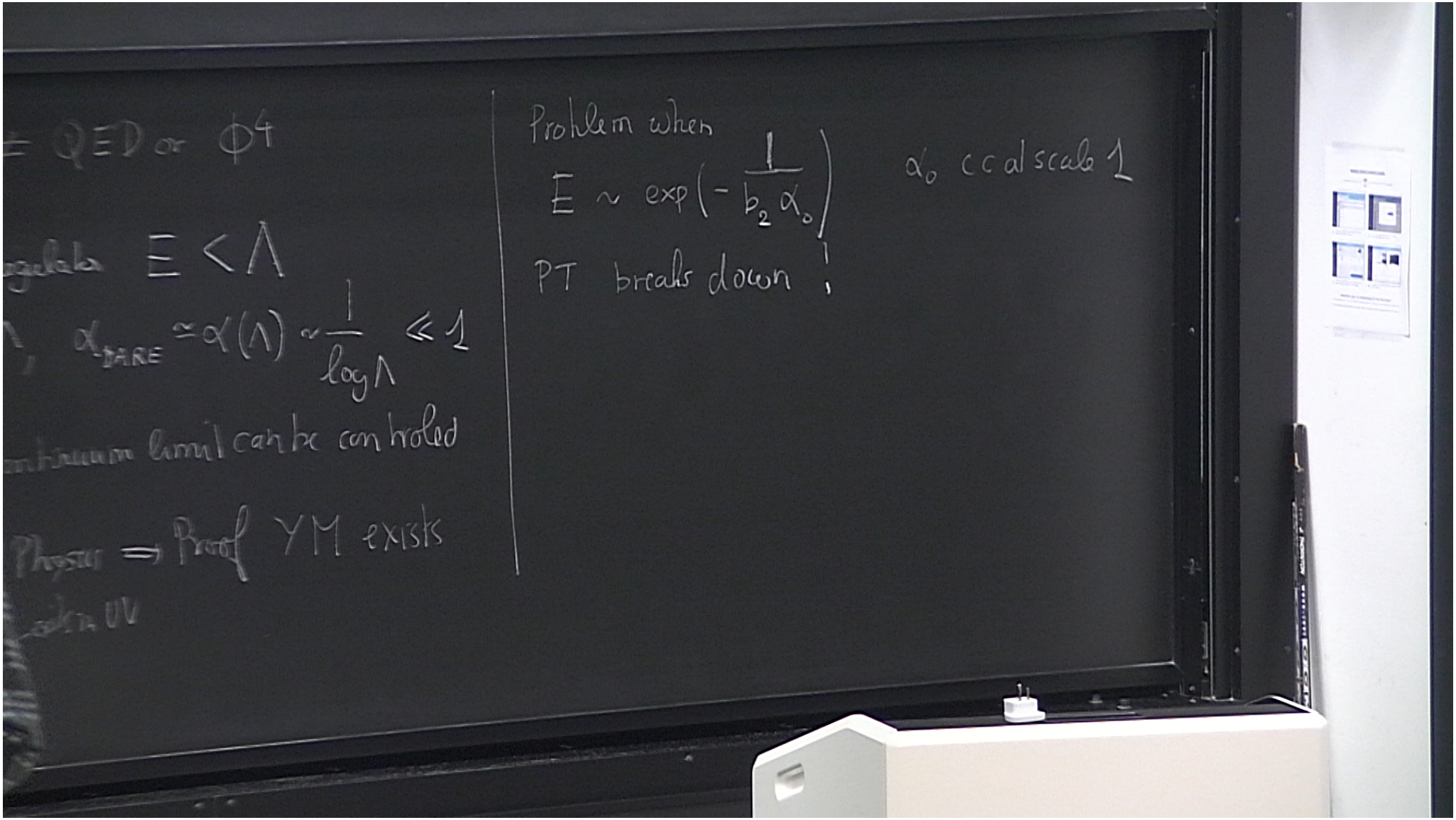
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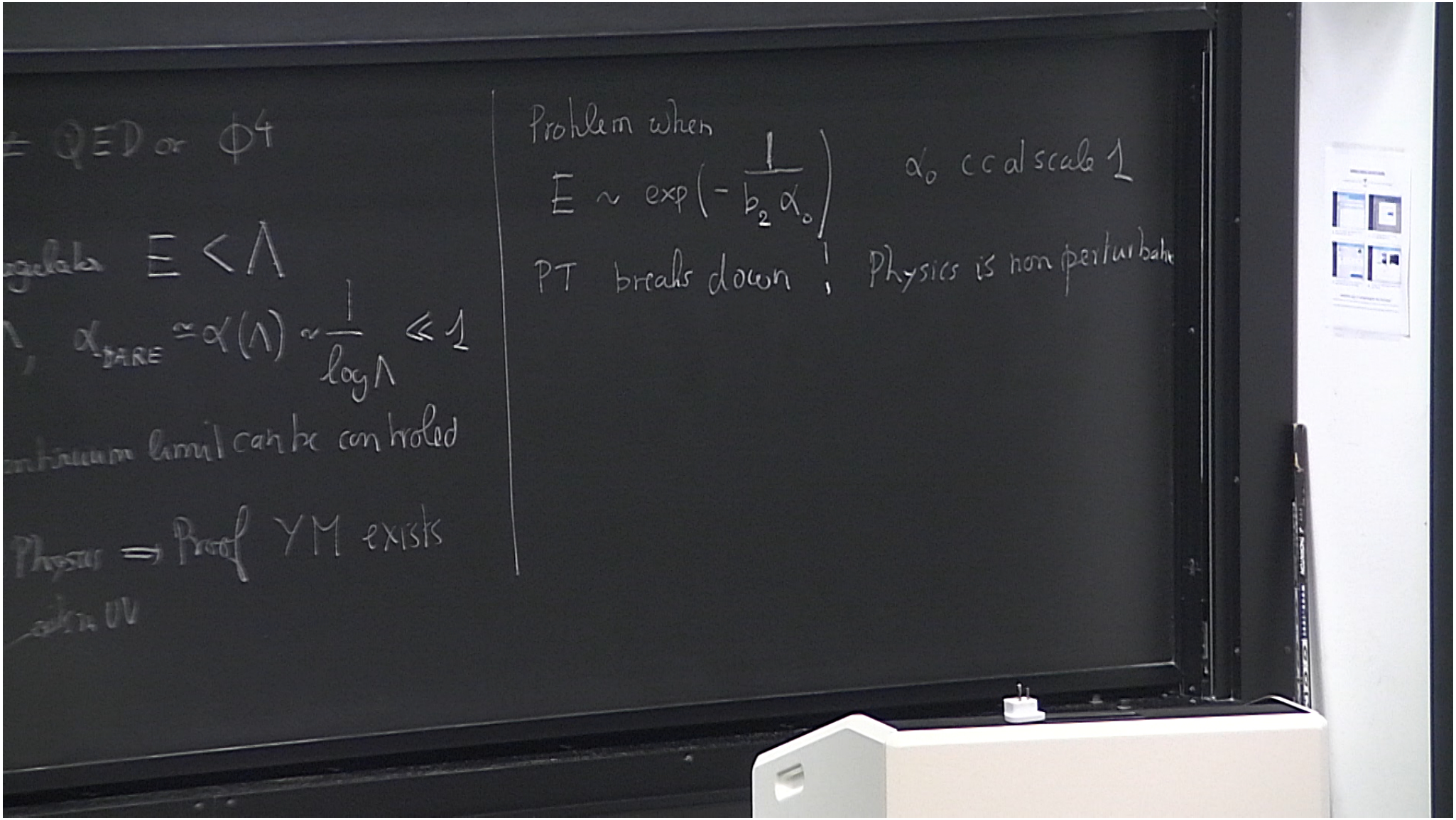


Problem when

$$E \sim \exp\left(-\frac{1}{b_2\,\alpha_0}\right.$$


= QED or ϕ^4
$E < \Lambda$
$\alpha_{BARE} \simeq \alpha(\Lambda) \sim \frac{1}{\log \Lambda} \ll 1$
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Physics ⇒ Proof YM exists
UV
Problem when
$E \sim \exp\left(-\frac{1}{b_2 \alpha_0}\right)$
α_0 ccal scale 1


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PT breaks down !


= QED or ϕ^4
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Physics ⇒ Proof YM exists
UV
Problem when
$E \sim \exp\left(-\frac{1}{b_2 \alpha_0}\right)$
α_0 ccal scale 1
PT breaks down ! Physics is non perturbative

= QED or ϕ^4

$E < \Lambda$

$\alpha_{BARE} \simeq \alpha(\Lambda) \sim \frac{1}{\log \Lambda} \ll 1$

continuum limit can be controlled

Physics ⇒ Proof YM exists

with UV

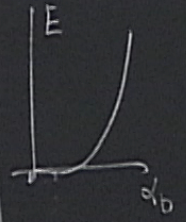
Problem when

$$E \sim \exp\left(-\frac{1}{b_2 \alpha_0}\right)$$

α_0 ccal scale 1

PT breaks down ; Physics is non perturbative

Mass scale $\sim \exp\left(-\frac{1}{b_2 \alpha_0}\right)$ dynamical generation of a mass

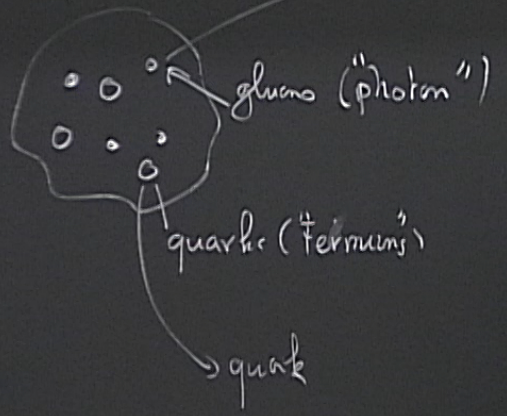


$0 + \alpha_0 \times 0 + \alpha_0^2 \cdot 0 + \cdots$

70's ⇒ H.E Physics

WS model $SU(2)\times U(1)\times U(1)$ (up to the Higgs field)

"Confinement":



only states which are color singlets are asymptotic states

Pure YM



glueballs

WS model $SU(2)\times U(1)\times U(1)$ (up to the Higgs field)

gluon

Pure YM

QCD



balls



mesons



hadrons



tetra quarks or penta quarks ?

$$\text{mass} \sim M \sim \exp\left(-\frac{1}{b_2 \alpha_0}\right) \times \mu$$

α_0 coupling at ren. scale μ

WS model $SU(2) \times U(1) \times U(1)$ (up to the Higgs field)

gluon

Pure YM

QCD



glueballs



mesons



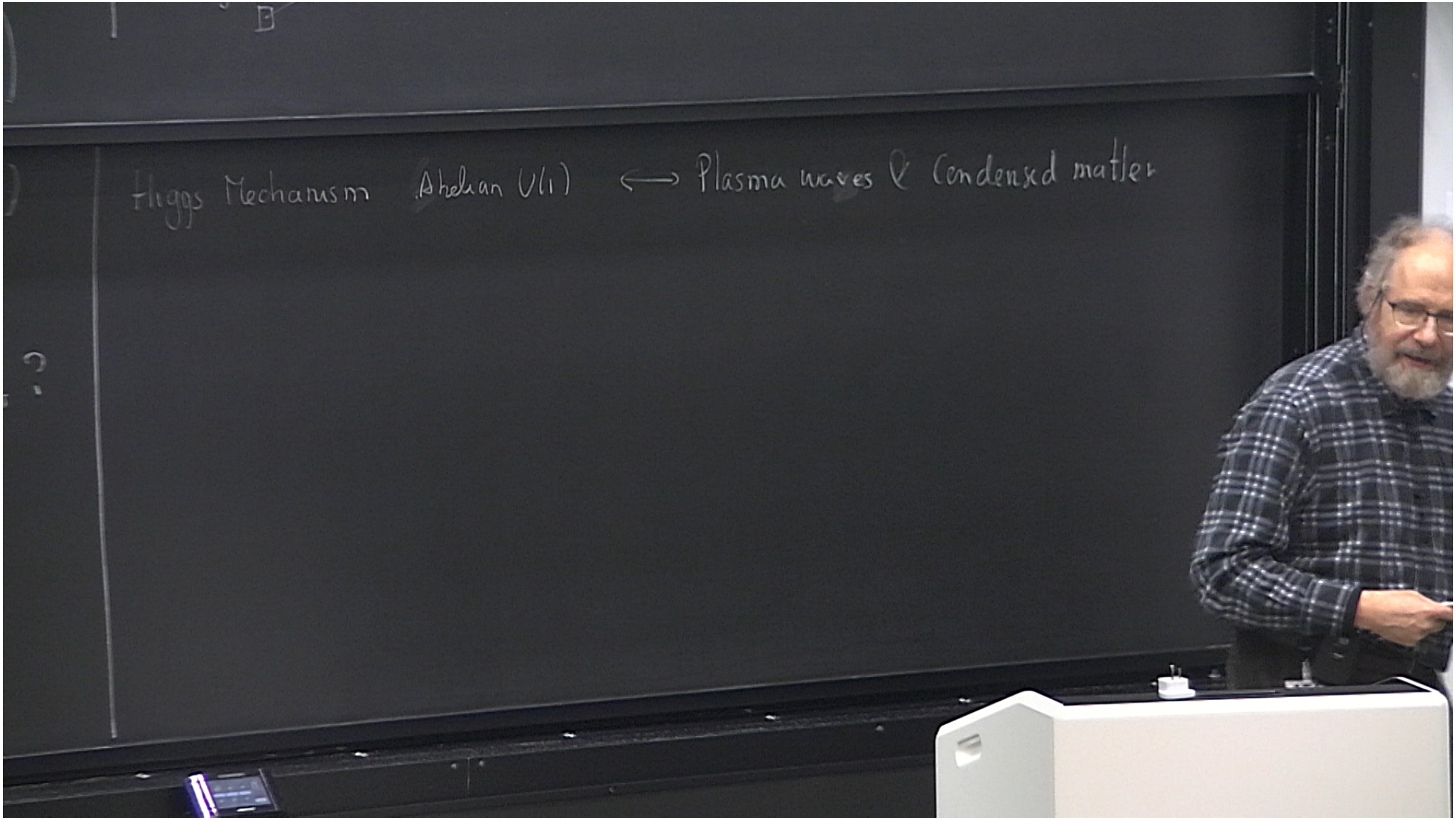
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Higgs Mechanism Abelian U(1) ⟷ Plasma waves & Condensed matter