Title: The Chiral anomaly without Landau levels: from the quantum to the classical regime

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Abstract: <p>We study the chiral anomaly in disordered Weyl semimetals, where the broken translational symmetry prevents the direct application of Nielsen and Ninomiyaâ€™s mechanism and disorder is strong enough that quantum effects are important. In the weak disorder regime, there exists rare regions of the random potential where the disorder strength is locally strong, which gives rise to quasi-localized resonances and their effect on the chiral anomaly is unknown. We numerically show that these resonant states do not affect the chiral anomaly only in the case of a single Weyl node. At energies away from the Weyl point, or with strong disorder where one is deep in the diffusive regime, the chiral Landau level itself is not well defined and the semiclassical treatment is not justified. In this limit, we analytically use the supersymmetry method and find that the Chern-Simons (CS) term in the effective action which is not present in non-topological systems gives rise to a non- zero average level velocity which implies chiral charge pumping. We numerically establish that the non-zero average level velocity serves as an indicator of the chiral anomaly in the diffusive limit. </p>

Chiral anomaly without Landau levels

Junhyun Lee

2018. 11. 26.
Perimeter Institute

Junhyun Lee, J. H. Pixley, and Jay D. Sau, *arXiv:1805.00485*

Collaborators

J. H. Pixley
(UMD → Rutgers)

Jay D. Sau
(UMD)

Junhyun Lee, J. H. Pixley, and Jay D. Sau, *arXiv:1805.00485*

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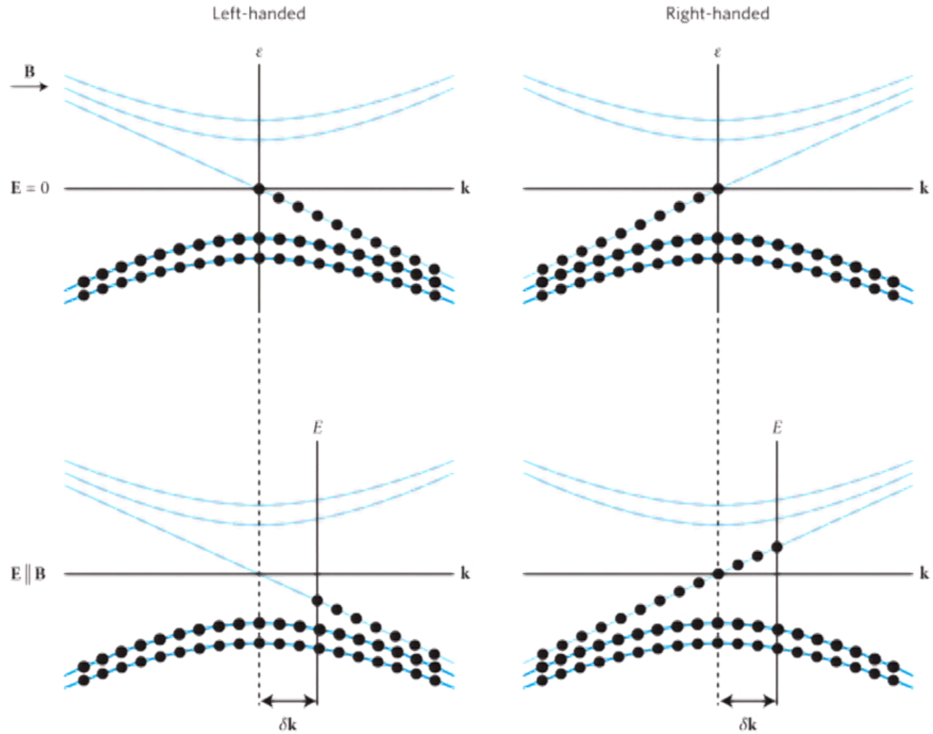
Chiral anomaly in lattice model



Burkov, *Nat. Mater.* (2016)

Adler, *Phys. Rev.* (1969); Bell and Jackiw *Nuovo Cimento A (1969)*
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1

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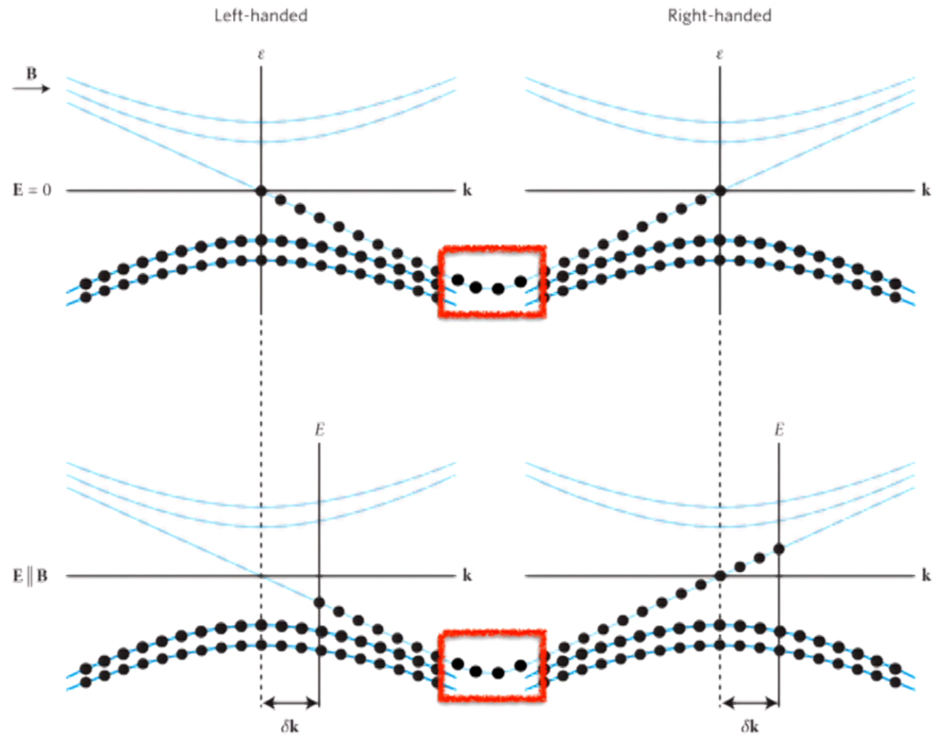

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Negative magnetoresistance

$$\sigma_{zz} = \frac{e^2}{4\pi^2 \hbar c} \frac{v}{c} \frac{(eB)^2 v^2}{\mu^2} \tau$$

Nielson and Ninomiya, *Phys. Lett. B* (1983)
Son and Spivak, *Phys. Rev. B* (2013)
Goswami *et al*, *Phys. Rev. B* (2015)

2

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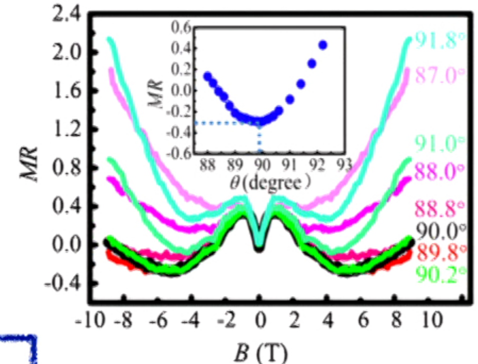
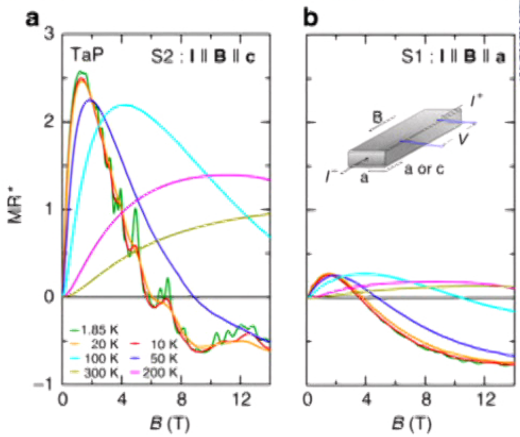
TaAs



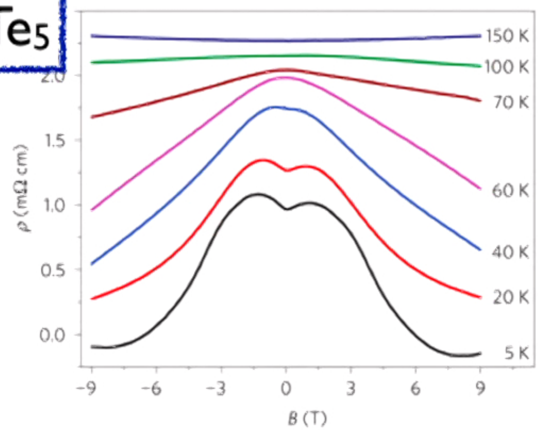
Huang *et al*, *PRX* (2015)

TaP



Arnold *et al*, *Nat. Commun.* (2016)

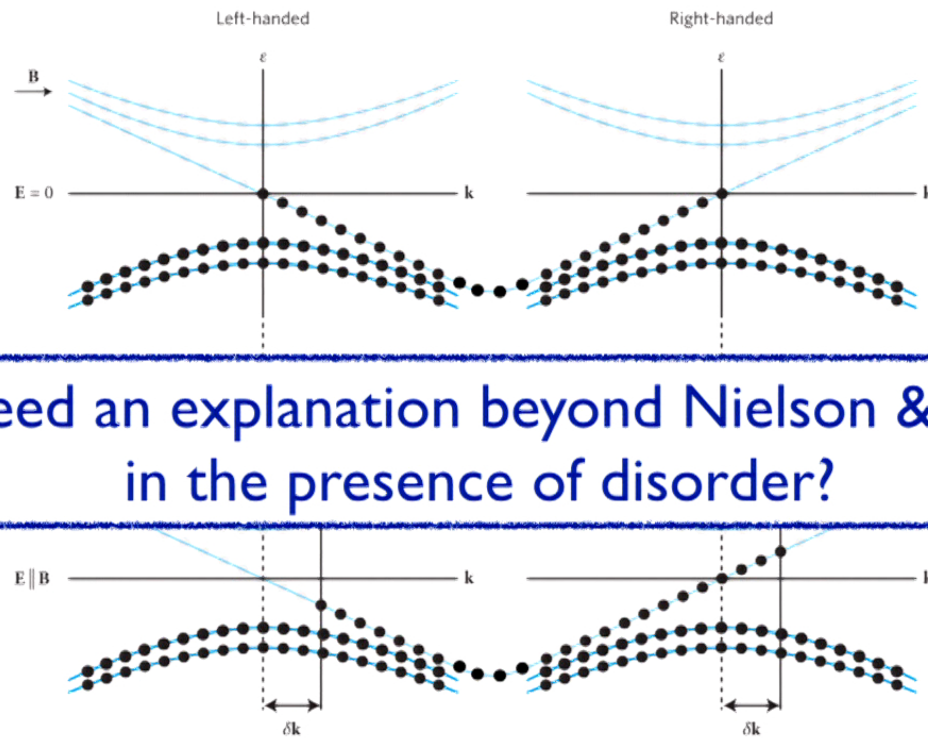
$ZrTe_5$



Li *et al*, *Nat. Phys.* (2016)

2

Chiral anomaly in lattice model



Do we need an explanation beyond Nielson & Ninomiya in the presence of disorder?

Burkov, *Nat. Mater.* (2016)

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I

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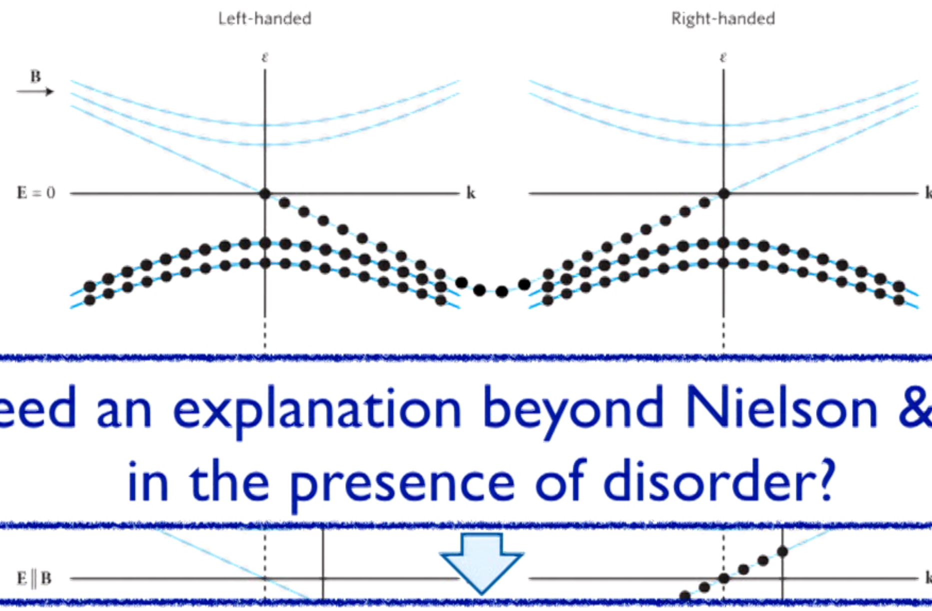

Do we need an explanation beyond Nielson & Ninomiya in the presence of disorder?

Yes!

Case I: Non-perturbative effect of disorder (a.k.a. rare-states)

Case II: Diffusive limit due to strong disorder

Adler, *Phys. Rev.* (1969); Bell and Jackiw *Nuovo Cimento A (1969)*
Nielson and Ninomiya, *Phys. Lett. B* (1983)

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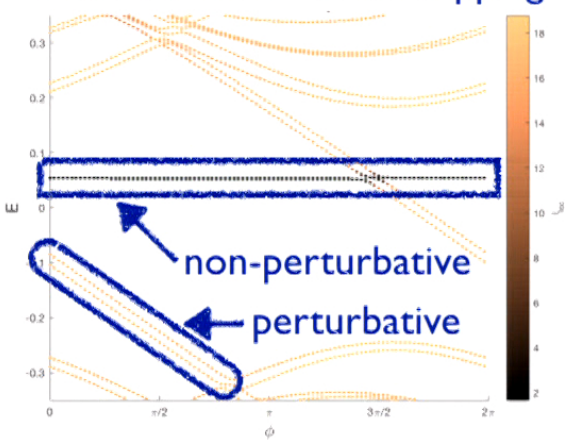
Main Q: Explain chiral anomaly beyond the Nielson & Ninomiya picture

Case I: Non-perturbative effect of disorder (a.k.a. rare-states)

Disorder of 0.7 times hopping



Fate of chiral anomaly in the presence of RS

- With one Weyl node: always survives
- With two Weyl nodes (realistic case): modifies the $E \cdot B$ coefficient

3

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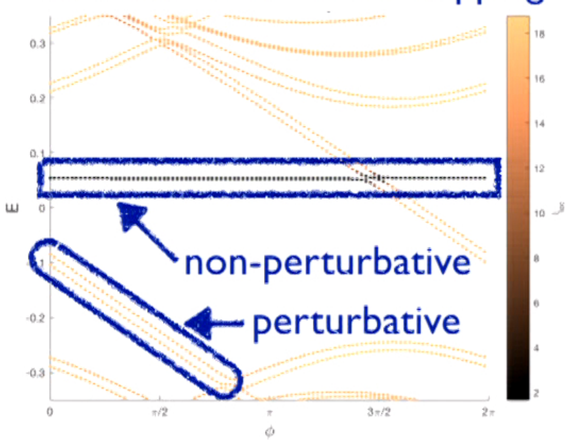

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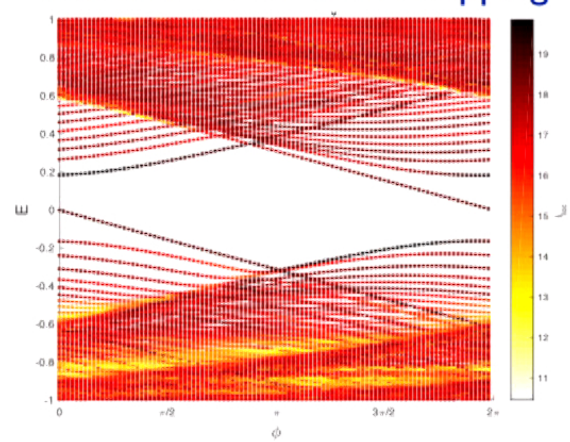
Disorder of 6.0 times hopping



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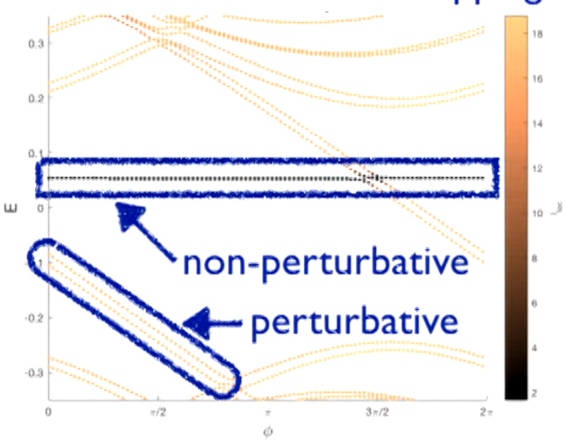

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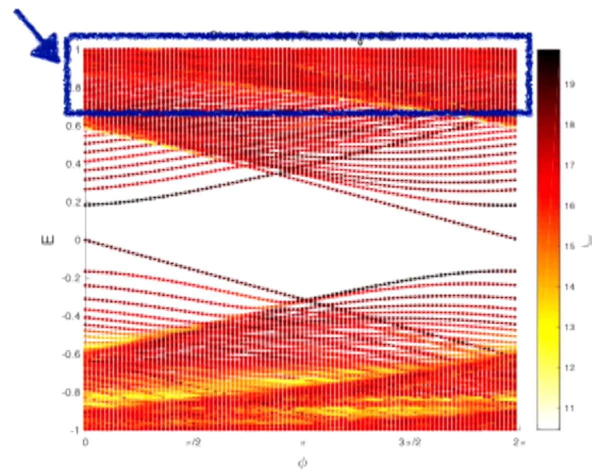
Case II: Diffusive limit due to strong disorder

Where is the chiral Landau level?



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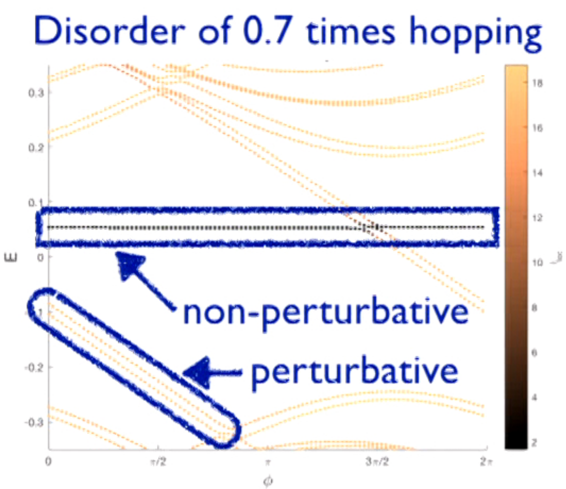

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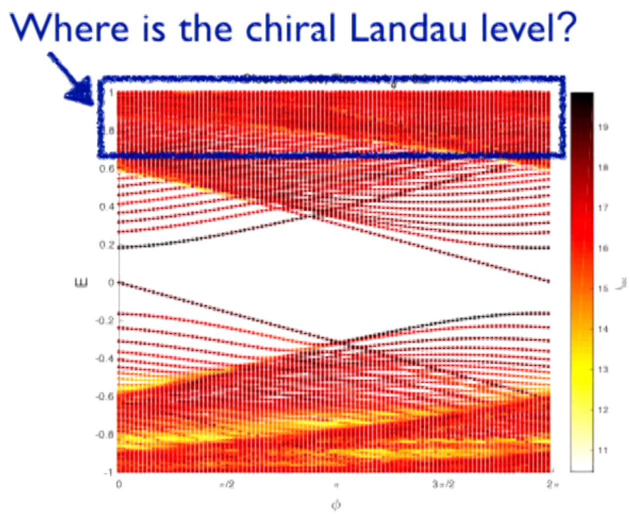

Chern-Simons term in SUSY effective action

- Leads to non-zero average level velocity ($\langle v_\phi \rangle \propto \Phi\Delta$)
- Can be numerically 'measured' in dispersions as above

Toy model with Weyl fermions

$$H = \sum_{i,\eta=x,y,z} t\, c_i^\dagger \sigma_z c_{i+\eta} + \sum_{i,\alpha=x,y} i\, t'\, c_i^\dagger \sigma_\alpha c_{i+\alpha} + \text{h.c.}$$
$$+ \sum_i c_i^\dagger \left(\quad - m\sigma_z \right) c_i$$

Without disorder ($V_i = 0$)

$$H = [2t(\cos k_x + \cos k_y + \cos k_z) - m]\, \sigma_z$$
$$+ 2t' \sin k_x \sigma_x + 2t' \sin k_y \sigma_y$$

When $1 < \dfrac{m}{2t} < 3$:

two Weyl nodes at $\vec{k} = (0, 0, \pm\cos^{-1}(m/2t - 2))$

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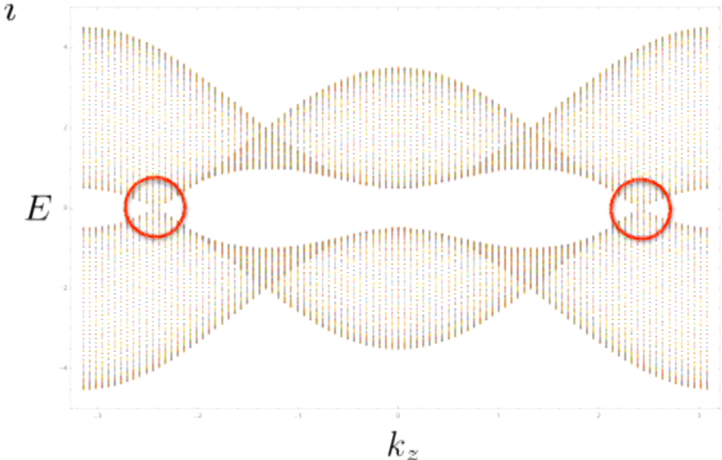

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$$+ \sum_i c_i^\dagger \left(V_i - m\sigma_z\right) c_i$$

- Magnetic flux through z-direction: Peierls phase
- Twisted boundary condition: $\psi(z + l_z) = e^{i\phi}\psi(z)$
- Gaussian correlated disorder:

$$\langle V(k)V(-k)\rangle = W^2 e^{-k^2/k_0^2}$$

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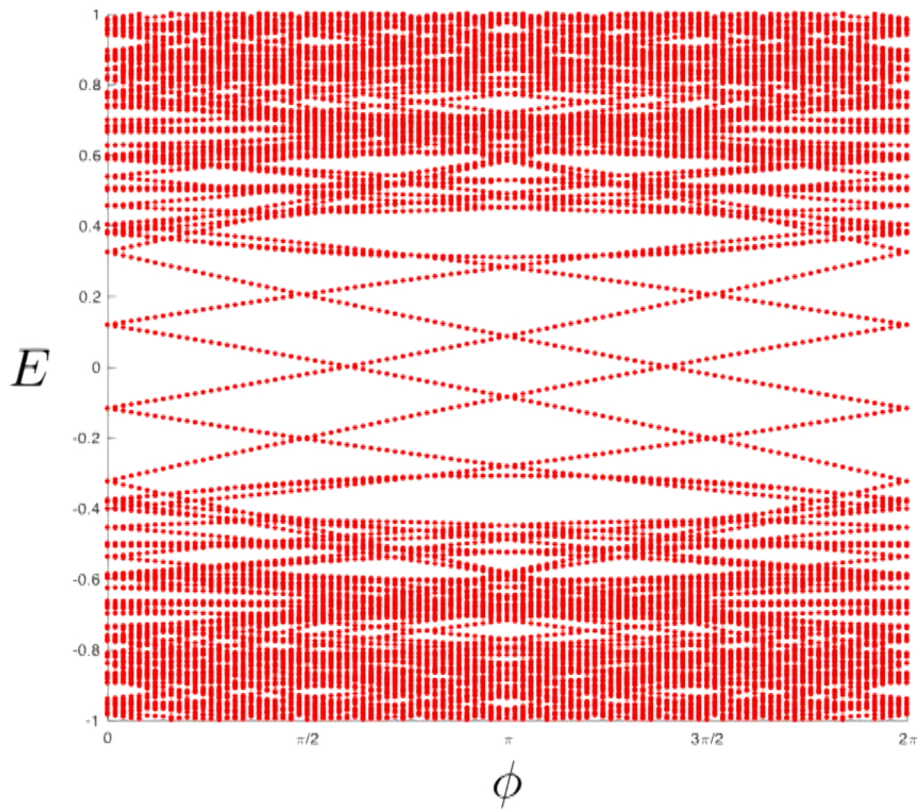
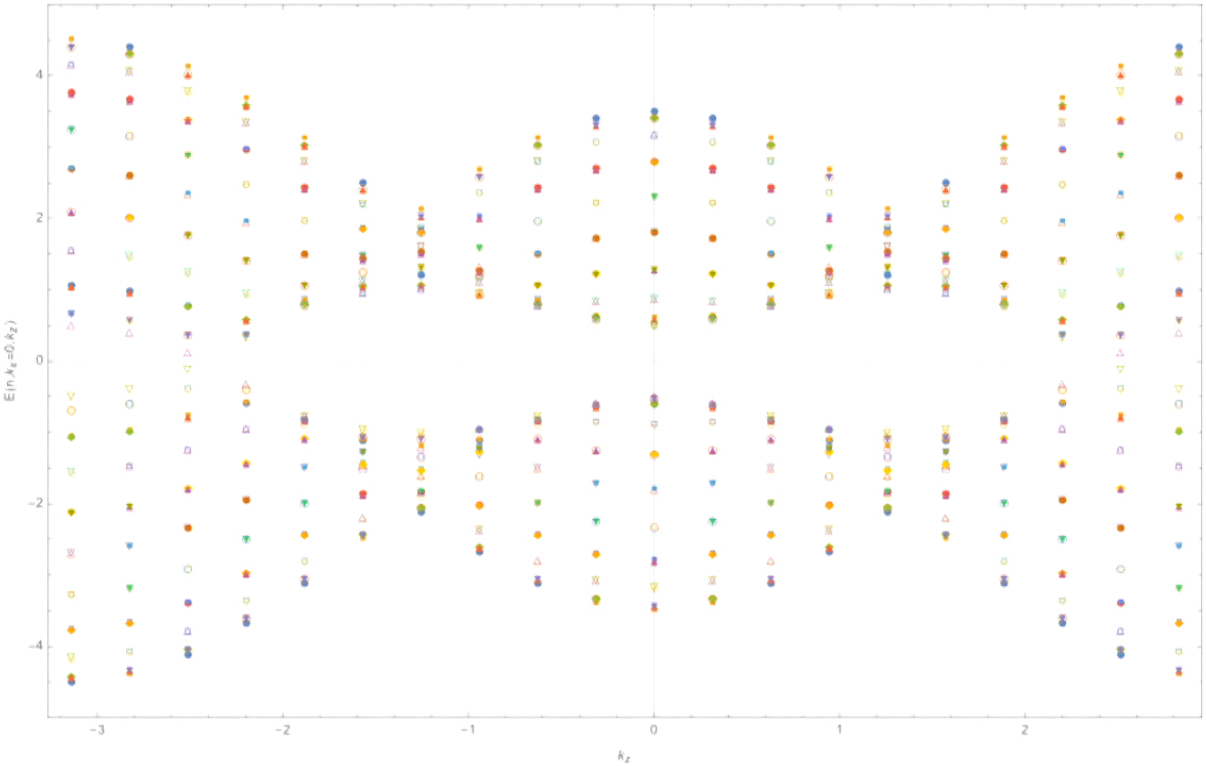
$\phi-$dispersion relation



$$\langle \Phi = 0, W = 0 \rangle$$

5

$\phi-$dispersion relation



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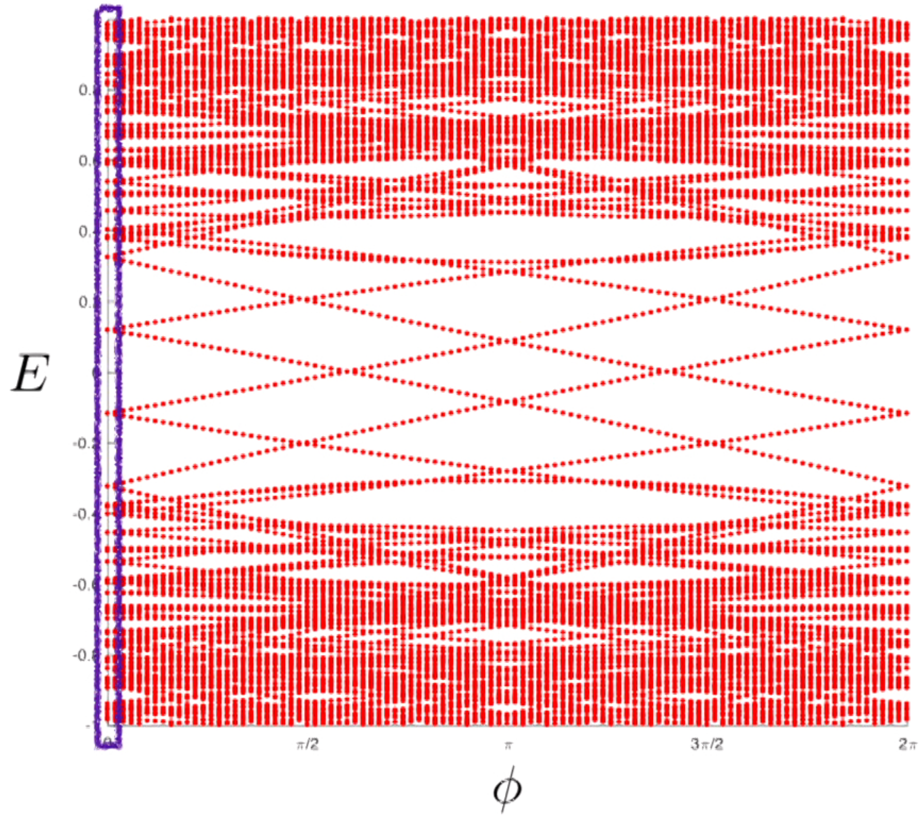

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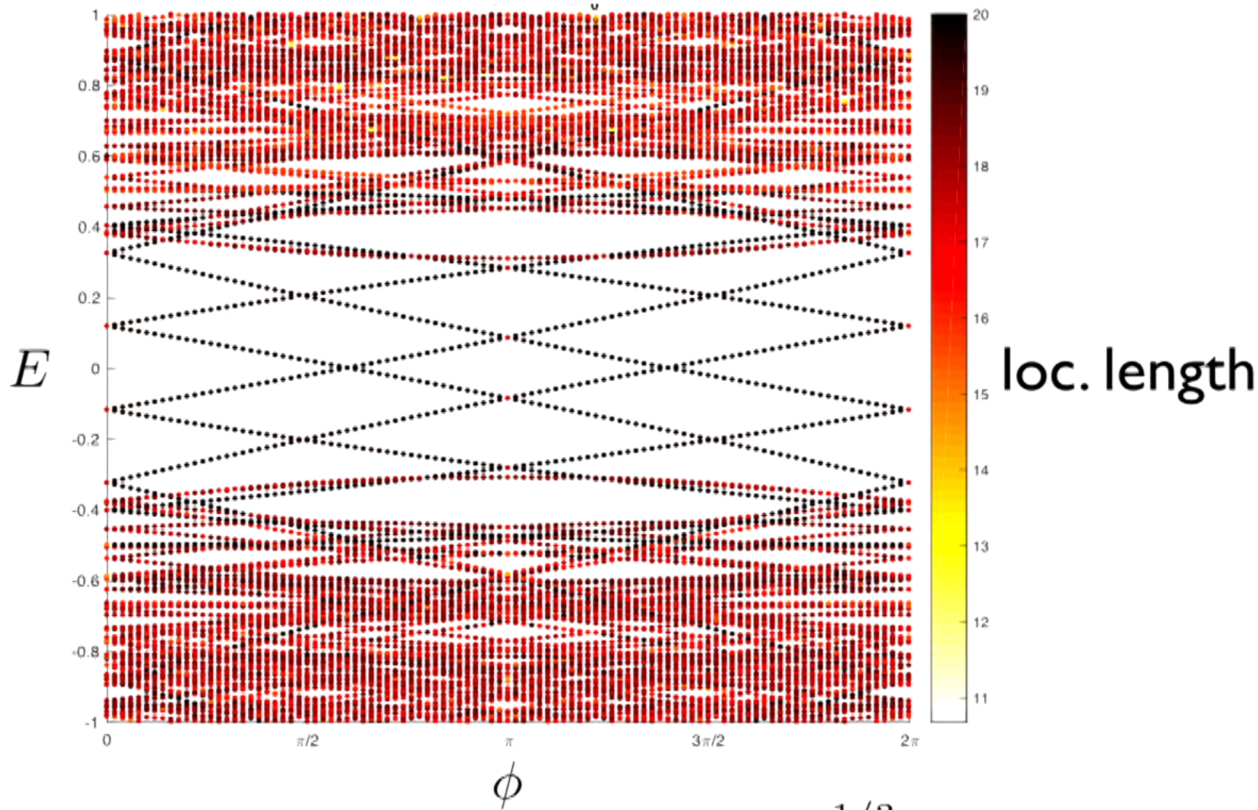
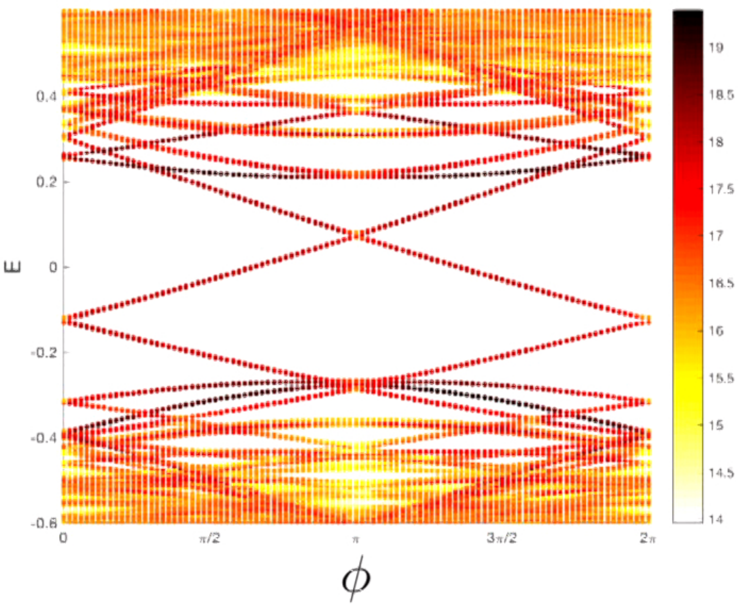
$\phi-$dispersion relation



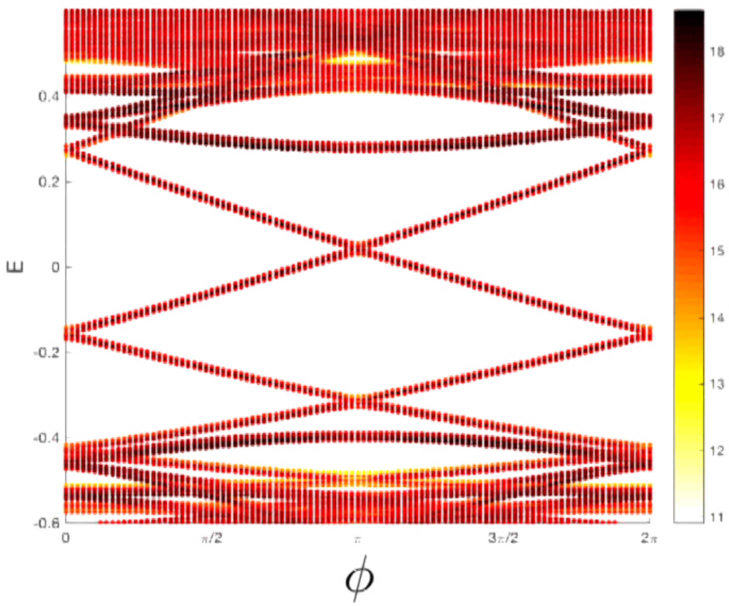
$$\xi = \left(\int d^3r (|\psi_\uparrow|^2 + |\psi_\downarrow|^2)^2 \right)^{-1/3}$$

5

Chiral Landau level in $\phi-$dispersion



$\langle \Phi = 2\Phi_0, W = 0.3t, k_0 = \infty \rangle$

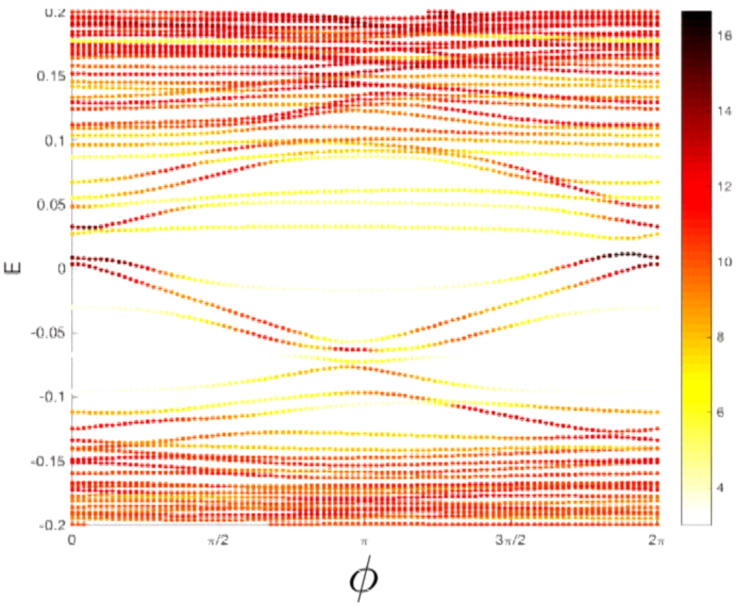


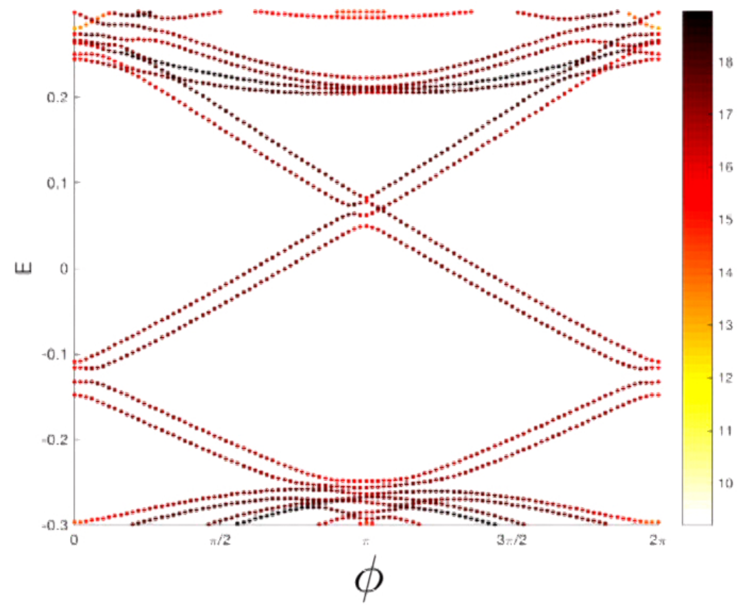
$\langle \Phi = 4\Phi_0, W = 0.3t, k_0 = \infty \rangle$

Degeneracy = number of flux

6

Effects of inter-valley scattering



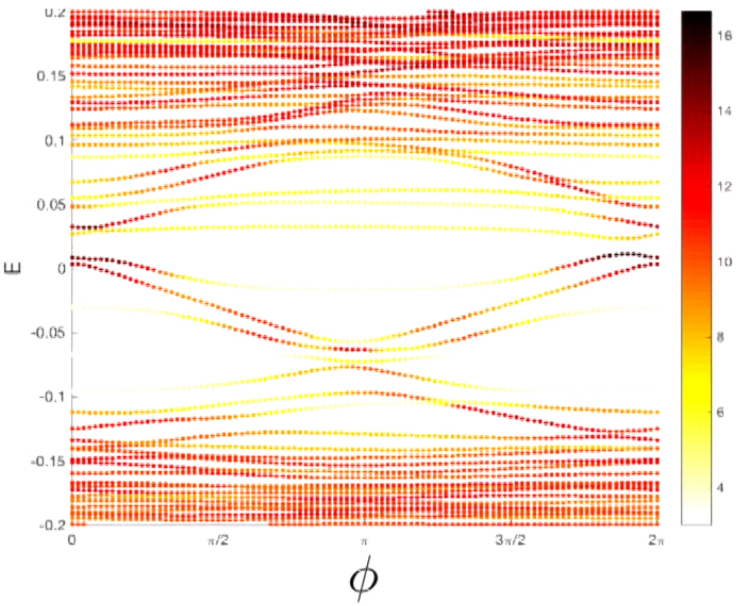


$\langle \Phi = 2\Phi_0, W = t, k_0 = \infty \rangle$ $\langle \Phi = 2\Phi_0, W = t, k_0 = k_F/2 \rangle$

$$\langle V(k)V(-k)\rangle = W^2 e^{-k^2/k_0^2}$$

7

Effects of inter-valley scattering



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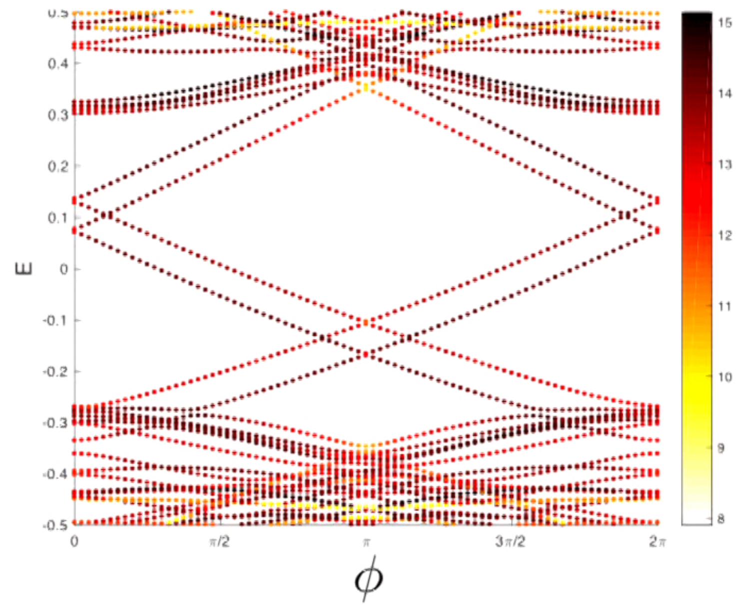
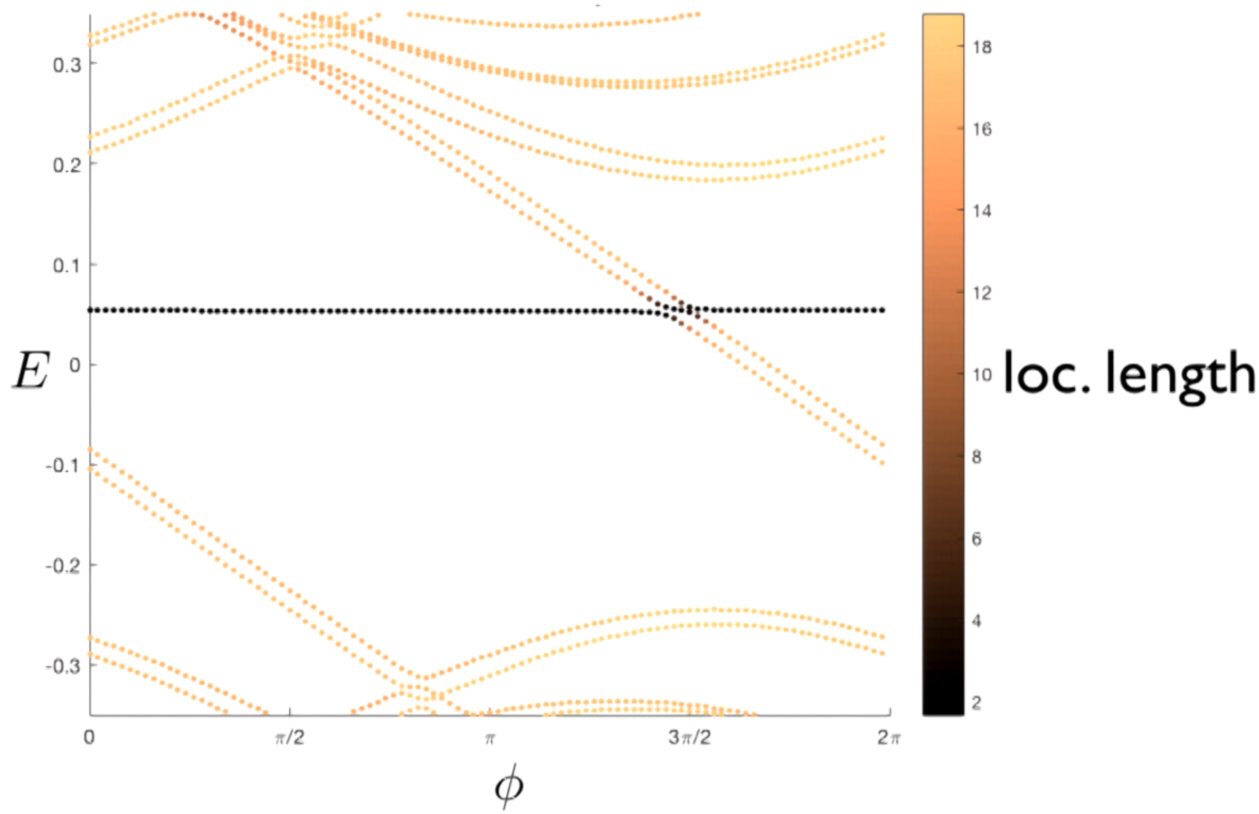


$$\langle \Phi = 2\Phi_0, W = 5t, k_0 = k_F/10 \rangle$$

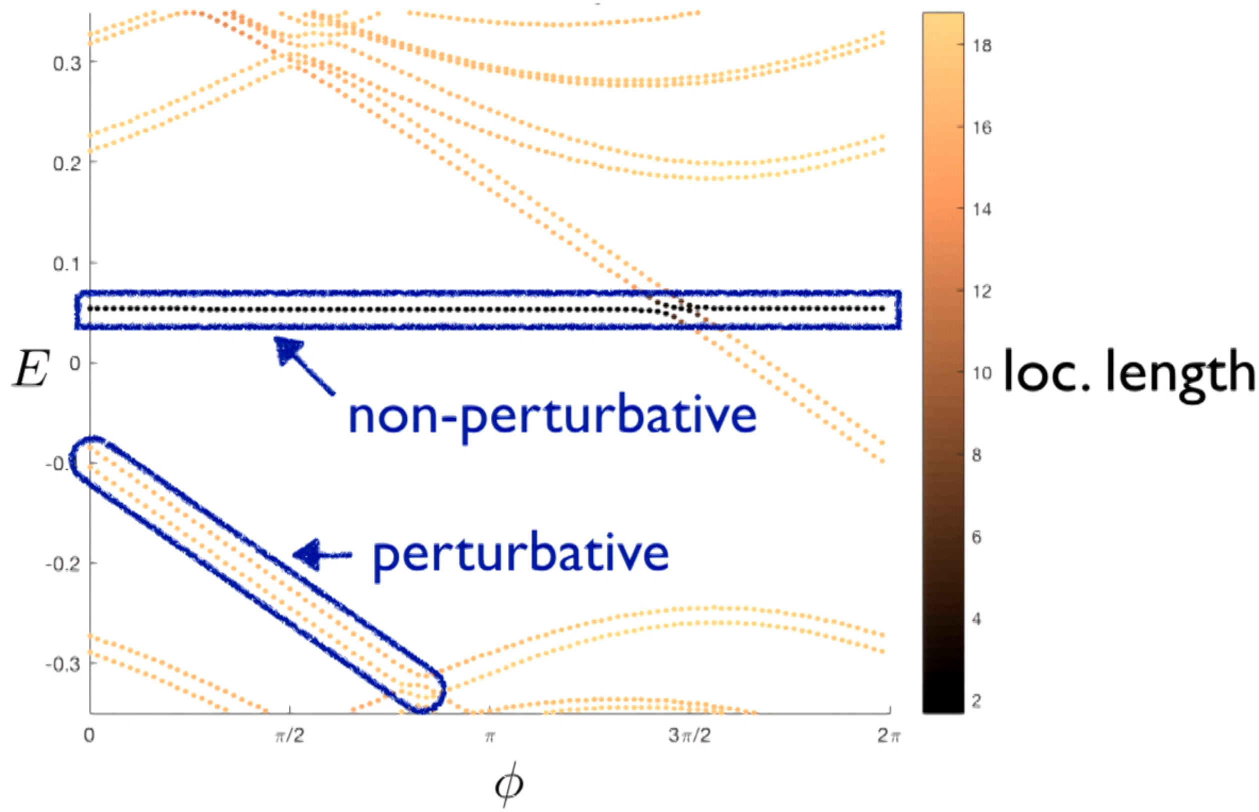
Most natural way of eliminating chiral anomaly!

Weak disorder: non-perturbative effect



$$\langle \Phi = 2\Phi_0, W = 0.7t, k_0 = \infty \rangle$$

Weak disorder: non-perturbative effect



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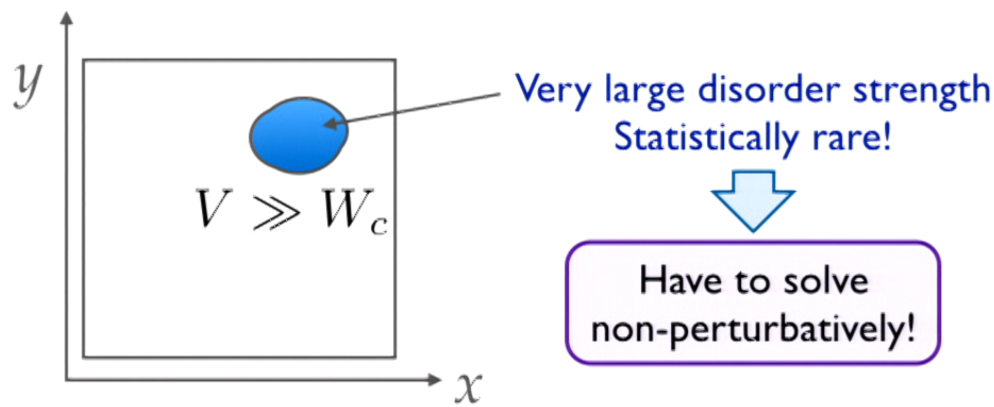
8

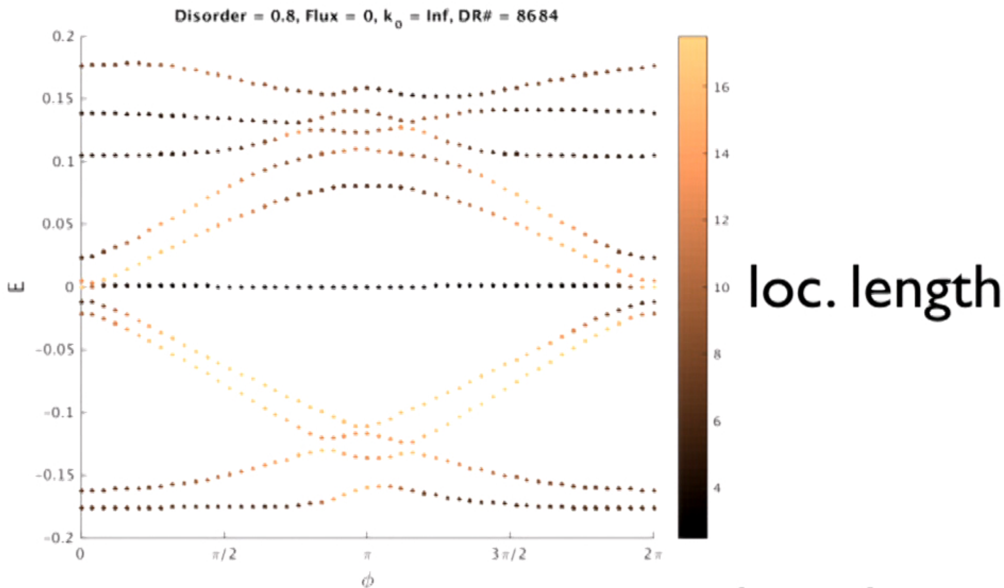
Rare regions, rare states



Nandkishore, Huse, and Sondhi, *PRB* (2014)

- Rare regions are areas in space where disorder is much stronger than typical disorder strength
- Rare states, are the bound states in rare regions with power law quasi-local wavefunction $\psi \sim 1/r^2$

Rare regions, rare states



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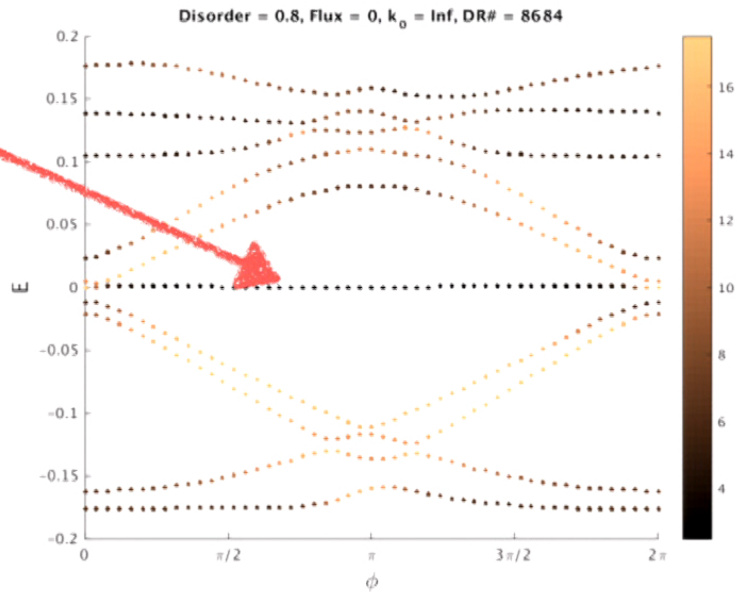
9

Rare regions, rare states

- Low energy
- Localized
- Flat band



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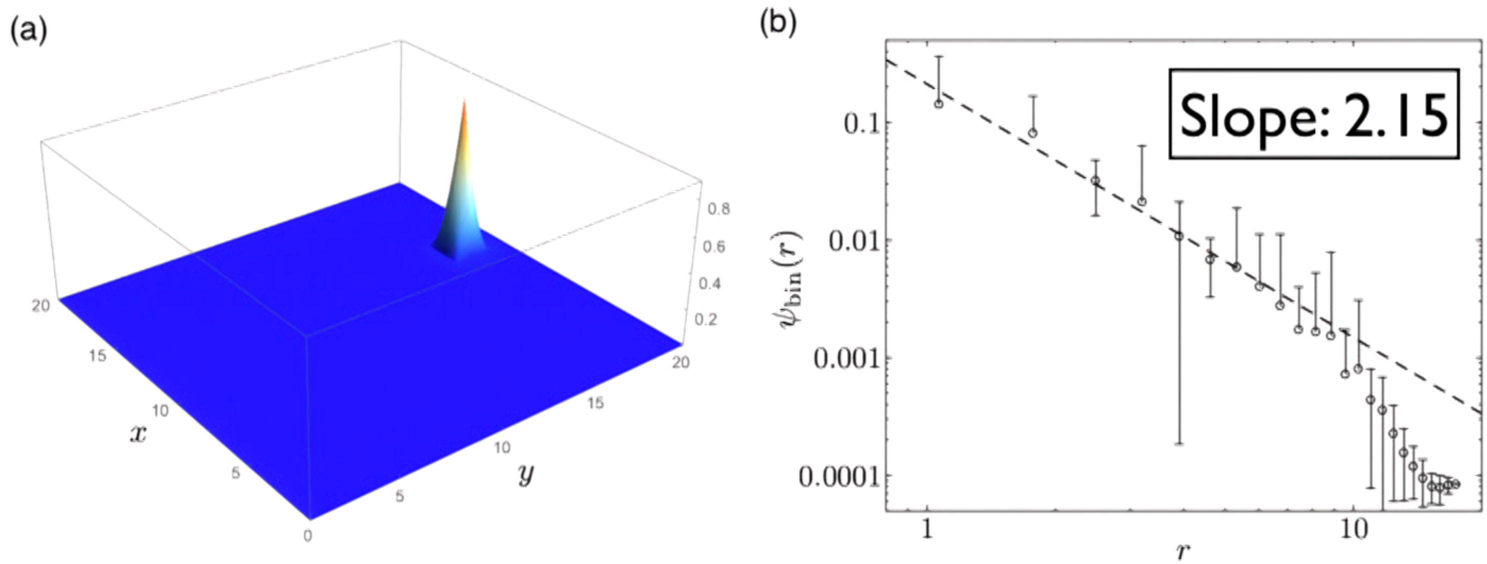

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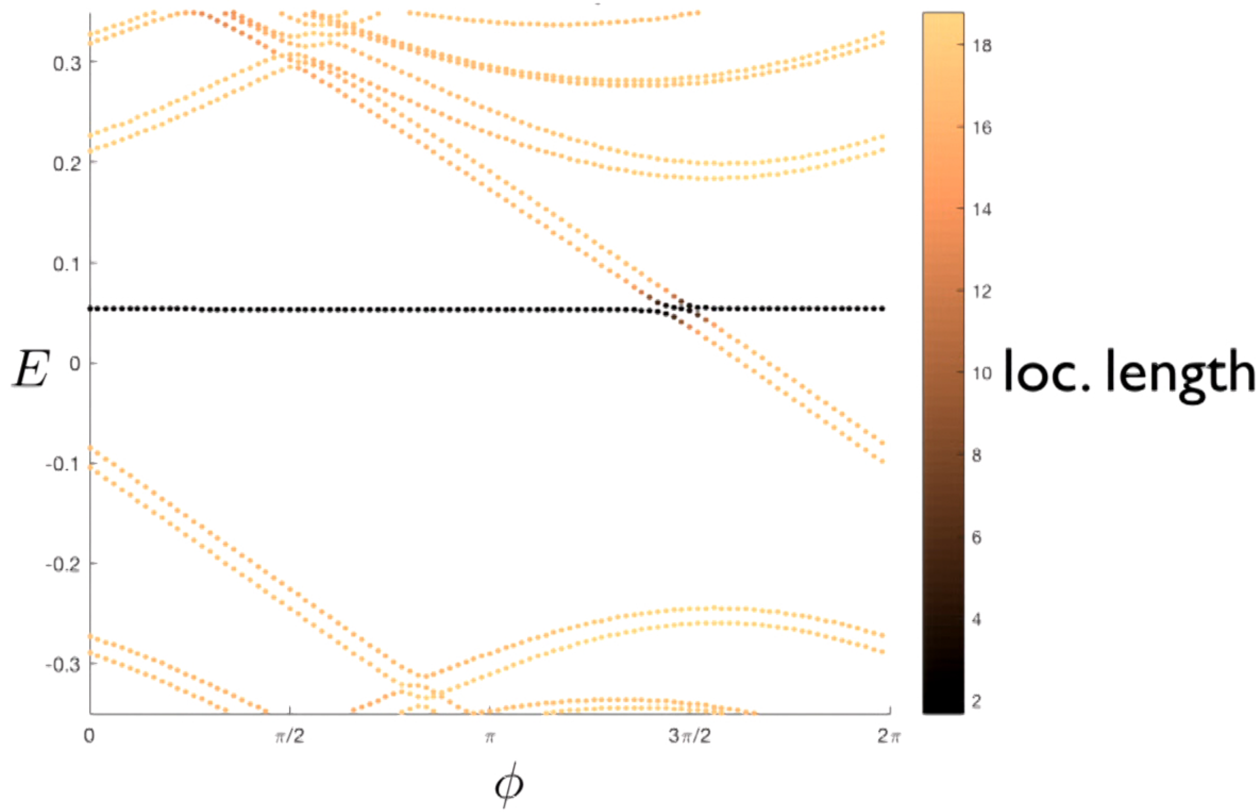
Single Weyl node with rare states



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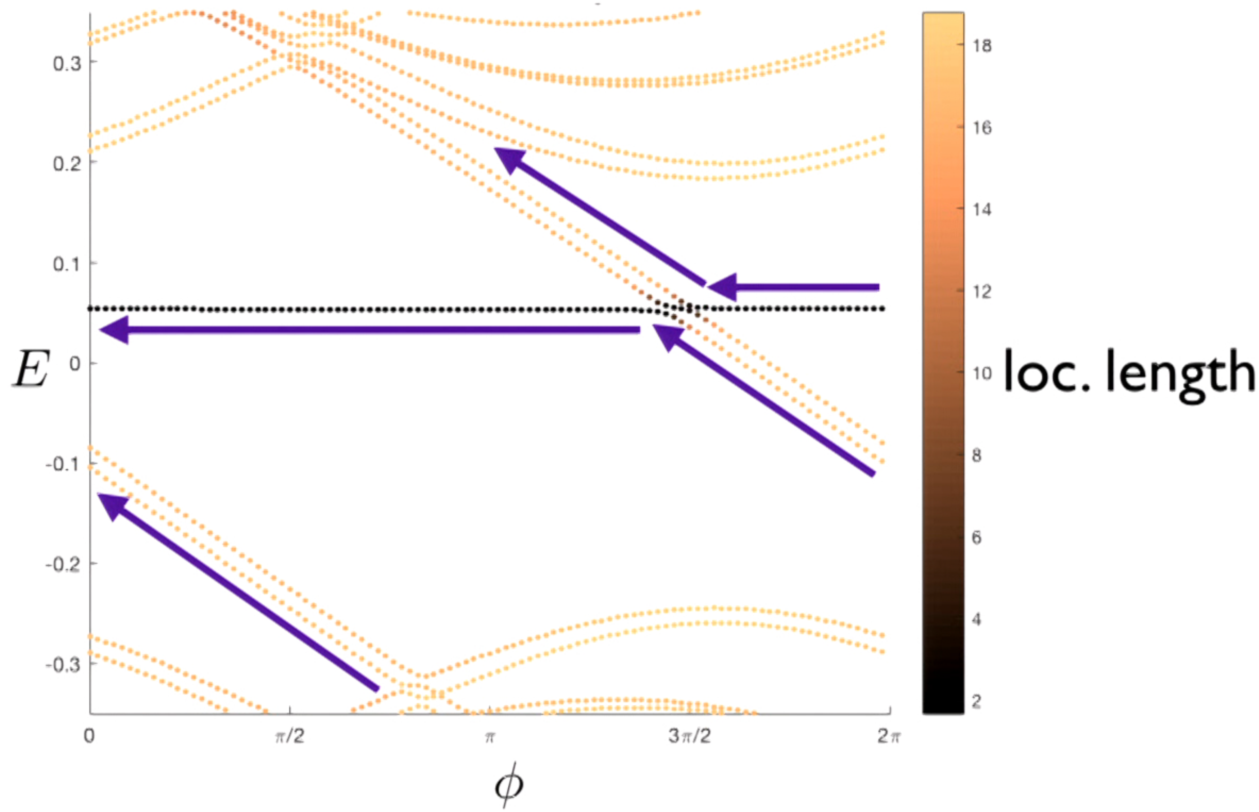

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10

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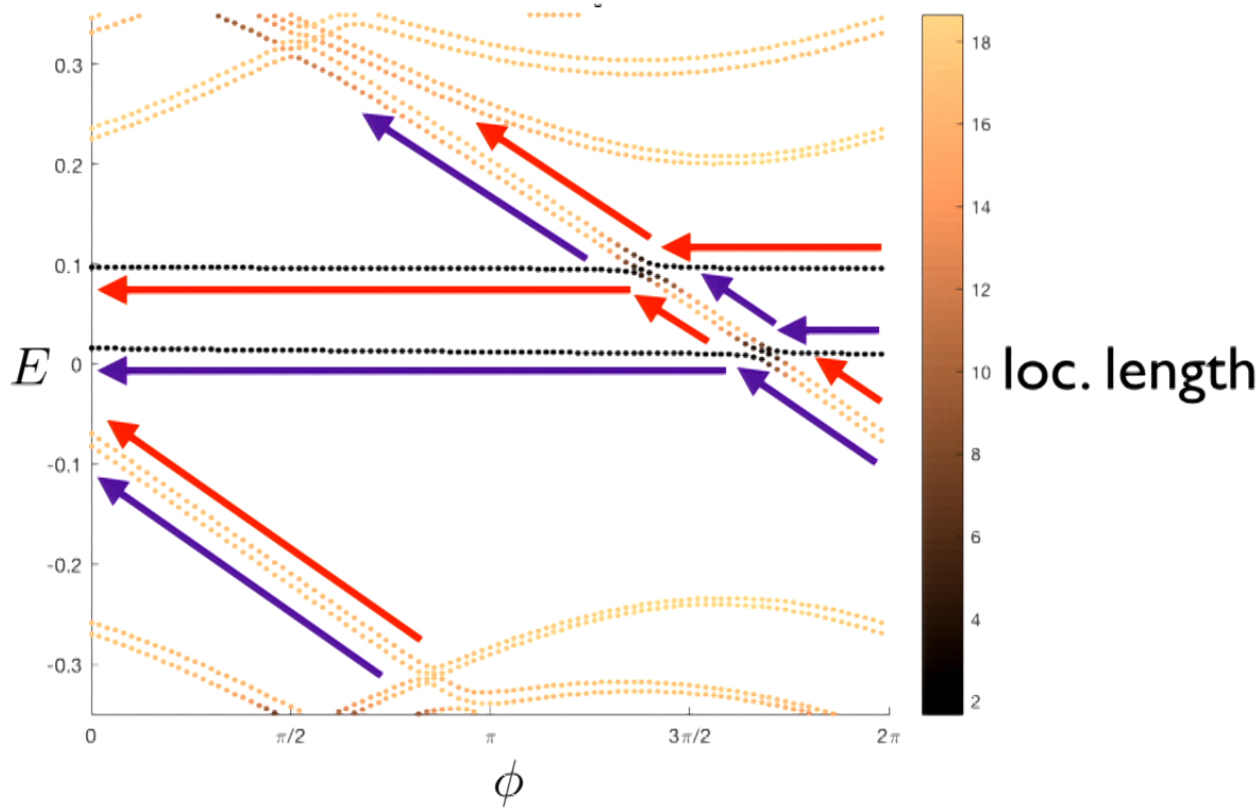
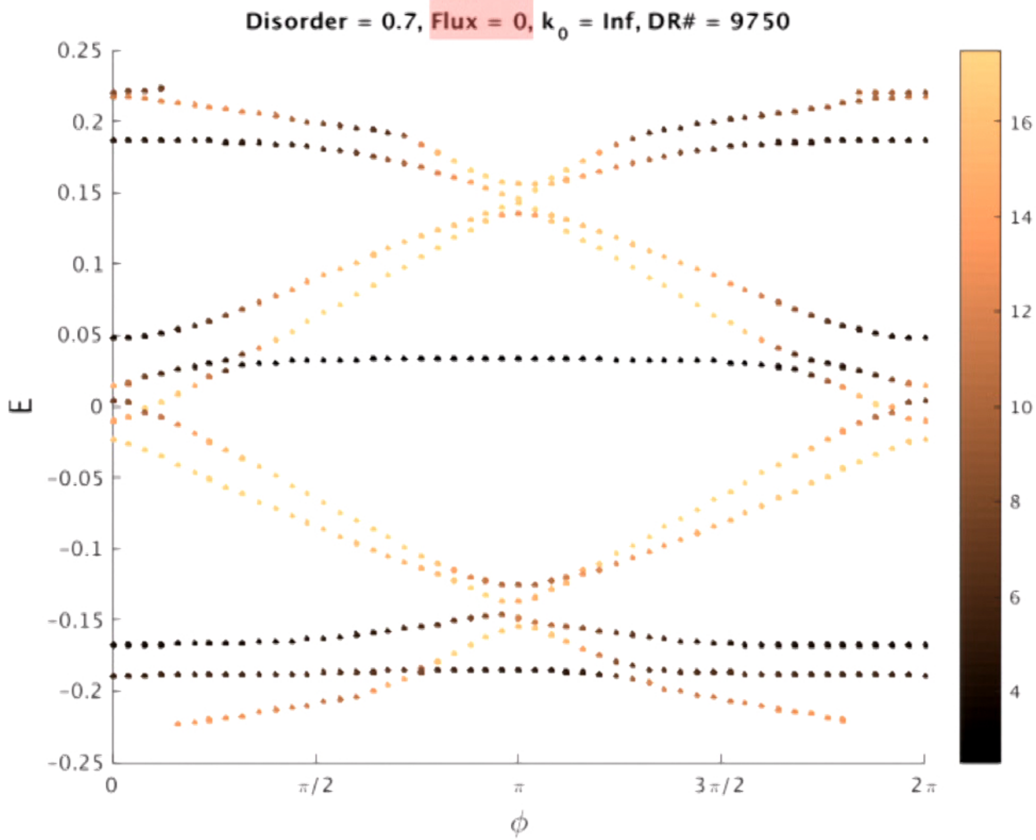


Anomaly always survive rare states!!

10

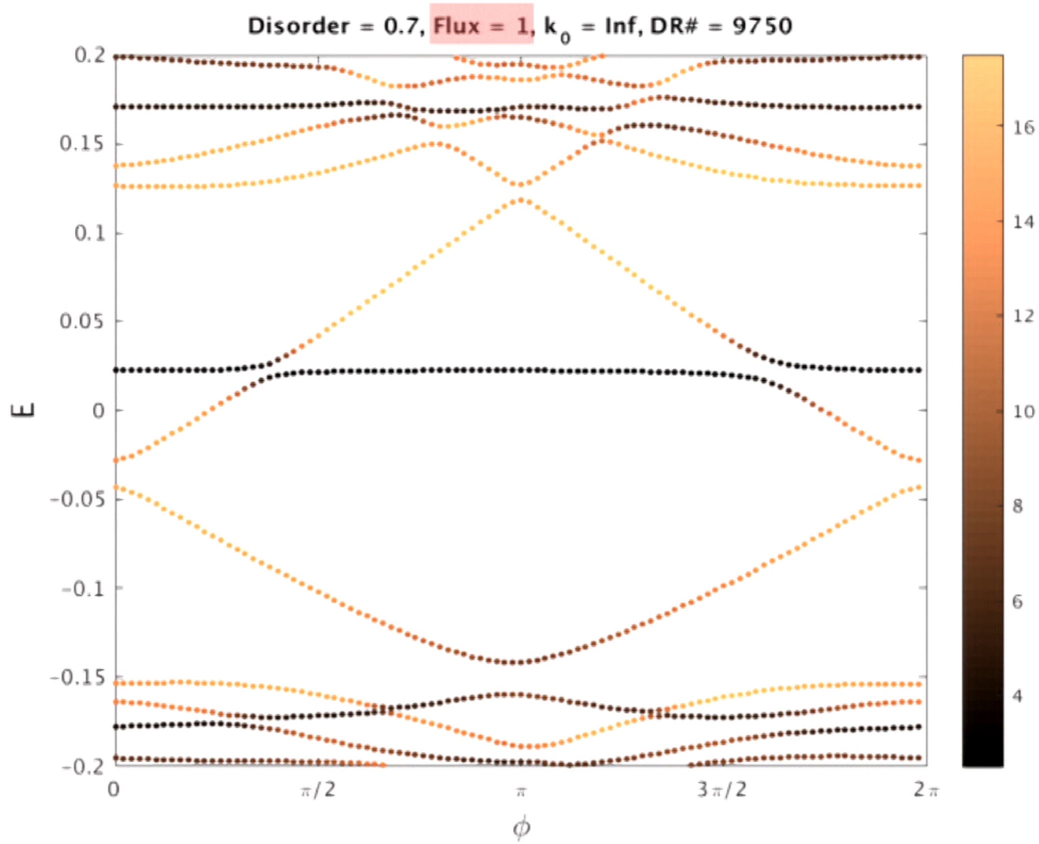
Two Weyl nodes with rare states



loc. length

11

Two Weyl nodes with rare states



loc. length

11

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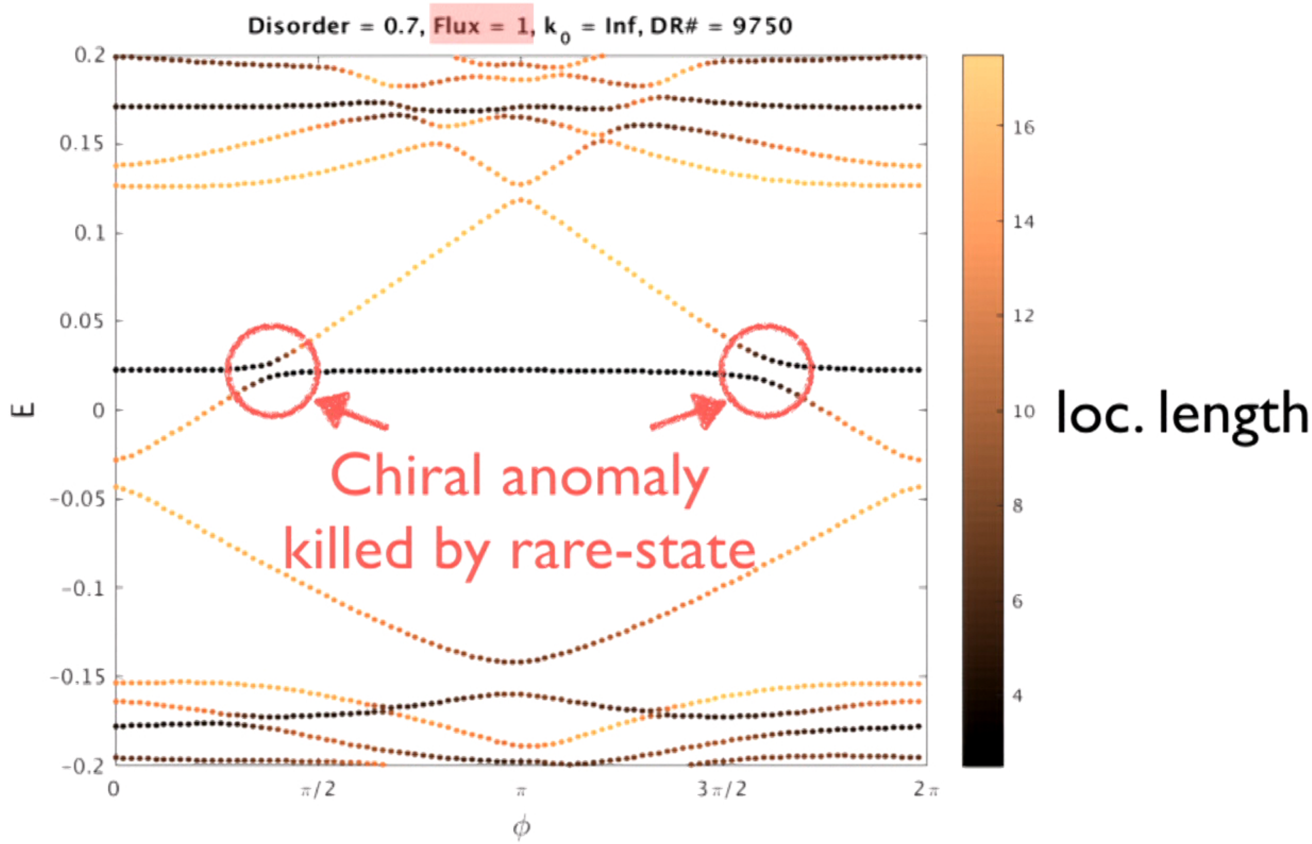

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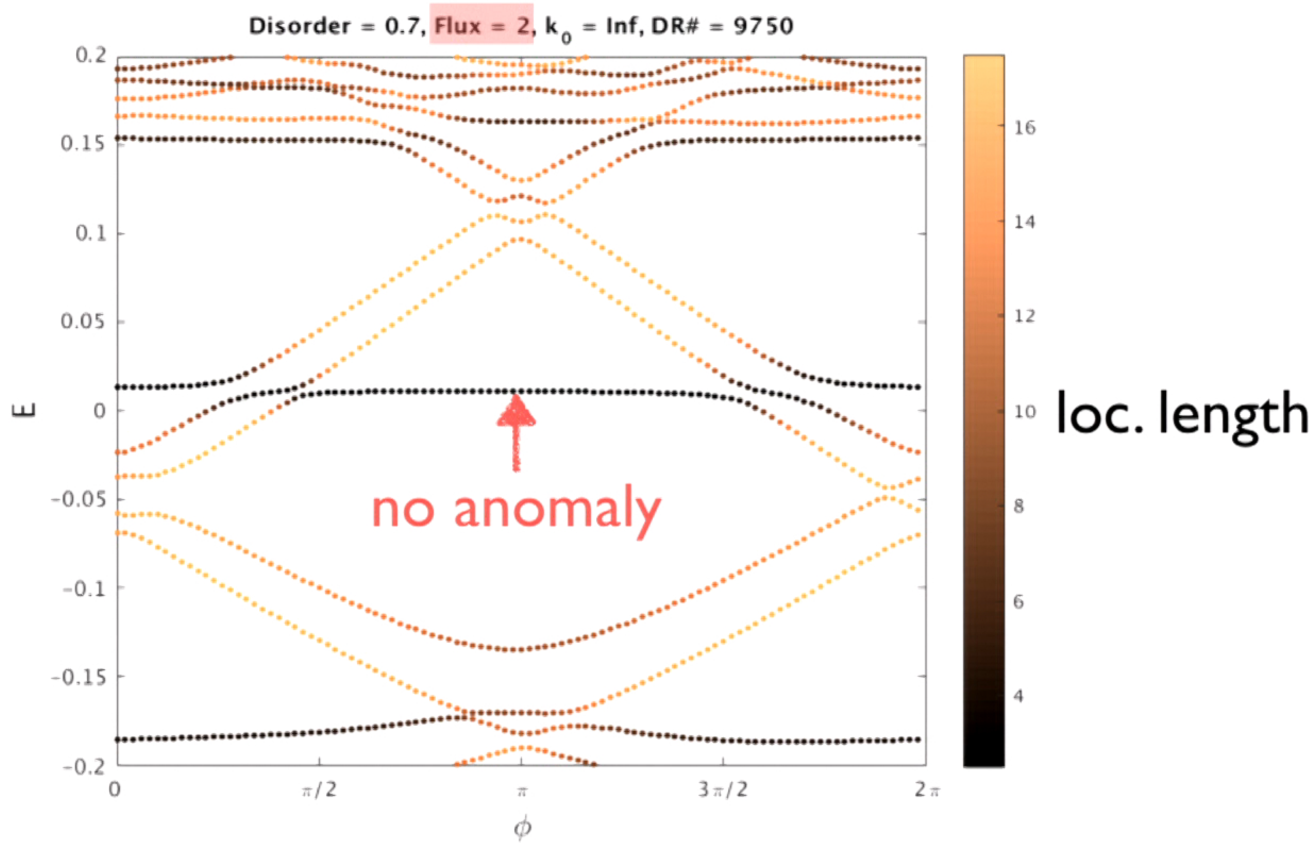

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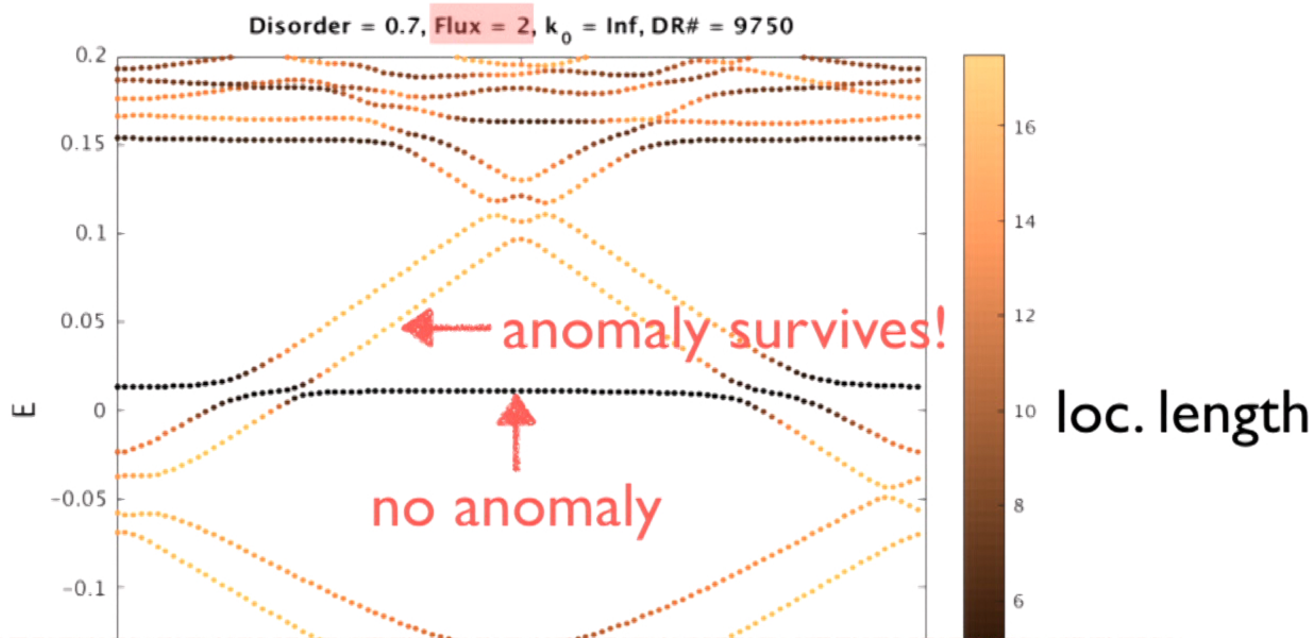
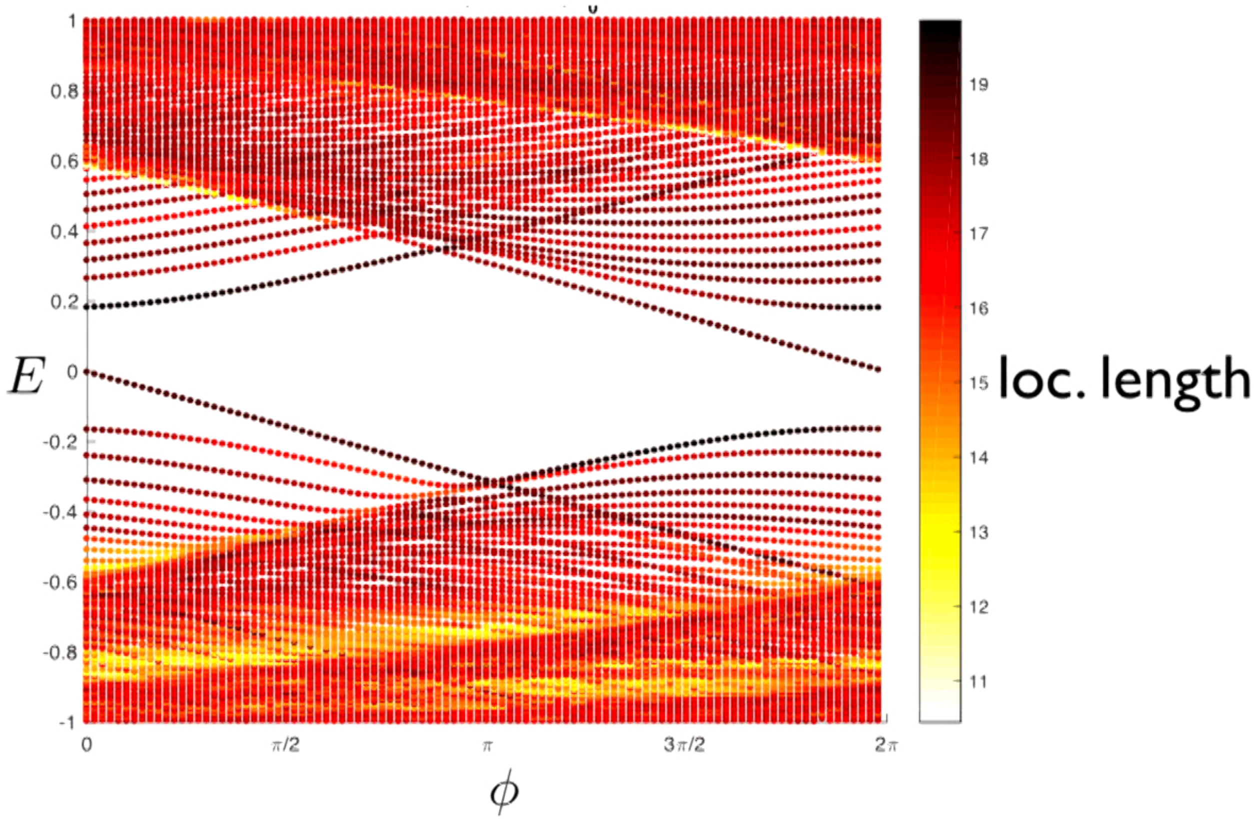


Chiral anomaly weakened in quantized way

(# of fluxes - # of rare states)

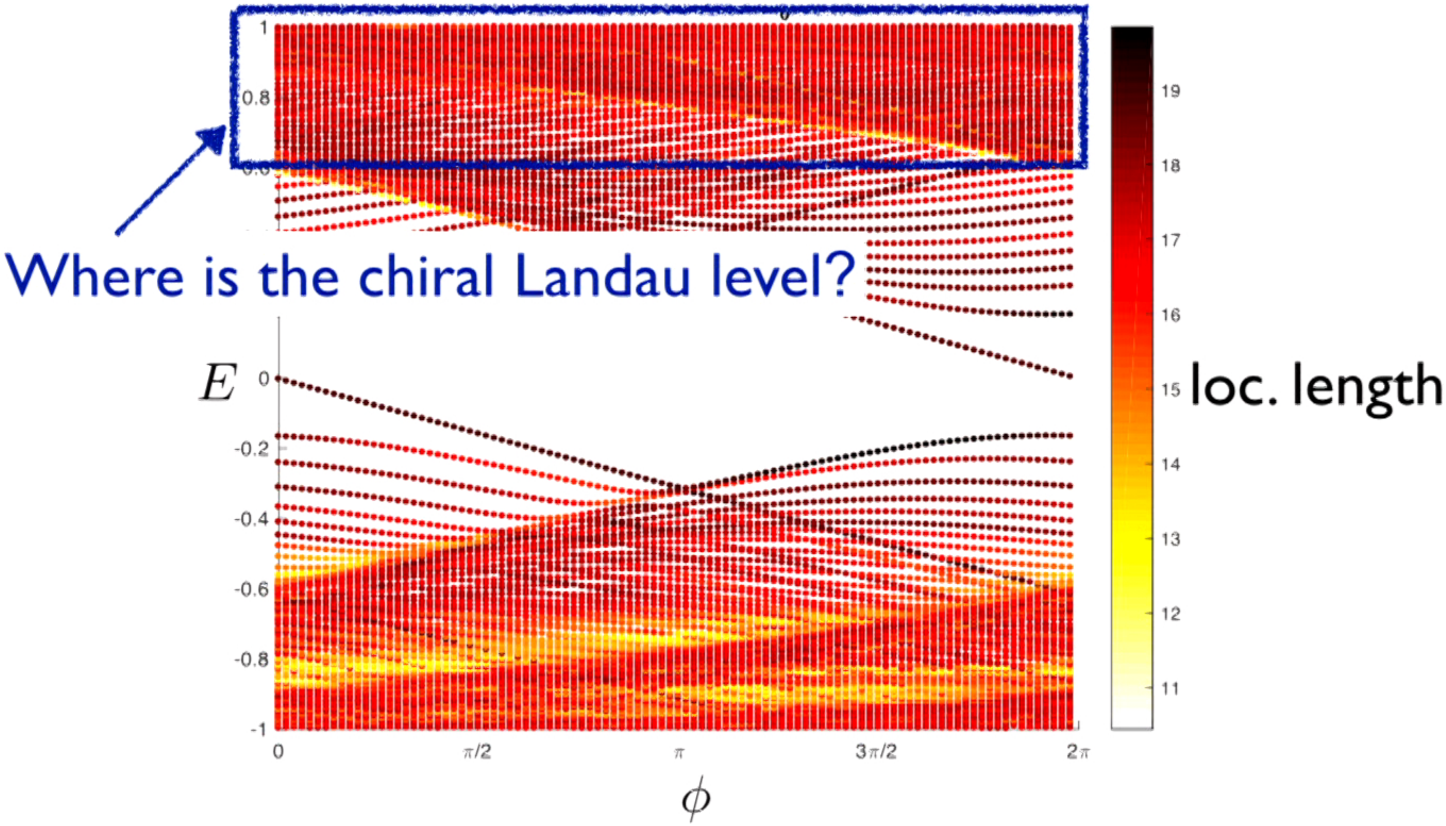
11

Strong disorder: diffusive limit



$$\langle \Phi = \Phi_0, W = 6.0t, k_0 = k_F/5 \rangle$$

Strong disorder: diffusive limit



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12

SUSY method on disordered system

$$K(\omega, \delta\phi) = \langle \nu(E, \phi)\, \nu(E + \omega, \phi + \delta\phi) \rangle$$
$$= -\frac{\Delta^2 \bar{\nu}^2}{4\pi^2} \int \left\langle \left(G^A - G^R\right) \left(G^A - G^R\right) \right\rangle$$

Efetov, *Adv. Phys.* (1983); Simons and Altshuler, *Phys. Rev. B* (1993)
Altland and Bagrets, *Phys. Rev. B* (2016)

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↓ Path integral form of $\langle G^A G^R \rangle$ in supermatrix Q

$$F[Q] = \frac{\pi\nu}{8} \int dr\, \mathrm{str} \left[D \left(\nabla Q - \frac{ie}{c}[Q, \vec{A}\tau_3] \right)^2 \right.$$

$$\left. +2i \left(\omega \qquad\qquad \right) \Lambda Q \right]$$

Efetov, *Adv. Phys.* (1983); Simons and Altshuler, *Phys. Rev. B* (1993)
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↓ Path integral form of $\langle G^A G^R \rangle$ in supermatrix Q

$$F[Q] = \frac{\pi\nu}{8} \int dr\, \mathrm{str}\left[D \left(\quad - \frac{ie}{c}[Q, \vec{A}\tau_3] \right)^2 + 2i \left(\omega - \frac{B\delta\phi}{4\pi^2 L_z \nu} \right) \Lambda Q \right]$$

Chern-Simons term for Weyl Hamiltonian

Efetov, *Adv. Phys.* (1983); Simons and Altshuler, *Phys. Rev. B* (1993)
Altland and Bagrets, *Phys. Rev. B* (2016)

Non-zero average level velocity

$$\langle \frac{\omega}{\delta\phi} \nu(E, \phi)\, \nu(E + \omega, \phi + \delta\phi) \rangle = \frac{B}{4\pi^2 L_z \nu} K$$

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Non-zero average level velocity

$$\langle \frac{\omega}{\delta\phi} \nu(E,\phi)\, \nu(E+\omega, \phi+\delta\phi) \rangle = \frac{B}{4\pi^2 L_z \nu} K$$

$$\int_0^{2\pi} d\phi \langle v_\phi \rangle = \frac{\Phi}{h/e} \Delta$$

Δ : level spacing

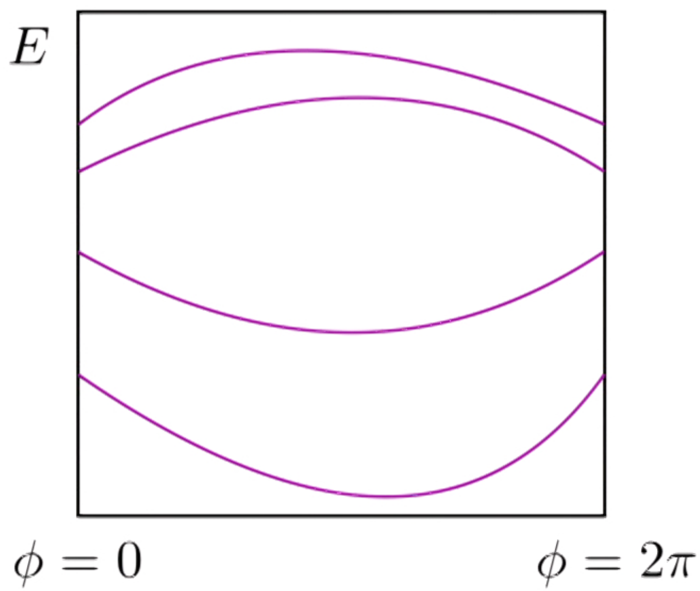
Φ : total flux

Due to CS term, average level velocity is non-zero

→ Indication of chiral anomaly in strong disorder

Level velocity as a signature of anomaly

$$v_\phi = \frac{\partial E}{\partial \phi} = \left\langle \frac{\partial H}{\partial \phi} \right\rangle \quad \Rightarrow \quad \int_0^{2\pi} d\phi \; v_\phi$$



Trivial bands

$$\int_0^{2\pi} d\phi \; v_\phi = 0$$

Level velocity as a signature of anomaly

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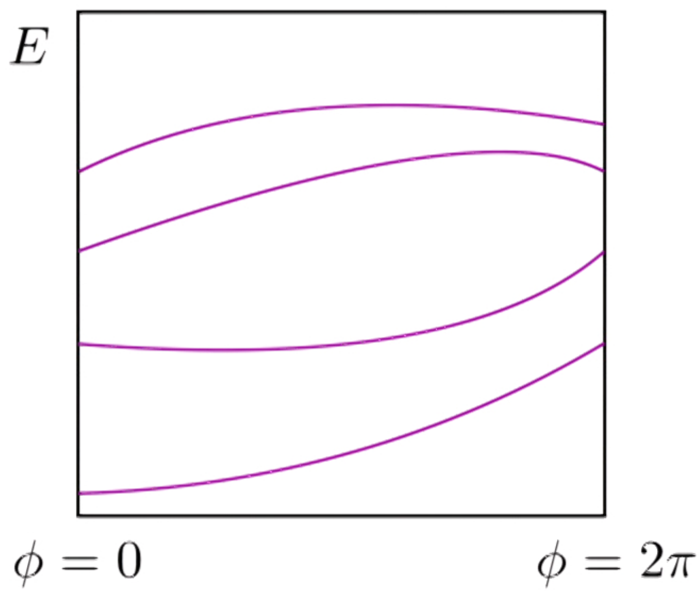

Chiral bands

$$\int_0^{2\pi} d\phi \; v_\phi = \Delta E$$

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Level velocity as a signature of anomaly

Trivial bands

$$\int_0^{2\pi} d\phi \; v_\phi = 0$$

Chiral bands

$$\int_0^{2\pi} d\phi \; v_\phi = \Delta E$$

$$f(E) \equiv \int_{-\infty}^{E} dE \int_0^{2\pi} d\phi \; v_\phi = \alpha(E - E_0)$$

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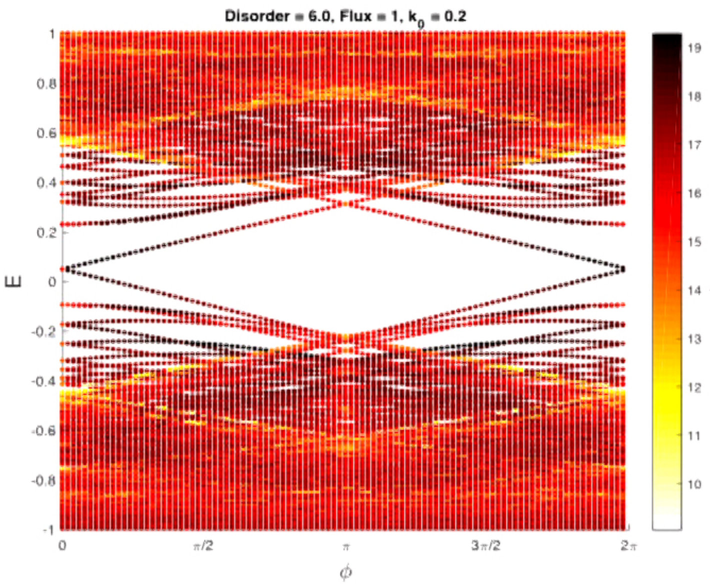
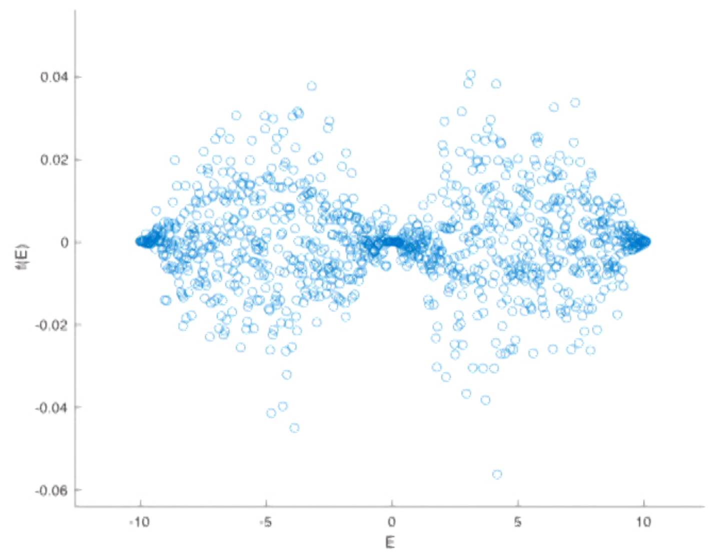
Level velocity as a signature of anomaly

$$f(E) \equiv \int_{-\infty}^{E} dE \int_{0}^{2\pi} d\phi\ v_\phi = \alpha(E - E_0)$$





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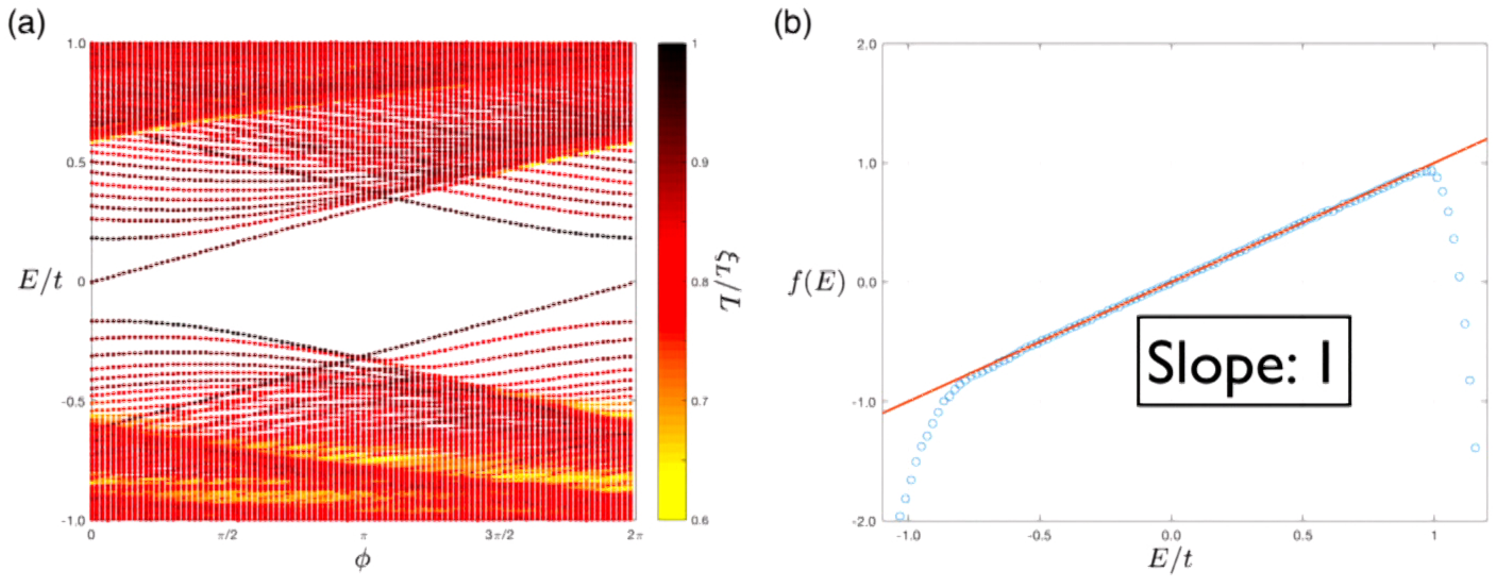

16

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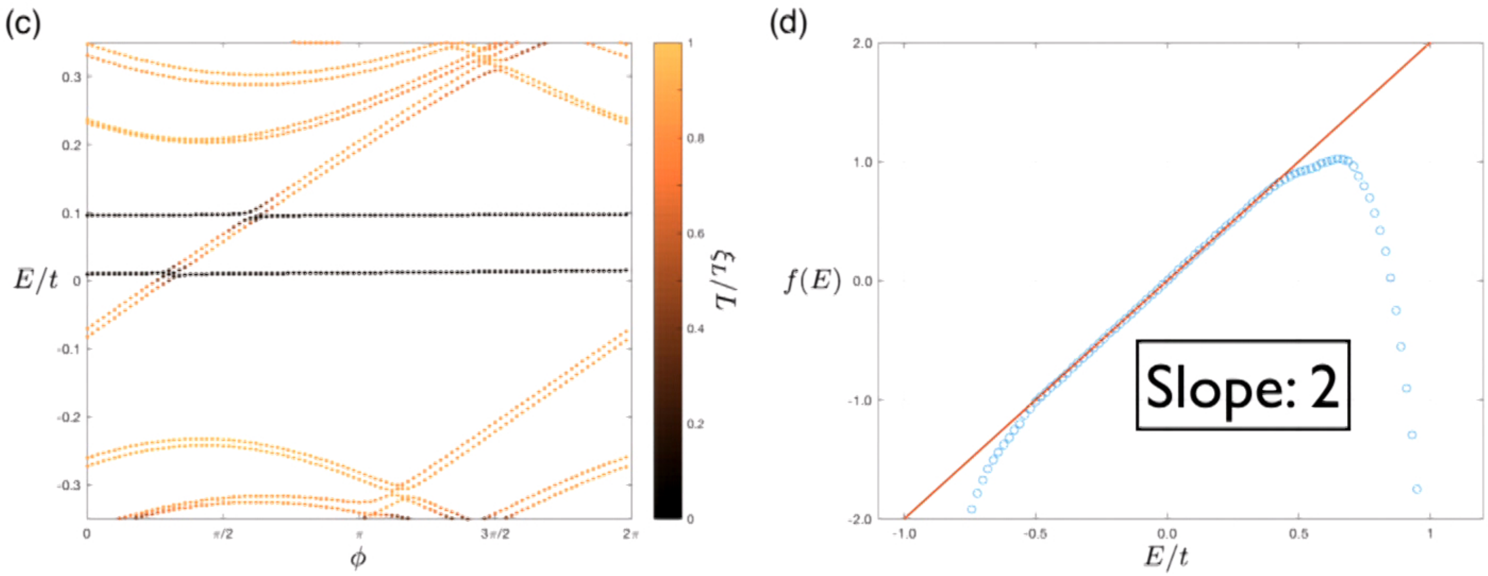

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Outlook

- Chiral anomaly is immune to rare states in single node, but is weakens in a quantized way in two nodes
- Chiral anomaly also survives in the diffusive limit, and a non-zero average level velocity can be an indicator of the anomaly