Title: PSI 2018/2019 - Quantum Field Theory I - Lecture 14

Date: Dec 04, 2018 10:00 AM

URL: http://pirsa.org/18120001

Abstract:

Gauge Fixing, Ghosts, Gauge Fixed Action

$G = SU(2)$ Gauge Field A $\in Adj(G)$

vector

$S[A] = \int d^4x \; tr(F^2)$ $F = [D, D]$ Field Strength

$\int \mathcal{D}[A] \, e^{-S[A]}$

$\mathcal{A} = \{A\}$

Gauge Field Configuration Space

$\mathcal{G}$ Group of local Gauge Transformations

Gauge transf

$A \longrightarrow A_g =$

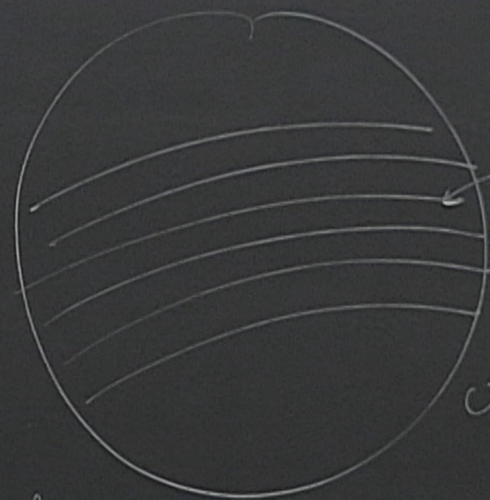
$\mathcal{A}[A] = \mathcal{A}[$

Fixed Action

$A \in \mathrm{Adj}(G)$

$= [D, D]$ Field Strength

$\{A\}$



$\mathcal{G}$ acts on $\mathcal{A}$

orbits of physically equivalent configurations

$\mathcal{A}$

Gauge transf

$$A \longrightarrow A_g = g \cdot A g^{-1} + ig(\partial g^{-1})$$

$$\mathcal{A}[A] = \mathcal{A}[A_g]$$

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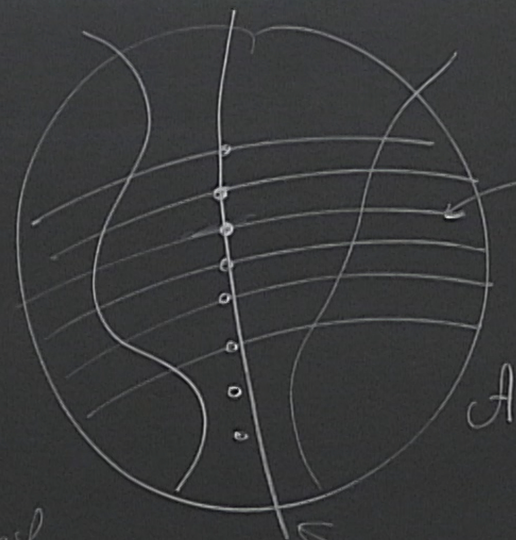
$[D, D]$ Field Strength

A]

d

Space



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$$\mathcal{A}[A] = \mathcal{A}[A_g]$$

$\mathcal{G}$ acts on $\mathcal{A}$

orbits of physically equivalent configurations

$\mathcal{C} = \mathcal{A}/\mathcal{G}$ = space of orbits

Gauge Fixing = choose a representative in each orbit

gauge slice = choose a slice

different slices $\Longrightarrow$ same result

Gau

ally

gurations

= space of orbits

oose a representative

each orbit

hoose a slice

same result

Gauge Fixing Condition. = choose a function $F[A]$

such that $\{A\,;F[A]=0\}$ = the gauge Fixing slice

F such that for any x, $a=1,2,3$

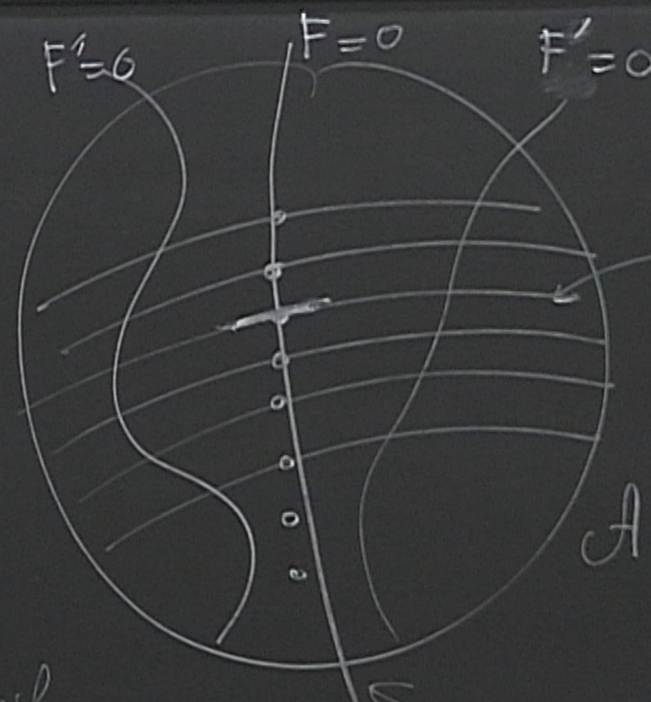
$$F^a[A(x), \partial_\mu A(x), \dots] = 0$$ gauge fixing condition

F is a function from $\mathcal{A} \longrightarrow \mathrm{Lie}(\mathcal{G})$ = tangent space to a slice at a given point A in $\mathcal{A}$

ed Action

$\in \mathrm{Adj}(G)$

$[D,D]$ Field Strength



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Example: Lorentz Landau Gauge

$$\partial^\mu A^a_\mu(x) = 0$$

$$F[A] = \partial^\mu A_\mu$$

Feynman

chose a function

$\epsilon \quad M \to \mathrm{Lie}(G)$

$$\partial^\mu A^a_\mu(x) = \epsilon^a(x)$$

, Gauge Fixed Action

Gauge Field $A \in Adj(G)$

vector

$F = [D, D]$ Field Strength

$\mathcal{A} = \{A\}$

Gauge Field

Configuration Space

Gauge Transf

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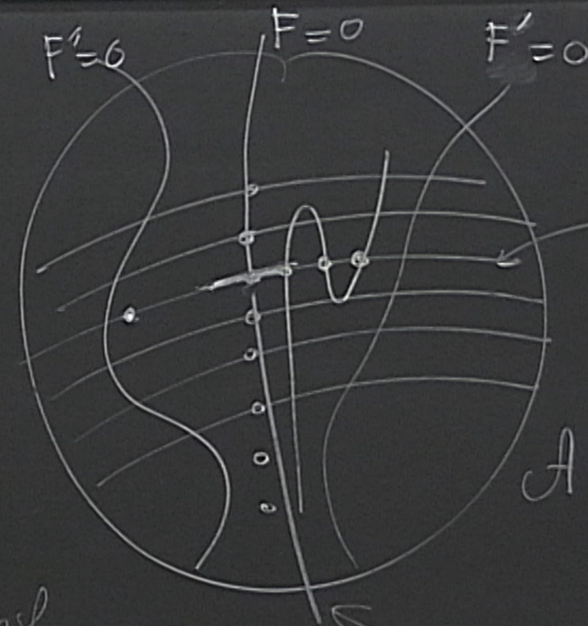

$\mathcal{G}$ acts on

orbits of phys equivalent conf

$\mathcal{E} = \mathcal{A}/\mathcal{G}$

Gauge Fixing =

gauge slice =

different slices =

$\epsilon(x) = \epsilon^a(x)\, t_a \,, \quad \epsilon(x) \in Lie(G)$

ϵ as a function $\in Lie(\mathcal{G})$

Good Gauge Fixing

- For <u>any A</u>, there si a g such thal $F[A_g] = 0$
- Better if there is only <u>one</u> such g

If not: <u>Gribov copies problem</u>

Serious problem for $SU(2)$ and Lorentz Gauge

But not in perturbation theory

Gauge Fixing. Ghosts, Gauge Fixed Action

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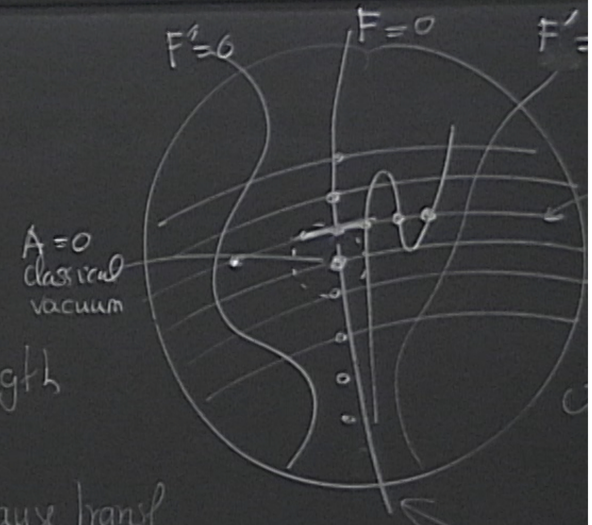


Gauge transf

$A \longrightarrow A_g = g \cdot A g^{-1} + i g(\partial g^{-1})$

$\mathcal{A}[A] = \mathcal{A}[A_g]$

Gauge Invariance $S[A_g] = S[A]$

$$D[A_g] = D[A]$$

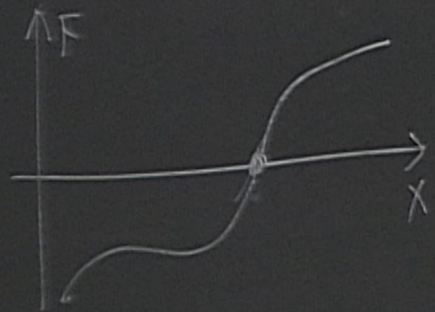
Which measure on the slice?

$F[A_g] = 0$

Faddeev, Popov, DeWitt, Feynman

How to write 1 as an integral?

Integral over $\mathbb{R}$: $F(x) = 0 \Rightarrow x = x_0$



$$\int_{\mathbb{R}} dx\, \delta(x - x_0) = 1 = \int dx\, \delta(F(x)) \cdot |F'(x)|$$

Dirac δ-function

Gauge

$= S[A]$

$= D[A]$

Integral over $\mathbb{R}^n$

$\vec{x} = (x_1 \dots x_n)$

$\vec{F}(x) = (F_1(\vec{x}), \dots F_n(\vec{x})) \quad \mathbb{R}^n \to \mathbb{R}^n$

$\cdot$ x_0

a single $\vec{x}_0$ such that $\vec{F}(x_0) = \vec{0}$

Jacobian

$$\boxed{1 = \int d^n\vec{x}\, \delta^{(n)}(\vec{x} - \vec{x}_0) = \int d^n x\, \delta^{(n)}(\vec{F}(\vec{x})) \left| \det(\vec{F}'(\vec{x})) \right|}$$

$\delta(x_1 - (x_0)_1) \dots \delta(x_n - x_{0,n})$

$F'(x) = \left(\frac{\partial F_a}{\partial x_b} \right)_{a,b=1,n}$

$N \times N$ matrix

x_0

$(\dots - x_0) = 1 = \int dx\, \delta(F(x)) \cdot |F'(x)|$

x_a

$\mathbb{R}^n \to \mathbb{R}^n$

$\vec{F}(x_a) = \vec{0}$

Jacobian

$$) = \int d^n x \, \delta^{(n)}(\vec{F}(\vec{x})) \, \boxed{\left| \det(\vec{F}'(\vec{x})) \right|}$$

$$F'(x) = \left(\frac{\partial F_a}{\partial x_b} \right)_{a,b=1,n}$$

N×N matrix

$$Z = \int D[A] \exp(-S[A])$$

For a given A, there is a unique g_0 depends on A and F

such that $F[A_{g_0}] = 0$ denote $g_0 = g_F[A]$

$$\mathbb{R}^n \to \mathbb{R}^n$$

$$\vec{F}(x_a) = \vec{0}$$

Jacobian

$$\ldots) = \int d^n x\, \delta^{(n)}\big(\vec{F}(\vec{x})\big) \boxed{\left|\det\big(\vec{F}'(\vec{x})\big)\right|}$$

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$N \times N$ matrix

$$Z = \int D[A] \underbrace{\;}_{1} \exp(-S[A])$$

For a given A, there is a unique g_0 depends on A and F such that $F[A_{g_0}] = 0$ denote $g_0 = g_F[A]$

$$1 = \int_{\mathcal{G}} D[g]\, \delta(g, g_F(A))$$

measure on the group $\mathcal{G}$

Dirac δ-function

Gauge Transformations

Configuration Space

$\mathcal{A}[A] = \mathcal{A}[A_g]$

$\mathcal{G}$ is a Lie Group, it has a unique Haar Measure

it is invariant under Left or right action

$D_{Haar}[g]$

A is fixed

Matrix of derivatives of F with respect to variations of g

$g_F[A]$ is given by $F\left[A_{g_F[A]}\right] = 0$

$g \to hg$ or gh

$$1 = \int_{\mathcal{G}} D[g]\, \delta\left[g - g_F[A]\right] = \int_{\mathcal{G}} D[g]\, \delta\left[F[A_g]\right] \times \left|\det\left[F'[A_g]\right]\right|$$

$F \in Lie[\mathcal{G}]$ = tangent space to $\mathcal{G}$, what is F'? take a $g \to g(1 + i\alpha)$

... ⟹ same result | $F[A] = \partial^\mu A_\mu$ | $\in$ Mink → Lie(G) $\partial A_\mu(x) = c$

$F[A_g] \longrightarrow F = \{ F^a[A_g](x) \}$ for all $x \in$ Space, $a = 1, 2, 3$

generator $t_a = \frac{1}{2}\sigma_a$

$g = \{ g(x) \in G \}$ $g \rightarrow g(1 + i\,\alpha(x)) = g(Id + i\,\alpha^a(x)\, t_a)$

$\alpha(x) = \alpha^a(x)\, t_a$

Functional derivative of $F^a[A_g(x)]$ w.r.t $\alpha^b(y)$ for all a, b, x, y

F is non local
involves ∂A

infinitesimal deformation
so $\alpha \in Lie(G)$

$$\frac{\delta}{\delta\alpha(x)} \rightarrow g(x)\frac{\delta}{\delta g(x)}$$

$F[A] = \partial^\nu A_\nu$ | $\in$ Mink $\to$ Lie(G)

$F[A_g] \longrightarrow F = \{ F^a[A_g](x) \}$ for all $x \in$ Space, $a = 1,2,3$

generator $t_a = \frac{1}{2}\sigma_a$

$g = \{ g(x) \in G \}$ $\quad g(x) \to g(x)(1 + i\,\alpha(x)) = g\,(\mathrm{Id} + i\,\alpha^a(x)\, t_a)$

$\alpha(x) = \alpha^a(x)\, t_a$

Functional derivative of $F^a[A_g(x)]$ w.r.t. $\alpha^b(y)$ for all a, b, x, y

F is non local involves ∂A

$$\boxed{F[A_g] \longrightarrow J(x,y)^a{}_b = \frac{\delta F^a[A_{g(1+i\alpha)}](x)}{\delta \alpha^b(y)}}$$

$\uparrow$ depends on A and g

Kernel of an operator

Kernel of an operator acting on functions

Space $\longrightarrow$ Lie(G)

infinitesimal deformation $\alpha \in$ Lie($\mathfrak{g}$)

...ie Group, it has a unique Haar Measure

it is invariant under Left or right action

$g \to hg$ or gh

Functional determinant

A is fixed

Matrix of derivatives of F with respect to variations of g

$g_F[A]$ is given by $F[A_{g_F[A]}]=0$

$$1 = \int_g D[g]\,\delta[g-g_F[A]] = \int_g D[g]\,\delta[F[A_g]] \times \left|\det[F'[A_g]]\right|$$

$F \in \mathrm{Lie}[g]$ = tangent space to g, what is F'? take a $g \to g(1+i\alpha)$ infinitesimal

So $\alpha \in$ Lie...

careful...

$F'[A$...

Function...

$$Z = \int_{\mathcal{A}} D[A] \cdot \int_{\mathcal{G}} D[g]\, \delta\left[F[A_g]\right] \left| \det\left[F'[A_g]\right] \right| \exp(-S[A])$$

$$= \int_{\mathcal{G}} D[g] \int_{\mathcal{A}} \underline{D[A]}\, \delta\left[F[A_g]\right] \left| \det\left[F'[A_g]\right] \right| \exp(-S[\underline{A}]) = \int_{\mathcal{G}} D$$

? Yes because you can discretize

$$D[A] \exp(-S[A]) = D[A_g] \exp[-S[A_g]]$$

Now use gauge invariance then A_g is a dummy variable

Minkowski or
Euclidean

Faddeev Popov
Determinant

Gauge Fixed Functional Integral

$$\ldots [A_g]] \Big| \exp(-S[A])$$

$$\ldots \Big| \exp(-S[A])\Big] = \int_{g} D[g] \times \boxed{\int_{A} D[A]\, \delta[F[A]] \cdot \Big|\det[F'[A]]\Big| \exp(-S[A])}$$

Volume(g)

$$\ldots p[-S[A_g]]$$

Faddeef - Popov Ghosts Trick

$$\left|\det(F')\right|^{-1} = \det(F')$$

if no copies problem

$$\det[\text{Operator}] = \int d\bar{c}\, dc\, e^{-\bar{c}\cdot \text{Operator}\, c}$$

c and $\bar{c}$ are Grassman variables

c and $\bar{c}$ are going to be functions from $M \to \text{Lie}(G)$

$$c = \{ c^a(x) \}_{a=1,3}, \quad \bar{c} = \{ \bar{c}_a(x) \}_{1,3}$$

no Lorentz/Dirac indices

spin 0 fields "Ghost"

Explicit calculation : Lorentz-Landau, Feynman

$$F^a[A](x) = \partial^\mu A^a_\mu(x) - \epsilon^a(x)$$

↑ through derivative of A at point x

gauge transformation

$$A \longrightarrow A_g \quad \text{with } g = 1 + i\alpha(x)$$

$$A^a_\mu(x) \longrightarrow A^a_\mu(x) + D_\mu \alpha_a(x)$$

$$A^a_\mu(x) + \partial_\mu \alpha^a(x) + \epsilon_{abc} A^b_\mu(x) \alpha^c(x)$$

SU(2) J

Operator

$M \to Lie(G)$

Lorentz/Dirac indices

variation

δF

Explicit calculation: Lorentz-Landau, Feynman

$$F^a[A](x) = \partial^\mu A^a_\mu(x) - \cancel{\epsilon\, \alpha^a(x)}$$

additional parameter

not important

↑ through derivative of A at point x

gauge transformation

$$A \longrightarrow A_g \quad \text{with } g = 1 + i\alpha(x)$$

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SU(2)

variation of F[A]

$$\delta F[A]^a(x) = (\partial_\mu D^\mu \alpha)^a(x)$$

Kernel operator of $F'[A]$

$$\boxed{\hat{F}[A]^a_c = \left[\Delta_x \delta^{ac} + \epsilon_{abc} A^b_\mu(x) \ldots\right.}$$

ac reduces

"Ghost"

SU(2)

$$\delta(F[A]) = \prod_x \prod_a \delta\left[F^a[A](x)\right] = \prod_x \prod_a \int_{-\infty}^{+\infty} d\Lambda_a(x)\ \exp\left(i \int \Lambda_a(x) F^a[A](x)\right) = \int D[A]\ \exp\left(i \int dx\ \Lambda_a(x) F^a[A](x)\right)$$

Auxiliary Field $\in \mathrm{Lie}(g)$

$\bar{c} = \{\bar{c}^a(x)\}_{a=1,3}$, $c = \{c_a(x)\}_{1,3}$ spin 0 fields "Ghost"

$$\delta(F[A]) = \prod_x \prod_a \delta\left[F^a[A](x)\right] = \prod_x \prod_a \int_{-\infty}^{+\infty} d\Lambda_a$$

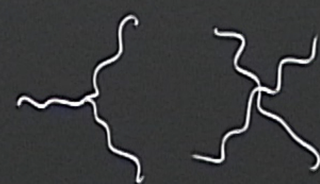
$$\int D[A] \int D[\bar{c}, c] \int d[\Lambda]\ \exp\Big(-\frac{1}{2g^2}\underbrace{\int d^4x\, \mathrm{tr}[FF]}_{\text{YM action}}\Big) \times \exp\Big(-\int d^4x \big[\bar{c}_a(x)\, \Delta_x c_a(x) + \bar{c}_a(x)\epsilon_a$$

ghost + gauge action

gauge field

Maxwell theory

with long modes projected out



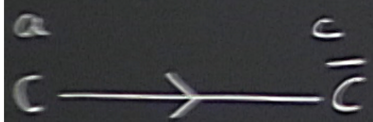
"Ghost"

SU(2)

$$\delta(F[A]) = \prod_x \prod_a \delta\left[F^a[A](x)\right] = \prod_x \prod_a \int_{-\infty}^{+\infty} d\Lambda_a(x)\ \exp\left(i\,\Lambda_a(x)\, F^a[A](x)\right) = \int D[A]\ ex$$

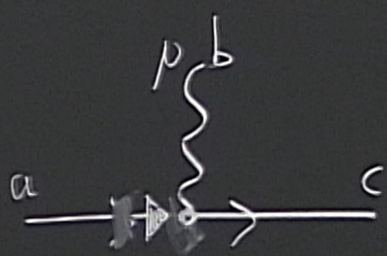
$$\mathrm{tr}[FF]\Big) \times \exp\left(-\int d^4x \left[\bar{C}_a(x)\, \Delta_x C_a(x) + \bar{C}_a(x)\,\epsilon_{abc}\, \partial_\mu\left(A^b_\mu(x)\cdot C_c(x)\right)\right]\right) \times \exp \int i \int dx\ \Lambda\ F^a[A](x)$$

ghost + gauge action



ghost propagator

$$\frac{\delta^{ac}}{p^2}$$



$p^\mu\, \epsilon_{abc}$

ghost carry charges

interact with gauge bosons

$$A^a_\mu(x) + \partial_\mu \alpha^a(x) + \epsilon_{abc} A^b_\mu(x)\alpha^c(x)$$

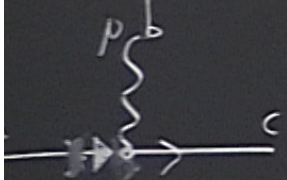
SU(2)

$$\delta\left[F^a[A](x)\right] = \prod_x \prod_a \int_{-\infty}^{+\infty} d\Lambda_a(x)\ \exp\left(i \Lambda_a(x) F^a[A](x)\right) = \int D[A]\ \exp\left(i \int dx\ \Lambda_a(x) F^a[A](x)\right)$$

Auxiliary Field $\in$ Lie(g)

$$\times\left[\bar{C}_a(x)\left(\Delta_x C_a(x) + \bar{C}_a(x)\epsilon_{abc}\,\partial_\mu\left(A^b_\mu(x)\, C_c(x)\right)\right)\right] \times \exp\left[i\int dx\ \Lambda\, F^a[A](x)\right]$$

ghost + gauge action



ghost carry charges

interact with gauge bosons

$$p^\mu \epsilon_{abc}$$

$c = \{ c^a(x) \}_{a=1,3}$, $\bar{c} = \{ \bar{c}_a(x) \}_{1,3}$ no Lorentz/Dirac indices

spin 0 fields "Ghost"

SU(2)

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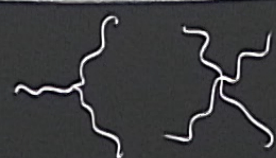
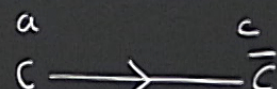
ghost + gauge action

gauge field

Maxwell theory with long modes projected out





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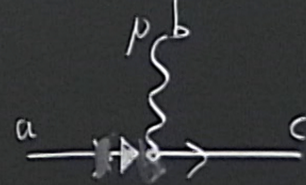

$p^\mu \epsilon_{abc}$

ghost carry charges

interact with gauge bosons